5.3 Derivatives

(Previous Year Questions from solutions 20015-2025)

McQ's :

1.

The derivative of log x with respect to $\frac{1}{x}$ is

(A) $-\frac{1}{x^3}$

(B) $-\frac{1}{x}$

(C) $-x$

(D) $\frac{1}{x}$

2.

If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x-1}{y-1}$

(B) $\frac{x-1}{y+1}$

(C) $\frac{y-1}{x+1}$

(D) $\frac{y+1}{x-1}$

3.

The primitive of $\frac{2}{1+\cos 2x}$ is

(a) $\sec^2 x$

(b) $2\sec^2 x \tan x$

(c) $\tan x$

(d) $-\cot x$

4.

If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{-y}{x}$ (B) $\frac{y}{x}$

(C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$

5.

If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to :

(a) $2\sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2(x^3)$

6.

If $y = \log(\sin e^x)$, then $\frac{dy}{dx}$ is :

(a) $\cot e^x$ (b) $\operatorname{cosec} e^x$

(c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

7.

The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x}\log x$

(c) $2x^{2x}(1+\log x)$ (d) $2x^{2x}(1-\log x)$

8.

If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

(a) x (b) $-x$

(c) $16x$ (d) $-16x$

9.

If $y = \log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$, then $\frac{dy}{dx}$ is :

(a) $\sec x$ (b) $\operatorname{cosec} x$

(c) $\tan x$ (d) $\sec x \tan x$

10.

If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ?

(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

(b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

(d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

11.

Derivative of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$ is :

(a) $-\frac{1}{4}$

(b) $\frac{1}{2}$

(c) 2

(d) $-\frac{1}{2}$

12.

If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{y\tan x + \log(\cos y)}{x\tan y - \log(\cos x)}$

(b) $\frac{x\tan y + \log(\cos x)}{y\tan x + \log(\cos y)}$

(c) $\frac{y\tan x - \log(\cos y)}{x\tan y - \log(\cos x)}$

(d) $\frac{y\tan x + \log(\cos y)}{x\tan y + \log(\cos x)}$

13.

If $y = 3\log\sqrt{\sin x}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is :

(a) 3

(b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{-3}{2}$

14.

If $y = \sin^{-1}(2x\sqrt{1-x^2}), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, then $\frac{dy}{dx}$ is :

(a) 2

(b) $\frac{2}{\sqrt{1-x^2}}$

(c) $\frac{-2}{\sqrt{1-x^2}}$

(d) $\sqrt{1-x^2}$

15.

If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to :

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) 1

(d) $\frac{1}{2}$

16.

If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is:

a) e^{y-x}	b) e^{x+y}
c) $-e^{y-x}$	d) $2e^{x-y}$

17.

If y = 5 cos x – 3 sin x, then $\frac{d^2y}{dx^2}$ is equal to:

a) - y	b) y
c) 25y	d) 9y

18.

If $y = \log(\cos e^x)$, then $\frac{dy}{dx}$ is:

a) $\cos e^{x-1}$	b) $e^{-x}\cos e^x$
c) $e^x \sin e^x$	d) $-e^x \tan e^x$

19.

The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t $\sin^{-1}x$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, is:

a) 2	b) $\frac{\pi}{2} - 2$
c) $\frac{\pi}{2}$	d) -2

2024 march :

1.

The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is :

(A) 1 (B) -1

(C) $-2\sqrt{\pi}$ (D) $2\sqrt{\pi}$

2.

Derivative of x^2 with respect to x^3, is :

(A) $\frac{2}{3x}$ (B) $\frac{3x}{2}$

(C) $\frac{2x}{3}$ (D) $6x^5$

3.

If $x = at,\ y = \frac{a}{t}$, then $\frac{dy}{dx}$ is :

(A) t^2 (B) $-t^2$

(C) $\frac{1}{t^2}$ (D) $-\frac{1}{t^2}$

The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :

(A) $\frac{x}{1+x^4}$ (B) $\frac{2x}{1+x^4}$

(C) $-\frac{2x}{1+x^4}$ (D) $\frac{1}{1+x^4}$

4.

$\frac{d}{dx}[\cos(\log x + e^x)]$ at $x = 1$ is :

(A) $-\sin e$ (B) $\sin e$

(C) $-(1 + e)\sin e$ (D) $(1 + e)\sin e$

5.

If $y = \cos^{-1}(e^x)$, then $\frac{dy}{dx}$ is :

(A) $\frac{1}{\sqrt{e^{-2x}+1}}$ (B) $-\frac{1}{\sqrt{e^{-2x}+1}}$

(C) $\frac{1}{\sqrt{e^{-2x}-1}}$ (D) $-\frac{1}{\sqrt{e^{-2x}-1}}$

6.

If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to :

(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$

(C) $\frac{y}{x}$ (D) $-\frac{y}{x}$

7.

The derivative of 2^x w.r.t. 3^x is :

(A) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$ (B) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$

(C) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$ (D) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

8.

The derivative of 5^x w.r.t. e^x is :

(A) $\left(\frac{5}{e}\right)^x \frac{1}{\log 5}$ (B) $\left(\frac{e}{5}\right)^x \frac{1}{\log 5}$

(C) $\left(\frac{5}{e}\right)^x \log 5$ (D) $\left(\frac{e}{5}\right)^x \log 5$

9.

If $y = \sin^{-1} x$, then $\frac{d^2y}{dx^2}$ is :

(A) $\sec y$ (B) $\sec y \tan y$

(C) $\sec^2 y \tan y$ (D) $\tan^2 y \sec y$

10.

Derivative of x^2 with respect to x^3, is :

(A) $\frac{2}{3x}$ (B) $\frac{3x}{2}$

(C) $\frac{2x}{3}$ (D) $6x^5$

11.

Derivative of e^{2x} with respect to e^x, is :

(A) e^x (B) $2e^x$

(C) $2e^{2x}$ (D) $2e^{3x}$

12.

Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

(A) $\sin x\, e^{\sin^2 x}$ (B) $\cos x\, e^{\sin^2 x}$

(C) $-2 \cos x\, e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x\, e^{\sin^2 x}$

13.

If $e^{x^2 y} = c$, then $\frac{dy}{dx}$ is :

(A) $\frac{x e^{x^2 y}}{2y}$ (B) $\frac{-2y}{x}$

(C) $\frac{2y}{x}$ (D) $\frac{x}{2y}$

14.

If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :

(A) -1 (B) 1

(C) $-e$ (D) $-\frac{1}{e}$

2025 March :

1.

If $y = \sin^{-1} x$, then $(1 - x^2)\dfrac{d^2y}{dx^2}$ is equal to :

(A) $x\dfrac{dy}{dx}$ (B) $-x\dfrac{dy}{dx}$

(C) $x^2\dfrac{dy}{dx}$ (D) $-x^2\dfrac{dy}{dx}$

2.

If $y = \log_{2x}(\sqrt{2x})$, then $\dfrac{dy}{dx}$ is equal to :

(A) 0 (B) 1

(C) $\dfrac{1}{x}$ (D) $\dfrac{1}{\sqrt{2x}}$

3.

If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\dfrac{dy}{dx}$ is :

(A) $\dfrac{x}{y}$ (B) $-\dfrac{x}{y}$

(C) $\dfrac{a}{x}$ (D) $\dfrac{a}{y}$

4.

If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y_2 + xy_1$ is :

(A) $\cot(\log x)$ (B) y

(C) $-y$ (D) $\tan(\log x)$

5.

If $f(x) = -2x^8$, then the correct statement is :

(A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

(C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

I.Composite function (chain rule) :

1.

Differentiate $\cos\{\sin(x)^2\}$ with respect to x.

Sol.

$$\frac{dy}{dx} = -2x \cdot \cos x^2 \cdot \sin(\sin x^2)$$

2.

Differentiate $\sqrt{\sin(e^x)}$ with respect to x.

Sol.

$$\frac{dy}{dx} = \frac{e^x \cdot \cos(e^x)}{2\sqrt{\sin(e^x)}}$$

3. 2024

If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

Sol.

$$y = \cos^3(\sec^2 2t)$$

$$\Rightarrow \frac{dy}{dt} = 3\cos^2(\sec^2 2t)[-\sin(\sec^2 2t)] \times \frac{d(\sec^2 2t)}{dt}$$

$$\Rightarrow \frac{dy}{dt} = -3\cos^2(\sec^2 2t).\sin(\sec^2 2t) \times 2\sec 2t.\sec 2t \tan 2t.2$$

$$\therefore \frac{dy}{dt} = -12\cos^2(\sec^2 2t) \times \sin(\sec^2 2t) \times \sec^2 2t \times \tan 2t.$$

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4.

Differentiate $\sin^2(x^2)$ w.r.t x^2.

5.

Differentiate $\sin^2\left(\sqrt{x}\right)$ with respect to x.

Ans: $\frac{\sin\left(2\sqrt{x}\right)}{2\sqrt{x}}$ or $\frac{\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

6.a

If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\,\dfrac{dy}{dx} = \dfrac{1+y^4}{4y}$.

Sol.

$$y = \sqrt{tan\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{sec^2\sqrt{x}}{2\sqrt{tan\sqrt{x}}} \times \frac{1}{2\sqrt{x}}$$

$$\sqrt{x}\frac{dy}{dx} = \frac{sec^2\sqrt{x}}{4\sqrt{tan\sqrt{x}}}$$

$$= \frac{1+(tan\sqrt{x})^2}{4\sqrt{tan\sqrt{x}}} = \frac{1+y^4}{4y}$$

6.b

If $f(x) = \sqrt{\tan\sqrt{x}}$, then find $f'\left(\dfrac{\pi^2}{16}\right)$.

Sol.

$$f'(x) = \frac{\sec^2\sqrt{x}}{4\sqrt{x}\sqrt{\tan(\sqrt{x})}}$$

$$f'\left(\frac{\pi}{16}\right) = \frac{2}{4\cdot\frac{\pi}{4}} = \frac{2}{\pi}$$

7.a 2024

If $y = \sin(\tan^{-1} e^x)$, then find $\dfrac{dy}{dx}$ at $x = 0$.

Sol.

$$\frac{dy}{dx} = \cos(tan^{-1}(e^x)) \times \frac{e^x}{1+e^{2x}}$$

$$\left(\frac{dy}{dx}\right)_{x=0} = cos\frac{\pi}{4} \times \frac{1}{2} = \frac{1}{2\sqrt{2}}$$

7.b

If $y = \text{cosec}(\cot\sqrt{x})$, then find $\dfrac{dy}{dx}$.

Sol.

$$\frac{dy}{dx} = +\frac{\text{cosec}(\cot\sqrt{x})\cot(\cot\sqrt{x})\,\text{cosec}^2\sqrt{x}}{2\sqrt{x}}$$

8.

If $y = \log(\cos e^{x})$, then find $\frac{dy}{dx}$.

Sol.

$$\frac{dy}{dx} = \frac{-\sin e^{x}}{\cos e^{x}} \cdot e^{x} \text{ or } -e^{x} \cdot \tan e^{x}$$

9.

If $y = \sqrt{a + \sqrt{a + x}}$, then find $\frac{dy}{dx}$.

Sol.

$$y = \sqrt{a + \sqrt{a + x}}, \text{ let } u = a + \sqrt{a + x}$$

$$y = \sqrt{u} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{u}} \cdot \frac{du}{dx}, \frac{du}{dx} = \frac{1}{2\sqrt{a + x}}$$

$$= \frac{1}{2\sqrt{a + \sqrt{a + x}}} \cdot \frac{1}{2\sqrt{a + x}}$$

$$= \frac{1}{4\sqrt{a + \sqrt{a + x}}\sqrt{a + x}}$$

10.2023

If $y=\sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2=0$.

Sol.

$$y=\sqrt{ax+b}\Rightarrow y^2=ax+b$$

Differentiate with respect to 'x', $2y\frac{dy}{dx}=a$

Differentiate with respect to 'x', $2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=0\Rightarrow y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

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11. 2025

Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x.

Sol.

Let $y=\frac{\sin x}{\sqrt{\cos x}}$

$$\frac{dy}{dx}=\frac{\sqrt{\cos x}.\cos x-\sin x.\left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$$

$$\Rightarrow\frac{dy}{dx}=\frac{2\cos^2x+\sin^2x}{2(\cos x)^{3/2}}\text{ or }\frac{1+\cos^2x}{2(\cos x)^{3/2}}$$

prepared by : BALAJI KANCHI

12. 2025

Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

Sol.

$$\text{Let, } y=\frac{5^x}{x^5}=5^x\cdot x^{-5}\Rightarrow\frac{dy}{dx}=\left(5^x\right)'\cdot x^{-5}+5^x\cdot\left(x^{-5}\right)'$$

$$=\frac{5^x}{x^5}\log 5-\frac{5^{x+1}}{x^6}$$

prepared by : BALAJI KANCHI

13. 2025

Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.

