@kanchibalaji7
+91-8099454846
@ikbmaths7

9. Differential Equations

(Previous years Questions 2017 -2025 Solutions)

2022 March :

1.

The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

(A) 0

(B) 1

(C) 2

(D) 3

2.

The integrating factor of the differential equation $(x + 3y^2)\dfrac{dy}{dx} = y$ is

(A) y

(B) $-y$

(C) $\dfrac{1}{y}$

(D) $-\dfrac{1}{y}$

3.

If $y = Ae^{5x} + Be^{-5x}$, then $\dfrac{d^2y}{dx^2}$ is equal to

(a) $25\ y$ (b) $5\ y$ (c) $-25\ y$ (d) $15\ y$

4.

The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is

(A) 1, 1

(B) 1, 2

(C) 2, 1

(D) 2, 2

5.

The order and the degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^3 + 5x = 0$ are

(A) 3; 6

(B) 3; 3

(C) 3; 9

(D) 6; 3

2023 March:

1.

The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(a) 5 (b) 2

(c) 3 (d) 4

2.

The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2 \log x + \frac{x^3}{3} + C$

(c) $y = \frac{x^2}{2} + C$ (d) $y = 2 \log x + \frac{x^2}{2} + C$

3.

Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is

(A) 2 (B) 1

(C) not defined (D) 0

4.

The integrating factor of the differential equation

$$(1-y^2)\frac{dx}{dy} + yx = ay, \ (-1 < y < 1) \text{ is}$$

(A) $\frac{1}{y^2-1}$

(B) $\frac{1}{\sqrt{y^2-1}}$

(C) $\frac{1}{1-y^2}$

(D) $\frac{1}{\sqrt{1-y^2}}$

5.

The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is :

(a) $\frac{1}{x} + \frac{1}{y} = C$

(b) $\log x - \log y = C$

(c) $xy = C$

(d) $x + y = C$

6.

What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3

(b) 2

(c) 6

(d) not defined

7.

The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is :

(a) e^{-y}

(b) e^{-x}

(c) x

(d) $\frac{1}{x}$

8.

The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are :

(a) 2, 2 (b) 1, 3

(c) 2, 3 (d) 2, degree not defined

9.

The number of solutions of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$, is :

(a) zero (b) one

(c) two (d) infinite

10.

The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is

(a) 2 (b) 3

(c) 5 (d) 0

11.

The order and the degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$ respectively are :

(a) $1, \frac{2}{3}$ (b) 3, 1

(c) 3, 3 (d) 1, 2

12.

The integrating factor of the differential equation $(3x^2 + y)\dfrac{dx}{dy} = x$ is

(a) $\dfrac{1}{x}$ (b) $\dfrac{1}{x^2}$ (c) $\dfrac{2}{x}$ (d) $-\dfrac{1}{x}$

13.

The difference of the order and the degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + x^4 = 0$ is :

(a) 1 (b) 2 (c) -1 (d) 0

12.

Solution of the differential equation $(1 + y^2)(1 + \log x)\,dx + x\,dy = 0$ is :

(a) $\tan^{-1} y + \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(b) $\tan^{-1} y - \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(c) $\tan^{-1} y - \log |x| - \dfrac{(\log |x|)^2}{2} = C$

(d) $\tan^{-1} y + \log |x| - \dfrac{(\log |x|)^2}{2} = C$

13.

Integrating factor of the differential equation $x\dfrac{dy}{dx} - 2y = 4x^2$ is :

(a) x^2 (b) $-\dfrac{1}{x^2}$

(c) $\dfrac{1}{x^2}$ (d) $-x^2$

14.

The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is :

(a) $\frac{1}{x} + \frac{1}{y} = C$

(b) $xy = C$

(c) $\log x \log y = C$

(d) $x + y = C$

15.

Assertion (A) : The differential equation representing the family of parabolas $y^2 = 4ax$, where 'a' is a parameter, is $x\frac{dy}{dx} - 2y = 0$.

Reason (R) : If the given family of curves has n parameters, then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.

16.

The sum of the order and the degree of the differential equation

$3x^2\left(\frac{d^2y}{dx^2}\right)^3 + 5\frac{d^3y}{dx^3} + 9 = 0$ is :

(a) 5

(b) 6

(c) 4

(d) 2

17.

A particular solution of the differential equation $dy = y \tan x \, dx$; $y = 1$ when $x = 0$, is :

(a) $y = 0$

(b) $y = 1 + \sec x$

(c) $y = \sec x$

(d) $y \cos x = 0$

18.

The general solution of the differential equation $\frac{dy}{dx} = e^{x-y}$ is :

(a) $e^x + e^y = C$

(b) $e^x - e^{-y} = C$

(c) $-e^x - e^y = C$

(d) $e^x - e^y = C$

19.

The sum of the order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + 3x\left(\frac{d^2y}{dx^2}\right)^4 = \log x$, is :

(a) 5

(b) 6

(c) 7

(d) 4

20.

For what value of n is the following a homogeneous differential equation:

$$\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + xy^2}$$

2024 March:

1.

The integrating factor of the differential equation $(1 - x^2)\dfrac{dy}{dx} + xy = ax$, $-1 < x < 1$, is :

(A) $\dfrac{1}{x^2 - 1}$ (B) $\dfrac{1}{\sqrt{x^2 - 1}}$

(C) $\dfrac{1}{1 - x^2}$ (D) $\dfrac{1}{\sqrt{1 - x^2}}$

2.

The order and degree of the differential equation $\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^3 = \dfrac{d^2y}{dx^2}$ respectively are :

(A) 1, 2 (B) 2, 3

(C) 2, 1 (D) 2, 6

3.

The integrating factor of the differential equation $x\dfrac{dy}{dx} - y = x^4 - 3x$ is :

(A) x (B) $-x$

(C) x^{-1} (D) $\log(x^{-1})$

4.

The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1}{\log y}$ is :

(A) $\log y = x + c$ (B) $y \log y - y = x + c$

(C) $\log y - y = x + c$ (D) $y \log y + y = x + c$

@kanchibalaji7
+91-8099454846
@ikbmaths7

5.

The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is :

(A) $\cos x - \sin\left(\frac{y}{x}\right)$

(B) $\frac{y}{x}$

(C) $\frac{x^2 + y^2}{xy}$

(D) $\cos^2\left(\frac{x}{y}\right)$

6.

The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is :

(A) 1

(B) 2

(C) 3

(D) not defined

7.

The number of solutions of differential equation $\frac{dy}{dx} - y = 1$, given that $y(0) = 1$, is :

(A) 0

(B) 1

(C) 2

(D) infinitely many

8.

The degree and order of differential equation $y''^2 + \log(y') = x^5$ respectively are :

(A) not defined, 5

(B) not defined, 2

(C) 5, not defined

(D) 2, 2

9.

