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8. Application of Integrals

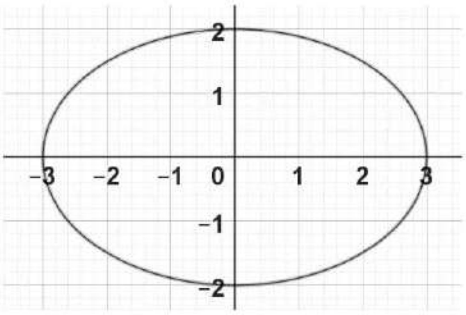
(Class XII CBSE Board Exam Models from 2022-2025 with solutions)

I. Area of circle/ellipse :

1.

Using integration, find the area enclosed by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$.

Sol.



$$\text{Required area } = 4\cdot\frac{2}{3}\int_0^3\sqrt{9-x^2}\,dx$$

$$=\frac{8}{3}\left[\frac{x\sqrt{9-x^2}}{2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$=\frac{8}{3}\left[0+\frac{9\pi}{4}\right]=6\pi$$

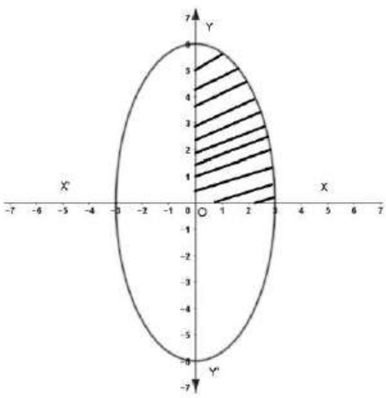
prepared by : BALAJI KANCHI

2.

Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

Sol.

The given equation can be written as: $\frac{x^2}{9}+\frac{y^2}{36}=1$, which is an ellipse.



Area of the region bounded by the curve

$$=4\times\frac{6}{3}\int_0^3\sqrt{9-x^2}\,dx$$

$$=8\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$=18\pi$$

prepared by : BALAJI KANCHI

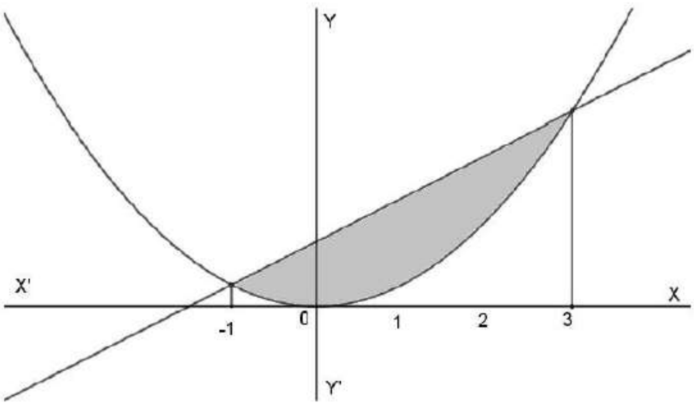
II.Area bounded by parabola and lines :

a.Parabola and two variable line :

1.

Find the area of the region bounded by curve $4x^2 = y$ and the line $y = 8x + 12$, using integration.

Sol.



Point of intersection

$$4x^2 = 8x + 12$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1$$

$$\text{Area} = \int_{-1}^{3}[(8x+12) - 4x^2]dx$$

$$= 4x^2 + 12x - \frac{4}{3}x^3\Big]_{-1}^{3}$$

$$= 36 + 36 - 36 - \left(4 - 12 + \frac{4}{3}\right)$$

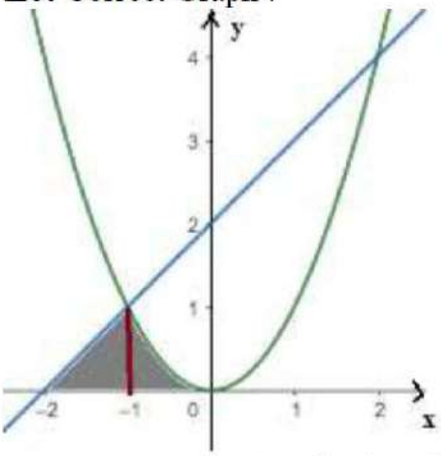
$$= 44 - \frac{4}{3} = \frac{128}{3}$$

2.

Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.

Sol.

Let Correct Graph :



x coordinates of point of intersection are – 1, 2

$$\text{Required area} = \int_{-2}^{-1} (x+2)\,dx + \int_{-1}^{0} x^2\,dx$$

$$= \left.\frac{(x+2)^2}{2}\right|_{-2}^{-1} + \left.\frac{x^3}{3}\right|_{-1}^{0}$$

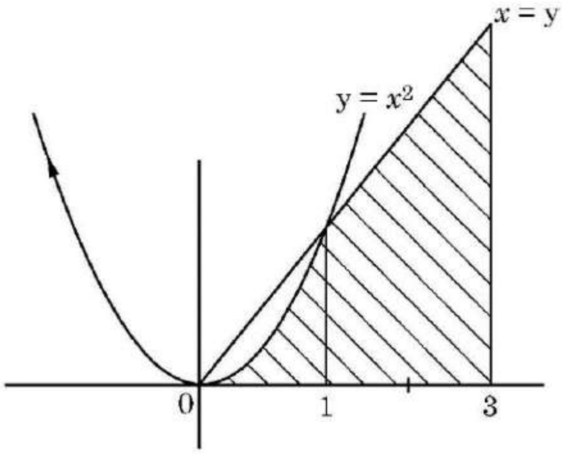
$$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

3.

Using integration, find the area of the region

$\left\{(x, y): 0 \leq y \leq x^2,\ 0 \leq y \leq x,\ 0 \leq x \leq 3\right\}$.

Sol.



Required Area

$$= \int_0^1 x^2\,dx + \int_1^3 x\,dx$$

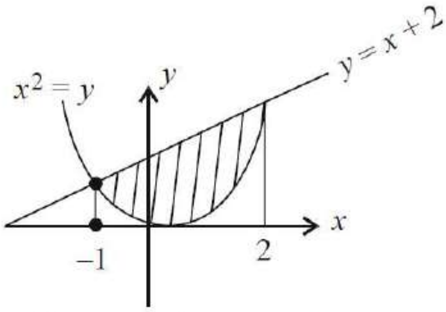
$$= \left.\frac{x^3}{3}\right]_0^1 + \left.\frac{x^2}{2}\right]_1^3$$

$$= \frac{1}{3} + 4 = \frac{13}{3}$$

4.

Find the area of the region $\{(x, y) : x^2 \leq y \leq x + 2\}$, using integration.

So



x-coordinates of points of intersection are $-1, 2$.

$$\text{Required area} = \int_{-1}^{2} [(x+2) - x^2]\,dx$$

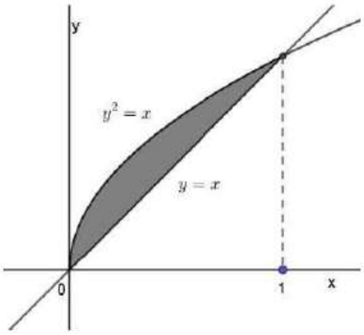
$$= \left.\frac{(x+2)^2}{2} - \frac{x^3}{3}\right|_{-1}^{2}$$

$$= \frac{16}{3} - \frac{5}{6} = \frac{9}{2}$$

5.

Using integration, find the area of the region $\{(x, y) : y^2 \le x \le y\}$.

Sol.



Clearly x coordinates of point of intersection are 0, 1

Required area $= \int_0^1 (\sqrt{x} - x)\, dx$

$$= \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{6}$$

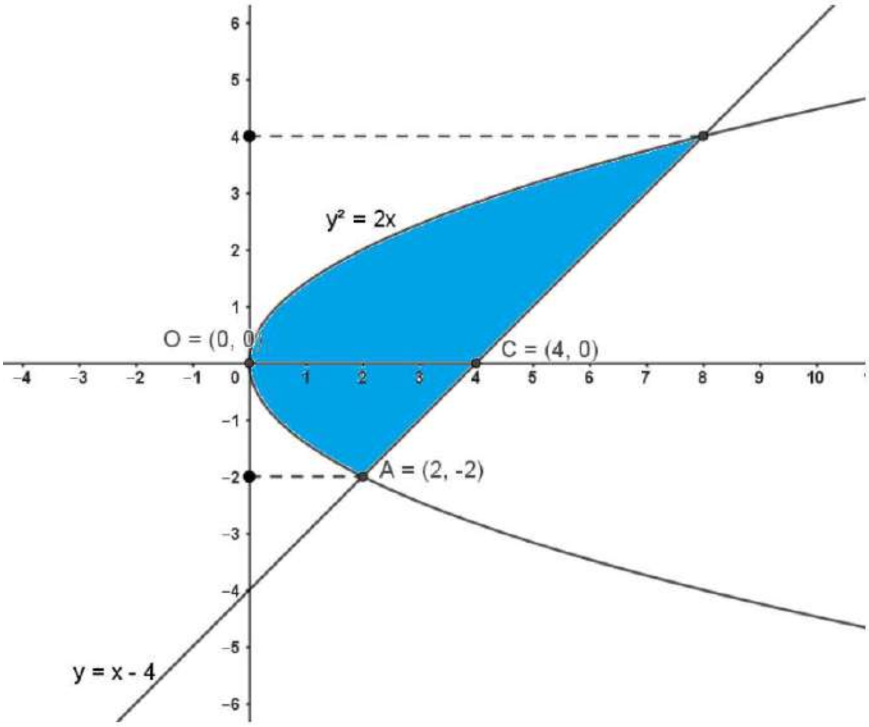
prepared by : BALAJI KANCHI

6.

Find the area of the following region using integration :

$$\{(x, y) : y^2 \leq 2x \text{ and } y \geq x - 4\}$$

Sol.



Solving $y^2 = 2x$ and $y = x - 4$, we get

$y = 4$ or -2

Required area $= \int_{-2}^{4} \left[(y + 4) - \frac{y^2}{2}\right] dy$

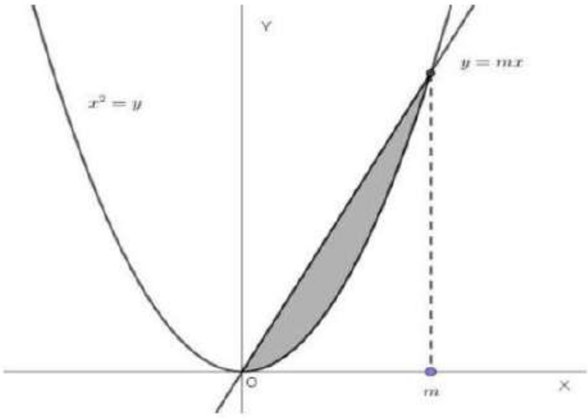
$$= \left| \frac{y^2}{2} + 4y - \frac{1}{6}y^3 \right|_{-2}^{4}$$

$= \mathbf{18}$

prepared by : BALAJI KANCHI

7.

If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.



x-coordinates of points of intersection are $0, m$.

