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5.2 Differentiability

(Previous Year Questions solutions from 2008-2025)

MCQ's :

1.

The value of k for which function $f(x)=\begin{cases} x^2, & x\geq 0 \\ kx, & x<0 \end{cases}$ is differentiable at $x=0$ is :

(a) 1 (b) 2

(c) any real number (d) 0

2.

If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of $f(x)$ at $x = 0$ is :

(a) 6 (b) 5

(c) 3 (d) 2

3.

The function $f(x) = x\,|x|$, $x \in R$ is differentiable

(a) only at $x = 0$ (b) only at $x = 1$

(c) in R (d) in $R - \{0\}$

4. 2024

The number of points, where $f(x) = [x]$, $0 < x < 3$ ($[\cdot]$ denotes greatest integer function) is not differentiable is :

(A) 1 (B) 2

(C) 3 (D) 4

5.

If $f(x) = |x| + |x - 1|$, then which of the following is correct ?

(A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.

(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.

(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.

(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

6.

If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is :

(A) f is continuous but not differentiable at $x = 2$.

(B) f is neither continuous nor differentiable at $x = 2$.

(C) f is continuous as well as differentiable at $x = 2$.

(D) f is not continuous but differentiable at $x = 2$.

7.

Let $f(x) = |x|$, $x \in R$. Then, which of the following statements is ***incorrect*** ?

(A) f has a minimum value at $x = 0$.

(B) f has no maximum value in R.

(C) f is continuous at $x = 0$.

(D) f is differentiable at $x = 0$.

8.

Let $f(x) = x^2$, $x \in R$. Then, which of the following statements is ***incorrect*** ?

(A) Minimum value of f does not exist.

(B) There is no point of maximum value of f in R.

(C) f is continuous at $x = 0$.

(D) f is differentiable at $x = 0$.

a. Modulus function :

1.a

Check for differentiability of the function f defined by

$f(x) = |x - 5|$, at the point $x = 5$.

Sol.

$$\text{LHD} = \lim_{x \to 5^-} \frac{|x-5| - 0}{x-5} = \lim_{x \to 5^-} \frac{-(x-5)}{x-5} = -1$$

$$\text{RHD} = \lim_{x \to 5^+} \frac{|x-5| - 0}{x-5} = \lim_{x \to 5^+} \frac{(x-5)}{x-5} = 1$$

$\text{LHD} \neq \text{RHD}, \ \therefore f$ is not differentiable at $x = 5$

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1.b

Show that the function $f(x) = |x - 3|, x \in R$, is continuous but not differentiable at $x = 3$.

1.c

Check the differentiability of $f(x) = |x - 3|$ at $x = 3$.

2.a

Discuss the continuity and differentiability of the function $f(x) = |x| + |x - 1|$ in the interval $(-1, 2)$.

2.a

The number of points of discontinuity of f defined by $f(x) = |x| - |x + 1|$ is ____________.

2.b

If function $f(x) = |x - 3| + |x - 4|$, then show that $f(x)$ is not differentiable at $x = 3$ and $x = 4$.

2.c

Show that the function f(x) = |x – 1| + |x + 1|, for all $x \in \mathbb{R}$, is not differentiable at the points x = – 1 and x = 1.

If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of $f(x)$ at $x = 0$ is ________.

3.a

Show that the function f(x) = $|x|^3$ is differentiable at all points of its domain.

Sol.

(b) $f(x) = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x \leq 0 \end{cases}$

At x = 0

$$\text{LHD} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\left(\frac{h^3}{-h}\right) = \lim_{h\to 0}(-h^2) = 0$$

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\left(\frac{h^3}{h}\right) = \lim_{h\to 0}(h^2) = 0$$

$\because$ LHD = RHD at x = 0; when $x \neq 0$, f(x) is a polynomial and hence differentiable.

$\therefore$ f(x) is differentiable at all points.

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3.b

Check whether the function $f(x) = x^2\ |x|$ is differentiable at $x = 0$ or not.

Sol.

$$f(x) = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x \leq 0 \end{cases}$$

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} h^2 = 0$$

$$\text{LHD} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} (-h^2) = 0$$

$$\because RHD = LHD = 0, \qquad So\ f(x)\ is\ differentiable\ at\ x = 0$$

3.c 2025

Check the differentiability of function $f(x) = x|x|$ at $x = 0$.

Sol.

$$f(x) = x|x| = \begin{cases} -x^2 & , x \leq 0 \\ x^2 & , x > 0 \end{cases}$$

$$\text{LHD} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{-h^2-0}{-h} = 0$$

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2-0}{h} = 0$$

Since LHD = RHD, f is differentiable at $x = 0$

3.d

Check the differentiability of $f(x) = |\cos x|$ at $x = \frac{\pi}{2}$.