The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is :

(A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$

(C) $\frac{x^2+y^2}{xy}$ (D) $\cos^2\left(\frac{x}{y}\right)$

10.

$x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

11.

The general solution of the differential equation $x\,dy + y\,dx = 0$ is :

(A) $xy = c$ (B) $x + y = c$

(C) $x^2 + y^2 = c^2$ (D) $\log y = \log x + c$

12.

The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y\ (y > 0)$ is :

(A) $\frac{1}{x}$ (B) x

(C) y (D) $\frac{1}{y}$

13.

The number of arbitrary constants in the particular solution of the differential equation

$\log\left(\frac{dy}{dx}\right) = 3x + 4y;\ y(0) = 0$ is/are

(A) 2 (B) 1
(C) 0 (D) 3

14.

The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0,\ x \neq 0$ is :

(A) $\frac{2}{x}$ (B) x^2

(C) $e^{\frac{2}{x}}$ (D) $e^{\log(2x)}$

15.

The order of the following differential equation

$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^5 = 4\log\left(\frac{d^4y}{dx^4}\right)$ is :

(A) not defined (B) 3
(C) 4 (D) 5

16.

The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is :

(A) 4 (B) 3
(C) 2 (D) not defined

17.

The general solution of the differential equation

$\frac{dy}{dx} = e^{x+y}$ is :

(A) $e^x + e^{-y} = c$ (B) $e^{-x} + e^{-y} = c$
(C) $e^{x+y} = c$ (D) $2e^{x+y} = c$

2025 Mark :

1.

Which of the following is <u>not</u> a homogeneous function of x and y ?

(A) $y^2 - xy$ (B) $x - 3y$

(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$

2.

The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is

(A) $e^{\frac{y^2}{2}}$ (B) $\frac{1}{\sqrt{y}}$

(C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y^2}}$

3.

The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(A) xe^x (B) $\frac{e^x}{x}$

(C) $\frac{x}{e^x}$ (D) $xe^{\frac{1}{x}}$

4.

The order and degree of differential function $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$ are

(A) order 1, degree 1 (B) order 1, degree 2

(C) order 2, degree 1 (D) order 2, degree 2

5.

If p and q are respectively the order and degree of the differential equation

$\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is

(A) 0 (B) 1

(C) 2 (D) 3

6.

The order and degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$ are :

(A) order 2, degree 2 (B) order 2, degree 1

(C) order 2, degree not defined (D) order 1, degree not defined

7.

The integrating factor of the differential equation

$\frac{dy}{dx} + y \tan x - \sec x = 0$ is :

(A) $-\cos x$ (B) $\sec x$

(C) $\log \sec x$ (D) $e^{\sec x}$

8.

The solution of the differential equation $\frac{dy}{dx} = \frac{-x}{y}$ represents family of :

(A) parabolas (B) circles

(C) ellipses (D) hyperbolas

9.

The integrating factor of the differential equation

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1 \text{ is :}$$

(A) $e^{-1/\sqrt{x}}$ (B) $e^{2/\sqrt{x}}$

(C) $e^{2\sqrt{x}}$ (D) $e^{-2\sqrt{x}}$

10.

The sum of the order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2} \text{ is :}$$

(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4

11.

The integrating factor of the differential equation

$$\frac{dx}{dy} = \frac{x\log x}{\frac{2}{x}\log x - y} \text{ is :}$$

(A) $\frac{1}{8x}$ (B) e

(C) $e^{\log x}$ (D) $\log x$

12.

The integrating factor of the differential equation

$$\frac{dx}{dy} = \frac{-(1+\sin x)}{x+y\cos x} \text{ is :}$$

(A) $\log \cos x$ (B) $1 + \sin x$

(C) $e^{(1+\sin x)}$ (D) $e^{\log \cos x}$

13.

The order and degree of the following differential equation are, respectively :

$$-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$$

(A) $-4, 1$ (B) 4, not defined

(C) 1, 1 (D) 4, 1

14.

The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

15.

The order and degree of the differential equation

$\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right) = x\log\left(\frac{d^2y}{dx^2}\right)$ are respectively :

(A) 0, 3 (B) 2, 1

(C) 2, not defined (D) 1, not defined

I. Degree and order Based Problems:

1.

Find the sum of the order and the degree of the differential equation :

$$\left(x+\frac{dy}{dx}\right)^2=\left(\frac{dy}{dx}\right)^2+1$$

Sol.

Given differential equation can be written as

$$x^2+\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}=\left(\frac{dy}{dx}\right)^2+1$$

i.e. $x^2+2x\dfrac{dy}{dx}=1$; Order = 1, degree = 1

Sum of order and degree = 1 + 1 = 2

2.

Find the value of (2a – 3b), if a and b represent respectively the order and the degree of the differential equation

$$x\left[y\left(\frac{d^2y}{dx^2}\right)^3+x\left(\frac{dy}{dx}\right)^2-\frac{y}{x}\frac{dy}{dx}\right]=0.$$

Ans.

order = 2, degree = 3

$\therefore$ 2a – 3b = 4 – 9 = –5

a. Logical/indirect equations degreee :

1.

Find the order and the degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$.

Sol. $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$

Order of the highest derivative = 2

and power of $\frac{d^2y}{dx^2}$ is 1

So, order of differential equation = 2

Degree of differential equation = 1

2.

Find the product of the order and the degree of the differential equation $\left[\frac{d}{dx}(xy^2)\right] \cdot \frac{dy}{dx} + y = 0$.

Ans.

Given differential equation can be written as

$$2xy\left(\frac{dy}{dx}\right)^2 + y^2\frac{dy}{dx} + y = 0 \qquad \text{Order} = 1, \text{Degree} = 2$$

Order × degree = 1 × 2 = 2

II. Simple general/particular solution :

1.

Find the general solution of the differential equation $\frac{dy}{dx} = a$, where a is an arbitrary constant.

Ans. $\frac{dy}{dx} = a, \quad \int dy = \int a.dx$

$$\Rightarrow y = ax + c$$

2.

Find the general solution of the differential equation $e^{y-x}\frac{dy}{dx} = 1$.

Ans: Given differential equation is $e^{y}dy = e^{x}\,dx$

Integrating to get $e^{y} = e^{x} + C$

3.

Find the particular solution of the differential equation $\frac{dy}{dx} = y \tan x$, when $y(0) = 1$.

Ans: $\frac{dy}{dx} = y \cdot \tan x \Rightarrow \int \frac{1}{y}dy = \int \tan x dx \Rightarrow y = c \cdot \sec x$

$y(0) = 1 \Rightarrow c = 1 \;\therefore$ particular solution is $y = \sec x$

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4.