Sol.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}} \cdot \frac{1}{\sin\left(\frac{x^3}{3}-1\right)} \cdot \cos\left(\frac{x^3}{3}-1\right) \cdot \frac{3x^2}{3}$$

$$= \frac{x^2 \cot\left(\frac{x^3}{3}-1\right)}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}}$$

14. 2025

Differentiate $\log(x^x + \operatorname{cosec}^2 x)$ with respect to x.

Sol.

$$\frac{d}{dx}\log\left(x^x + \operatorname{cosec}^2 x\right) = \frac{1}{x^x + \operatorname{cosec}^2 x}\frac{d}{dx}\left(e^{x\log x} + \operatorname{cosec}^2 x\right) \quad (\because x^x = e^{x\log x})$$

$$= \frac{1}{x^x + \operatorname{cosec}^2 x}\left[e^{x\log x}(1+\log x) - 2\operatorname{cosec}^2 x \cot x\right]$$

$$= \frac{1}{x^x + \operatorname{cosec}^2 x}\left[x^x(1+\log x) - 2\operatorname{cosec}^2 x \cot x\right]$$

15.

Differentiate $e^{\sqrt{3x}}$, with respect to x.

Ans.

$$\frac{d}{dx}(e^{\sqrt{3x}}) = \frac{\sqrt{3}}{2\sqrt{x}} e^{\sqrt{3x}}$$

16.

If $y = \cos(\sqrt{3x})$, then find $\frac{dy}{dx}$.

Sol.

$$\frac{dy}{dx} = -\frac{\sqrt{3}\sin(\sqrt{3x})}{2\sqrt{x}}$$

II.Inverse Trigonometric Functions :

1.

If $y = \sin^{-1} x + \cos^{-1} x$, find $\frac{dy}{dx}$.

Sol.

$$y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$$

2.

If $y = \tan^{-1} x + \cot^{-1} x$, $x \in R$, then $\frac{dy}{dx}$ is equal to __________ .

Ans.

$$y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$$

3.a

If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1 + x^2}\frac{dy}{dx} - x = 0$.

Sol.

$$y = \sqrt{1 + cot^2(cot^{-1}x)} = \sqrt{1 + x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{1 + x^2}}$$

$$\Rightarrow \sqrt{1 + x^2}\frac{dy}{dx} - x = 0$$

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3.b

Find the differential of the function $\cos^{-1}(\sin 2x)$ w.r.t. x.

Sol.

Let $y = \cos^{-1}(\sin 2x)$, then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\sin^2 2x}}(2\cos 2x)$

$$= -2 \quad \text{or} \quad 2$$

4.

If $y = \tan^{-1}\left[\frac{x}{\sqrt{a^2 - x^2}}\right]$, $|x| < a$, then find $\frac{dy}{dx}$.

Sol.

Substituting $x = a \sin\theta$

$$y = \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right) = \theta = \sin^{-1}\frac{x}{a}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\cdot\frac{1}{a} = \frac{1}{\sqrt{a^2 - x^2}}$$

5.a 2025

Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Sol.

$x = \sin t$ gives $y = \sin^{-1}(\sin 3t) = 3t = 3\sin^{-1}x$

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$$

Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$

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5.b 2025

Differentiate $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

If $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, $0 < x < 1$, then find $\dfrac{dy}{dx}$.

Sol.

$x = \tan t$ gives $y = \cos^{-1}(\cos 2t) = 2t = 2\tan^{-1}x$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

Aliter: $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \dfrac{-4x}{(1+x^2)^2}$.

prepared by : BALAJI KANCHI

OR

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \text{ Put } x = \tan\theta \Rightarrow \theta = \tan^{-1}x$$

$$\Rightarrow y = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x,$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

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5.c 2025

Differentiate $\tan^{-1}\dfrac{3x-x^3}{1-3x^2}$, $|x| < \dfrac{1}{\sqrt{3}}$ w.r.t. $\tan^{-1}\dfrac{x}{\sqrt{1-x^2}}$.

6.

If $y = \sin^{-1}(6x\sqrt{1-9x^2}), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

Sol.

$$y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$$

put $3x = \sin\theta = \Rightarrow \theta = \sin^{-1} 3x$

$$y = \sin^{-1}(\sin 2\theta)$$

$$= 2\theta = 2\sin^{-1} 3x$$

$$\therefore \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

7.

Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

Sol.

Let $2x = \sin\theta$

$$\therefore y = \sin^{-1}\left(\frac{6x - \sqrt{1-4x^2}}{5}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\sin\theta - \frac{4}{5}\cos\theta\right)$$

$= \sin^{-1}(\cos\alpha \sin\theta - \sin\alpha \cos\theta)$ $[\cos\alpha = \frac{3}{5}; \sin\alpha = \frac{4}{5}]$

$= \sin^{-1}(\sin(\theta - \alpha))$

$= \theta - \alpha$

$= \theta - \alpha$

$= \sin^{-1}(2x) - \alpha$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

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8.

Differentiate the following with respect of x:

$$y=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

9.a

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

9.b

If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $x^2\le 1$, then find $\frac{dy}{dx}$.

Sol.

Putting $x^2=\cos\theta$, we get

$$y=\tan^{-1}\left(\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\cos\theta/2+\sin\theta/2}{\cos\theta/2-\sin\theta/2}\right)=\tan^{-1}\left(\frac{1+\tan\theta/2}{1-\tan\theta/2}\right)$$

$$y=\frac{\pi}{4}+\theta/2=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

$$\frac{dy}{dx}=-\frac{1}{2}\frac{1}{\sqrt{1-x^4}}\cdot 2x=-\frac{x}{\sqrt{1-x^4}}$$

prepared by : BALAJI KANCHI

10.

If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

Sol.

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1} x$

$$\therefore y = \sin^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{2}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$$

$$\Rightarrow y = \frac{\pi}{4}+\theta = \frac{\pi}{4}+\frac{1}{2}\cos^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$$

11.

Find :

$$\frac{d}{dx}\cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$$

Sol.

$$\text{Let } y = \cos^{-1}\left(\frac{x-x^{-1}}{x-x^{-1}}\right) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$$

$$= \pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \pi - 2\tan^{-1} x$$

$$\therefore \frac{dy}{dx} = -\frac{2}{1+x^2}$$

12.

Find $\frac{dy}{dx}$, if $y = e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

13.

Differentiate $\tan^{-1}\left(\dfrac{1+\cos x}{\sin x}\right)$ with respect to x.

Sol.

$$f(x) = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2\ \cos x/2}\right)$$

$$= \tan^{-1}\left(\cot x/2\right) = \frac{\pi}{2} - \frac{x}{2}$$

$$\therefore\ f'(x) = -\frac{1}{2}$$

14.

Differentiate $\tan^{-1}\left(\dfrac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x.

Sol.

$$\text{Let } y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

15. 2025

Show that the derivative of $\tan^{-1}(\sec x + \tan x), \left[-\frac{\pi}{2} < x < \frac{\pi}{2}\right]$ with respect to x is equal to $\frac{1}{2}$.

Sol.

$$\tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right)$$

$$= \tan^{-1}\left(\frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$$

$$= \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

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$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) = \frac{\pi}{4}+\frac{x}{2}$$

$$\therefore \left(\tan^{-1}(\sec x + \tan x)\right)' = \frac{1}{2}$$

16.

Differentiate $\cot^{-1}\left(\sqrt{1+x^2}+x\right)$ w.r.t. x.

Sol.

$y = \cot^{-1}(\sqrt{1+x^2}+x)$, (Put $x = \cot\theta$)

$$= \cot^{-1}(\operatorname{cosec}\theta + \cot\theta) = \cot^{-1}\left(\frac{1+\cos\theta}{\sin\theta}\right)$$

$$= \cot^{-1}\left(\cot\frac{\theta}{2}\right) = \frac{1}{2}\cot^{-1}x$$

$$\frac{dy}{dx} = -\frac{1}{2(1+x^2)}$$

prepared by : BALAJI KANCHI

17.

Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{2^{x+1}}{1+4^{x}}\right]$.

Sol.

$$y = \sin^{-1}\left(\frac{2 \cdot 2^{x}}{1+\left(2^{x}\right)^{2}}\right) = \sin^{-1}\left(\frac{2t}{1+t^{2}}\right), \text{ where } t = 2^{x}$$

$$\Rightarrow y = 2\tan^{-1} t$$

$$\frac{dy}{dt} = \frac{2}{1+t^{2}} \text{ and } \frac{dt}{dx} = 2^{x} \cdot \log 2.$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{1+t^{2}} \cdot 2^{x} \cdot \log 2 = \frac{2^{x+1} \cdot \log 2}{1+4^{x}}$$

prepared by : **BALAJI KANCHI**

III. Derivative of the function using logarithm (variable power variable form, power form) :

a. $y = u + v$ form , then $y' = u' + v'$:

1.a

If $y = (\cos x)^x + \tan^{-1}\sqrt{x}$, find $\dfrac{dy}{dx}$.

Sol.

$$y = (\cos x)^x + \tan^{-1}\sqrt{x} \ = u + v, \therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\log u = x \log(\cos x) \Rightarrow \frac{du}{dx} = (\cos x)^x \left[-x \tan x + \log(\cos x)\right]$$

$$\text{and } \frac{dv}{dx} = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = (\cos x)^x \left[-x \tan x + \log(\cos x)\right] + \frac{1}{2\sqrt{x}\ (1+x)}$$

1.b

If $y = (\cos x)^x + \sin^{-1}\sqrt{3x}$, find $\dfrac{dy}{dx}$.

Sol.

$$\text{Let } u = (\cos x)^x \Rightarrow \log u = x.\log \cos x$$

$$\Rightarrow \quad \frac{du}{dx} = (\cos x)^x.[-x \tan x + \log \cos x]$$

$$\therefore \quad y = (\cos x)^x + \sin^{-1}\sqrt{3x} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-3x}} \cdot \frac{\sqrt{3}}{2\sqrt{x}}$$

$$\therefore \quad \frac{dy}{dx} = (\cos x)^x[-x \tan x + \log \cos x] + \frac{\sqrt{3}}{2\sqrt{x}} \cdot \frac{1}{\sqrt{1-3x}}$$

prepared by : BALAJI KANCHI

1.c 2024

Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.

Sol.

$$\text{Let } y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\cos x)^x \Rightarrow \log u = x \log \cos x$$

$$\Rightarrow \frac{du}{dx} = (\cos x)^x [\log \cos x - x \tan x]$$

$$v = \cos^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = -\frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$\therefore \frac{dy}{dx} = (\cos x)^x [\log \cos x - x \tan x] - \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

prepared by : BALAJI KANCHI

1.d

Find $\frac{dy}{dx}$, if $y = x^{\cos x} - 2^{\sin x}$.

Sol.

$$\text{Let } y = u - v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx},$$

$$\text{where } u = x^{\cos x}, v = 2^{\sin x}$$

$$\log u = \cos x \log x$$

$$\Rightarrow \frac{du}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right]$$

$$\frac{dv}{dx} = 2^{\sin x} \cos x \log 2$$

$$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] - 2^{\sin x} \cos x \log 2$$

prepared by : BALAJI KANCHI

2.a

Differentiate $x^{\sin x}+(\sin x)^{\cos x}$ with respect to x.

Sol.

$$\text{Let, } y = x^{\sin x} + (\sin x)^{\cos x} = u + v; \quad \therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = x^{\sin x} \Rightarrow \log u = \sin x \cdot \log x \Rightarrow$$

$$\frac{du}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\}$$

$$v = (\sin x)^{\cos x} \Rightarrow \log v = \cos x \cdot \log(\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x \cdot \log \sin x\}$$

$$\frac{dy}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x \,.\, \log \sin x\}$$

prepared by : BALAJI KANCHI

2.b

Differentiate the function $(\sin x)^x + \sin^{-1}\sqrt{x}$ with respect to x.

Find the derivative of $(\sin x)^x + \sin^{-1}\sqrt{x}$ w.r.t. x

Sol.

$$y = (\sin x)^x + \sin^{-1}\sqrt{x}$$

$$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin x)^x$$

$$\Rightarrow \log u = x \log \sin x$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x [x \cot x + \log \sin x]$$

$$v = \sin^{-1}\sqrt{x}$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{1}{2\sqrt{x - x^2}}$$

$$\therefore \quad \frac{dy}{dx} = (\sin x)^x[x \cot x + \log \sin x] + \frac{1}{2\sqrt{x - x^2}}$$

2.c

If $y = x^{\sin x} + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

Sol.