According to question $= \int_0^m (mx - x^2)\,dx = \frac{32}{3}$

$$\Rightarrow m\frac{x^2}{2} - \frac{x^3}{3}\Big|_o^m = \frac{32}{3}$$

$$\frac{m^3}{6} = \frac{32}{3} \Rightarrow m^3 = 64$$

$$\Rightarrow m = 4$$

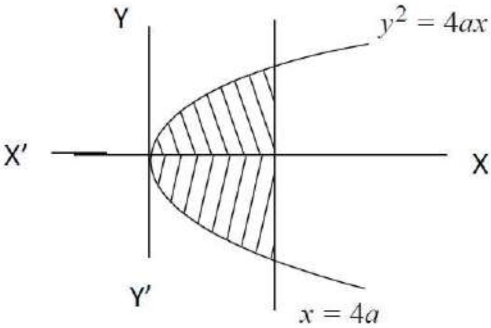
b.Parabola and single variable lines :

1.

If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

Sol.

Given area $= \frac{256}{3}$



Correct Figure

Area of Shaded region $= 2\int_0^{4a} \sqrt{4ax}\, dx$

$$= 8\sqrt{a}\,\frac{x^{3/2}}{3}\Bigg|_0^{4a}$$

$$= \frac{64a^2}{3}$$

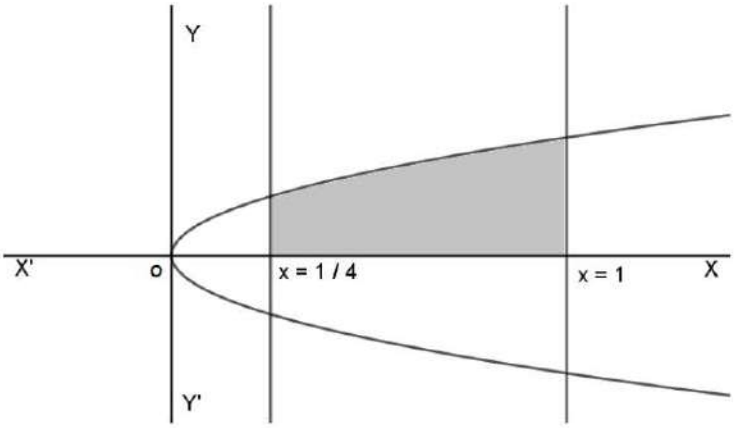
$$\frac{64a^2}{3} = \frac{256}{3}$$

$\Rightarrow a^2 = 4$ gives $a = 2$ (as $a > 0$)

2.

Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

Sol.



$$I = \int_{1/4}^{1} \sqrt{x}dx$$

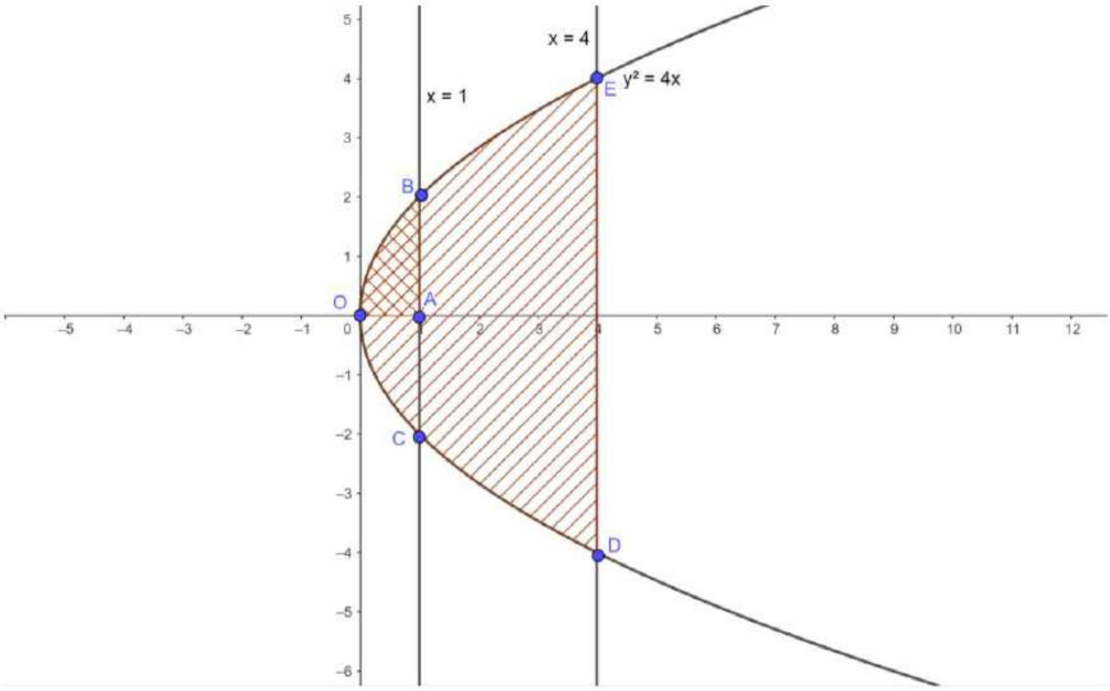
$$= \frac{2}{3}x^{3/2}\Big]_{1/4}^{1}$$

$$= \frac{2}{3}\left[1 - \frac{1}{8}\right] = \frac{7}{12}$$

3.

If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

Sol.



$$A_1 = \text{Area (region OABO)} = \int_0^1 2\sqrt{x}\,dx = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{4}{3}$$

$$A_2 = \text{Area (region ODEO)} = 2\int_0^4 2\sqrt{x}\,dx = 4 \times \frac{2}{3}[2^3] = \frac{64}{3}$$

$$A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1 : 16$$

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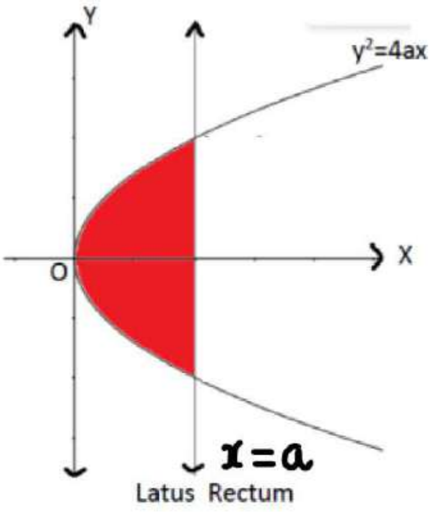
4.

Using integration, find the area of the parabola $y^2 = 4ax$ bounded by its latus rectum.

Sol.



Required area $= 2\int_0^a 2\sqrt{ax}\ \ dx$

$= 4\sqrt{a}\int_0^a \sqrt{x}\ \ dx$

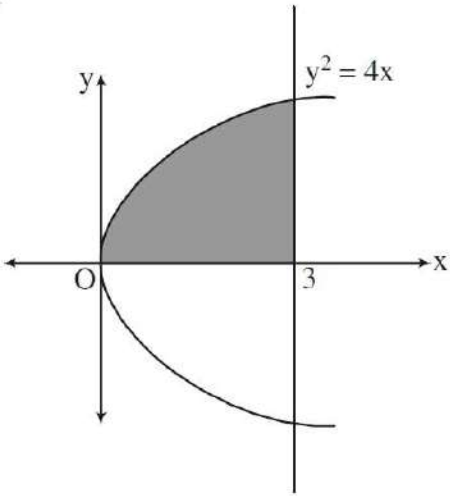
$= 4\sqrt{a}\left|\frac{2x^{3/2}}{3}\right|_0^a$

$= \frac{8}{3}a\sqrt{a}\ \sqrt{a}$

$= \frac{8}{3}a^2$

5.

Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines $x = 0$ and $x = 3$ and the x-axis.



$$\text{Required Area} = \int_0^3 2\sqrt{x}\, dx$$

$$= 2 \times \frac{2}{3}[x^{3/2}]_0^3$$

$$= \frac{4}{3} \times 3^{3/2} = 4\sqrt{3}$$

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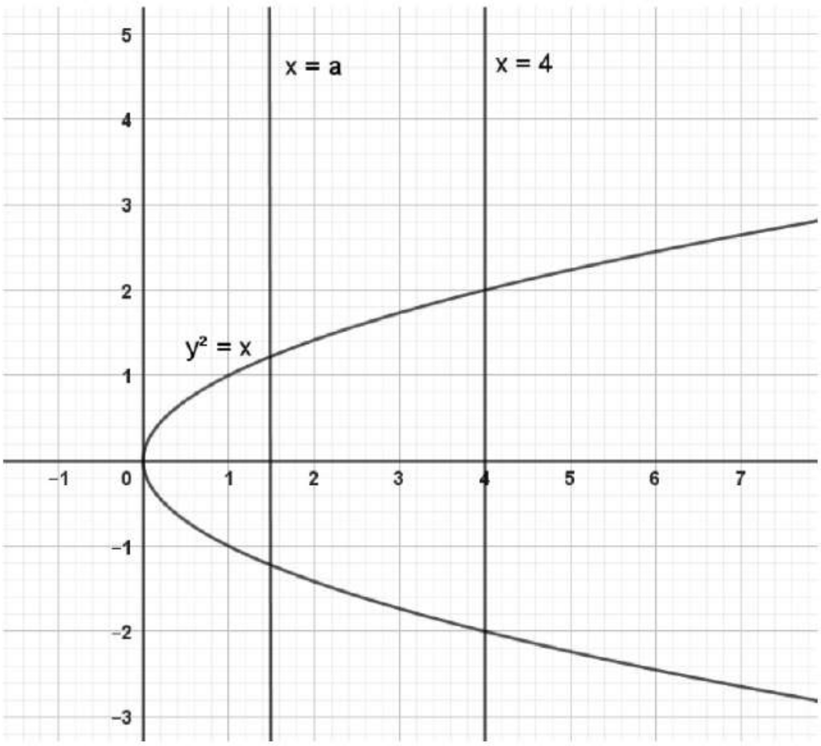
6.

The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$. Find the value of a.

Sol.



$$2\int_0^a \sqrt{x}\,dx = \int_0^4 \sqrt{x}\,dx$$

$$\Rightarrow 2 \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^a = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4$$

$$\Rightarrow \frac{4}{3}\left[a^{\frac{3}{2}}\right] = \frac{2}{3}[8]$$

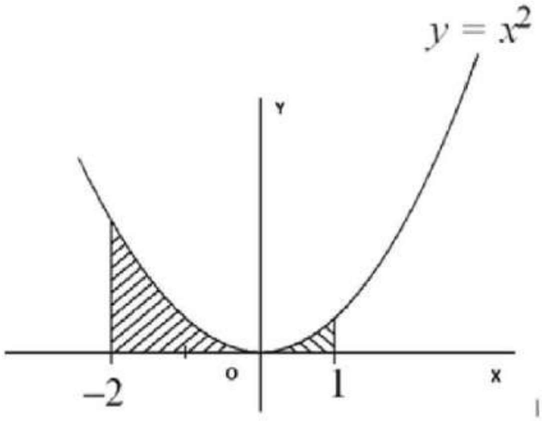
$$\Rightarrow a^{\frac{3}{2}} = 4$$

$$\Rightarrow a = 4^{\frac{2}{3}}$$

7.