Solve the differential equation $\cos\left(\frac{dy}{dx}\right) = a, \ (a \in \mathbb{R})$.

Sol.

$$\frac{dy}{dx} = \cos^{-1} a \;\Rightarrow\; \int dy = \cos^{-1} a \cdot \int dx$$

$$y = x\cos^{-1} a + c$$

5.

Solve the differential equation :

$\log\left(\frac{dy}{dx}\right) = x - y.$

Sol.

Given differential equation is

$$\frac{dy}{dx} = e^{x-y}$$

i.e. $e^y dy = e^x dx$

integrating both sides, we get

$$e^y = e^x + C$$

6.

Find the particular solution of the differential equation $\frac{dy}{dx} = 2y^2$, given $y = 1$ when $x = 1$.

Sol.

Given differential equation can be written as

$$\frac{dy}{y^2} = 2dx$$

$$\int \frac{dy}{y^2} = 2\int dx$$

$$\frac{-1}{y} = 2x + C$$

putting $x = 1,\ y = 1$ gives $C = -3$

$$\therefore -\frac{1}{y} = 2x - 3$$

Or $y = \frac{-1}{2x-3}$

7.

Find the general solution of the differential equation : $\log\left(\frac{dy}{dx}\right) = ax + by$.

Sol.

$$\log\left(\frac{dy}{dx}\right) = ax + by$$

$$\frac{dy}{dx} = e^{ax+by}$$

$$\frac{dy}{dx} = e^{ax} \cdot e^{by}$$

$$\int \frac{dy}{e^{by}} = \int e^{ax}\,dx$$

$$\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$$

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8.

Solve the following homogeneous differential equation :

$$x\frac{dy}{dx} = x + y$$

Ans: Let $y = vx \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore x\left(v + x\frac{dv}{dx}\right) = x + vx \Rightarrow x\frac{dv}{dx} = 1$$

$$\therefore \int dv = \int \frac{1}{x}dx \Rightarrow v = \log|x| + c$$

$$\Rightarrow y = x(\log|x| + c)$$

9.

Find the general solution of the differential equation : $e^{dy/dx} = x^2$.

Sol.

$$e^{dy/dx} = x^2$$

$$\frac{dy}{dx} = \log x^2 = 2\log x$$

$$\int dy = 2\int \log x \, dx$$

$$y = 2[\log x \times x - \int 1 dx]$$

$$y = 2[x \log x - x] + C$$

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@kanchibalaji7

10.

Show that the function $y = ax + 2a^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$.

Answer:

$$y = ax + 2a^2 \Rightarrow \frac{dy}{dx} = a$$

$$\text{LHS} = 2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y$$

$$= 2(a)^2 + x(a) - (ax + 2a^2) = 0 = \text{RHS}$$

11.

Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

Ans.

Given differential equation can be written as:

$$\frac{dy}{dx} = e^x \cdot e^y \Rightarrow e^{-y} dy = e^x dx$$

Integrating both sides, we get

$$-e^{-y} = e^x + c$$

III. Integral factor (IF) based problems:

1.

Find the integrating factor of $x\dfrac{dy}{dx} + (1 + x\cot x)\,y = x$.

Sol.

The differential equation can be written as: $\dfrac{dy}{dx} + \left(\dfrac{1}{x + \cot x}\right)y = 1$

Integrating factor $= e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{(\log x + \log \sin x)} = e^{\log(x\,\sin x)} = x.\sin x$

2.

Find the integrating factor of the differential equation $y\dfrac{dx}{dy} - 2x = y^3 e^{-y}$.

Sol.

$$\frac{dx}{dy} - \frac{2}{y}\cdot x = y^2 e^{-y}$$

$$\text{I.F.} = e^{\int -\frac{2}{y}dy} = e^{-2\log y} = \frac{1}{y^2}$$

3.

Find the integrating factor of the differential equation $x\dfrac{dy}{dx} - 2y = 2x^2$.

Sol.

$$\frac{dy}{dx} - \frac{2}{x}\cdot y = 2x \Rightarrow \text{I.F.} = e^{-2\log x} = \frac{1}{x^2}$$

I. Variable separable method : [x&y terms under multiplication]

a. Direct model :

1.

Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

Sol.

$\frac{dy}{dx} = y\cot 2x \Rightarrow \int \frac{dy}{y} = \int \cot 2x dx$

$\Rightarrow \log|y| = \frac{1}{2}\log|\sin 2x| + \log c$

Thus, $y = c.\sqrt{\sin 2x}$

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when $y(\frac{\pi}{4}) = 2$, gives $c = 2$

$\therefore y = 2\sqrt{\sin 2x}$ is the required Particular solution of given D.E.

2. 2023

Solve the following differential equation:

$e^x \tan y \, dx + \left(1 - e^x\right)\sec^2 y \, dy = 0$

Sol.

The given D.E. is

$\frac{\sec^2 y}{\tan y} dy = -\frac{e^x}{1 - e^x} dx$

Integrating

$\Rightarrow \log|\tan y| = \log|1 - e^x| + \log C$

$\Rightarrow \tan y = C\,(1 - e^x)$

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2.a

Find the particular solution of the differential equation $e^x \tan y\, dx + (2 - e^x) \sec^2 y\, dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

Sol.

Separating the variables, we get:

$$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{e^x}{e^x - 2} dx$$

$\Rightarrow \quad \log |\tan y| = \log |e^x - 2| + \log C$

$\Rightarrow \quad \tan y = C(e^x - 2)$, for $x = 0$, $y = \pi/4$, $C = -1$

$\therefore \quad$ Particular solution is: $\tan y = 2 - e^x$.

2.b 2022

Find the general solution of the differential equation

$$\sec^2 x \,.\, \tan y\, dx + \sec^2 y \,.\, \tan x\, dy = 0.$$

Sol.

Given differential equation can be written as $\frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$

Integrating, $\log|\log x| + \log|\tan y| = \log C$

$\tan x \cdot \tan y = C$

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2.c

Find the particular solution of the differential equation

$$\cos y\, dx + (1 + e^{-x}) \sin y\, dy = 0$$

given that $y = \frac{\pi}{4}$ when $x = 0$.

Ans: $\frac{dx}{1+e^{-x}} = -\frac{\sin y}{\cos y} dy$ (Separating variables)

$$\Rightarrow \int \frac{e^x}{1+e^x} dx = -\int \tan y\, dy$$

$$\Rightarrow \log\left|e^x + 1\right| = \log\left|\cos y\right| + c$$

whern $x = 0, \ y = \frac{\pi}{4}$

$$\log 2 = \log \frac{1}{\sqrt{2}} + c$$

$$\Rightarrow c = \frac{3}{2} \log 2$$

$$\therefore \log\left|e^x + 1\right| = \log\left|\cos y\right| + \frac{3}{2} \log 2$$

or $y = \cos^{-1}\left(\frac{e^x + 1}{2\sqrt{2}}\right)$


@kanchibalaji7
+91-8099454846
@ikbmaths7

3

Find the particular solution of the differential equation

$x\,dx - y\,e^y\sqrt{1+x^2}\,dy = 0$, given that $y = 1$ when $x = 0$.