Let $u = x^{\sin x} \quad \therefore\ y = u + \sin^{-1}\sqrt{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{2\sqrt{x}\,\sqrt{1-x}} \quad \ldots (i)$$

Also, $\log u = \sin x \cdot \log x$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \cdot \log x + \frac{\sin x}{x}$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x} \cdot \left(\cos x \cdot \log x + \frac{\sin x}{x}\right) \quad \ldots (ii)$$

Putting (ii) in (i) we get

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \log x + \frac{\sin x}{x}\right) + \frac{1}{2\sqrt{x}\,\sqrt{1-x}}$$

2.d

If $y = (\sin x)^x + \sin^{-1}(\sqrt{1-x^2})$, then find $\frac{dy}{dx}$.

Sol.

$$y = (\sin x)^x + \sin^{-1}\sqrt{1-x^2}$$

$$\text{Let } A = (\sin x)^x \Rightarrow \log A = x \log \sin x$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dx} = \log \sin x + x \cot x$$

$$\Rightarrow \frac{dA}{dx} = (\sin x)^x \left(\log \sin x + x \cot x\right)$$

$$B = \sin^{-1}\sqrt{1-x^2} = \cos^{-1} x$$

$$\frac{dB}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$y = A + B \Rightarrow \frac{dy}{dx} = \frac{dA}{dx} + \frac{dA}{dx}$$

$$= (\sin x)^x \left(\log \sin x + x \cot x\right) - \frac{1}{\sqrt{1-x^2}}$$

prepared by : **BALAJI KANCHI**

2.e

If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$.

Sol.

Let $u = x^{\cos x}$, $v = (\cos x)^{\sin x}$

$$y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$$\log u = \cos x \log x \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{\cos x}{x} - \sin x \log x$$

$$\frac{du}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right)$$

$$\log v = \sin x \log \cos x$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \tan x + \cos x \log \cos x$$

$$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$$

So, $\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right) + (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$

prepared by : BALAJI KANCHI

2.f

Differentiate $(\sin 2x)^x + \sin^{-1}\sqrt{3x}$ with respect to x.

Sol.

$y = (\sin 2x)^x + \sin^{-1}(\sqrt{3x}) = u + v$

$$\therefore \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin 2x)^x \Rightarrow \log u = x \log \sin 2x$$

$$\frac{1}{u}\frac{du}{dx} = 2x \cdot \cot 2x + \log \sin 2x$$

$$\therefore \quad \frac{du}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x]$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-3x}} \frac{\sqrt{3}}{2\sqrt{x}}$$

$$\therefore \quad \frac{dy}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x] + \frac{\sqrt{3}}{2\sqrt{x}\sqrt{1-3x}}$$

3.

If $y = (\log x)^x + x^{\log x}$, then find $\frac{dy}{dx}$.

Sol.

$$y=(\log x)^x+x^{\log x}=u+v\Rightarrow\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$$

$$\therefore \log u=x\log(\log x)\text{ and }\log v=(\log x)^2$$

$$\frac{du}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]\text{and }\frac{dv}{dx}=x^{\log x}.\frac{2\log x}{x}$$

$$\Rightarrow\frac{dy}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]+x^{\log x}.\frac{2\log x}{x}$$

prepared by : BALAJI KANCHI

OR

Sol.

Let $u = (\log x)^x \quad v = x^{\log x}$

$$y = u + v \Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$$

$$\log u = x \log x$$

$$\frac{1}{u}\frac{du}{dx}=\frac{x}{\log x}\cdot\frac{1}{x}+\log\ \log x$$

$$\frac{du}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log\log x\right]$$

$$\log v = (\log x)^2$$

$$\frac{1}{v}\frac{dv}{dx}=\frac{2\log x}{x}$$

$$\therefore\ \frac{dv}{dx}=x^{\log x}\left(\frac{2\log x}{x}\right)$$

So $\frac{dy}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log\log x\right]+2x^{(\log x-1)}\cdot\log x$

4.

If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$.

Sol.

Let $u = (\cos x)^x \Rightarrow y = e^{x^2 \cdot \cos x} + u$

$$\therefore \frac{dy}{dx} = e^{x^2 \cdot \cos x}(2x \cdot \cos x - x^2 \cdot \sin x) + \frac{du}{dx}$$

$\log u = \log(\cos x)^x \Rightarrow \log u = x \,.\, \log(\cos x)$

Differentiate w.r.t. "x"

$$\frac{1}{u}\frac{du}{dx} = \log(\cos x) - x\tan x \Rightarrow \frac{du}{dx} = (\cos x)^x \{\log(\cos x) - x\tan x\}$$

Therefore,

$$\frac{dy}{dx} = e^{x^2 \cdot \cos x}(2x \cdot \cos x - x^2 \cdot \sin x) + (\cos x)^x \{\log(\cos x) - x\tan x\}$$

prepared by : BALAJI KANCHI

5.

If $y = x^3 (\cos x)^x + \sin^{-1}\sqrt{x}$, find $\dfrac{dy}{dx}$.

Sol.

Let $u = x^3(\cos x)^x$ and $v = \sin^{-1}\sqrt{x}$ so that $y = u + v$

$$\log u = 3\log x + x\log(\cos x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{3}{x} - x\tan x + \log\cos x$$

$$\Rightarrow \frac{du}{dx} = x^3(\cos x)^x\left[\frac{3}{x} - x\tan x + \log\cos x\right] \quad \text{.... (i)}$$

and $v = \sin^{-1}\sqrt{x} \Rightarrow \dfrac{dv}{dx} = \dfrac{1}{2\sqrt{x}\sqrt{1-x}}$ ... (ii)

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{dy}{dx} = x^3(\cos x)^x\left[\frac{3}{x} - x\tan x + \log\cos x\right] + \frac{1}{2\sqrt{x-x^2}}$$

6. 2024

Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find $\dfrac{dy}{dx}$.

Sol.

As, $y = (\sin x)^x . x^{\sin x} + a^x = u + a^x \Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{d(a^x)}{dx}$

where $u = (\sin x)^x . x^{\sin x} \Rightarrow \log u = x\log(\sin x) + \sin x . \log x$

on differentiating both sides with respect to x, we get

$$\Rightarrow \frac{du}{dx} = (\sin x)^x . x^{\sin x}\left[\log(\sin x) + x\cot x + \frac{\sin x}{x} + \log x . \cos x\right]$$

$$\text{Thus, } \frac{dy}{dx} = (\sin x)^x . x^{\sin x}\left[\log(\sin x) + x\cot x + \frac{\sin x}{x} + \log x . \cos x\right] + a^x \log a$$

7. 2025

Find $\frac{dy}{dx}$, if $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$.

Sol.

Let $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$, where $u = x^{\tan x}, v = \frac{\sqrt{x^2+1}}{2}$

$u = x^{\tan x} \Rightarrow \log u = \tan x \log x$, differentiating with respect to 'x', we get

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{\tan x}{x} + \sec^2 x \log x$$

$$\Rightarrow \frac{du}{dx} = u\left(\frac{\tan x}{x} + \sec^2 x \log x\right) = x^{\tan x}\left(\frac{\tan x}{x} + \sec^2 x \log x\right)$$

$$v = \frac{\sqrt{x^2+1}}{2} \Rightarrow \frac{dv}{dx} = \frac{2x}{4\sqrt{x^2+1}} = \frac{x}{2\sqrt{x^2+1}}$$

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$$\Rightarrow \frac{dy}{dx} = x^{\tan x}\left(\frac{\tan x}{x} + \sec^2 x \log x\right) + \frac{x}{2\sqrt{x^2+1}}$$

8.

Find the derivative of the following function f(x) w.r.t. x, at x = 1 :

$$\cos^{-1}\left[\sin\sqrt{\frac{1+x}{2}}\right] + x^x$$

Sol.

$$\text{Let } y = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} + x^x$$

$$\text{Let } u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\}; \quad v = x^x$$

$$\therefore \quad y = u + v$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} = \cos^{-1}\left[\cos\cdot\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right]$$

$$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}}$$

$$\therefore \frac{du}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$v = x^x$$

$$\therefore \log v = x \log x$$

$$\frac{1}{v}\frac{dv}{dx} = x \cdot \frac{1}{x} + 1 \log x = 1 + \log x$$

$$\frac{dv}{dx} = x^x (1 + \log x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} + x^x (1 + \log x)$$

$$\left(\frac{dy}{dx}\right)_{at\ x=1} = -\frac{1}{4} + 1 = \frac{3}{4}$$

prepared by : BALAJI KANCHI

9. 2025

For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number.

Sol.

$$\text{Let} \quad u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}}.\log a.\left(1 - \frac{1}{t^2}\right)$$

$$v = \left(t + \frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a\left(t + \frac{1}{t}\right)^{a-1}.\left(1 - \frac{1}{t^2}\right)$$

$$\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}}.\log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$$

prepared by : BALAJI KANCHI

10.

If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\dfrac{dy}{dx}$.

Sol.

$$y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$$

$$\frac{d}{dx}(\cos x^2) = -2x \sin x^2$$

$$\frac{d}{dx}(\cos^2 x) = 2 \cos x\,(-\sin x) = -2\sin x\cos x$$

$$\frac{d}{dx}(\cos^2(x^2)) = 2 \cos(x^2)\,(-\sin(x^2))(2x) = -4x \sin x^2 \cos x^2$$

$$\frac{d}{dx}(\cos(x^x)) = -\sin(x^x)\,[x^x(1 + \log x)]$$

$$\frac{dy}{dx} = -2x \sin x^2 - 2\sin x\cos x - 4x \sin x^2 \cos x^2 - \sin(x^x)\,[x^x(1 + \log x)]$$

prepared by : **BALAJI KANCHI**

b. $u + v = w$ form , then $u' + v' = w'$ (variable power variable):

1.

If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

Sol.

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx} - \frac{dv}{dx} = 0$$

Now, $\log u = y.\log x$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} \Rightarrow \frac{du}{dx} = x^y \left(\frac{y}{x} + \log x \cdot \frac{dy}{dx} \right)$$

Again, $\log v = x \cdot \log y$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1 \Rightarrow \frac{dv}{dx} = y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right)$$

From (1), (2) and (3)

$$x^y \left(\frac{y}{x} + \log x \frac{dy}{dx} \right) - y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) = 0$$

prepared by : BALAJI KANCHI

2. 2024

Given that $x^y + y^x = a^b$,

where a and b are positive constants, find $\dfrac{dy}{dx}$.

Sol.

Let $u = x^y \Rightarrow \dfrac{du}{dx} = x^y\left(\dfrac{y}{x} + \log x.\dfrac{dy}{dx}\right)$, $v = y^x \Rightarrow \dfrac{dv}{dx} = y^x\left(\dfrac{x}{y}\dfrac{dy}{dx} + \log y\right)$

Since, $u + v = a^b \Rightarrow \dfrac{du}{dx} + \dfrac{dv}{dx} = 0$

$$\therefore \frac{dy}{dx} = \frac{-\left(x^{y-1}y + y^x \log y\right)}{\left(x^y \log x + y^{x-1}x\right)}$$

OR

$x^y + y^x = a^b$

Let $u + v = a^b$, where $x^y = u$ and $y^x = v$.

$$\therefore \quad \frac{du}{dx} + \frac{dv}{dx} = 0 \qquad \ldots(i)$$

$$y \log x = \log u \Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right]$$

$$x \log y = \log v \Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$$

Putting in (i) $x^y\left[\dfrac{y}{x} + \log x\dfrac{dy}{dx}\right] + y^x\left[\dfrac{x}{y}\dfrac{dy}{dx} + \log y\right] = 0$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^x \log y + y \cdot x^{y-1}}{x^y \cdot \log x + x \cdot y^{x-1}}$$

3. 2025

Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

sol.

Let $u = y^x$, $v = x^y$ and $w = x^x$

$$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \text{ ....(i)}$$

$$u = y^x \Rightarrow \log u = x.\log y \Rightarrow \frac{1}{u}.\frac{du}{dx} = \frac{x}{y}.\frac{dy}{dx} + \log y$$

$$\Rightarrow \frac{du}{dx} = y^x\left(\frac{x}{y}.\frac{dy}{dx} + \log y\right) = xy^{x-1}\frac{dy}{dx} + y^x \log y$$

$$v = x^y \Rightarrow \log v = y.\log x \Rightarrow \frac{1}{v}.\frac{dv}{dx} = \frac{y}{x} + \log x.\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) = yx^{y-1} + x^y \log x \frac{dy}{dx}$$

$$w = x^x \Rightarrow \log w = x.\log x \Rightarrow \frac{1}{w}.\frac{dw}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dw}{dx} = x^x.(1 + \log x)$$

∴ From (i), we get

$$xy^{x-1}.\frac{dy}{dx} + y^x.\log y + yx^{y-1} + x^y.\log x.\frac{dy}{dx} + x^x.(1 + \log x) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^x.(1 + \log x) + y^x.\log y + yx^{y-1}}{x.y^{x-1} + x^y.\log x}$$

prepared by : BALAJI KANCHI

4.