Using integration, find the area of the region enclosed by the curve y = x^2, the x-axis and the ordinates $x = -2$ and $x = 1$.

Sol.



Correct figure

$$A = \int_{-2}^{1} x^2 dx$$

$$= \left.\frac{x^3}{3}\right]_{-2}^{1}$$

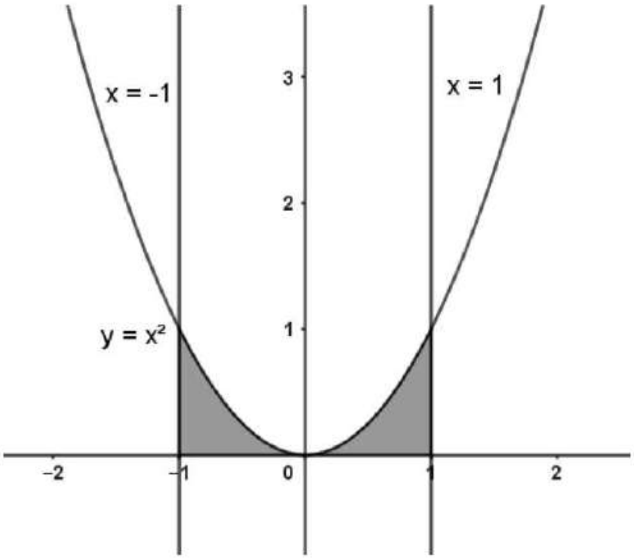
$$= \frac{1}{3} - \left(\frac{-8}{3}\right) = 3$$

8.

Using integration, find the area of the region bounded by the curve $y = x^2$, $x = -1$, $x = 1$ and the x-axis.

Sol.

$y = x^2,\ x = 1, x = 0$ and $x = -1$

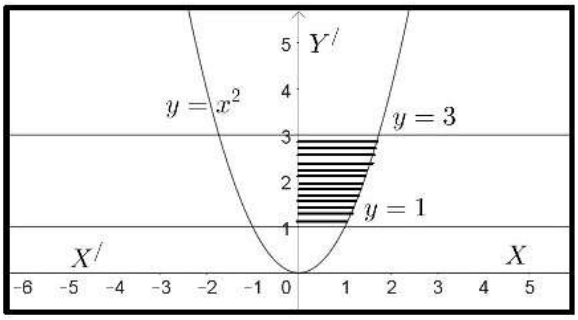


$$\text{Required area } = 2\int_0^1 x^2 dx$$
$$= 2\left[\frac{\mathbf{x^3}}{\mathbf{3}}\right]_{\mathbf{0}}^{\mathbf{1}}$$
$$= 2\left(\frac{1}{3}\right) = \frac{2}{3}$$

9.

Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines $y = 1$ and $y = 3$ and the y-axis.

Sol.



Area of the region bounded by the curve

$$= \int_1^3 \sqrt{y}\,dy$$

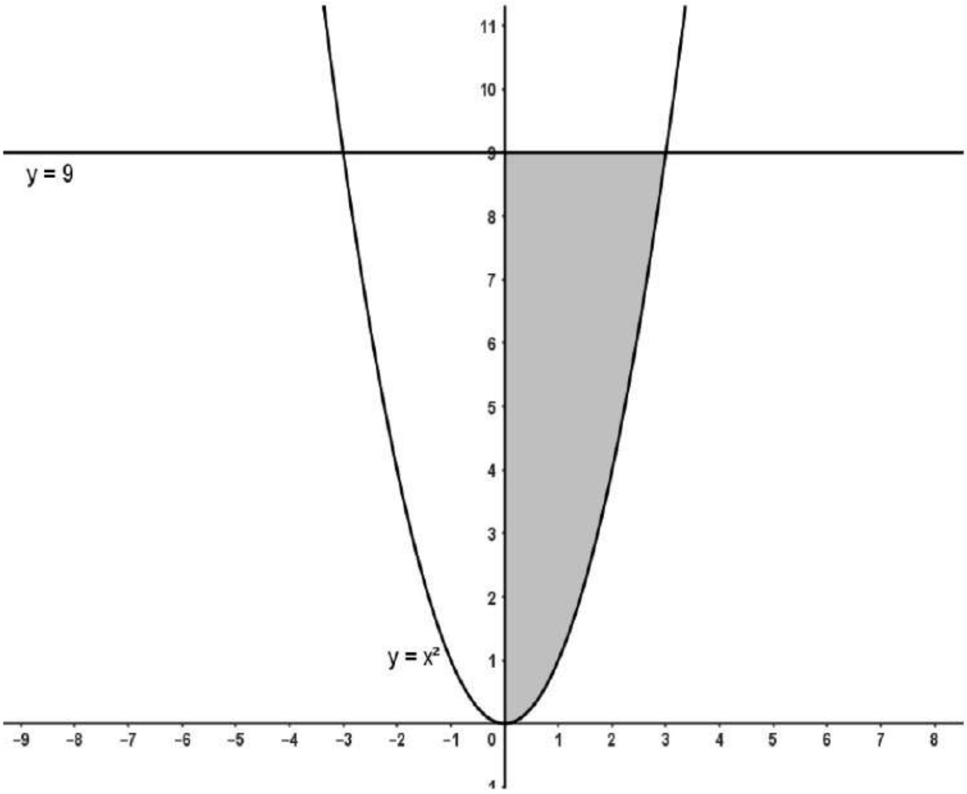
$$= \left[\frac{2y^{3/2}}{3}\right]_1^3 = \frac{2}{3}\left(3\sqrt{3} - 1\right)$$

prepared by : BALAJI KANCHI

10.

Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.

Sol.



$$\text{Required area} = \int_0^9 \sqrt{y}\, dy$$

$$= \frac{2}{3}\left[y^{3/2}\right]_0^9$$

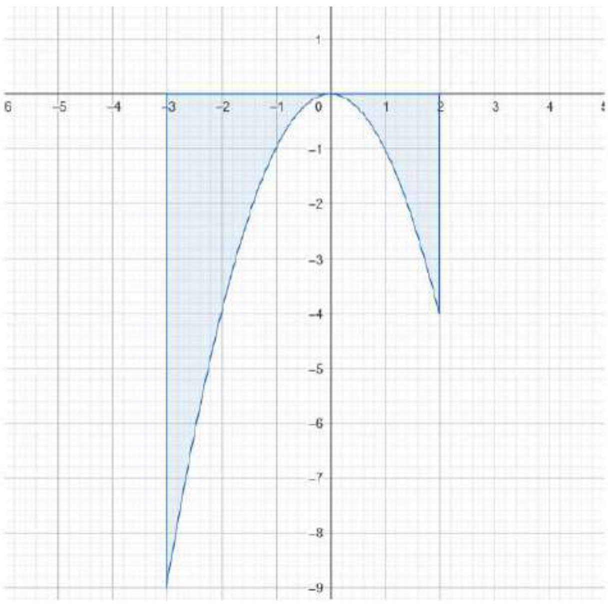
$$= 18$$

Note: If area is found in second quadrant, may be considered.

11.

Use integration to find the area of the region enclosed by curve $y = -x^2$ and the straight lines $x = -3$, $x = 2$ and $y = 0$. Sketch a rough figure to illustrate the bounded region.

Sol.



$$\text{Required area} = \left|\int_{-3}^{2} -x^2\,dx\right|$$

$$= \left|\left[-\frac{1}{3}x^3\right]_{-3}^{2}\right|$$

$$= \left|-\frac{1}{3}(8-(-27))\right|$$

$$= \frac{35}{3}$$

prepared by : BALAJI KANCHI

c. Semi parabola and two variables Line:

1.

Find the area of the region bounded by the curve $y = \sqrt{x}$, the line $x = 2y + 3$ and the x-axis, using integration.

Sol.

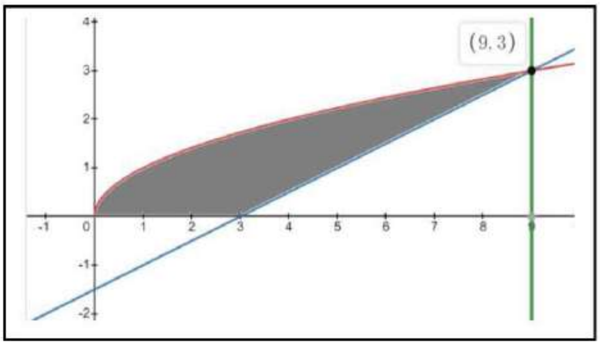
Given curves are $y = \sqrt{x}$ and $x = 2y + 3$

Point of intersection is $(9,3)$.

Required Area $= \int_0^9 \sqrt{x}\, dx - \int_3^9 \left(\frac{x-3}{2}\right) dx$

$= \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^9 - \frac{1}{4}\left[(x-3)^2\right]_3^9$

$= 9$



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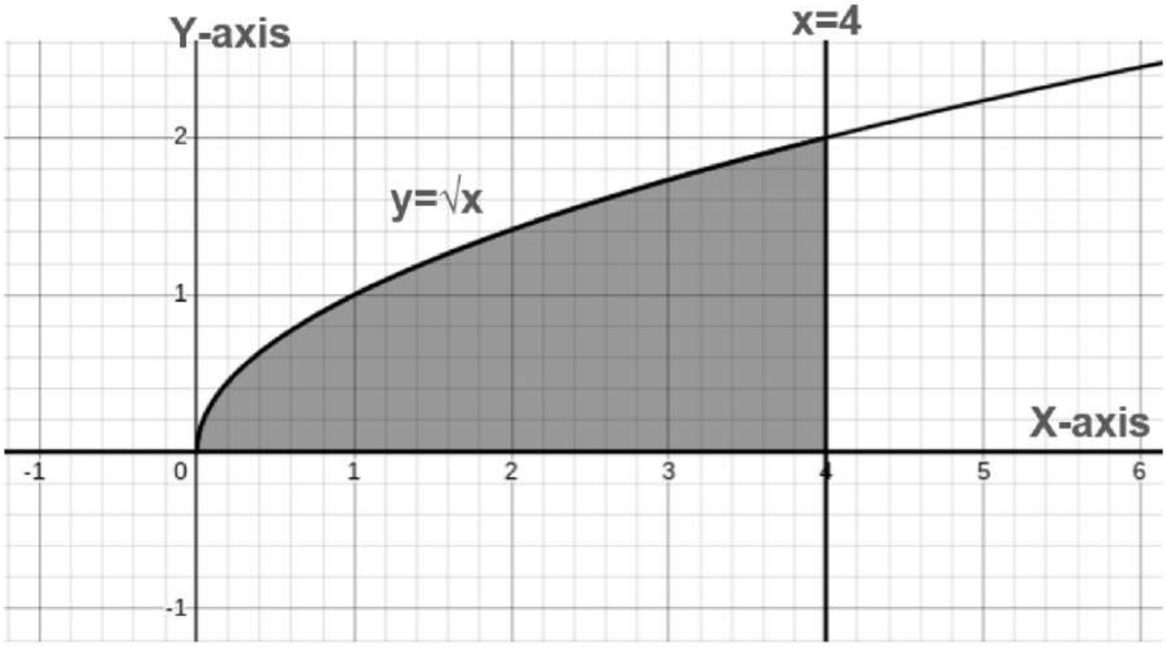
d. Semi parabola and single variables Line:

1.