Sol.

Given equation can be written as

$$x\,dx = ye^y\sqrt{1+x^2}\,dy$$

$$\Rightarrow \int \frac{x}{\sqrt{1+x^2}}dx = \int y \cdot e^y dy$$

$$\Rightarrow \sqrt{1+x^2} = e^y\,(y-1) + C$$

when $y = 1,\ x = 0 \Rightarrow C = 1$

$\therefore$ Required solution is $\sqrt{1+x^2} = e^y(y-1)+1$

3.a

Solve the differential equation $(e^x + 1)\,y\,dy = e^x\,(y + 1)\,dx$.

Ans: $\int \frac{e^x}{e^x+1}dx = \int \frac{y}{y+1}dy$

$$\Rightarrow \log\,(e^x + 1) = y - \log\,(y+1) + c$$

3.b

Find the particular solution of the differential equation :
$(1 + e^{2x})\, dy + (1 + y^2)\, e^x\, dx = 0$, given that $y(0) = 1$.

Sol.

$$(1+e^{2x})dy+(1+y^2)e^x\,dx=0$$

$$\Rightarrow \int\frac{dy}{1+y^2}=-\int\frac{e^x}{1+e^{2x}}dx$$

$$\Rightarrow \tan^{-1} y=-\int\frac{e^x}{1+e^{2x}}dx$$

Put $e^x = t$, so that $e^x\, dx = dt$

$$\tan^{-1} y=-\int\frac{dt}{1+t^2}\Rightarrow \tan^{-1} y=-\tan^{-1}(e^x)+C$$

Substituting $y = 1$, when $x = 0$ in equation (i)

$$\tan^{-1}(1)=-\tan^{-1}(1)+C\Rightarrow C=\frac{\pi}{2}$$

Substituting $C=\frac{\pi}{2}$ in equation (i) $\Rightarrow \tan^{-1} y+\tan^{-1}(e^x)=\frac{\pi}{2}$

3.c

Solve the differential equation : $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$; $y(0) = 0$.

Given equation can be written as

$$\int \frac{dy}{2e^{-y}-1} = \int \frac{dx}{x+1}$$

$$\Rightarrow \int \frac{e^y}{2-e^y} dy = \int \frac{dx}{x+1}$$

$$\Rightarrow -\log|2-e^y| + \log c = \log|x+1|$$

$$\Rightarrow (2 - e^y)(x + 1) = c$$

When $x = 0$, $y = 0 \Rightarrow c = 1$

$\therefore$ Solution is $(2 - e^y)(x + 1) = 1$

4.

Solve the differential equation : $(x+1)\frac{dy}{dx} = 2e^{-y} + 1$; $y(0) = 0$.

Answer:

Given differential equation can be written as

$$\frac{dy}{2e^{-y}+1} = \frac{dx}{x+1}$$

$$\Rightarrow \int \frac{e^y}{2+e^y} dy = \int \frac{dx}{x+1}$$

$$\Rightarrow \log\left|2+e^y\right| = \log|x+1| + \log C$$

$$\Rightarrow 2 + e^y = C(x+1)$$

when $x = 0, y = 0 \Rightarrow C = 3$

$\therefore$ Required solution is $2 + e^y = 3(x+1)$ or $e^y = 3x+1$

5.

Find the particular solution of the differential equation :

$$x \cos y \, dy = (x e^x \log x + e^x) \, dx \text{ given that } y = \frac{\pi}{2} \text{ when } x = 1.$$

Sol.

$$x \cos y \, dy = (x \log x + 1)e^x \, dx$$

$$\Rightarrow \int \cos y \, dy = \int \left(\log x + \frac{1}{x} \right) \cdot e^x \, dx$$

$$\Rightarrow \sin y = \log x . e^x + C \qquad \left(\because \int [f(x) + f'(x)] \cdot e^x dx = f(x) \cdot e^x + C \right)$$

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OR

Separating the variables and integrating:

$$\int \cos y \, dy = \int e^x \left(\log x + \frac{1}{x} \right) dx$$

$$\Rightarrow \sin y = e^x \log x + c$$

Using $y = \frac{\pi}{2}, x = 1$**, we get** $c = 1$

$\therefore$ **the particular solution is,** $\sin y = 1 + e^x \log x$

5.b

Find the general solution of the following differential equation :

$$(4 + y^2)\,(3 + \log x)\,dx + x\,dy = 0$$

Given diff. equation can be written as

$$\frac{(3+\log x)}{x}dx + \frac{1}{4+y^2}dy = 0$$

Integrating to get

$$\frac{1}{2}(3+\log x)^2 + \frac{1}{2}\tan^{-1}\frac{y}{2} = C_1$$

Or, $(3+\log x)^2 + \tan^{-1}\frac{y}{2} = C$

5.c 2024

Find the general solution of the differential equation $\frac{dy}{dx} = xy \log x \log y$.

Separating the variables and integrating, $\int \frac{dy}{y \log y} = \int x \log x \, dx$

$$\Rightarrow \log|\log y| = \log x\left(\frac{x^2}{2}\right) - \int \frac{1}{x}\cdot\frac{x^2}{2}dx + c$$

$$\Rightarrow \log|\log y| = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$

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6. 2025

Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.

Sol.

Given differential equation can be written as

$\frac{y}{y+3} dy = \frac{2}{x} dx$

$\Rightarrow \int \left(1 - \frac{3}{y+3}\right) dy = 2 \int \frac{1}{x} dx$

$\Rightarrow y - 3log|y + 3| = 2log|x| + C$

y = -2, when x = 1 $\Rightarrow C = -2$

Hence, the required particular solution is

$\Rightarrow y - 3log|y + 3| = 2log|x| - 2$ prepared by : BALAJI KANCHI

6.a

For the differential equation $xy \frac{dy}{dx} = (x + 2)(y + 2)$, find the solution curve passing through the point $(1, -1)$.

Sol.

Given differential equation can be written as

$$\frac{y}{y+2} dy = \frac{x+2}{x} dx$$

$$\int \left(1 - \frac{2}{y+2}\right) dy = \int \left(1 + \frac{2}{x}\right) dx$$

$y - 2 \log |y + 2| = x + 2 \log |x| + C$

It passes through $(1, -1)$

$-1 - 0 = 1 + C \Rightarrow C = -2$

$\therefore$ required solution curve is $y - 2 \log |y + 2| = x + 2 \log |x| - 2$

@kanchibalaji7
+91-8099454846
@ikbmaths7

b. Indirect Model:

[After simplification using Lcm,factorization,

Formulas : if log(x)=y then x = e^y , $e^{x+y}=e^x e^y$, sin(x+y), sin(x-y)]

1.