If $(\sin x)^y = x + y$, find $\dfrac{dy}{dx}$.

Sol.

$$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$$

differentiating w.r.t. x, we get

$$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\dfrac{1}{x+y} - y\cot x}{\log \sin x - \dfrac{1}{x+y}}$$

$$= \frac{1 - y(x+y)\cot x}{(x+y)\log \sin x - 1}$$

c. Taking log on both sides direct form (single term on both sides):

1

If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

Sol.

$$(\cos x)^y = (\sin y)^x$$

$$\Rightarrow y \cdot \log(\cos x) = x \cdot \log(\sin y)$$

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} \cdot \log(\cos x) + y(-\tan x) = \log(\sin y) + x \cdot \cot y \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \cdot \tan x + \log(\sin y)}{\log(\cos x) - x \cot y}$$

2.

Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

Sol.

Taking 'log' on both sides of $(\cos x)^y = (\cos y)^x$, we get

$$y \log \cos x = x \log \cos y$$

$$\Rightarrow \frac{dy}{dx} \log \cos x + y(-\tan x) = \log \cos y + x(-\tan y)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$$

3.

If $y = (\tan x)^x$, then find $\dfrac{dy}{dx}$.

Sol.

$$y = (\tan x)^x$$

$$\log y = x \log(\tan x)$$

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{\sec^2 x}{\tan x}\right) + \log(\tan x)$$

$$\frac{dy}{dx} = (\tan x)^x\left[\left(\frac{x\sec^2 x}{\tan x}\right) + \log(\tan x)\right]$$

prepared by : BALAJI KANCHI

4.

Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$.

Sol.

Taking log on both sides. $\log f(x) = \tan x \log \tan x$

differentiating to get $\dfrac{f'(x)}{f(x)} = \sec^2 x + \sec^2 x \log \tan x$

Thus, $f'(x) = (\tan x)^{\tan x} \cdot \sec^2 x(1 + \log \tan x)$

5.

Find $\frac{dy}{dx}$, if $x^y \cdot y^x = x^x$.

Sol.

$x^y \cdot y^x = x^x$

$\Rightarrow \quad y \log x + x \log y = x \log x$

differentiate both sides w.r.t. x,

$$\left(y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}\right) + \left(x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1\right) = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\Rightarrow \quad \frac{y}{x} + \log\left(\frac{y}{x}\right) - 1 = -\left(\log x + \frac{x}{y}\right) \cdot \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1 - \frac{y}{x} - \log\left(\frac{y}{x}\right)}{\log x + \frac{x}{y}} \text{ or } \frac{y}{x}\left[\frac{x + x \log x - y - x \log y}{y \log x + x}\right]$$

prepared by : BALAJI KANCHI

6. 2025

If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

Sol.

Taking log on both sides, we get y logx = x logy

Differentiating both sides w.r.t. x, we get

$$\frac{y}{x.} + \log x \frac{dy}{dx} = \frac{x}{y.}\frac{dy}{dx} + \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$$

prepared by : BALAJI KANCHI

OR

$$y^x = x^y \Rightarrow x\log y = y\log x$$

Differentiating with respect to 'x',

$$\frac{x}{y}\frac{dy}{dx} + \log y = \frac{y}{x} + \frac{dy}{dx}\log x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y - x\log y}{x - y\log x}\right)$$

7. 2023

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

Sol.

$$y = x^x \Rightarrow \log y = x\log x,$$

differentiating with respect to 'x', we get

$$\frac{dy}{dx} = y(1 + \log x),$$

differentiating with respect to 'x', we get

$$\frac{d^2y}{dx^2} = \frac{y}{x} + (1 + \log x)\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

8. 2023

If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

Sol.

$$y = x^{1/x}$$

$$\Rightarrow \log y = \frac{1}{x}\log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{\log x}{x^2} + \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}}\frac{(1-\log x)}{x^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 1$$

prepared by : BALAJI KANCHI

9.

If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Sol.

$$As, x^y = e^{x-y} \Rightarrow \log(x^y) = \log(e^{x-y})$$

$$\Rightarrow y\log x = (x-y) \Rightarrow y = \frac{x}{1+\log x}$$

Now, Differentiating both the sides wrt x

$$\frac{dy}{dx} = \frac{(\log x+1).1 - x(\frac{1}{x})}{(\log x+1)^2} = \frac{\log x}{(1+\log x)^2}$$

prepared by : BALAJI KANCHI

OR

Taking log on both sides

$y \log x = (x - y)$

$y(1 + \log x) = x$

$$y = \frac{x}{1+\log x}$$

$$\frac{dy}{dx} = \frac{(1+\log x)1 - x\frac{1}{x}}{(1+\log x)^2}$$

$$= \frac{\log x}{(1+\log x)^2},$$ Hence proved

10.

If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

Sol.

$$x^y = e^{x-y} \Rightarrow y \log x = x - y \Rightarrow y = \frac{x}{\log x + 1}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log x}{(\log x + 1)^2}$$

$$= \frac{\log x}{\{\log(xe)\}^2}$$

prepared by : BALAJI KANCHI

11.

Find the derivative of $x^{\log x}$ w.r.t. $\log x$.

Sol.

Let $u = x^{\log x}$ and $v = \log x$

Now, $\log u = (\log x)^2 \Rightarrow \frac{1}{u}\frac{du}{dx} = 2\log x.\frac{1}{x}$

$\Rightarrow \frac{du}{dx} = \frac{2\log x}{x}.x^{\log x}$

Again, $v = \log x \Rightarrow \frac{dv}{dx} = \frac{1}{x}$

$\therefore \frac{du}{dv} = 2x^{\log x}\log x$

12.

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$

Sol.

$y = x^x \Rightarrow \log y = x.\log x$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (1 + \log x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

VI.Parametric form :

1.

If $x = at^2$, $y = 2at$, then find $\frac{d^2y}{dx^2}$.

Sol.

Ans: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$

$$\frac{d^2y}{dx^2} = -\frac{1}{t^2} \cdot \frac{dt}{dx} = -\frac{1}{t^2} \cdot \frac{1}{2at} = -\frac{1}{2a\,t^3}$$

prepared by : BALAJI KANCHI

2.

If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$,

then show that $\frac{dy}{dx} = -\frac{x}{y}$

and hence show that

$$y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$

Sol.

$$\frac{dx}{d\theta} = -\ a\sin\theta + b\cos\theta$$

$$\frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{a\cos\theta + b\sin\theta}{a\sin\theta + b\cos\theta} = -\frac{x}{y}$$

$$\text{or}\quad y\frac{dy}{dx} + x = 0$$

$$\therefore y\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0$$

Using (i) we get $y\dfrac{d^2y}{dx^2} - \dfrac{x}{y}\dfrac{dy}{dx} + 1 = 0$

$$\therefore\ y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

3.a

Find the value of $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{3}$, if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$.

Sol.

$$\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$$

$$\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$$

$$\therefore \frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \sqrt{3}$$

3.b

If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then prove that

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

Sol.

Getting $\dfrac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$, $\dfrac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$$\frac{dy}{dx} = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$

$$= \frac{2\sin\frac{3\theta}{2}\cdot\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}} = \tan\frac{3\theta}{2}.$$

4.a

If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, then find $\frac{d^2y}{dx^2}$.

Sol.

$$\frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta)$$

$$= a\,\theta\cos\theta$$

$$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$$

$$= a\,\theta\sin\theta$$

$$\therefore \quad \frac{dy}{dx} = \tan\theta$$

prepared by : BALAJI KANCHI

$$\frac{d^2y}{dx^2} = \sec^2\theta \times \frac{d\theta}{dx} = \frac{\sec^3\theta}{a\theta}$$

4.b

If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

Sol.

$$\frac{dx}{dt} = a(-\sin t + t\cos t + \sin t) = at\cos t$$

$$\frac{dy}{dt} = a(\cos t + t\sin t - \cos t) = at\sin t$$

$$\frac{dy}{dx} = \frac{at\sin t}{at\cos t} = \tan t$$

$$\frac{d^2y}{dx^2} = \sec^2 t\,\frac{dt}{dx} = \sec^2 t\,\frac{1}{at\cos t} = \frac{\sec^3 t}{at}$$

prepared by : BALAJI KANCHI

5.

If $x = a\,(2\theta - \sin 2\theta)$ and $y = a\,(1 - \cos 2\theta)$, find $\dfrac{dy}{dx}$ when $\theta = \dfrac{\pi}{3}$.

Sol.

$$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta) = 4a\sin^2\theta$$

$$\frac{dy}{d\theta} = 2a\sin 2\theta = 4a\sin\theta\cdot\cos\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{4a\sin\theta\cos\theta}{4a\sin^2\theta} = \cot\theta$$

$$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{3}} = \frac{1}{\sqrt{3}}$$

6.

If $x = a\sin 2t\,(1 + \cos 2t)$ and $y = b\cos 2t\,(1 - \cos 2t)$, find the values of $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$ and $t = \dfrac{\pi}{3}$.

Sol.

Here $x = a\left(\sin 2t + \dfrac{1}{2}\sin 4t\right)$, $y = b\,(\cos 2t - \cos^2 2t)$

$$\frac{dx}{dt} = 2a[\cos 2t + \cos 4t],\ \frac{dy}{dt} = 2b\,[-\sin 2t + 2\cos 2t\sin 2t]$$

$$= 2b\,[\sin 4t - \sin 2t]$$

$$\frac{dy}{dx} = \frac{b}{a}\left[\frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t}\right]$$

$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \frac{b}{a}$$

and $\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{3}} = \sqrt{3}\,\dfrac{b}{a}$

$$RHL = \lim_{x \to \frac{\pi}{2}^+} \frac{q(1-\sin x)}{(\pi - 2x)^2} = \lim_{h \to 0} \frac{q(1-\cos h)}{(2h)^2}, \text{ where } x - \frac{\pi}{2} = h$$

$$= \lim_{h \to 0} \frac{2q \sin^2 \frac{h}{2}}{4.4. \frac{h^2}{4}} = \frac{q}{8}$$

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$$\therefore \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4$$

7.

Find $\frac{dy}{dx}$ at $t = \frac{2\pi}{3}$ when $x = 10\,(t - \sin t)$ and $y = 12\,(1 - \cos t)$.

Sol.

$$\frac{dy}{dt} = 12 \sin t, \quad \frac{dx}{dt} = 10\,(1 - \cos t)$$

$$\therefore \frac{dy}{dx} = \frac{6}{5} \times \frac{\sin t}{1 - \cos t}$$

$$\left.\frac{dy}{dx}\right|_{t = \frac{2\pi}{3}} = \frac{6}{5\sqrt{3}}$$

prepared by : BALAJI KANCHI

8.

If x = a (θ – sin θ), y = a (1 – cos θ), a > 0, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

Sol.

$$\frac{dy}{d\theta} = a\sin\theta, \quad \frac{dx}{d\theta} = a(1-\cos\theta)$$

$$\frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \cot\frac{\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}\cdot \operatorname{cosec}^2\frac{\theta}{2}\cdot\frac{d\theta}{dx} = -\frac{\operatorname{cosec}^2\frac{\theta}{2}}{2a(1-\cos\theta)}$$

$$\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{3}} = -\frac{1}{2}\times\frac{4}{a\left(1-\frac{1}{2}\right)} = -\frac{4}{a}$$

prepared by : BALAJI KANCHI

9.

If x = α sin 2t (1 + cos 2t) and y = β cos 2t (1 – cos 2t), show that $\frac{dy}{dx} = \frac{\beta}{\alpha}\tan t$.

Sol.

$$\frac{dx}{dt} = \alpha\left[-2\sin 2t\sin 2t + 2\cos 2t(1+\cos 2t)\right]$$

$$\frac{dy}{dt} = \beta\left[2\sin 2t\cos 2t - (1-\cos 2t)\cdot 2\sin 2t)\right]$$

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right)\bigg/\left(\frac{dx}{dt}\right) = \frac{\beta(2\sin 4t - 2\sin 2t)}{\alpha(2\cos 4t + 2\cos 2t)}$$

$$= \frac{\beta}{\alpha}\cdot\frac{2\cos 3t\sin t}{2\cos 3t\cos t} = \frac{\beta}{\alpha}\tan t$$

prepared by : BALAJI KANCHI

10.