Draw a rough sketch of the curve $y = \sqrt{x}$. Using integration, find the area of the region bounded by the curve $y = \sqrt{x}$, $x = 4$ and x-axis, in the first quadrant.

Sol.



$$\text{Required area} = \int_0^4 \sqrt{x}\ \ dx$$

$$= \frac{2}{3}\left[x^{3/2}\right]_0^4$$

$$= \frac{2}{3} \times 8 = \frac{16}{3}$$

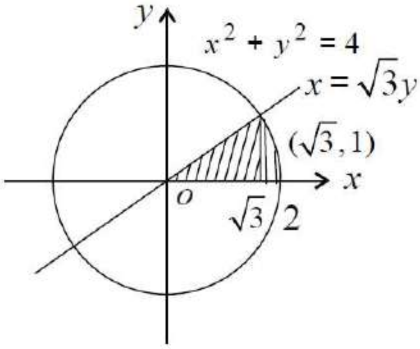
III. Area bounded by circle and line :

a. circle and two variables line :

1.

Using integration, find the area of the region bounded by the curves $x^2 + y^2 = 4$, $x = \sqrt{3}\,y$ and x-axis lying in the first quadrant.

Sol.



For Point of intersection $$x^2 + \frac{x^2}{3} = 4$$

$$x^2 = 3 \Rightarrow x = \sqrt{3}$$

$$I = \int_0^{\sqrt{3}} \frac{1}{\sqrt{3}} x\, dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$$

$$= \frac{1}{\sqrt{3}} \left[\frac{x^2}{2} \right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} \right]_{\sqrt{3}}^{2}$$

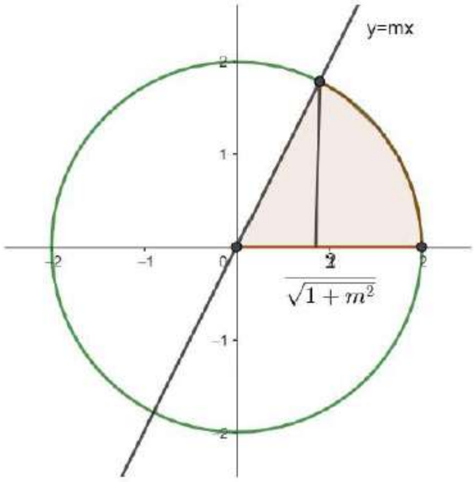
$$= \frac{\sqrt{3}}{2} + 2\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{3}$$

$$= \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

2.

The area of the region bounded by the line y = mx (m > 0), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

Sol.



$x^2 + y^2 = 4$ and $y = mx$

$\Rightarrow x^2 + m^2x^2 = 4 \Rightarrow x = \frac{2}{\sqrt{1+m^2}}$

x- coordinate of the required point of intersection is $\frac{2}{\sqrt{1+m^2}}$.

According to question,

$$\int_0^{\frac{2}{\sqrt{1+m^2}}} mx\,dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} \sqrt{4-x^2}\,dx = \frac{\pi}{2}$$

$$\Rightarrow m\left.\frac{x^2}{2}\right|_0^{\frac{2}{\sqrt{1+m^2}}} + \left.\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right|_{\frac{2}{\sqrt{1+m^2}}}^{2} = \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{1+m^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+m^2}} \Rightarrow m^2 + 1 = 2$$

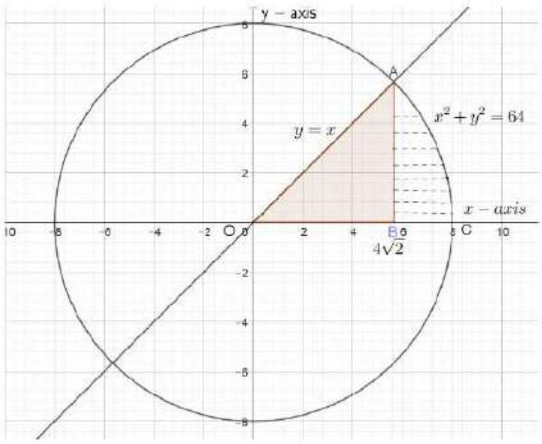
$$\Rightarrow m = 1 \text{ (as } m > 0)$$

prepared by : BALAJI KANCHI

3.

A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

Sol.



Equation of the circular tabletop:

$$x^2+y^2=64$$

Equation of line (scratch): $x=y$

The line and circle intersect at $x=4\sqrt{2}$

Area of the shaded region

$$=\int_0^{4\sqrt{2}} x\,dx+\int_{4\sqrt{2}}^{8}\sqrt{64-x^2}\,dx$$

$$=\left.\frac{x^2}{2}\right]_0^{4\sqrt{2}}+\left[\frac{x}{2}\sqrt{64-x^2}+32\sin^{-1}\frac{x}{8}\right]_{4\sqrt{2}}^{8}$$

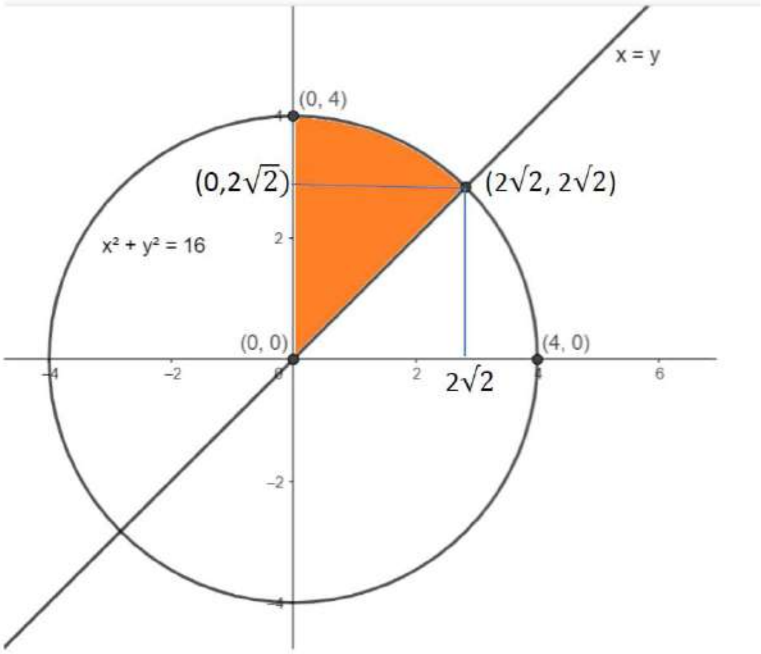
$$=\frac{32}{2}+32\sin^{-1}1-2\sqrt{2}\cdot4\sqrt{2}-32\sin^{-1}\frac{1}{\sqrt{2}}$$

$$=16+16\pi-16-8\pi=8\pi\ \text{cm}^2$$

4.

Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$, line $y = x$ and y-axis, but lying in the 1^{st} quadrant.

Sol.



$$\text{Required area} = \int_0^{2\sqrt{2}} y \, dy + \int_{2\sqrt{2}}^{4} \sqrt{16 - y^2} \, dy$$

$$= \left|\frac{y^2}{2}\right|_0^{2\sqrt{2}} + \left|\frac{y}{2}\sqrt{16 - y^2} + \frac{16}{2}\sin^{-1}\left(\frac{y}{4}\right)\right|_{2\sqrt{2}}^{4}$$

$$= 4 + \left\{8\sin^{-1}(1) - \sqrt{2}\,.\,\sqrt{8} - 8\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= 4 + 8\left(\frac{\pi}{2}\right) - 4 - 8\left(\frac{\pi}{4}\right)$$

$$= 8\left(\frac{\pi}{4}\right) \text{ or } 2\pi$$

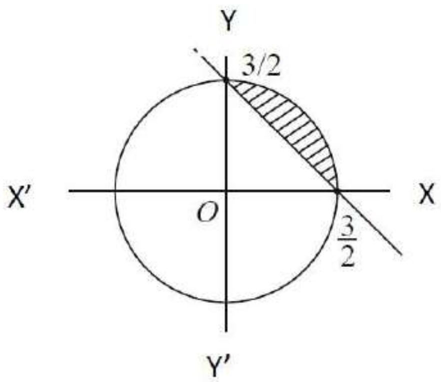
prepared by : **BALAJI KANCHI**

5.

Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

Sol. Clearly point of intersection are

$\left(\frac{3}{2}, 0\right) \& \left(0, \frac{3}{2}\right)$



$$\text{Required area} = \int_0^{3/2} \sqrt{\frac{9}{4} - x^2}\, dx - \int_0^{3/2} \left(\frac{3}{2} - x\right) dx$$

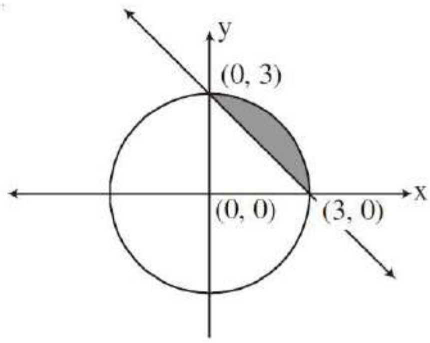
$$= \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Bigg|_0^{3/2} + \frac{\left(\frac{3}{2} - x\right)^2}{2}\Bigg|_0^{3/2}$$

$$= \frac{9\pi}{16} - \frac{9}{8}$$

6.

Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 9,\ x + y \geq 3\}$.

Ans. Point of intersection (3, 0) and (0, 3)



Correct figure

Required Area

$$= \int_0^3 \sqrt{9 - x^2}\, dx - \int_0^3 (3 - x) dx$$

$$= \left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 - \left[\frac{(3 - x)^2}{-2}\right]_0^3$$

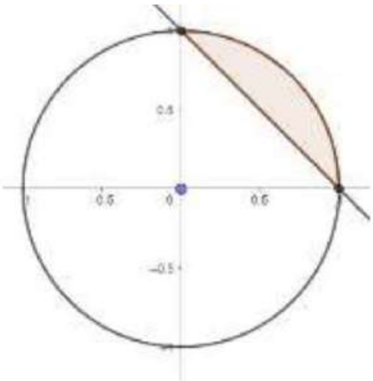
$$= \frac{9}{2}\sin^{-1} 1 - \frac{9}{2} = \frac{9}{2}\left(\frac{\pi}{2} - 1\right)$$

prepared by : BALAJI KANCHI

7.

Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$, using integration.