Find the general solution of the differential equation $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x}$.

Answer:

Given differential equation can be written as

$$\frac{dy}{dx}=\frac{1}{x}\left(e^y-1\right)\Rightarrow\frac{dy}{e^y-1}=\frac{dx}{x}$$

$$\Rightarrow\int\frac{dy}{e^y-1}=\int\frac{dx}{x}$$

$$\Rightarrow\int\frac{e^{-y}}{1-e^{-y}}dy=\int\frac{dx}{x}$$

$$\Rightarrow\log\left|1-e^{-y}\right|=\log\left|x\right|+\log C$$

$$\Rightarrow 1-e^{-y}=Cx$$

2. 2022

Find the general solution of the following differential equation :

$$\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$$

Sol.

$$\int\frac{dy}{e^{-y}}=\int(x^2+e^x)dx$$

$$e^y=\frac{x^3}{3}+e^x+C$$

3.

Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

$$\int \frac{dy}{1+y^2} = \int (x^2 + 1)\,dx$$

$$\tan^{-1} y = \frac{x^3}{3} + x + C$$

$$x = 0,\ y = 1 \Rightarrow C = \frac{\pi}{4}$$

So particular solution is $\tan^{-1} y = \frac{x^3}{3} + x + \frac{\pi}{4}$

4.

Find the general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$.

Ans: Separating the variables and integrating as :

$$\int e^{-by}\,dy = \int e^{ax}\,dx$$

$$\Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c \quad \left(\text{or } be^{ax} + ae^{-by} + k = 0\right)$$

prepared by : **BALAJI KANCHI**

5.

Find the particular solution of the differential equation : $\frac{dy}{dx} = \sin(x + y) + \sin(x - y)$, given that when $x = \frac{\pi}{4}$, $y = 0$.

Sol.

Given differential equation becomes

$$\frac{dy}{dx} = 2 \sin x \cos y$$

$\Rightarrow \sec y \, dy = 2 \sin x \, dx$

Integrating we get

$\log |\sec y + \tan y| = -2 \cos x + C$

$x = \frac{\pi}{4}$, $y = 0$ gives $C = \sqrt{2}$

$\therefore$ particular solution is $\log |\sec y + \tan y| = -2 \cos x + \sqrt{2}$

prepared by : BALAJI KANCHI

II. . Homogeniuos equation in terms of only x&y (degree of the terms same): then $y = vx$

1. 2022

Solve the differential equation : $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$.

Sol.

Let $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Gives $x\frac{dv}{dx} = \frac{1}{v}$

$$v\,dv = \frac{dx}{x}$$

Integrating we get

$$\frac{v^2}{2} = \log|x| + C$$

$$\frac{y^2}{2x^2} = \log|x| + C$$

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1.a 2024

Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$.

Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2xvx}$$

$$\int \frac{1}{x}\, dx = \int \frac{2v}{1-v^2}\, dv$$

$$\Rightarrow \log|x| = -\log|1 - v^2| + \log C$$

$$\log|x(1 - v^2)| = \log C$$

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$$\Rightarrow x\left(1 - \frac{y^2}{x^2}\right) = C \text{ or } x^2 - y^2 = Cx$$

1.b

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.

Sol.

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2} \quad \ldots(1)$$

$$\text{Put } \frac{y}{x} = v \text{ i.e. } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\text{Equation}(1) \text{ gives } v + x\frac{dv}{dx} = \frac{v}{1+v^2}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{v^3}{1+v^2}$$

$$\Rightarrow \int \frac{1+v^2}{v^3} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{-1}{2v^2} + \log|v| = -\log|x| + \log c$$

$$\text{putting } v = \frac{y}{x} \text{ and simplifying gives}$$

$$-\frac{x^2}{2y^2} = \log\left|\frac{c}{y}\right|$$

$$\text{now, } x = 0, y = 1 \text{ gives } c = 1$$

$$\text{required solution is : } \frac{x^2}{2y^2} = \log|y|$$

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1. c

Solve the following differential equation:
$(x^2 - y^2)\,dx + 2xy\,dy = 0$
given that y = 1 when x = 1

Sol.

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{\frac{y^2}{x^2} - 1}{\frac{2y}{x}}$$

Put $\frac{y}{x} = v \Rightarrow y = vx$ and so $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} \Rightarrow \frac{xdv}{dx} = -\frac{(1+v^2)}{2v}$$

$$\int \frac{dx}{x} = -\int \frac{2vdv}{1+v^2} \Rightarrow \log x = -\log(1+v^2) + \log C$$

$$\Rightarrow x(1 + v^2) = C \text{ so } x\left(1 + \frac{y^2}{x^2}\right) = C \text{ or } x^2 + y^2 = Cx$$

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1.d 2025

Solve the differential equation $(x^2 + y^2)\,dx + xy\,dy = 0$, $y(1) = 1$.

Sol.

$$\frac{dy}{dx} = -\frac{x^2+y^2}{xy} = -\frac{1+\left(\frac{y}{x}\right)^2}{\frac{y}{x}}, \text{ Put } y = vx, \ \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{1+v^2}{v} \Rightarrow x\frac{dv}{dx} = -\frac{1+2v^2}{v}$$

$$\Rightarrow \frac{1}{4}\int \frac{4v}{1+2v^2}\,dv = -\int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{4}\log\left(1+2v^2\right) = -\log|x| + \log C$$

$$\Rightarrow \log\left(1+2v^2\right) = \log\frac{C^4}{x^4} \Rightarrow 1 + 2\left(\frac{y}{x}\right)^2 = \frac{D}{x^4}, \ D = C^4$$

$$\text{For } x = 1, y = 1, D = 3,$$

$$\therefore \text{The solution of the differential equation is, } \left(2y^2 + x^2\right)x^2 = 3$$

@kanchibalaji7
+91-8099454846
@ikbmaths7

1.f

Show that the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is homogeneous and solve it.

Sol.

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} = \frac{1 + 3\left(\frac{y}{x}\right)^2}{2\,\frac{y}{x}}$$

$$\text{Put } \frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1+3v^2}{2v} \Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\Rightarrow \int \frac{2v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \log(1+v^2) = \log x + \log c$$

$$\Rightarrow 1 + \left(\frac{y}{x}\right)^2 = cx, x = 1, y = 0 \Rightarrow c = 1$$

$$\Rightarrow x^2 + y^2 = x^3 \text{ is the required solution}$$

1.g

If the solution of the differential equation

$\frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$ is $\frac{ax}{y} = b \log |x| + C$, find the value of a and b.