Sol.

$$f(x) = |\cos x| = \begin{cases} \cos x & 0 \le x \le \frac{\pi}{2} \\ -\cos x & \frac{\pi}{2} \le x \le \pi \end{cases}$$

$$\text{LHD at } \frac{\pi}{2} = \lim_{h \to 0} \frac{f\left(\frac{\pi}{2} - h\right) - f\left(\frac{\pi}{2}\right)}{-h} = \lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2} - h\right) - 0}{-h} = \lim_{h \to 0} \frac{\sin h}{-h} = -1$$

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$$\text{RHD at } \frac{\pi}{2} = \lim_{h \to 0} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h} = \lim_{h \to 0} \frac{-\cos\left(\frac{\pi}{2} + h\right) - 0}{h} = \lim_{h \to 0} \frac{\sin h}{h} = 1$$

$\text{LHD} \neq \text{RHD}$

$\therefore$ f is not differentiable at $x = \frac{\pi}{2}$

b. Greatest Integer function :

4.a

Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x = 1$

4.b

Check the differentiability of function f(x) = [x] at x = – 3, where [·] denotes greatest integer function.

Sol.

$f(x) = [x]$ at $x = -3$

$$RHD = \lim_{h \to 0} \frac{f(-3+h) - f(-3)}{h}$$

$$= \lim_{h \to 0} \frac{-3-(-3)}{h} = 0$$

$$LHD = \lim_{h \to 0} \frac{f(-3-h) - f(-3)}{-h}$$

$$= \lim_{h \to 0} \frac{-4-(-3)}{h} = \lim_{h \to 0} \left(\frac{-1}{h}\right)$$

$=$ not defined

$\because$ LHD $\neq$ RHD

So f is not differentiable at $x = -3$

4.c

The greatest integer function defined by f(x) = [x], 0 < x < 2 is not differentiable at x = __________ .

5.

If f(x) = $|\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

Sol.

$$f(x) = -tan2x,\ \frac{\pi}{4} < x < \frac{\pi}{2}$$
$$f'(x) = -2sec^2 2x,\ \frac{\pi}{4} < x < \frac{\pi}{2}$$
$$f'(\frac{\pi}{3}) = -2(-2)^2 = -8$$

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C. Algebraic function :

5. 2024

Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at x = 1.

Sol.

LHD at $x = 1$

$$= \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{\left[(1-h)^2 + 1\right] - 2}{-h} = 2$$

RHD at $x = 1$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\left[3 - (1+h)\right] - 2}{h} = -1$$

as LHD $\neq$ RHD, so $f(x)$ is not differentiable at $x = 1$

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5.a 2023

If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

Sol.

Here

$$\text{RHD} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 2$$

$$\text{LHD} = \lim_{h \to 0} \left[\frac{f(1-h) - f(1)}{-h}\right] = 1$$

Since RHD $\neq$ LHD

$\therefore$ f is not differentiable at x = 1.

5.b 2025

$$\text{If } f(x) = \begin{cases} 2x - 3, & -3 \leq x \leq -2 \\ x + 1, & -2 < x \leq 0 \end{cases}$$

Check the differentiability of f(x) at $x = -2$.

Sol.

$$Lf'(-2) = \lim_{h \to 0} \frac{f(-2-h) - f(-2)}{-h} \qquad (h > 0)$$

$$= \lim_{h \to 0} \frac{2(-2-h) - 3 - (-7)}{-h}$$

$$= \lim_{h \to 0} 2 = 2$$

$$Rf'(-2) = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} \qquad (h > 0)$$

$$= \lim_{h \to 0} \frac{-2 + h + 1 - (-7)}{h}$$

$= \lim_{h \to 0} \frac{6+h}{h}$, which does not exist, i.e., RHD does not exist.

Therefore, the function is not differentiable at -2.

Note: (1) If a student finds only RHD and concludes the result, full marks may be awarded.

(2) If a student proves that the function is discontinuous at -2 and hence not differentiable at -2, full marks may be awarded.

5.c

Find whether the following function is differentiable at x = 1 and x = 2 or not :

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

Sol.

$$\text{L H D at } x = 1 : \lim_{x \to 1^-} \left(\frac{x-1}{x-1} \right) = 1$$

$$\text{R H D at } x = 1 , \lim_{x \to 1^+} \frac{2 - x - 1}{x - 1} = -1$$

$$\therefore \text{ f is not differentiable at } x = 1$$

$$\text{L H D at } x = 2 , \lim_{x \to 2^-} \frac{2 - x - 0}{x - 2} = -1$$

$$\text{R H D at } x = 2 , \lim_{x \to 2^+} \frac{-2 + 3x - x^2}{(x - 2)} = \lim_{x \to 2^-} - \frac{(x-1)(x-2)}{(x-2)} = -1$$

$$\therefore \text{ f is diff. at } x = 2$$

5.d

Examine the following function f(x) for continuity at x = 1 and differentiability at x = 2.

$$f(x) = \begin{cases} 5x - 4 & , \quad 0 < x < 1 \\ 4x^2 - 3x & , \quad 1 < x < 2 \\ 3x + 4 & , \quad x \geq 2 \end{cases}$$

d. finding unknown coefficients a & b :

6.a

Find the values of a and b so that the following function is differentiable for all values of x :

$$f(x) = \begin{cases} ax + b & , \quad x > -1 \\ bx^2 - 3 & , \quad x \leq -1 \end{cases}$$

sol.