Sol.



x coordinates of point of intersection are 1, 0

Required area = $\int_0^1 \sqrt{1 - x^2}\, dx - \int_0^1 (1 - x)dx$

$= \frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x \Big|_0^1 + \frac{(1 - x)^2}{2}\Big|_0^1$

$= \frac{\pi}{4} - \frac{1}{2}$

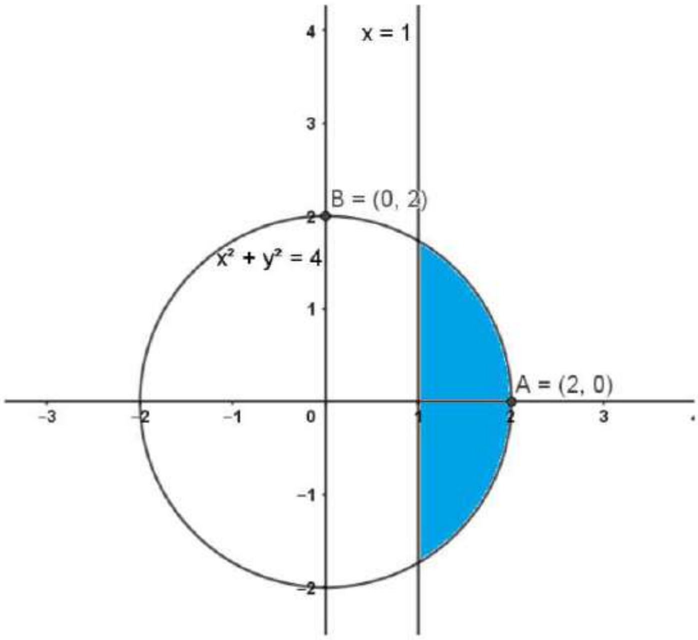
prepared by : BALAJI KANCHI

b. circle and single variable lines :

1.

Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.

Sol.



$$\text{Required area} = 2\int_1^2 \sqrt{4 - x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$$

$$= 2\left[\left\{0 + 2\left(\frac{\pi}{2}\right)\right\} - \left\{\frac{1}{2}\sqrt{3} + 2\cdot\frac{\pi}{6}\right\}\right]$$

$$= 2\left(\pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right)$$

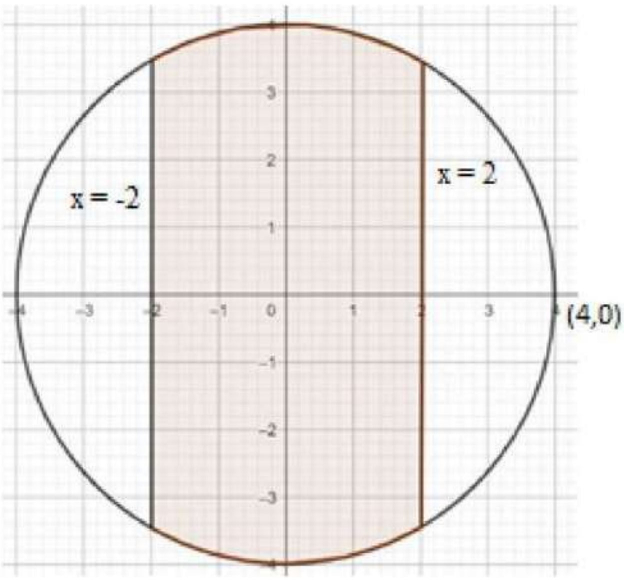
$$= \left(\frac{4\pi}{3} - \sqrt{3}\right)$$

prepared by : BALAJI KANCHI

2.

Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$.

Sol.



$$\text{Required area} = 4\int_0^2 \sqrt{16-x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{16-x^2} + 8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$$

$$= 8\sqrt{3} + \frac{16\pi}{3}$$

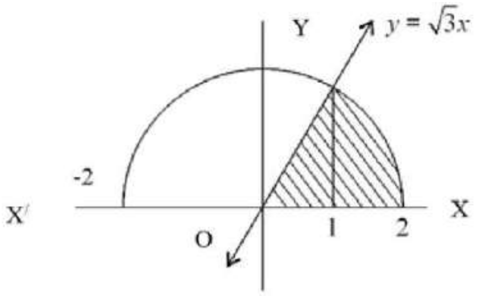
prepared by : BALAJI KANCHI

c. semi Circle and line :

1.

Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$, semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

Sol.



Correct figure

x coordinate of point of intersection is 1

Required area $= \int_0^1 \sqrt{3}\, x\, dx + \int_1^2 \sqrt{4 - x^2}\, dx$

$$= \sqrt{3}\frac{x^2}{2}\Bigg|_0^1 + \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\Bigg|_1^2$$

$$= \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

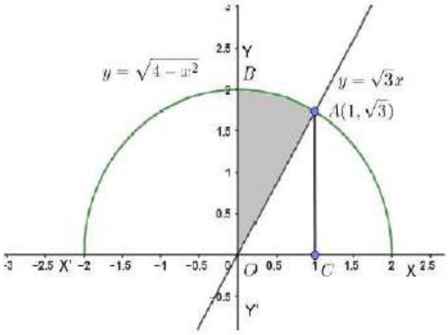
$$= \frac{2\pi}{3}$$

prepared by : BALAJI KANCHI

2.

Using integration, find the area of region bounded by line y $=\sqrt{3}x$, the curve y $=\sqrt{4-x^2}$ and y–axis in first quadrant.

Sol.



Point of intersection at x = 1

$$ar(OAB)=\int_0^1 \sqrt{4-x^2}\,dx-\int_0^1 \sqrt{3}x\,dx$$

$$=\left(\frac{x\sqrt{4-x^2}}{2}+2\sin^{-1}\left(\frac{x}{2}\right)\right)\Bigg]_0^1-\frac{\sqrt{3}}{2}x^2\Big]_0^1$$

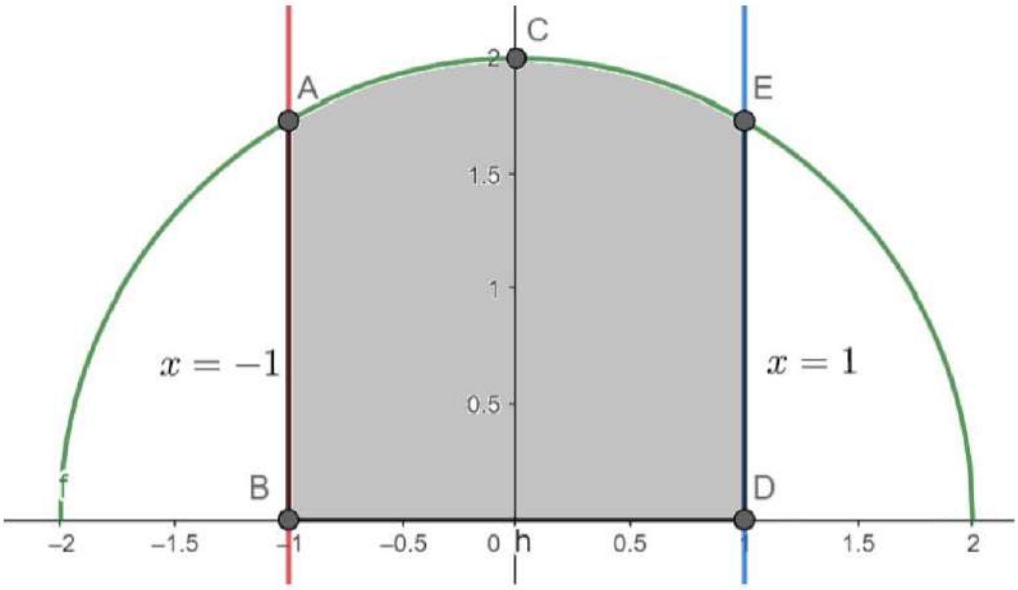
$$=\frac{\sqrt{3}}{2}+2\times\frac{\pi}{6}-\frac{\sqrt{3}}{2}=\frac{\pi}{3}$$

prepared by : BALAJI KANCHI

3.

Using integration, find the area of the region enclosed between the curve $y = \sqrt{4 - x^2}$ and the lines $x = -1$, $x = 1$ and the x-axis.

Sol.



$$\text{Required area} = 2\int_0^1 \sqrt{4 - x^2}\, dx$$

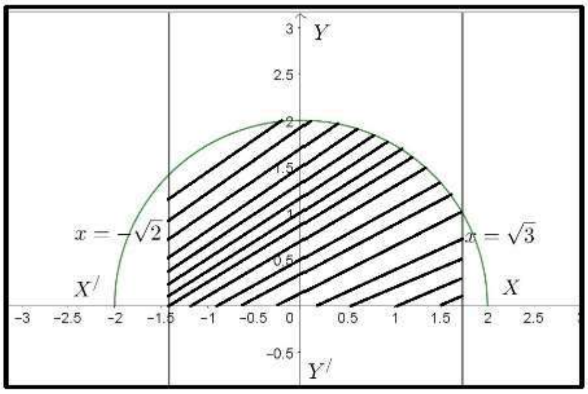
$$= 2\left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_0^1$$

$$= \sqrt{3} + \frac{2\pi}{3}$$

4.

Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

Sol.



Area of the region bounded by the curve

$$= \int_{-\sqrt{2}}^{\sqrt{3}} \sqrt{4 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_{-\sqrt{2}}^{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2} + 2\cdot\frac{\pi}{3} + 1 + 2\cdot\frac{\pi}{4} = \frac{\sqrt{3}}{2} + 1 + \frac{7\pi}{6}$$

prepared by : BALAJI KANCHI

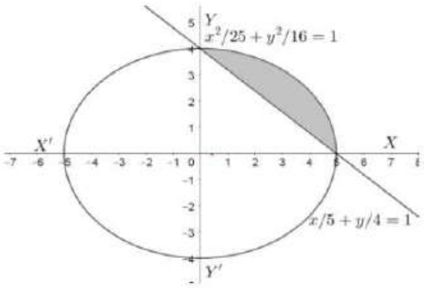
IV. Area bounded by ellipse and line :

a. ellipse and two variable line

1.

Find the area of the smaller region bounded by the curves $\frac{x^2}{25}+\frac{y^2}{16}=1$ and $\frac{x}{5}+\frac{y}{4}=1$, using integration.

Sol.



$$\text{Required Area} = \frac{4}{5}\int_0^5 \sqrt{25-x^2}\,dx - \frac{4}{5}\int_0^5 (5-x)\,dx$$

$$= \frac{4}{5}\left[\left(\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\frac{x}{5}\right)\right]_0^5 + \frac{2}{5}\left.(5-x)^2\right]_0^5$$

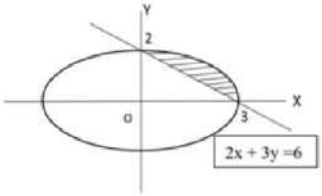
$$= \frac{4}{5}\left(\frac{25\pi}{4}\right) - 10 = 5\pi - 10$$

prepared by : BALAJI KANCHI

2.

Using integration, find the area of the region $\{(x, y) : 4x^2 + 9y^2 \le 36, 2x + 3y \ge 6\}$.

Sol.