Sol.

Ans. The given differential equation can be written as.

$$\frac{dy}{dx} = \frac{y}{x} - \frac{1}{2}\left(\frac{y}{x}\right)^2$$

Put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = v - \frac{v^2}{2}$$

$$\Rightarrow \quad \frac{1}{v^2}dv = \left(-\frac{1}{2}\right)\frac{1}{x}dx \text{, integrating both sides}$$

$$\Rightarrow \quad -\frac{1}{v} = -\frac{1}{2}\log|x| + c$$

$$\Rightarrow \quad -\frac{x}{y} = -\frac{1}{2}\log|x| + c, \quad a = -1, \ b = -\frac{1}{2}$$

$$\text{or} \quad \frac{x}{y} = \frac{1}{2}\log|x| + c, a = 1, b = \frac{1}{2}$$

1.h 2024

Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2} = \frac{y}{x} + \frac{y^2}{2x^2}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

The equation becomes

$$x\frac{dv}{dx} = \frac{1}{2}v^2$$

$$\Rightarrow \frac{dv}{v^2} = \frac{1}{2} \times \frac{dx}{x}$$

Integrating both sides, we get

$$\frac{-1}{v} = \frac{1}{2}log|x| + C$$

$$\Rightarrow -\frac{x}{y} = \frac{1}{2}log|x| + C$$

$y = 2, x = 1$ gives $C = -\frac{1}{2}$

The particular solution is

$$-\frac{x}{y} = \frac{1}{2}log|x| - \frac{1}{2} \; or, y = \frac{2x}{1 - log|x|}$$

1.i 2024

Solve the following differential equation :

$x^2\,dy + y(x + y)\,dx = 0$

Sol.

$$x^2dy + y\left(x+y\right)dx = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} - \left(\frac{y}{x}\right)^2$$

Putting $\frac{y}{x} = v \Rightarrow y = vx,\ \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = -v - v^2,$$

separating the variable and integrating

$$\int \frac{1}{v^2+2v}dv = -\int \frac{1}{x}dx$$

$$\Rightarrow \int \frac{1}{\left(v+1\right)^2 - 1}dv = -\int \frac{1}{x}dx$$

$$\Rightarrow \frac{1}{2}\log\left|\frac{v}{v+2}\right| = \log\left|\frac{C}{x}\right|$$

The solution of the differential equation is,

$$\left|\frac{y}{y+2x}\right| = \frac{C^2}{x^2} \text{ or } x^2y = k\left(y+2x\right)$$

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1.k 2023

Find the general solution of the differential equation :

$(xy - x^2)\, dy = y^2\, dx.$

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2} = \frac{\left(\frac{y}{x}\right)^2}{\frac{y}{x} - 1}$$

$$\text{Put } y = ux, \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\therefore u + x\frac{du}{dx} = \frac{u^2}{u-1} \Rightarrow x\frac{du}{dx} = \frac{u}{u-1}$$

Separating the variables and integrating

$$\int\left(1 - \frac{1}{u}\right)du = \int\frac{dx}{x} \Rightarrow u - \log u = \log|x| + c \Rightarrow \frac{y}{x} - \log\frac{y}{x} = \log|x| + c$$

prepared by : BALAJI KANCHI

@kanchibalaji7
+91-8099454846
@ikbmaths7

1.i 2025

Find the general solution of the differential equation

$x^2 \frac{dy}{dx} = x^2 + xy + y^2.$

Sol.

$\frac{dy}{dx} = \frac{y^2}{x^2} + \frac{y}{x} + 1$

Put $\frac{y}{x} = v$ i.e. $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Equation becomes

$v + x\frac{dv}{dx} = v^2 + v + 1$

$\frac{1}{v^2+1} dv = \frac{dx}{x}$

Integrating, we get

$\tan^{-1} v = \log|x| + C \quad \Rightarrow \tan^{-1}\frac{y}{x} = \log|x| + C$

2.a 2022,2025

Find the general solution of the differential equation

$$x^2y\,dx - (x^3 + y^3)\,dy = 0.$$

Sol.

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

Put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^3} \Rightarrow x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

$$\int\frac{1+v^3}{v^4}dv = -\int\frac{dx}{x}$$

$$\int\frac{1}{v^4}dv + \int\frac{1}{v}dv = -\log|x| + C$$

$$\frac{1}{-3v^3} + \log|v| = -\log|x| + C$$

prepared by : BALAJI KANCHI

$$\frac{-x^3}{3y^3} + \log\left|\frac{y}{x}\right| = -\log|x| + C \text{ or } \frac{-x^3}{3y^3} + \log|y| = C$$

2.b

Find the general solution of the differential equation $x^2y\,dx-\left(x^3+y^3\right)dy=0$

Ans: $\dfrac{dx}{dy}=\dfrac{x^3+y^3}{x^2y}$

Put $x=vy \Rightarrow \dfrac{dx}{dy}=v+y\cdot\dfrac{dv}{dy}$

$\therefore\quad v+y\dfrac{dv}{dy}=\dfrac{y^3(v^3+1)}{y^3v^2}$

$\Rightarrow\quad y\dfrac{dv}{dy}=\dfrac{1}{v^2}$

$\Rightarrow\quad v^2dv=\dfrac{dy}{y}$

Integrating both sides, we get

$\dfrac{v^3}{3}=\log y+c\Rightarrow\dfrac{x^3}{3y^3}=\log y+c$

$\Rightarrow x^3=3y^3\log y+3cy^3$

2.c

Find the general solution of the differential equation
$x\,(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$.

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{2y^4 + 5x^3y}{xy^3 + x^4} \qquad \ldots(1)$$

Let, $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

Equation (1) becomes $v + x\dfrac{dv}{dx} = \dfrac{2v^4 + 5v}{v^3 + 1}$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^4 + 4v}{v^3 + 1}$$

$$\frac{v^3 + 1}{v^4 + 4v}\,dv = \frac{dx}{x}$$

$$\int \frac{4v^3 + 4}{v^4 + 4v}\,dv = 4\int \frac{dx}{x}$$

$$\log|v^4 + 4v| = \log(x)^4 + \log C$$

$$\log\left|\frac{y^4 + 4yx^3}{x^4}\right| = \log Cx^4$$

$$y^4 + 4yx^3 = Cx^8$$

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2.d

Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ is the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)\,dy$, where C is a parameter.