If x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 – cos 2t), then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

Sol.

$$\frac{dx}{dt} = 2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t$$

$$\frac{dy}{dt} = -2b\sin 2t\,(1-\cos 2t) + 2b\cos 2t\cdot\sin 2t$$

$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \left.\frac{2b\cos 2t\cdot\sin 2t - 2b\sin 2t\,(1-\cos 2t)}{2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t}\right]_{t=\frac{\pi}{4}} = \frac{b}{a}$$

prepared by : BALAJI KANCHI

11.

If $x = 3\cos t - 2\cos^3 t$ and $y = 3\sin t - 2\sin^3 t$ then find $\frac{d^2y}{dx^2}$.

Sol.

$$\frac{dx}{dt} = -3\sin t + 6\cos^2 t\sin t$$

$$= +3\sin t\cos 2t$$

$$\frac{dy}{dt} = 3\cos t - 6\sin^2 t - \cos t$$

$$= 3\cos t\cos 2t$$

$$\frac{dy}{dx} = \cot t$$

prepared by : BALAJI KANCHI

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 t\,\frac{dt}{dx} = \frac{-1}{3}\,\frac{\operatorname{cosec}^3 t}{\cos 2t}$$

12.

If $x = \cos t + \log \tan\left(\frac{t}{2}\right)$, $y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

Sol.

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times \left(\sec^2\frac{t}{2} \times \frac{1}{2}\right) = \frac{\cos^2 t}{\sin t}$$

$$\frac{dy}{dt} = \cos t$$

$$\frac{d^2y}{dt^2} = -\sin t \Rightarrow \left.\frac{d^2y}{dt^2}\right]_{t=\frac{\pi}{4}} = -\frac{1}{\sqrt{2}}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \sec^4 t \cdot \sin t$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = 2\sqrt{2}$$

13. 2023

If $x = a \sin 2t,\ y = a(\cos 2t + \log \tan t)$, then find $\dfrac{dy}{dx}$.

Sol.

$$\frac{dx}{dt} = 2a \cos 2t$$

$$\frac{dy}{dt} = 2a\left(- \sin 2t + \frac{\sec^2 t}{2 \tan t}\right) = 2a\,\frac{cos^2\ 2t}{\sin 2t}$$

$$\frac{dy}{dx} = \cot 2t$$

14. 2025

If $x = a\left(\cos\theta + \log \tan\dfrac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{4}$.

Sol.

$$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$$

$$\Rightarrow \frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\frac{\theta}{2}} \times \sec^2\frac{\theta}{2} \times \frac{1}{2}\right)$$

$$= a\left(-\sin\theta + \frac{1}{\sin\theta}\right) = a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$$

$$\frac{dx}{d\theta} = a\cot\theta\cos\theta$$

$$\text{Also, } y = \sin\theta \Rightarrow \frac{dy}{d\theta} = \cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\tan\theta}{a}$$

Differetiating wrt x,

$$\frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{d\theta}{dx}$$

$$=\frac{\sec^3\theta\tan\theta}{a^2}$$

$$\left.\frac{d^2y}{dx^2}\right]_{\text{at }\theta=\frac{\pi}{4}}=\frac{2\sqrt{2}}{a^2}$$

15. 2023

If $x = a \cos t$ and $y = b \sin t$, then find $\frac{d^2y}{dx^2}$.

Sol.

Given $x = a \cos t$ and $y = b \sin t$, we have

$$\frac{dx}{dt} = -a \sin t \text{ and } \frac{dy}{dt} = b \cos t$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{b \cos t}{-a \sin t} = -\frac{b}{a} \cot t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(-\frac{b}{a} \cot t\right) \cdot \frac{dt}{dx}$$

$$= \frac{b}{a} \operatorname{cosec}^2 t \cdot \frac{1}{-a \sin t}$$

$$= -\frac{b}{a^2} \cdot \frac{1}{\sin^3 t} \text{ or } -\frac{b}{a^2} \operatorname{cosec}^3 t$$

16.

If $x = a\cos\theta; y = b\sin\theta$, then find $\dfrac{d^2y}{dx^2}$.

Sol.

$$\frac{dx}{d\theta} = -a\sin\theta, \frac{dy}{d\theta} = b\cos\theta \Rightarrow \frac{dy}{dx} = -\frac{b}{a}\cot\theta$$

$$\frac{d^2y}{dx^2} = \frac{b}{a}\operatorname{cosec}^2\theta\left(\frac{-1}{a\sin\theta}\right) = -\frac{b}{a^2}\operatorname{cosec}^3\theta$$

17.

If $x = a\cos\theta$ and $y = b\sin\theta$, then prove that $\dfrac{d^2y}{dx^2} = -\dfrac{b^4}{a^2y^3}$.

Sol.

$$\frac{dx}{d\theta} = -a\sin\theta, \frac{dy}{d\theta} = b\cos\theta \Longrightarrow \frac{dy}{dx} = -\frac{b}{a}\cot\theta$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a}(-\operatorname{cosec}^2\theta).\frac{d\theta}{dx}$$

$$= -\frac{b^4}{a^2y^3}$$

$$= \frac{b}{a}\operatorname{cosec}^2\theta\left(-\frac{1}{a\sin\theta}\right) = \frac{-b}{a^2\sin^3\theta}$$

prepared by : **BALAJI KANCHI**

18.

If $x = a\sec\theta,\ y = b\tan\theta,$ then find $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{3}$.

Sol.

$$\frac{dx}{d\theta} = a\sec\theta\tan\theta$$

$$\frac{dy}{d\theta} = b\sec^2\theta$$

$$\frac{dy}{dx} = \frac{b}{a}\sec\theta\cot\theta$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{2b}{a\sqrt{3}} \quad \text{or} \quad \frac{2\sqrt{3}b}{3a}$$

prepared by : BALAJI KANCHI

19.

If $x = a\sec^3\theta,\ y = a\tan^3\theta,$ then find $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{4}$.

Sol.

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta} = \sin\theta$$

$$\frac{d^2y}{dx^2} = \cos\theta\cdot\frac{d\theta}{dx} = \frac{1}{3a\sec^4\theta\cdot\tan\theta}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\frac{\pi}{4}} = \frac{1}{12a}$$

prepared by : BALAJI KANCHI

20.

If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\dfrac{d^2y}{dx^2}$.

Sol.

$$\frac{dx}{d\theta} = 3a\sec^2\theta \cdot \sec\theta\tan\theta$$

$$\frac{dy}{d\theta} = 3a\tan^2\theta \cdot \sec^2\theta$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \sin\theta$$

Also, $$\frac{d^2y}{dx^2} = \cos\theta \cdot \frac{d\theta}{dx}$$

$$= \frac{\cos\theta}{3a\tan\theta\sec^3\theta} \text{ or } \frac{\cos^5\theta}{3a\sin\theta}$$

prepared by : BALAJI KANCHI

21.

If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$

Sol.

Writing $\frac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta$

$$\frac{dx}{d\theta} = 3a\sec^3\theta\tan\theta$$

$$\frac{dy}{dx} = \frac{\tan\theta}{\sec\theta} = \sin\theta$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx} = \cos\theta \times \frac{1}{3a\sec^3\theta\tan\theta}$$

$$\left.\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{3}} = \frac{\frac{1}{2}}{3a \times 8 \times \sqrt{3}} = \frac{1}{48\sqrt{3}a}$$

prepared by : **BALAJI KANCHI**

22. 2024

If $x = a\sin^3\theta$, $y = b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Sol.

$$\frac{dx}{d\theta} = 3a\sin^2\theta\ \cos\theta,\ \frac{dy}{d\theta} = -3b\cos^2\theta\ \sin\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3b\cos^2\theta\ \sin\theta}{3a\sin^2\theta\ \cos\theta} = -\frac{b}{a}\cot\theta$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a}\operatorname{cosec}^2\theta\ \frac{d\theta}{dx}$$

$$= \frac{b}{a}\ \operatorname{cosec}^2\theta \cdot \frac{1}{3a\sin^2\theta\ \cos\theta} = \frac{b}{3a^2}\sec\theta\ \operatorname{cosec}^4\theta$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{4}} = \frac{4\sqrt{2}b}{3a^2}$$

prepared by : BALAJI KANCHI

23.

If $x = \sin t$ and $y = \sin pt$, prove that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + p^2y = 0$.

Sol.

$$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = p\cos pt \Rightarrow \frac{dy}{dx} = \frac{p\cos pt}{\cos t}$$

$$\frac{dy}{dx} = \frac{p\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = p^2(1-y^2)$$

differentiating both sides w.r.t x

$$\Rightarrow 2(1-x^2)\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -2p^2y\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

prepared by : BALAJI KANCHI

24.

If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\dfrac{dy}{dx} = -\dfrac{y\log x}{x\log y}$.

Sol.

$$\frac{dx}{dt} = e^{\cos 2t}(-2\sin 2t) \text{ or } -2x\sin 2t$$

$$\frac{dy}{dt} = e^{\sin 2t}2\cos 2t \text{ or } 2y\cos 2t$$

$$\frac{dy}{dx} = \frac{-e^{\sin 2t}2\cos 2t}{e^{\cos 2t}2\sin 2t} \text{ or } -\frac{y}{x}\frac{\cos 2t}{\sin 2t}$$

$$= \frac{-y\log x}{x\log y}$$

prepared by : BALAJI KANCHI

25. 2024

If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

Sol.

$$\frac{dx}{dt} = e^{cos3t} \times (-sin3t) \times 3$$
$$\frac{dy}{dt} = e^{sin3t} \times (cos3t) \times 3$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^{sin3t} \times (cos3t)}{-e^{cos3t} \times (sin3t)}$$

$$x = e^{cos3t} \Rightarrow cos3t = logx$$
$$y = e^{sin3t} \Rightarrow sin3t = logy$$
$$\therefore \frac{dy}{dx} = \frac{-ylogx}{xlogy}$$

prepared by : **BALAJI KANCHI**

26.

If $x = ae^t (\sin t + \cos t)$ and $y = ae^t (\sin t - \cos t)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

Sol.

$x = ae^t (\sin t + \cos t)$; $y = ae^t (\sin t - \cos t)$

then $\frac{dy}{dt} = ae^t(\sin t - \cos t) + ae^t(\cos t + \sin t)$

$= y + x$

$\frac{dx}{dt} = ae^t(\sin t + \cos t) + ae^t(\cos t - \sin t)$

$= x - y$

$$\therefore \frac{dy}{dx} = \frac{y+x}{x-y} \text{ or } \frac{x+y}{x-y}$$

prepared by : **BALAJI KANCHI**

27.

If $x = \sqrt{a^{\tan^{-1} t}}$, $y = \sqrt{a^{\cot^{-1} t}}$, then show that $x\dfrac{dy}{dx} + y = 0$.

Sol.

$$\frac{dx}{dt} = \frac{\sqrt{a^{\tan^{-1} t}}}{2}\,\frac{\log a}{1+t^2}$$

$$\frac{dy}{dt} = -\frac{\sqrt{a^{\cot^{-1} t}}}{2}\,\frac{\log a}{1+t^2}$$

$$\frac{dy}{dx} = -\frac{\sqrt{a^{\cot^{-1} t}}}{\sqrt{a^{\tan^{-1} t}}} = -\frac{y}{x}$$

$$\Rightarrow x\frac{dy}{dx} + y = 0$$

prepared by : BALAJI KANCHI

V. Derivative of f(x) with respect to g(x) :

1.

Differentiate $\sec^2(x^2)$ with respect to x^2.

Sol.

Let $x^2 = u$, differentiating $\sec^2 u$ w.r.t u, putting $u = x^2$

$2\sec^2 x^2 \tan x^2$

2.

Find the derivative of $x^{\log x}$ w.r.t. $\log x$.

3.

Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.

Sol.

Let $y = \sin^2 x$ and $z = e^{\cos x}$ $\therefore \frac{dy}{dx} = 2\sin x \cos x$ and $\frac{dz}{dx} = -\sin x . e^{\cos x}$

$\therefore \frac{dy}{dz} = \frac{2\sin x \cos x}{-\sin x\, e^{\cos x}} = \frac{-2\cos x}{e^{\cos x}}$ or $-2\cos x e^{-\cos x}$

4.a

Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

Sol.

Let $y = \sin^2 x$ and $z = e^{\cos x}$ $\therefore \frac{dy}{dx} = 2\sin x \cos x$ and $\frac{dz}{dx} = -\sin x . e^{\cos x}$

$\therefore \frac{dy}{dz} = \frac{2\sin x \cos x}{-\sin x\, e^{\cos x}} = \frac{-2\cos x}{e^{\cos x}}$ or $-2\cos x e^{-\cos x}$

4.b 2025

Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$.