Correct figure

Clearly points of intersection are (3, 0) and (0, 2)

Required area $= \frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx - \frac{2}{3}\int_0^3 (3-x)\,dx$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + \frac{(3-x)^2}{2}\right]_0^3$$

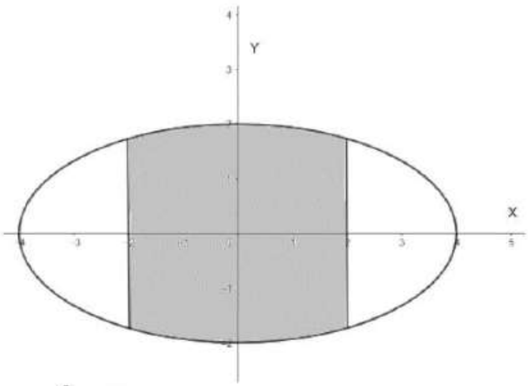
$$= \frac{2}{3}\left[\frac{9\pi}{4} - \frac{9}{2}\right] \ or \ \frac{3\pi}{2} - 3$$

b. ellipse and single variable line

1.

Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines x = – 2 and x = 2.

Sol.



$$\text{Area} = 4\int_0^2 y\,dx$$

$$= 4\left[\frac{1}{2}\int_0^2 \sqrt{4^2 - x^2}\,dx\right]$$

$$= 2\left[\frac{x}{2}\sqrt{4^2 - x^2} + 8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$$

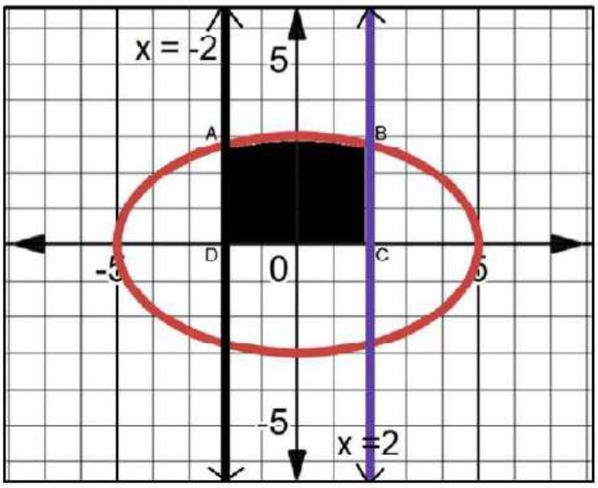
$$= 2\left[\sqrt{12} + \frac{8\pi}{6}\right] = 4\sqrt{3} + \frac{8\pi}{3}$$

prepared by : BALAJI KANCHI

2.

Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

Sol.



As, $9x^2 + 25y^2 = 225 \Rightarrow \boxed{y = \pm\frac{3}{5}\sqrt{5^2 - x^2}}$

$$\text{Required Area} = \int_{-2}^{2} \frac{3}{5}\sqrt{5^2 - x^2}\,dx = \frac{6}{5}\int_{0}^{2}\sqrt{5^2 - x^2}\,dx$$

$$= \frac{6}{5}\left(\frac{x\sqrt{5^2 - x^2}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right)_0^2$$

$$= \frac{6}{5}\left(\frac{2\sqrt{21}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)\right)$$

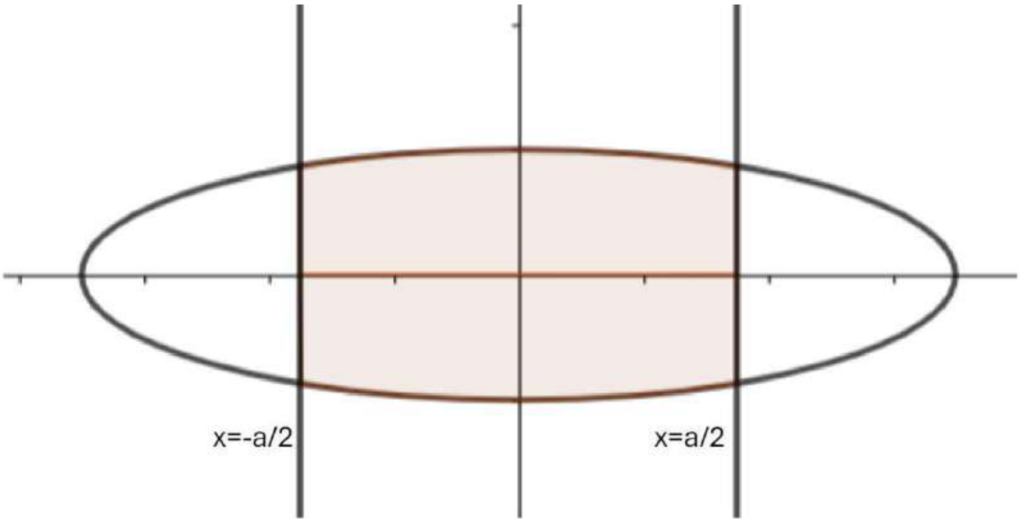
$$= \left(\frac{6\sqrt{21}}{5} + 15\sin^{-1}\left(\frac{2}{5}\right)\right)$$

prepared by : BALAJI KANCHI

3.

Using integration, find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bounded between the lines $x=-\frac{a}{2}$ to $x=\frac{a}{2}$.

Sol.



$$\text{Required area} = \frac{4b}{a}\int_0^{\frac{a}{2}}\sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{\frac{a}{2}}$$

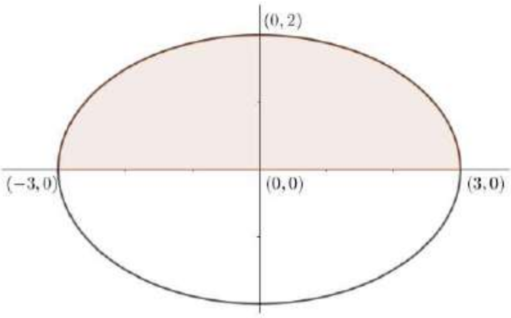
$$= \frac{4b}{a}\left[\frac{a}{4}\cdot\frac{\sqrt{3}a}{2}+\frac{a^2}{2}\cdot\frac{\pi}{6}\right]$$

$$= ab\left[\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right]$$

4.

Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x-axis using integration.

Sol.



$$A = 2 \times \frac{2}{3} \int_0^3 \sqrt{9 - x^2}\, dx$$

$$= \frac{4}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$$

$$= \frac{4}{3}\left[\left(0 + \frac{9}{2}\sin^{-1}1\right) - 0\right]$$

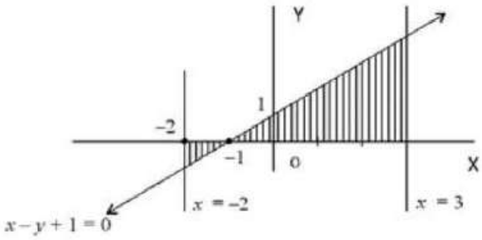
$$= 3\pi$$

V. Area bounded by lines, x-axis/y-axis :

1.

Using integration, find the area of the region bounded by lines $x - y + 1 = 0$, $x = -2$, $x = 3$ and x-axis.

Sol.



Correct figure

Required area $= -\int_{-2}^{-1}(x+1)\,dx + \int_{-1}^{3}(x+1)\,dx$

$$= -\left.\frac{(x+1)^2}{2}\right|_{-2}^{-1} + \left.\frac{(x+1)^2}{2}\right|_{-1}^{3}$$

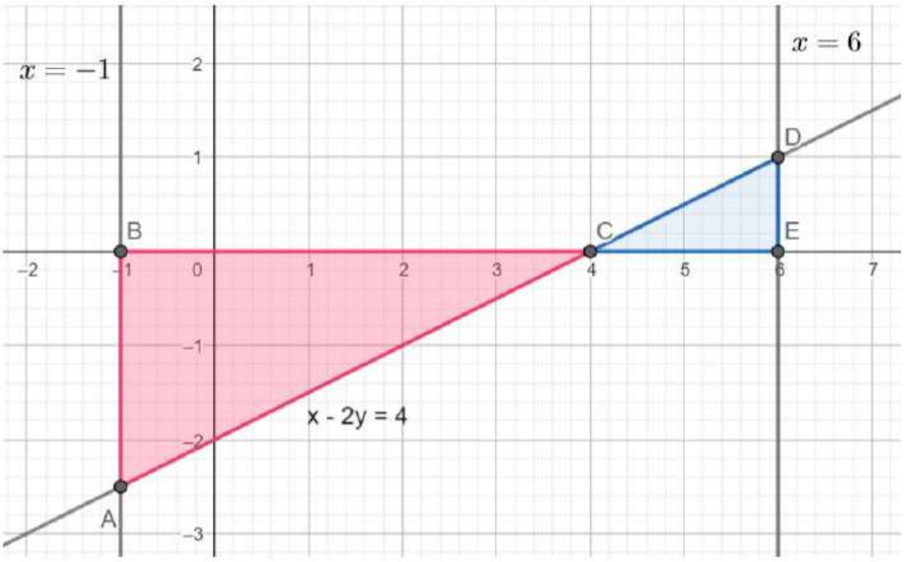
$$= \frac{1}{2} + 8$$

$$= \frac{17}{2}$$

2.

Find the area of the region bounded by the lines x – 2y = 4, x = –1, x = 6 and x-axis, using integration.

Sol.



$$\text{Required area} = \left| \int_{-1}^{4} \left(\frac{x-4}{2} \right) dx \right| + \int_{4}^{6} \left(\frac{x-4}{2} \right) dx$$

$$= \left| \left[\frac{(x-4)^2}{4} \right]_{-1}^{4} \right| + \left[\frac{(x-4)^2}{4} \right]_{4}^{6}$$

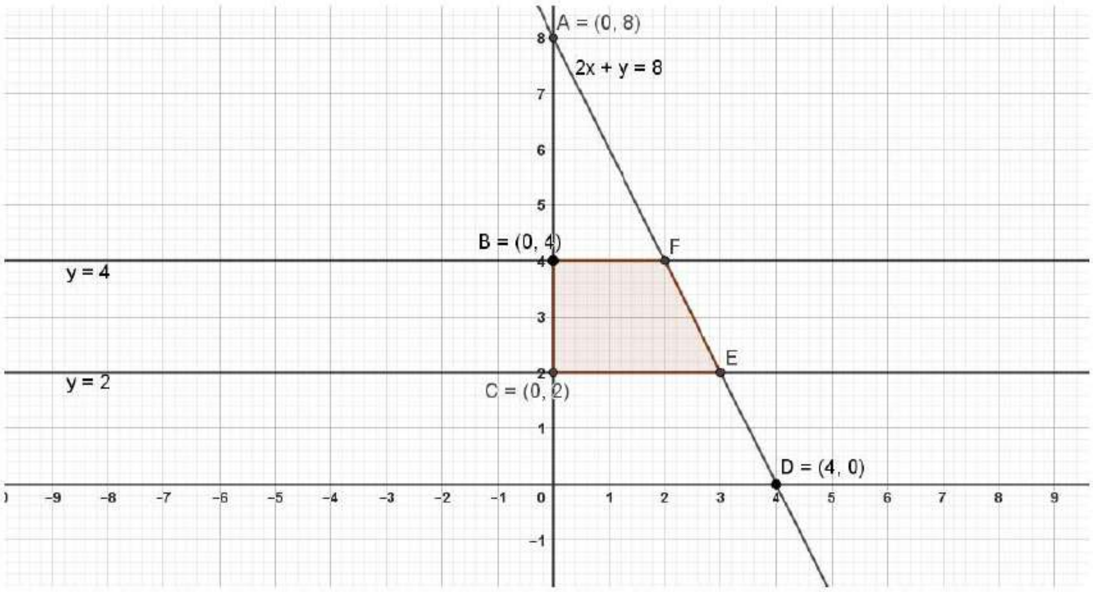
$$= \frac{25}{4} + 1 = \frac{29}{4}$$

prepared by : BALAJI KANCHI

3.

Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

Sol.



$$\text{Required area} = \int_2^4 \frac{1}{2}\,(8 - y)dy$$

$$= \frac{1}{2}\left|8y - \frac{y^2}{2}\right|_2^4$$

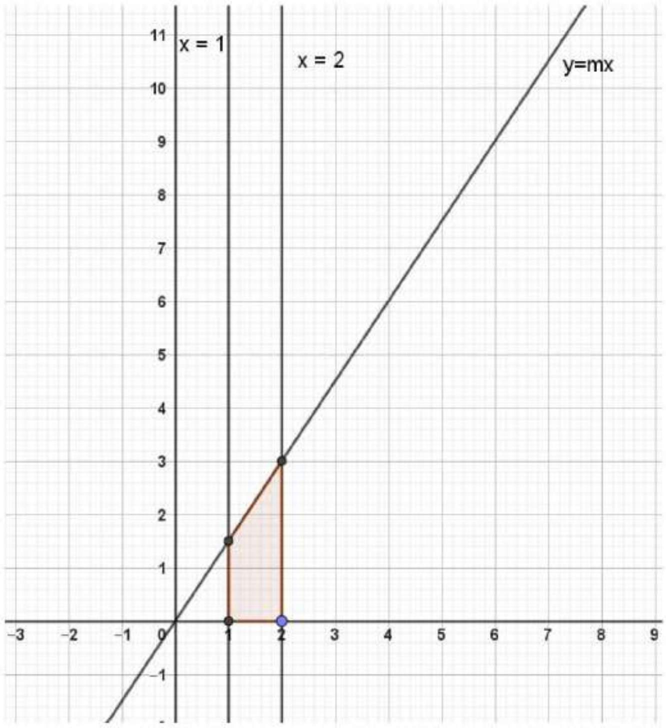
$$= 5$$

prepared by : BALAJI KANCHI

4.

Using integration, find the area of the region bounded by $y = mx$ $(m > 0)$, $x = 1$, $x = 2$ and the x-axis.

Sol.



$$\text{Required area} = \int_1^2 (mx)dx$$

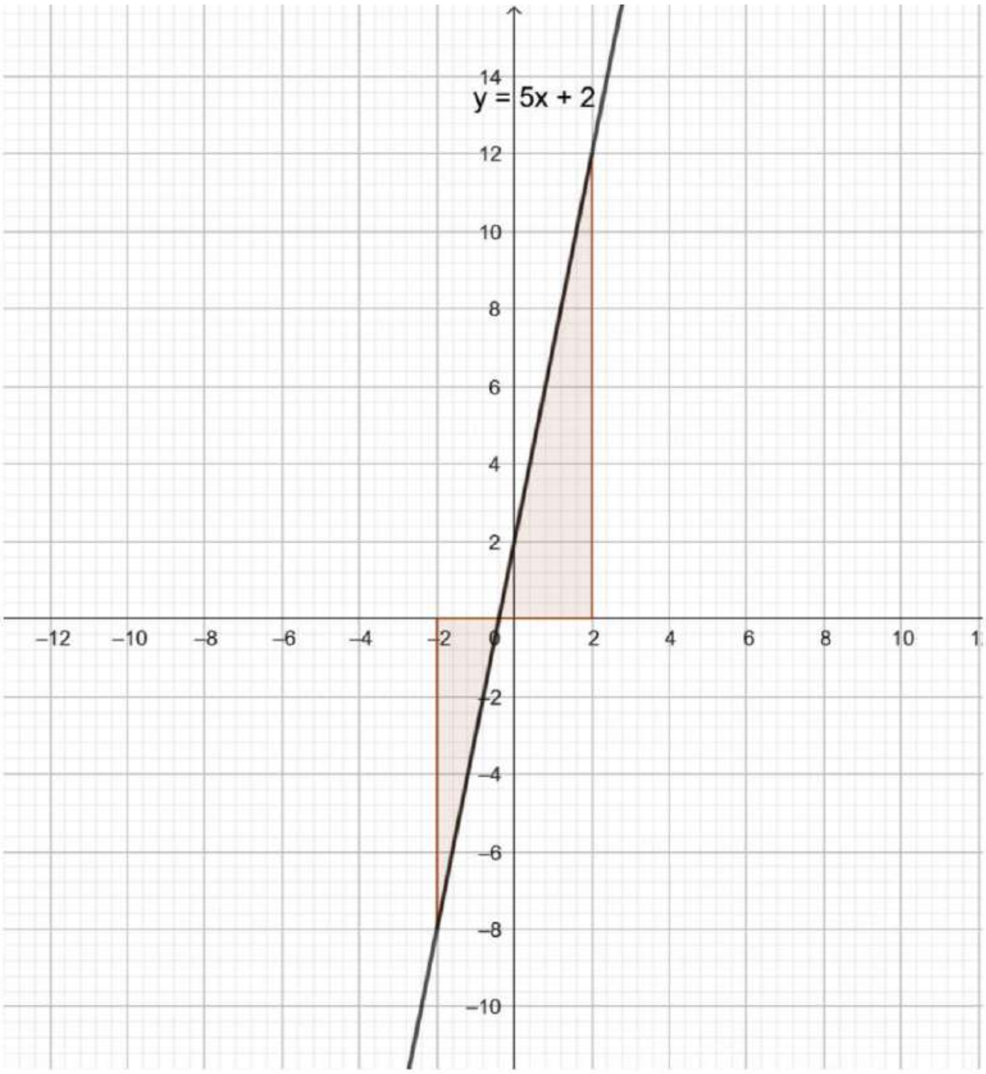
$$= m\left[\frac{x^2}{2}\right]_1^2$$

$$= \frac{3}{2}m$$

5.

Using integration, find the area of the region bounded by the line y = $5x + 2$, the $x-$ axis and the ordinates $x = -2$ and $x = 2$.

Sol.



The required area

$$= \left|\int_{-2}^{-\frac{2}{5}}(5x+2)dx\right| + \int_{-\frac{2}{5}}^{2}(5x+2)dx$$

$$= \left|\left[\frac{(5x+2)^2}{10}\right]_{-2}^{-\frac{2}{5}}\right| + \left[\frac{(5x+2)^2}{10}\right]_{-\frac{2}{5}}^{2}$$

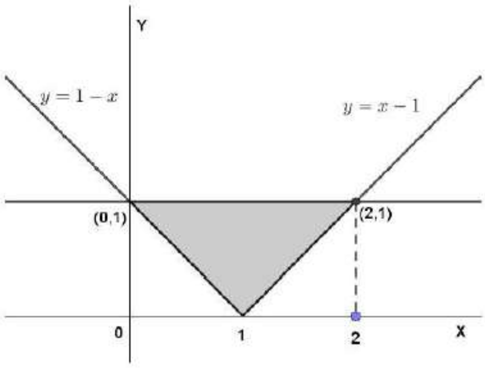
$$= \frac{64}{10} + \frac{144}{10} = \frac{104}{5}$$

VI. Modulus function :

1.

Find the area bounded by the curves $y = |x-1|$ and $y = 1$, using integration.

Sol.



(for correct figure)

Area of the bounded region is

$$\int_0^1 [1-(1-x)]\,dx + \int_1^2 [1-(x-1)]\,dx$$

$$\int_0^1 x\,dx + \int_1^2 (2-x)\,dx$$

$$= \left.\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{2} + \left[4 - \frac{4}{2} - 2 + \frac{1}{2}\right]$$

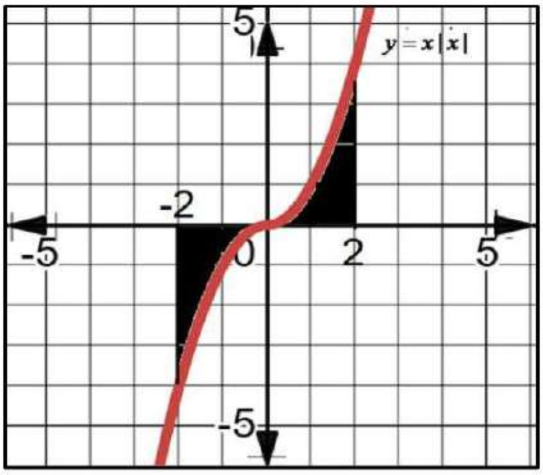
$$= -1 + 2 = 1$$

prepared by : BALAJI KANCHI

2.

Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

Sol.



As, $y = x|x| = \begin{cases} -x^2, x < 0 \\ x^2, x \geq 0 \end{cases}$

\

$$\text{Area of the shaded region} = \int_{-2}^{2} y\, dx = 2\int_{0}^{2} y\, dx = 2\int_{0}^{2} x^2\, dx$$

$$= 2\left(\frac{x^3}{3}\right)_0^2$$

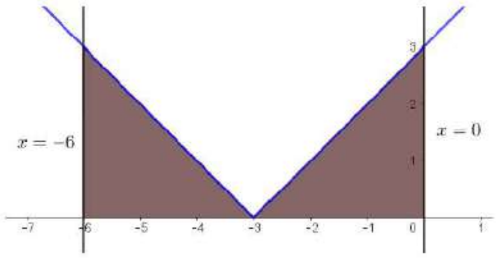
$$= 2\left(\frac{8}{3}\right) = \frac{16}{3}$$

prepared by : BALAJI KANCHI

3.

Sketch the graph of y = | $x + 3$ | and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

Sol.