Sol.

$$x^2 - y^2 = C(x^2 + y^2)^2 \Rightarrow 2x - 2yy' = 2C(x^2 + y^2)(2x + 2yy')$$

$$\Rightarrow \quad (x - yy') = \frac{x^2 - y^2}{y^2 + x^2}(2x + 2yy') \Rightarrow (y^2 + x^2)(x - yy') = (x^2 - y^2)(2x + 2yy')$$

$$\Rightarrow \quad [-2y(x^2 - y^2) - y(y^2 + x^2)]\frac{dy}{dx} = 2x(x^2 - y^2) - x(y^2 + x^2)$$

$$\Rightarrow \quad (y^3 - 3x^2y)\frac{dy}{dx} = (x^3 - 3xy^2)$$

$$\Rightarrow \quad (y^3 - 3x^2y)dy = (x^3 - 3xy^2)dx$$

Hence $x^2 - y^2 = C(x^2 + y^2)^2$ is the solution of given differential equation.

3.a

Find the particular solution of the differential equation $(x-y)\dfrac{dy}{dx} = (x+2y)$, given that $y = 0$ when $x = 1$.

Sol.

$$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+\frac{2y}{x}}{1-\frac{y}{x}}$$

$$\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \qquad \therefore\ v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$\Rightarrow\ x\frac{dv}{dx} = -\frac{1+2v-v+v^2}{v-1} \Rightarrow \int \frac{v-1}{v^2+v+1}dv = -\frac{dx}{x}$$

$$\Rightarrow\ \int \frac{2v+1-3}{v^2+v+1}dv = \int -\frac{2}{x}dx$$

$$\Rightarrow \int \frac{2v+1}{v^2+v+1}dv - 3\int \frac{1}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv = -\int \frac{2}{x}dx$$

$$\Rightarrow\ \log|v^2+v+1| - 3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log|x|^2 + c$$

$$\Rightarrow\ \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = c$$

$$x = 1,\ y = 0 \Rightarrow c = -2\sqrt{3}\cdot\frac{\pi}{6} = -\frac{\sqrt{3}}{3}\pi$$

$$\log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) + \frac{\sqrt{3}}{3}\pi = 0$$

3.b

Solve the differential equation :

$$\frac{dy}{dx} = \frac{x + y}{x - y}$$

Sol.

$$\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow \quad x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$$

$$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$

$$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$$

$$\text{or } \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$$

3.c 2023

Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x + y}{x}, \quad y(1) = 0.$$

Sol.

$$\frac{dy}{dx} = \frac{x+y}{x} \Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x}$$

Let $\frac{y}{x} = v$. Then $x\frac{dv}{dx} + v = \frac{dy}{dx}$

So, Differential equation becomes.

$$x\frac{dv}{dx} + v = 1 + v$$

$$\Rightarrow dv = \frac{dx}{x}$$

$$\Rightarrow v = \log|x| + c$$

$$\Rightarrow y = x\log|x| + cx$$

$$\Rightarrow x = 1, y = 0 \Rightarrow c = 0, y = x \log |x|$$

[can also be solved, taking first order linear diff. eqn]

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4.

Solve the differential equation given by
$x\,dy - y\,dx - \sqrt{x^2 + y^2}\,dx = 0.$

Ans.

Writing $\dfrac{dy}{dx} = \dfrac{y + \sqrt{x^2 + y^2}}{x} = \dfrac{y}{x} + \sqrt{1 + \left(\dfrac{y}{x}\right)^2}$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

Differential equation becomes $v + x\dfrac{dv}{dx} = v + \sqrt{1 + v^2}$

$$\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log c$$

$$\Rightarrow v + \sqrt{1 + v^2} = cx \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$$

when $x = 1,\ y = 0 \Rightarrow c = 1$

$\therefore\ y + \sqrt{x^2 + y^2} = x^2$ prepared by : BALAJI KANCHI

III. Diffrentail equation contains

$\sin\frac{y}{x}$, $\cos\frac{y}{x}$, $\tan\frac{y}{x}$, $e^{\frac{y}{x}}$, $\log\frac{y}{x}$ terms : $\frac{y}{x} = v$, $y = vx$:

a. Trigometric :

1.a

Find the particular solution of the differential equation $x\frac{dy}{dx} + x\cos^2\left(\frac{y}{x}\right) = y$; given that when $x = 1$, $y = \frac{\pi}{4}$.

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x} \qquad \ldots \quad (1)$$

Let $y = vx$ $\therefore$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

(1) Becomes $\cancel{v} + x\frac{dv}{dx} = \cancel{v} - \cos^2 v$

$$\sec^2 v\, dv = -\frac{dx}{x}$$

Integrating both sides we get

$$\tan v = -\log|x| + c$$

$$\tan\frac{y}{x} = -\log|x| + c$$

$$x = 1,\ y = \frac{\pi}{4} \Rightarrow c = 1$$

$\therefore$ Particular solution is $\tan\frac{y}{x} = -\log|x| + 1$ prepared by : BALAJI KANCHI

1.b 2024

Find the particular solution of the differential equation given by $x^2 \frac{dy}{dx} - xy = x^2 \cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1, y = \frac{\pi}{2}$.

Sol.

$\frac{dy}{dx} = \frac{y}{x} + \cos^2(\frac{y}{2x})$

Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v + \cos^2(\frac{v}{2})$

$\Rightarrow \int \sec^2\left(\frac{v}{2}\right) dv = \int \frac{1}{x} dx$

$\Rightarrow 2\tan\left(\frac{v}{2}\right) = \log|x| + C$

$\Rightarrow 2\tan\left(\frac{y}{2x}\right) = \log|x| + C$

$2\tan\frac{\pi}{4} = \log 1 + C \Rightarrow C = 2 \Rightarrow 2\tan\left(\frac{y}{2x}\right) = \log|x| + 2$

1.c 2025

$$\left(x\sin^2\left(\frac{y}{x}\right)-y\right)dx+x\,dy=0 \text{ given } y=\frac{\pi}{4} \text{ when } x=1.$$

Sol.

$$\left[x.\sin^2\frac{y}{x}-y\right].dx+x.dy=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\sin^2\frac{y}{x}$$

$$\text{Put } y=vx \implies \frac{dy}{dx}=x\frac{dv}{dx}+v$$

$$\therefore\ x\frac{dv}{dx}+v=v-\sin^2 v$$

$$\implies -\int \operatorname{cosec}^2 v\,dv=\int\frac{dx}{x}$$

$$\implies \cot v=\log|x|+c$$

$$\implies \cot\frac{y}{x}=\log|x|+c$$

$$x=1, y=\frac{\pi}{4} \implies c=1$$

$$\implies \cot\frac{y}{x}=\log|x|+1$$

2.a 2022

Find the particular solution of the differential equation :

$$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0, \text{ given that } y(1) = \frac{\pi}{2}.$$

Ans.

$$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin\left(\frac{y}{x}\right)} \qquad \ldots(i)$$

Put $\frac{y}{x} = v$ i.e., $y = vx$ in (i) so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v - \frac{1}{\sin v}$$

$$\int \sin v \, dv = \int -\frac{1}{x} dx$$

$$\Rightarrow -\cos v = -\log|x| + C \Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + C \qquad \ldots(ii)$$

Substituting $y = \frac{\pi}{2}$ when $x = 1$ in (ii)

$$\cos\left(\frac{\pi}{2}\right) = \log 1 + C \Rightarrow C = 0$$

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Required solution is $\cos\left(\frac{y}{x}\right) = \log|x|$

2.b

Solve the following differential equation :

$x\frac{dy}{dx} - y + x\,sin\frac{y}{x} = 0$

Sol.