Sol.

$$\text{Let } u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x}(-2\cos x \sin x)\log 2$$

$$\text{Let } v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2\cos x \sin x$$

$$\text{Now } \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$$

4.c 2025 Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

sol.

Let $u = \sqrt{e^{\sqrt{2x}}}$ and $v = e^{\sqrt{2x}}$

Derivative of $\sqrt{v}$ w.r.t. $v = \frac{1}{2\sqrt{v}}.$

Required derivative $= \frac{1}{2\sqrt{e^{\sqrt{2x}}}}.$

5.

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1} x^2$.

Sol.

Let $\theta = \cos^{-1} x^2 \Rightarrow x^2 = \cos\theta$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}\right)$$

$$= \frac{\pi}{4} - \frac{1}{2}\theta$$

$$\therefore \quad \frac{dy}{d\theta} = -\frac{1}{2}$$

6.

Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ w.r.t. $\sqrt{1 - x^2}$ at $x = \frac{1}{2}$.

7.

Differentiate $\tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$ w.r.t. $\sin^{-1}\frac{2x}{1 + x^2}$, if $x \in (-1, 1)$

Sol.

Let $\quad u = \tan^{-1}\frac{\sqrt{1+x^2} - 1}{x}$

Put $\quad x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$$\therefore \quad u = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{1 - \cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow \quad \frac{du}{dx} = \frac{1}{2(1+x^2)}$$

$$v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= 2\tan^{-1} x$$

$$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4}$$

8. 2023

Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

Sol.

Let $x = \sin\theta$. Then

$$U = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$

$$= \sec^{-1}(\sec\theta) = \theta = \sin^{-1}x$$

$$\Rightarrow \frac{dU}{dx} = \frac{1}{\sqrt{1-x^2}}$$

and $V = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$

$$= \sin^{-1}[2\sin\theta\cos\theta] = 2\theta = 2\sin^{-1}x$$

$$\Rightarrow \frac{dV}{dx} = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dU}{dV} = \frac{dU/dx}{dV/dx} = \frac{1}{2}$$

Note: If the substitution is made as $x = \cos\theta$,

answer will be $-\frac{1}{2}$

9. 2025

Differentiate $\tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2}),\ x \in \left(\dfrac{1}{\sqrt{2}},1\right)$

Sol.

Put $x=\cos\theta \Rightarrow \theta=\cos^{-1}x$

Let $u=\tan^{-1}\dfrac{\sqrt{1-x^2}}{x}=\tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)=\tan^{-1}(\tan\theta)=\theta=\cos^{-1}x$

$$\Rightarrow \frac{du}{dx}=-\frac{1}{\sqrt{1-x^2}}$$

Let $v=\cos^{-1}\left(2x\sqrt{1-x^2}\right)=\cos^{-1}(\sin 2\theta)=\cos^{-1}\left(\cos\left(\dfrac{\pi}{2}-2\theta\right)\right)=\dfrac{\pi}{2}-2\cos^{-1}x$

$$\Rightarrow \frac{dv}{dx}=\frac{2}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dv}=\frac{du/dx}{dv/dx}=-\frac{1}{2}$$

10.

Differentiate $\tan^{-1}\dfrac{3x-x^3}{1-3x^2},\ |x|<\dfrac{1}{\sqrt{3}}$ w.r.t. $\tan^{-1}\dfrac{x}{\sqrt{1-x^2}}$.

Sol.

Let $y=\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$, Put $x=\tan\theta$

$$y=\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)\Rightarrow y=\tan^{-1}(\tan 3\theta)=3\theta$$

$$y=3\tan^{-1}x \Rightarrow \frac{dy}{dx}=\frac{3}{1+x^2} \qquad \text{...(i)}$$

Let $z = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$, put $x = \sin\phi$

$$z = \tan^{-1}\left(\frac{\sin\phi}{\sqrt{1-\sin^2\phi}}\right) \Rightarrow z = \tan^{-1}(\tan\phi) = \phi$$

$$z = \phi = \sin^{-1} x \Rightarrow \frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{...(ii)}$$

Using (i) & (ii), $\dfrac{dy}{dz} = \dfrac{dy/dx}{dz/dx} = \dfrac{3\sqrt{1-x^2}}{1+x^2}$

prepared by : BALAJI KANCHI

VI. Derivative of the equation in terms of x and y { in the form of f(x,y)=k } :

1.a 2024

If $x^{1/3} + y^{1/3} = 1$, find $\dfrac{dy}{dx}$ at the point $\left(\dfrac{1}{8}, \dfrac{1}{8}\right)$.

Sol.

$$\frac{1}{3}x^{\frac{-2}{3}} + \frac{1}{3}y^{\frac{-2}{3}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{\frac{-2}{3}}}{y^{\frac{-2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{\left(\frac{1}{8},\frac{1}{8}\right)} = \frac{-4}{4} = -1$$

prepared by : BALAJI KANCHI

1.b 2025

For the curve $\sqrt{x} + \sqrt{y} = 1$, find the value of $\frac{dy}{dx}$ at the point $\left(\frac{1}{9}, \frac{1}{9}\right)$.

Sol.

Differentiating both sides w.r.t. x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\frac{dy}{dx} \text{ at } \left(\frac{1}{9}, \frac{1}{9}\right) = -1$$

1.c 2025

If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

Sol.

Differentiating $-2x^2 - 5xy + y^3 = 76$, with respect to 'x'

$$-4x - 5y - 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{4x + 5y}{3y^2 - 5x}$$

2.

If $y = \sqrt{\cos x + y}$, prove that $\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$.

Sol.

$$y^2 = \cos x + y$$

$$(2y - 1)\frac{dy}{dx} = -\sin x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x}{1 - 2y}$$

3.

If $y^2 \cos\left(\frac{1}{x}\right) = a^2$, then find $\frac{dy}{dx}$.

4.a

If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi$, $n \in Z$, then $\frac{dy}{dx}$ is equal to __________ .

4.b

Find $\frac{dy}{dx}$ at $x = 1$, $y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

Sol.

From the given equation

$$2\sin y\cos y\cdot\frac{dy}{dx} - \sin xy\cdot\left[x\cdot\frac{dy}{dx} + y\cdot 1\right] = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin(xy)}$$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{x=1,\, y=\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$$

4.c.

Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$.

Sol.

$\sin^2 x + \cos^2 y = 1$

differentiate wrt x,

$$2\sin x\cdot\cos x + 2\cos y\cdot(-\sin y)\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \sin 2x = \sin 2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

5.a

If $e^y(x+1)=1$, then show that $\dfrac{d^2y}{dx^2}=\left(\dfrac{dy}{dx}\right)^2$.

Sol.

$$e^y\cdot(x+1)=1 \Rightarrow e^y\cdot 1+(x+1)e^y\cdot\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\frac{1}{x+1}$$

$$\Rightarrow \frac{d^2y}{dx^2}=+\frac{1}{(x+1)^2}$$

$$\therefore \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$$

prepared by : BALAJI KANCHI

5.b

If $e^y(x+1)=1$, prove that $\dfrac{dy}{dx}=-e^y$.

Sol.

$$e^y(x+1)=1\Rightarrow e^y=\frac{1}{x+1}$$

$$\Rightarrow y=-\log(x+1)$$

$$\Rightarrow \frac{dy}{dx}=-\frac{1}{x+1}$$

$$=-e^y \qquad \left[\because \frac{1}{x+1}=e^y\right]$$

prepared by : BALAJI KANCHI

6.a 2023

If $xy = e^{(x-y)}$, then show that $\dfrac{dy}{dx} = \dfrac{y(x-1)}{x(y+1)}$

Sol.

Given $xy = e^{x-y}$, gives $x - y = \log x + \log y$

$$\Rightarrow 1 - \frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} + 1\right)\frac{dy}{dx} = 1 - \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x-1}{x} \times \frac{y}{1+y} = \frac{y(x-1)}{x(1+y)}$$

OR

$$x\frac{dy}{dx} + y = e^{x-y}\left(1 - \frac{dy}{dx}\right)$$

$$= xy\left(1 - \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{xy - y}{x + xy} = \frac{y(x-1)}{x(1+y)}$$

6.b

If $e^{y-x} = y^x$, prove that

$$\frac{dy}{dx} = \frac{y(1 + \log y)}{x \log y}$$

7.a 2024

If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

Sol.

$$x = e^{\frac{x}{y}} \Rightarrow \log x = \frac{x}{y} \Rightarrow y = \frac{x}{\log x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(\log x)(1) - x\left(\frac{1}{x}\right)}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$$

7.c 2025

If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.

Sol.

$$x = e^{\frac{x}{y}}$$

$$\Rightarrow logx = \frac{x}{y}$$

$$\Rightarrow ylogx = x$$

Differentiating both sides w.r.to x, we get

$$\frac{y}{x} + logx\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{x - y}{xlogx}$$

8.

If $(ax + b)\, e^{y/x} = x$, then show that

$$x^3\left(\frac{d^2y}{dx^2}\right) = \left(x\frac{dy}{dx} - y\right)^2$$

Sol.

$$e^{y/x} = \frac{x}{a+bx},$$ taking log, on both sides,

we get $\dfrac{y}{x} = \log x - \log(a+bx)$

Differentiating with respect to ‘x’

$$\frac{x \cdot y' - y}{x^2} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{(a+bx)\,x}$$

$$\Rightarrow \quad x \cdot y' - y = \frac{ax}{a+bx} \qquad \text{...(i)}$$

Differentiating with respect to ‘x’

$$\Rightarrow \quad x \cdot y'' + y' - y' = \frac{(a+bx)\cdot a - ax \cdot b}{(a+bx)^2} = \left(\frac{a}{a+bx}\right)^2$$

$$\Rightarrow \; x \cdot y'' = \left(\frac{a}{a+bx}\right)^2 \Rightarrow \; x^3 \cdot y'' = \left(\frac{ax}{a+bx}\right)^2$$

$$\Rightarrow \; x^3\frac{d^2y}{dx^2} = \left\{x \cdot \frac{dy}{dx} - y\right\}^2 \qquad \text{(Using (i))}$$

prepared by : **BALAJI KANCHI**

9.a

If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

Sol.

$$\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2 + y^2}\left(2x + 2y\frac{dy}{dx}\right) = 2 \cdot \frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \left(\frac{x \cdot \frac{dy}{dx} - y \cdot 1}{x^2}\right)$$

$$\Rightarrow \frac{2}{x^2 + y^2}\left(x + y\frac{dy}{dx}\right) = \frac{2x^2}{x^2 + y^2} \cdot \frac{1}{x^2} \cdot \left(x\frac{dy}{dx} - y\right)$$

$$\Rightarrow (x + y) = (x - y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x + y}{x - y}$$

prepared by : BALAJI KANCHI

9.

If $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

Sol.

Differentiating both sides w.r.t. x to get

$$\frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{2x + 2y\frac{dy}{dx}}{2\sqrt{x^2+y^2}}$$

Simplyfying we get $\quad x\frac{dy}{dx} - y = x + y\frac{dy}{dx}$

getting $\quad \frac{dy}{dx} = \frac{x+y}{x-y}$

prepared by : BALAJI KANCHI

10.

If $\frac{x}{x-y} = \log\frac{a}{x-y}$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$.

Sol.

Given $\frac{x}{x-y} = \log a - \log(x-y)$

Differentiating both sides and getting $\qquad [\because x \neq y]$

$$x - 2y + y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y - x}{y}$$

$$\Rightarrow \frac{dy}{dx} = 2 - \frac{x}{y}$$

prepared by : BALAJI KANCHI

11.a

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

Sol.

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$$

$$\Rightarrow x^2(1+y) = y^2(1+x)$$

$$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$$

$$\Rightarrow (x-y)(x+y+xy) = 0$$

$$x \neq y \Rightarrow x + y + xy = 0$$

$$\Rightarrow y = \frac{-x}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(x+1)^2}$$

OR

$$x\sqrt{1+y} + y\sqrt{1+x} = 0 \;\Rightarrow\; x\sqrt{1+y} = -y\sqrt{1+x}$$

Squaring to get: $x^2(1+y) = y^2(1+x)$

Simplifying to get: $(x-y)(x+y+xy) = 0$

As, $x \neq y \quad \therefore \quad y = -\frac{x}{1+x}$

Differentiating w.r.t. ‘x’, we get:

$$\frac{dy}{dx} = \frac{-1(1+x) - (-x)\cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

11.b 2025

If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

Sol.