Required Area

$$= \int_{-6}^{0} y\,dx$$

$$= 2\int_{-3}^{0} (x+3)\,dx$$

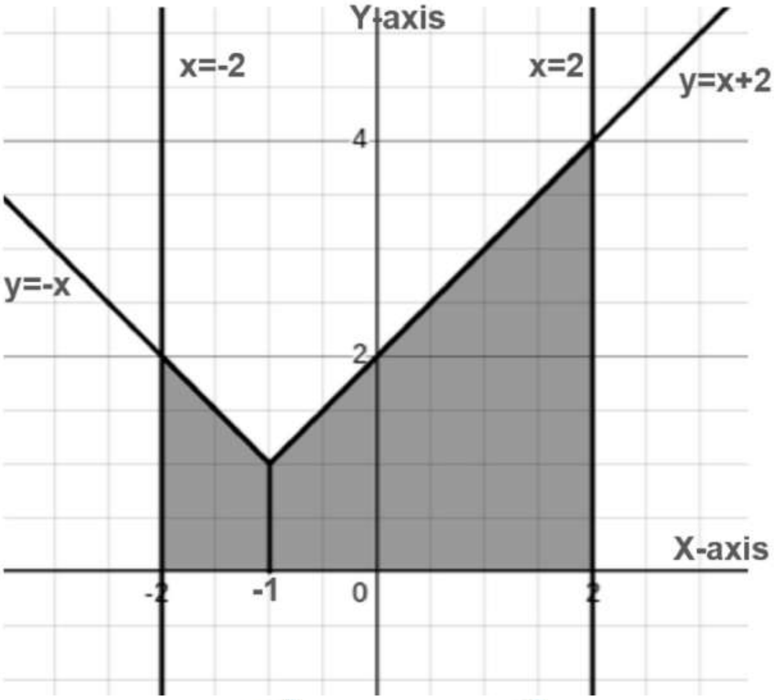
$$= 2\left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$$

$$= 9$$

4.

In a rough sketch, mark the region bounded by $y = 1 + |x + 1|$, $x = -2$, $x = 2$ and $y = 0$. Using integration, find the area of the marked region.

Sol.



$$\text{Required area} = \int_{-2}^{-1}(-x)\ dx + \int_{-1}^{2}(x+2)\ dx$$

$$= -\frac{1}{2}\left[x^2\right]_{-2}^{-1} + \left[\frac{1}{2}x^2 + 2x\right]_{-1}^{2}$$

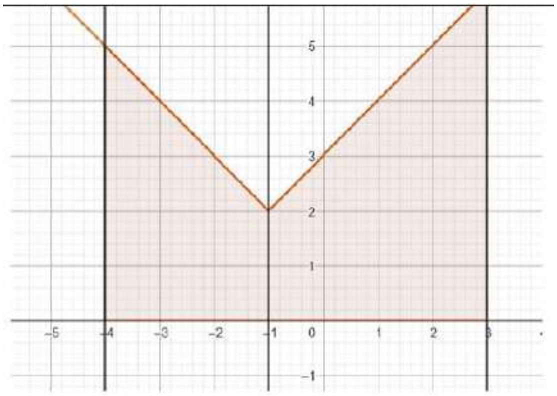
$$= 9$$

prepared by : BALAJI KANCHI

5.

Draw a rough sketch for the curve y = 2 + |x + 1|. Using integration, find the area of the region bounded by the curve y = 2 + |x + 1|, x = – 4, x = 3 and y = 0.

Sol.



$$\text{Required area} = \int_{-4}^{-1}(2+|x+1|)\,dx + \int_{-1}^{3}(2+|x+1|)\,dx$$

$$= \int_{-4}^{-1}(1-x)\,dx + \int_{-1}^{3}(3+x)\,dx$$

$$= -\frac{(1-x)^2}{2}\Big|_{-4}^{-1} + \frac{(3+x)^2}{2}\Big|_{-1}^{3}$$

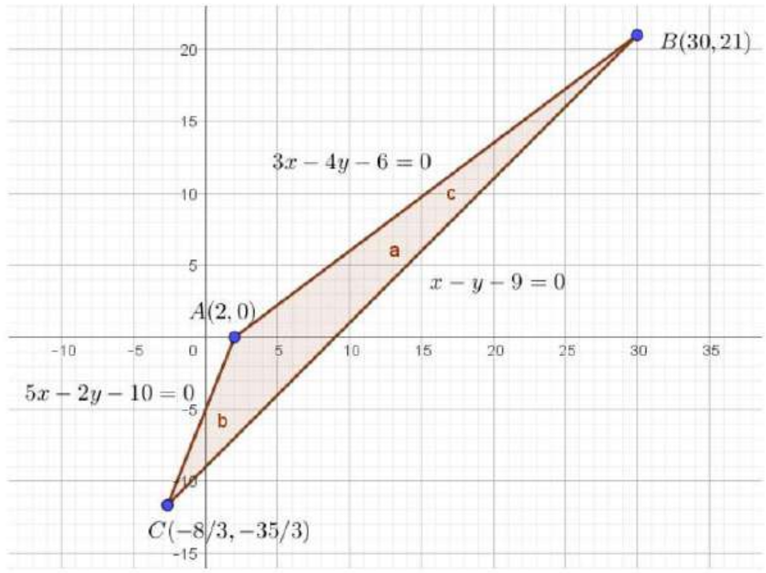
$$= \frac{21}{2} + 16 = \frac{53}{2}$$

VII. Area of triangle if sides were given :

1.

Find the area of the triangle ABC bounded by the lines represented by the equations $5x - 2y - 10 = 0$, $x - y - 9 = 0$ and $3x - 4y - 6 = 0$, using integration method.

Sol.



solving the given equations, the vertices of triangle are

$A(2,0), B(30,21) \text{ and } C\left(-\frac{8}{3}, -\frac{35}{3}\right)$

$$\text{ar}(\triangle ABC) = \frac{3}{4}\int_{2}^{30}(x-2)\,dx - \int_{9}^{30}(x-9)\,dx + \left|\int_{-\frac{8}{3}}^{9}(x-9)\,dx\right| - \left|\frac{5}{2}\int_{-\frac{8}{3}}^{2}(x-2)\,dx\right|$$

$$= \frac{3}{8}(x-2)^2\Big]_2^{30} - \frac{1}{2}(x-9)^2\Big]_9^{30} + \left|\frac{1}{2}(x-9)^2\Big]_{-\frac{8}{3}}^{9}\right| - \left|\frac{5}{4}(x-2)^2\Big]_{-\frac{8}{3}}^{2}\right|$$

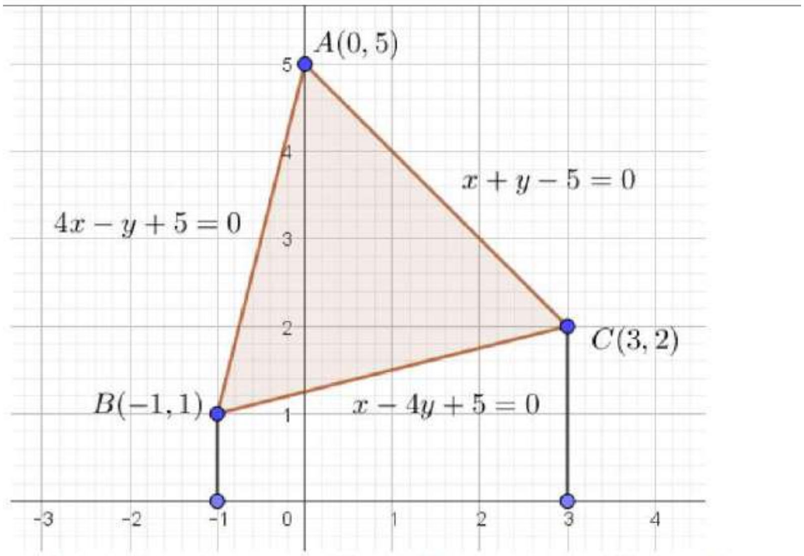
$$= 294 - \frac{441}{2} + \frac{1225}{18} - \frac{245}{9} = \frac{343}{3}$$

prepared by : BALAJI KANCHI

2.

Using integration, find the area of the region bounded by the triangle ABC when its sides are given by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.

sol.



solving the given equations, the vertices of triangle are $A(0,5), B(-1,1)$ and $C(3,2)$

$$\text{ar}(\triangle ABC) = \int_{-1}^{0} (4x+5)\,dx + \int_{0}^{3} (-x+5)\,dx - \int_{-1}^{3} \frac{5+x}{4}\,dx$$

$$= \left[\frac{4x^2}{2} + 5x\right]_{-1}^{0} + \left[\frac{-x^2}{2} + 5x\right]_{0}^{3} - \left[\frac{5x}{4} + \frac{x^2}{8}\right]_{-1}^{3}$$

$$= \frac{15}{2}$$

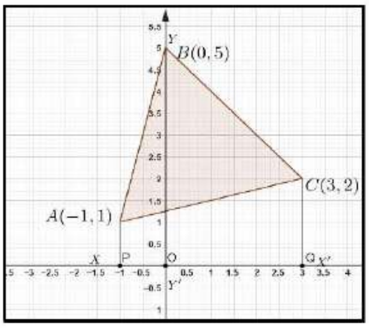
prepared by : **BALAJI KANCHI**

VIII. Area of triangle , vertices were given :

1.

Using Integration, find the area of triangle whose vertices are (–1, 1), (0, 5) and (3, 2).

Sol.



Equation of the lines AB, BC and AC are:

$$y = 4x + 5\ ;\ y = 5 - x\ ;\ y = \frac{x}{4} + \frac{5}{4} \text{ respectively,}$$

$$ar(\Delta ABC) = \int_{-1}^{0}(4x+5)\,dx + \int_{0}^{3}(5-x)\,dx - \frac{1}{4}\int_{-1}^{3}(5+x)\,dx$$

$$= \frac{1}{8}(4x+5)^2\Big]_{-1}^{0} - \frac{1}{2}(5-x)^2\Big]_{0}^{3} - \frac{1}{8}(5+x)^2\Big]_{-1}^{3}$$

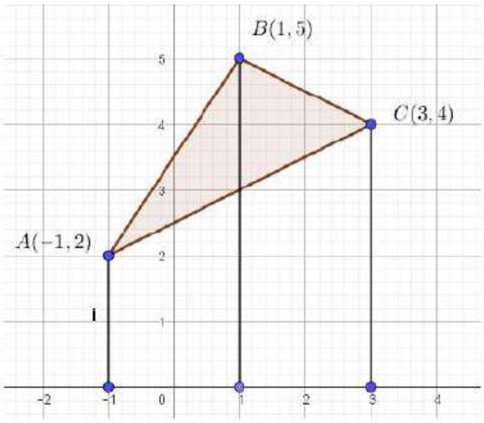
$$= 3 + \frac{21}{2} - 6 = \frac{15}{2}$$

prepared by : **BALAJI KANCHI**

2.

Using integration, find the area of the region bounded by the triangle whose vertices are $(-1, 2)$, $(1, 5)$ and $(3, 4)$.

Sol.



$$ar(ABC) = \int_{-1}^{1} y_{AB}\,dx + \int_{1}^{3} y_{BC}\,dx - \int_{-1}^{3} y_{AC}\,dx$$

$$= \int_{-1}^{1}\left(\frac{7+3x}{2}\right)dx + \int_{1}^{3}\left(\frac{11-x}{2}\right)dx - \int_{-1}^{3}\left(\frac{5+x}{2}\right)dx$$

$$= \frac{1}{2}\times\left(7x+\frac{3x^2}{2}\right)\Bigg]_{-1}^{1} + \frac{1}{2}\times\left(11x-\frac{x^2}{2}\right)\Bigg]_{1}^{3} - \frac{1}{2}\times\left(5x+\frac{x^2}{2}\right)\Bigg]_{-1}^{3}$$

$$= 7 + 9 - 12 = 4$$

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