Since Differentiability $\Rightarrow$ Continuity

f(x) is continuous at $x = -1 \Rightarrow \lim\limits_{x \to -1^-} \left(bx^2 - 3\right) = \lim\limits_{x \to -1^+} \left(ax + b\right)$

$$\Rightarrow b - 3 = -a + b, \therefore \ a = 3$$

f(x) is differentiable at $x = -1$

$$\Rightarrow \lim_{x \to -1^-} \frac{\left(bx^2 - 3\right) - \left(b - 3\right)}{x + 1} = \lim_{x \to -1^+} \frac{\left(ax + b\right) - \left(b - 3\right)}{x + 1}$$

$$\Rightarrow \lim_{x \to -1^-} b\left(x - 1\right) = \lim_{x \to -1^+} \frac{3\left(x + 1\right)}{x + 1}, \therefore \ b = -\frac{3}{2}$$

Alternately,

$$LHD=\lim_{h\to0}\frac{f(-1)-f(-1-h)}{h}=\lim_{h\to0}\frac{(b-3)-\left(b(-1-h)^2-3\right)}{h}=\lim_{h\to0}(-bh-2b)=-2b$$

$$RHD=\lim_{h\to0}\frac{f(-1)-f(-1-h)}{h}=\lim_{h\to0}\frac{\left[a(-1+h)+b\right]-(b-3)}{h}=\lim_{h\to0}\frac{(-a+3)+ah}{h}=a$$

and $-a+3=0\Rightarrow a=3,-2b=a\Rightarrow b={}^{-3}\!/_{2}$

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6.b

If $f(x)=\begin{cases}ax+b & ; \quad 0<x\le1\\ 2x^2-x & ; \quad 1<x<2\end{cases}$ is a differentiable function in (0, 2), then find the values of a and b.

Sol.

$f(x)$ is differentiable in $(0,2)$

$\Rightarrow f(x)$ is continuous on $(0,2)$

$\Rightarrow f(x)$ is continuous at $x=1$

$\therefore \lim_{x\to1^-}(ax+b)=\lim_{x\to1^+}(2x^2-x)\Rightarrow a+b=1$

Also, $f(x)$ is differentiable at $x=1$,

$\therefore L.H.D.(x=1)=R.H.D.(x=1)$

$\Rightarrow a=4(1)-1 \therefore a=3 \,\&\, b=1-a=-2$

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6.c

If the following function is differentiable at x = 2, then find the values of a and b.

$$f(x)=\begin{cases}x^2, \text{ if } x\le2\\ ax+b, \text{ if } x>2\end{cases}.$$

6.b

Find 'a' and 'b', if the function given by $f(x)=\begin{cases} ax^2+b, \text{ if } x<1 \\ 2x+1, \text{ if } x\geq 1 \end{cases}$

is differentiable at $x=1$

6.c

Find the values of a and b, if the function f defined by

$$f(x)=\begin{cases} x^2+3x+a\,, & x\leq 1 \\ bx+2 & , x>1 \end{cases}$$

is differentiable at $x = 1$.

Sol.

$f'_{1-} = 2x+3 = 5$

$f'_{1+} = b$

$f'_{1-} = f'_{1+} \Rightarrow \boxed{b=5}$

$\lim\limits_{x\to 1^-} f(x) = f(1) = \lim\limits_{x\to 1^+} f(x)$

$\Rightarrow \quad 4+a = b+2$

$\Rightarrow \quad \boxed{a=3}$

6.d

For what value of λ the function defined by $f(x) = \begin{cases} \lambda(x^2+2), & \text{if } x \le 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$.

Sol.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = 2\lambda$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = 6$$

$$f(0) = 2\lambda$$

$$\Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3$$

Differentiability

$$\text{LHD} = \lim_{h \to 0} \frac{f(0) - f(0-h)}{h} = \lim_{h \to 0} \frac{3(2) - 3((-h)^2 + 2)}{h} = \lim_{h \to 0} 3h = 0$$

$$\text{RHD} = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{(4h+6) - 3(2)}{h} = \lim_{h \to 0} 4 = 4$$

$\text{LHD} \neq \text{RHD}$ $\quad \therefore f(x)$ is not differentiable at $x = 0$

Case study :

1.

Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ?

(ii) What is L.H.D. of f(x) at x = 1 ?

Resource Materials by BALAJI KANCHI :

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