Let $x=\sin A, y=\sin B \Rightarrow A=\sin^{-1}x,\ B=\sin^{-1}y$

$\therefore \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \cot\left(\frac{A-B}{2}\right)=a \Rightarrow A-B=2\cot^{-1}a$

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$

differentiate both sides wrt x,

$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

prepared by : BALAJI KANCHI

OR

$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Put $x=\sin\theta$, $y=\sin\phi$

$\Rightarrow \cos\theta+\cos\phi=a(\sin\theta-\sin\phi)$

$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)=2a\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta+\phi}{2}\right)$

$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right)=a$

$\Rightarrow \theta-\phi=2\cot^{-1}a$

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

prepared by : BALAJI KANCHI

12.a

If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, $|x| < 1$, $|y| < 1$, show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$.

Sol.

v

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y), \text{ put } x = \sin\theta,\ y = \sin\phi$$

$$\sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\phi} = a(\sin\theta - \sin\phi)$$

$$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$$

$$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = 2a\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\Rightarrow \tan\left(\frac{\theta-\phi}{2}\right) = \frac{1}{a}$$

$$\Rightarrow \frac{\theta-\phi}{2} = \tan^{-1}\left(\frac{1}{a}\right) \Rightarrow \sin^{-1}x - \sin^{-1}y = 2\tan^{-1}\left(\frac{1}{a}\right)$$

Differentiating both sides w.r.t x

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \text{ or } \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

prepared by : BALAJI KANCHI

13.

If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b.

Sol.

$(x-a)^2+(y-b)^2=c^2,\ c>0$

Differentiating both sides with respect to ‘x’, we get

$$2(x-a)+2(y-b)\cdot\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x-a}{y-b}$$

Differentiating again with respect to ‘x’, we get;

$$\frac{d^2y}{dx^2}=-\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2}=\frac{-c^2}{(y-b)^3}\quad \text{(By substituting ‘}\frac{dy}{dx}\text{’)}$$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}=\frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}}=\frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}}=-c$$

Which is a constant independent of ‘a’ & ‘b’.

prepared by : BALAJI KANCHI

14.a

If $\sin y = x \sin(a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

Sol.

$$x = \frac{\sin y}{\sin(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\sin a}{\sin^2(a+y)},$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

OR

$$\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$$

differentiating w.r.t. y, we get

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

14.b

If $x\cos(a+y)=\cos y$ then prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$.

Sol.

$$\frac{dx}{dy}=\frac{\sin a}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$$

$$\frac{d^2y}{dx^2}=\frac{-2\cos(a+y)\sin(a+y)}{\sin a}\frac{dy}{dx}$$

$$=\frac{-\sin 2(a+y)}{\sin a}\frac{dy}{dx}$$

$$\Rightarrow \sin a\frac{d^2y}{dx^2}+\sin 2(a+y)\frac{dy}{dx}=0$$

14.c

If $\sin y = x\cos(a+y)$, then show that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\cos a}$.

Also, show that $\frac{dy}{dx}=\cos a$, when $x=0$.

Sol.

$$x=\frac{\sin y}{\cos(a+y)}$$

gives $$\frac{dx}{dy}=\frac{\cos(a+y)\cos y+\sin y\sin(a+y)}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\cos^2(a+y)}{\cos(a+y-y)}=\frac{\cos^2(a+y)}{\cos a}$$

Hence $\frac{dy}{dx}=\cos a$ when $x=0$ i.e. $y=0$

14.d

If $\cos y = x\cos(a+y)$, and $\cos a \neq \pm 1$, prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$.

14.e

If $x\cos(p+y) + \cos p \sin(p+y) = 0$, prove that $\cos p \dfrac{dy}{dx} = -\cos^2(p+y)$, where p is a constant.

Sol.

$$x\cos(p+y)+\cos p\sin(p+y)=0$$

$$\Rightarrow x=\frac{-\cos p\sin(p+y)}{\cos(p+y)} \Rightarrow x=-\cos p.\tan(p+y)$$

$$\Rightarrow \frac{dx}{dy}=-\cos p.\sec^2(p+y)$$

$$\Rightarrow \frac{dy}{dx}=\frac{-1}{\cos p.\sec^2(p+y)}$$

$$\Rightarrow \cos p\frac{dy}{dx}=-\cos^2(p+y)$$

15. 2023

If $(x^2+y^2)^2 = xy$, then find $\dfrac{dy}{dx}$.

Sol.

$$(x^2+y^2)^2 = xy \text{ gives}$$

$$2(x^2+y^2)\left[2x + 2y\frac{dy}{dx}\right] = x\frac{dy}{dx} + y$$

$$\Rightarrow [4y(x^2+y^2)-x]\frac{dy}{dx} = y - 4x(x^2+y^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}.$$

prepared by : BALAJI KANCHI

16.a

If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$.

Sol.

$x^p y^q = (x+y)^{p+q} \Rightarrow p\log x + q\log y = (p+q)\log(x+y)$

Differentaiting w.r.t x, $\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$

$$\Rightarrow \left(\frac{q}{y} - \frac{p+q}{x+y}\right)\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\Rightarrow \left[\frac{qx-py}{y(x+y)}\right]\frac{dy}{dx} = \frac{qx-py}{x(x+y)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Differentiating again w.r.t x

$$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y}{x^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{x\left(\frac{y}{x}\right) - y}{x^2} \Rightarrow \frac{d^2y}{dx^2} = 0$$

prepared by : BALAJI KANCHI

16.b

If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{d^2y}{dx^2} = 0$.

Sol.

$$x^m \cdot y^n = (x + y)^{m+n}$$

$$\Rightarrow \quad m \log x + n \log y = (m + n) \log (x + y)$$

$$\Rightarrow \quad \frac{m}{x} + \frac{n}{y} \cdot \frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} \qquad \ldots(i)$$

$$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y}{x^2} = 0 \qquad \ldots(ii) \ \ (\text{using (i)})$$

16.c 2024

If $x^{30} y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Sol.

Taking log of both sides, we get

$30 \log x + 20 \log y = 50 \log (x + y)$

Differentiating both sides w.r.t. x, we get

$$\frac{30}{x} + \frac{20}{y}\frac{dy}{dx} = \frac{50}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{20x - 30y}{y(x+y)}\right) = \frac{20x - 30y}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

prepared by : BALAJI KANCHI

17. 2024

Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.

Sol.

Differentiating both sides w.r.t. x, we get

$$5^x log5 + 5^y log5\frac{dy}{dx} = 5^{x+y}log5(1+\frac{dy}{dx})$$

$$\Rightarrow 5^x + 5^y\frac{dy}{dx} = (5^x + 5^y)(1+\frac{dy}{dx})$$

$$\Rightarrow 5^x + 5^y\frac{dy}{dx} = 5^x + 5^x\frac{dy}{dx} + 5^y + 5^y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{5^y}{5^x} = -5^{y-x}$$

prepared by : BALAJI KANCHI

Sol.

18. 2025

If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

Sol.

$$\tan^{-1}\left(x^2 + y^2\right) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$$

Differentiate both sides wrt x,

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

prepared by : BALAJI KANCHI

19.

If $y^2\cos\left(\frac{1}{x}\right) = a^2$, then find $\frac{dy}{dx}$.

Ans: $y^2\cos\left(\frac{1}{x}\right) = a^2$

Then $2y\frac{dy}{dx}\cdot\cos\left(\frac{1}{x}\right) - y^2\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = 0$

$$\Rightarrow \quad 2y\cdot\cos\left(\frac{1}{x}\right)\frac{dy}{dx} = -\frac{y^2}{x^2}\sin\left(\frac{1}{x}\right)$$

VII. Derivative of Composit function y= f(x) (function in terms of variable x): 2nd Order Derivatives

1.a

If $y = e^{a\cos^{-1}x}$, $-1 < x < 1$, then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

Sol.

$$\frac{dy}{dx} = e^{a\cos^{-1}x}\frac{-a}{\sqrt{1-x^2}} = -\frac{ay}{\sqrt{1-x^2}},$$

squaring we get

$$\left(\frac{dy}{dx}\right)^2(1-x^2) = a^2y^2,$$

differentiating again with respect to 'x'.

$$(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = 2a^2y\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

prepared by : BALAJI KANCHI

OR

$$\frac{dy}{dx} = \frac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\frac{dy}{dx} = -ae^{a\cos^{-1}x}$$

Differentiating again & getting

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\cdot\frac{dy}{dx} = \frac{a^2e^{a\cos^{-1}x}}{\sqrt{1-x^2}}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

prepared by : BALAJI KANCHI

OR

$$y = e^{a\cos^{-1}3x} \Rightarrow \frac{dy}{dx} = e^{a\cos^{-1}3x}.\frac{(-3a)}{\sqrt{1-9x^2}}$$

$$\Rightarrow \quad \sqrt{1-9x^2}\,\frac{dy}{dx} = -3ay$$

$$\sqrt{1-9x^2}\,\frac{d^2y}{dx^2} + \frac{-18x}{2\sqrt{1-9x^2}}\cdot\frac{dy}{dx} = -3a\frac{dy}{dx}$$

$$\Rightarrow \quad (1-9x^2)\frac{d^2y}{dx^2} - 9x\frac{dy}{dx} = -3a\sqrt{1-9x^2}\,\frac{dy}{dx}$$

$$= -3a\,(-3ay) = 9a^2y$$

$$\text{or} \quad (1-9x^2)\frac{d^2y}{dx^2} - 9x\frac{dy}{dx} - 9a^2y = 0$$

1.b

If $y = e^{\tan^{-1} x}$, prove that $(1 + x^2)\dfrac{d^2y}{dx^2} + (2x - 1)\dfrac{dy}{dx} = 0$.

Sol.

$$y = e^{\tan^{-1}} x$$

$$\frac{dy}{dx} = e^{\tan^{-1} x}\left(\frac{1}{1+x^2}\right) = \frac{y}{1+x^2}$$

$$(1+x^2)\frac{dy}{dx} = y \Rightarrow (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

prepared by : **BALAJI KANCHI**

1.c

If $\log y = \tan^{-1} x$, then show that $(1 + x^2)\dfrac{d^2y}{dx^2} + (2x - 1)\dfrac{dy}{dx} = 0$.

Sol.

Differentiating the given expression w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2)\frac{dy}{dx} = y$$

diff. again w.r.t. x,

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = \frac{dy}{dx}$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

prepared by : **BALAJI KANCHI**

2.

If $y = e^{ax} . \cos bx$, then prove that

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)\, y = 0$$

Sol.

$y = e^{ax} \cos bx$

$y_1 = ae^{ax} \cos bx - b\, e^{ax} \sin b\, x$

$y_1 = ay - b\, e^{ax} \sin b\, x$

$y_2 = ay_1 - b\, [ae^{ax} \sin bx + b\, e^{ax} \cos b\, x]$

$y_2 = ay_1 - a\, be^{ax} \sin bx - b^2\, e^{ax} \cos b\, x$

$y_2 = a\, y_1 - a\, (ay - y_1) - b^2\, y$

$y_2 - 2\, a\, y_1 + (a^2 + b^2)\, y = 0$

prepared by : **BALAJI KANCHI**

3.

If $y = \dfrac{x \cos^{-1} x}{\sqrt{1 - x^2}} - \log \sqrt{1 - x^2}$, then prove that $\dfrac{dy}{dx} = \dfrac{\cos^{-1} x}{(1 - x^2)^{3/2}}$

4.a

If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

Sol.

$$\frac{dy}{dx} = \frac{-2\sin(\log x)}{x} + \frac{3\cos(\log x)}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx} = -2\sin(\log x) + 3\cos(\log x), \text{ differentiate w.r.t 'x'}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-2\cos(\log x)}{x} - \frac{3\sin(\log x)}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \quad \Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

prepared by : BALAJI KANCHI

4.b

If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0.$

Sol.

$$y = a\cos(\log x) + b\sin(\log x)$$

$$\frac{dy}{dx} = \frac{-a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$$

$$\Rightarrow \quad x \cdot \frac{dy}{dx} = -a\sin(\log x) + b\cos(\log x)$$

differentiate both sides again w.r.t x,

$$x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 1 = \frac{-a\cos(\log x)}{x} - \frac{b\sin(\log x)}{x}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

prepared by : BALAJI KANCHI

4.c

If $y = 2\cos(\log x) + 3\sin(\log x)$, prove that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

5.a 2024

If $y = A\sin 2x + B\cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k.

Sol.

$$\frac{dy}{dx} = 2A\,cos2x - 2B\,sin2x$$

$$\Longrightarrow \frac{d^2y}{dx^2} = -4A\,sin2x - 4B\,cos2x = -4y$$

$$\Longrightarrow \frac{d^2y}{dx^2} + 4y = 0$$

$$\Longrightarrow k = -4$$

prepared by : BALAJI KANCHI

5.b 2025

If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

Sol.

$\mathbf{y = 5cosx - 3sinx,}$ **then** $\mathbf{\frac{dy}{dx} = -5.sinx - 3.cosx}$

$$\Rightarrow \frac{d^2y}{dx^2} = -5.cosx + 3.sinx = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

prepared by : BALAJI KANCHI

6.a

If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0.$

Sol.

$y = Ae^{mx} + Be^{nx} \Rightarrow mAe^{mx} + nBe^{nx}$

$$\frac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx}$$

$$LHS = \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$= m^2Ae^{mx} + n^2Be^{nx} - (m+n)\{mAe^{mx} + nBe^{nx}\} + mn\{Ae^{mx} + Be^{nx}\}$$

$$= Ae^{mx}(m^2 - m^2 - mn + mn) + Be^{nx}(n^2 - mn - n^2 + mn)$$

6.b

If $y = 5e^{7x} + 6e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y.$

Sol.

$$y = 5e^{7x} + 6e^{-7x} \Rightarrow \frac{dy}{dx} = 35e^{7x} - 42e^{-7x}$$

$$\frac{d^2y}{dx^2} = 245e^{7x} + 294e^{-7x} \Rightarrow \frac{d^2y}{dx^2} = 49y$$

6.c

If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$

Sol.

$$\frac{dy}{dx} = e^x - e^{-x}$$

$$= \sqrt{\left(e^x + e^{-x}\right)^2 - 4}$$

$$= \sqrt{y^2 - 4}$$

7.

If $y = \sin^{-1} x - \cos^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

Sol.

$y = \sin^{-1} x - \cos^{-1} x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2} \cdot \frac{dy}{dx} = 2$$

differentiate again wrt x,

$$\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{2\sqrt{1-x^2}}(-2x) = 0$$

$$\Rightarrow \quad (1-x^2) \cdot \frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

prepared by : BALAJI KANCHI

8.a

If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 2 = 0$.

Sol.

$$\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} \Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$

Differentiating with respect to 'x', we get

$$(1-x^2)\,2\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = 4\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

prepared by : **BALAJI KANCHI**

OR

$$y = (\sin^{-1}x)^2$$

$$\Rightarrow y' = 2\cdot\sin^{-1}x\cdot\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\cdot y' = 2\sin^{-1}x$$

$$\Rightarrow \sqrt{1-x^2}\cdot y'' + y'\cdot\frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\cdot y'' - xy' = 2 \text{ or } (1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0.$$

prepared by : **BALAJI KANCHI**

8.b

If $y = (\sec^{-1} x)^2$, $x > 0$, show that

$$x^2 (x^2 - 1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$$

Sol.

$$y = (\sec^{-1} x)^2, x > 0$$

$$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2 - 1}}$$

$$\Rightarrow \; x\sqrt{x^2 - 1}\frac{dy}{dx} = 2\sqrt{y}$$

squaring both sides, we get

$$x^2(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y \quad \text{or} \quad (x^4 - x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$

differentiating w.r.t. x.

$$(x^4 - x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3 - 2x)\left(\frac{dy}{dx}\right)^2 = 4 \cdot \frac{dy}{dx}$$

$$\Rightarrow \; x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$$

prepared by : BALAJI KANCHI

8.c

If $y = (3\cot^{-1} x)^2$, show that

$$\left(x^2+1\right)^2 \frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 18.$$

Sol.

$$y = (\cot^{-1} x)^2 \Rightarrow \frac{dy}{dx} = 2\cot^{-1} x \cdot \left(\frac{-1}{1+x^2}\right)$$

$$\Rightarrow (1+x^2)\frac{dy}{dx} = -2\cot^{-1} x = -2\sqrt{y}$$

squaring both sides, we get

$$(1+x^2)^2 \cdot \left(\frac{dy}{dx}\right)^2 = 4y$$

differentiating, w.r.t. x,

$$2(1+x^2)2x \cdot \left(\frac{dy}{dx}\right)^2 + 2(1+x^2)^2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = 4 \cdot \frac{dy}{dx}$$

$$\Rightarrow 2x(1+x^2)\frac{dy}{dx} + (1+x^2)^2\frac{d^2y}{dx^2} = 2.$$

prepared by : BALAJI KANCHI

8.d 2024

If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2.$

Sol.

$$y = \left(\tan^{-1} x\right)^2 \Rightarrow \frac{dy}{dx} = \frac{2\tan^{-1} x}{1+x^2}$$

$\Rightarrow \left(1+x^2\right)\frac{dy}{dx} = 2\tan^{-1} x$, **differentiating with respect to 'x'**

$$\Rightarrow \left(1+x^2\right)^2\frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 2$$

prepared by : BALAJI KANCHI

9. 2024

If $y = (\log x)^2$, prove that $x^2y'' + xy' = 2$.

Sol.

Differentiating both sides w.r.t. x, we get

$$y' = \frac{2\log x}{x}$$

$$\Rightarrow xy' = 2\log x$$

$$\Rightarrow xy'' + y' = \frac{2}{x}$$

$$\Rightarrow x^2y'' + xy' = 2$$

prepared by : BALAJI KANCHI

10.

If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$.

Sol.

$$y = \sin(\sin x) \Rightarrow \frac{dy}{dx} = \cos(\sin x)\cdot\cos x$$

and $$\frac{d^2y}{dx^2} = -\sin(\sin x)\cdot\cos^2 x - \sin x\cos(\sin x)$$

$$\text{LHS} = -\sin(\sin x)\cos^2 x - \sin x\cos(\sin x) + \frac{\sin x}{\cos x}\cos(\sin x)\cos x + \sin(\sin x)\cos^2 x$$

$$= 0 = \text{RHS}$$

prepared by : BALAJI KANCHI

11. 2023

If $y = \tan x + \sec x$, then prove that $\dfrac{d^2y}{dx^2} = \dfrac{\cos x}{(1-\sin x)^2}$.

Sol.

$$y = \tan x + \sec x = \frac{\sin x + 1}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x(\cos x) + (\sin x + 1)\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1 + \sin x}{1 - \sin^2 x} = \frac{1}{1 - \sin x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1 - \sin x).0 - 1(0 - \cos x)}{(1 - \sin x)^2} = \frac{\cos x}{(1 - \sin x)^2}$$

prepared by : **BALAJI KANCHI**

12.

If $y = x^3 \log\left(\dfrac{1}{x}\right)$, then prove that $x\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 3x^2 = 0$.

Sol.

$$y = -x^3 \log x$$

$$\frac{dy}{dx} = -x^2(1 + 3\log x)$$

$$\frac{d^2y}{dx^2} = -(5x + 6x\log x)$$

$$\text{L.H.S.} = x[-(5x + 6x\log x)] + 2x^2(1 + 3\log x) + 3x^2$$

$$= 0$$

$$= \text{R.H.S.}$$

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13. 2025

If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x + 1)^2\, y_2 + (x + 1)^2\, y_1 = 2$.

Sol.

The given function can be written as

$$y = 2\,log(x + 1) - logx$$

$$\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$$

$$\Rightarrow (x+1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$$

$$\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$$

$$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$$

$$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$$

$$\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$$

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14.

If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

Sol.

$$\frac{dy}{dx} = 2\left(x + \sqrt{x^2 - 1}\right)\left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) = \frac{2\left(x + \sqrt{x^2 - 1}\right)^2}{\sqrt{x^2 - 1}}$$

$$\sqrt{x^2 - 1}\,\frac{dy}{dx} = 2y$$

$$(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$$

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15.

If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that

$$\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y.$$

Sol.

$$\frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left[1+\frac{x}{\sqrt{1+x^2}}\right]$$

$$= \frac{n}{\sqrt{1+x^2}}\left[x+\sqrt{1+x^2}\right]^n = \frac{ny}{\sqrt{1+x^2}}$$

$$\therefore \sqrt{1+x^2}\,\frac{dy}{dx} = ny \quad \text{..........(i)}$$

$$\therefore \sqrt{1+x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{x}{\sqrt{1+x^2}} = n\frac{dy}{dx}$$

$$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\,\frac{dy}{dx} = n\cdot ny \quad (\text{from (i)})$$

$$= n^2y$$

16.

If $y = \sqrt{x+1} - \sqrt{x-1}$, prove that $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{1}{4}y = 0.$

Sol.

$$y = \sqrt{x+1} - \sqrt{x-1}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}}$$

$$= \frac{\sqrt{x-1}-\sqrt{x+1}}{2\sqrt{x^2-1}}$$

$$4\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2 = y^2$$

$$4\left(x^2-1\right)2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}+8x\left(\frac{dy}{dx}\right)^2=2y\frac{dy}{dx}$$

$$\left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=\frac{y}{4}$$

$$\left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-\frac{y}{4}=0$$

17.

If $y=\dfrac{x\cos^{-1}x}{\sqrt{1-x^2}}-\log\sqrt{1-x^2}$, then prove that $\dfrac{dy}{dx}=\dfrac{\cos^{-1}x}{(1-x^2)^{3/2}}$

Sol.

$$y=\frac{x\cos^{-1}x}{\sqrt{1-x^2}}-\log\sqrt{1-x^2}$$

$$\frac{dy}{dx}=\frac{\sqrt{1-x^2}\cdot\left(1\cos^{-1}x-\dfrac{x}{\sqrt{1-x^2}}\right)-\dfrac{x\cos^{-1}x(-2x)}{2\sqrt{1-x^2}}}{1-x^2}+\frac{2x}{2(1-x^2)}$$

$$=\frac{\sqrt{1-x^2}\cos^{-1}x-x+\dfrac{x^2\cos^{-1}x}{1-x^2}}{1-x^2}+\frac{x}{1-x^2}$$

$$=\frac{(1-x^2)\cos^{-1}x+x^2\cos^{-1}x}{(1-x^2)^{3/2}}=\frac{\cos^{-1}x}{(1-x^2)^{3/2}}$$

VIII. Derivative of misc function :

1.

Show that :

$$\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$$

Sol.

$$\frac{d(|x|)}{dx} = \frac{d(\sqrt{x^2})}{dx}, x \neq 0$$
$$= \frac{1}{2}(x^2)^{-\frac{1}{2}} \times \frac{d(x^2)}{dx}$$
$$= \frac{1}{2\sqrt{x^2}} 2x = \frac{x}{|x|}$$

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2.

If $y = x|x|$, find $\frac{dy}{dx}$ for $x < 0$.

Sol.

For $x < 0$, $y = x\,|x| = -x^2$

$$\therefore \quad \frac{dy}{dx} = -2x$$

2.b

If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

Sol.

$$f(x) = -tan2x, \ \frac{\pi}{4} < x < \frac{\pi}{2}$$
$$f'(x) = -2sec^2 2x, \ \frac{\pi}{4} < x < \frac{\pi}{2}$$
$$f'(\frac{\pi}{3}) = -2(-2)^2 = -8$$

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2.c

Write the derivative of $|x-5|$ at $x=2$.

Sol.

$$|x-5| = \begin{cases} x-5 & x \geq 5 \\ 5-x & x < 5 \end{cases}$$ So, derivative at $x=2$ is -1.

3.

If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$.

Sol.

$$\frac{dy}{dx} = f'(x^2)2x$$
$$= 2xe^{x}$$

4.

Find the derivative of $f(e^{\tan x})$ w.r.t. to x at $x = 0$. It is given that $f'(1) = 5$.

5.

If $f(x) = x + 1$, find $\frac{d}{dx}$ (fof) (x).

6.

If $f(x) = \sqrt{x^2+1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f'[h'\{g'(x)\}]$.

7.

If $f(x) = \sin 2x - \cos 2x$, find $f'\left(\frac{\pi}{6}\right)$.

Sol.

$f(x) = \sin 2x - \cos 2x$

$\Rightarrow \quad f'(x) = 2\cos 2x + 2\sin 2x$

$$f'\left(\frac{\pi}{6}\right) = 2\left[\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right] = (1+\sqrt{3})$$

7.b

If $f(x)=\sqrt{\dfrac{\sec x-1}{\sec x+1}}$, find $f'\left(\dfrac{\pi}{3}\right)$.

Ans.

$$f(x) = \sqrt{\dfrac{\dfrac{1}{\cos x}-1}{\dfrac{1}{\cos x}+1}} = \tan\frac{x}{2}$$

$$f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$$

$$f'\left(\frac{\pi}{3}\right) = \frac{2}{3}$$

8.

If $y=\log x$, then $\dfrac{d^2y}{dx^2}=$ ____________ .

Resource Materials by BALAJI KANCHI :

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