$x\frac{dy}{dx} - y + x\,sin\frac{y}{x} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - sin\frac{y}{x} \qquad (\frac{y}{x} = v)$

$\Rightarrow v + x\frac{dv}{dx} = v - sin\,v$

$cosec\,v\,dv = -\frac{dx}{x}$

$\Rightarrow log\,|\,cosec\,v - cot\,v\,| + log|x| = log\,C$

$\Rightarrow x\left(cosec\frac{y}{x} - cot\frac{y}{x}\right) = C$ prepared by : BALAJI KANCHI

3.a 2025

Find the particular solution of the differential equation

$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$, given that $y = \frac{\pi}{4}$, when $x = 1$.

Sol.

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$

Put y = vx

$\Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Longrightarrow v + x\frac{dv}{dx} = v + \sec v$

$\Longrightarrow \int \cos v\,dv = \int \frac{dx}{x}$

$\Longrightarrow \sin v = \log|x| + c$

$\Longrightarrow \sin\frac{y}{x} = \log|x| + c$ prepared by : BALAJI KANCHI

5. 2025

Find the particular solution, satisfying the given condition, for the following differential equation:

$$\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0; y=0 \text{ when } x=1$$

Sol.

$$\frac{dy}{dx}=\frac{y}{x}-\operatorname{cosec}\frac{y}{x}$$

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = v - \operatorname{cosec} v$$

$$\Rightarrow -\int \sin v \, dv = \int \frac{dx}{x}$$

$$\Rightarrow \cos v = \log|x| + c$$

$$\Rightarrow \cos\frac{y}{x} = \log|x| + c$$

$$x = 1, y = 0 \Rightarrow c = 1$$

$$\Rightarrow \cos\frac{y}{x} = \log|x| + 1$$

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6.

Solve the differential equation $x\frac{dy}{dx} = y - x\tan\frac{y}{x}$.

Sol.

The given differential equation can be written as,

$$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}$$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = v - \tan v$$

$$\Rightarrow \int \frac{dv}{\tan v} = -\int \frac{dx}{x}$$

$$\Rightarrow \log|\sin v| = -\log|x| + \log C$$

$$\Rightarrow \sin\frac{y}{x} = \frac{C}{x} \Rightarrow x\sin\frac{y}{x} = C$$

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b. Logarithmic :

1.a

Find the general solution of the differential equation $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$.

Sol.

$$x\frac{dy}{dx} = y[\log y - \log x + 1]$$

$$\frac{dy}{dx} = \frac{y}{x}\left[\log\frac{y}{x} + 1\right]$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v[\log v + 1]$$

$$x\frac{dv}{dx} = v\log v$$

$$\int\frac{dv}{v\log v} = \int\frac{1}{x}dx$$

$$\log|\log v| = \log|x| + \log C$$

$$\log\left(\frac{y}{x}\right) = Cx$$

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1.b

Find the general solution of the following differential equation :

$$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$$

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{2x - x\log\frac{y}{x}}$$

Clearly it is homogeneous let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v}{2 - \log v}$$

$$\Rightarrow x\frac{dv}{dx} \quad \frac{v \quad v\log v}{2 \quad \log v}$$

$$\Rightarrow \frac{\log v - 2}{v(\log v - 1)}dv = -\frac{dx}{x}$$

$$\left(\frac{1}{v} - \frac{1}{v(\log v - 1)}\right)dv = -\frac{dx}{x}$$

$$\Rightarrow \log v - \log|\log v - 1| = -\log|x| + \log C$$

$$\Rightarrow \frac{vx}{\log v - 1} = C$$

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$$\Rightarrow y = C\left(\log\frac{y}{x} - 1\right)$$

c. Exponential :

1.a 2022

Solve the following differential equation :

$$\left(1 + e^{y/x}\right) dy + e^{y/x}\left(1 - \frac{y}{x}\right) dx = 0 \ (x \neq 0).$$

Ans.

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{e^{y/x}\left(\frac{y}{x} - 1\right)}{1 + e^{y/x}}$$

put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{e^{v}(v-1)}{1+e^{v}}$$

$$x\frac{dv}{dx} = \frac{-e^{v} - v}{1+e^{v}}$$

$$\int \frac{(e^{v}+1)}{e^{v}+v} dv = -\int \frac{dx}{x}$$

$$\log|e^{v}+v| = -\log|x| + \log C$$

$$e^{v} + v = \frac{C}{x}$$

$$e^{y/x} + \frac{y}{x} = \frac{C}{x} \quad \text{or} \quad xe^{y/x} + y = C$$

prepared by : BALAJI KANCHI

@kanchibalaji7
+91-8099454846
@ikbmaths7

1.b 2023

Solve the following differential equation :

$$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

Sol.

Given differential equation is $\frac{dy}{dx} = \frac{y}{x} - e^{\frac{y}{x}}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

The given equation becomes $v + x\frac{dv}{dx} = v - e^{v}$

$\Rightarrow - e^{-v}dv = \frac{dx}{x}$

Integrating both sides, we get

$e^{-v} = \log|x| + C$

$\Rightarrow e^{-\frac{y}{x}} = \log|x| + C$

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1.c 2024

Find the particular solution of the differential equation

$$(xe^{\frac{y}{x}} + y)\,dx = x\,dy, \text{ given that } y = 1 \text{ when } x = 1.$$

Sol.

$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f(\frac{y}{x})$ so, its a homogeneous differential equation

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Now, $v + x\frac{dv}{dx} = e^{v} + v$

$\Rightarrow \int e^{-v}dv = \int \frac{1}{x}dx$

$\Rightarrow -e^{-v} = \log|x| + c \Rightarrow -e^{\frac{-y}{x}} = \log|x| + c......(1)$

Now, $x = 1,\ y = 1$,gives $c = -e^{-1}$

Thus, $\boxed{\log|x| + e^{\frac{-y}{x}} = e^{-1}}$

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1.d

Find the general solution of the following differential equation :

$$2x\, e^{y/x}\, dy + (x - 2y\, e^{y/x})\, dx = 0$$

Ans.

Given diff. equation is

$$\frac{dy}{dx} = \frac{2ye^{y/x} - x}{2xe^{y/x}}$$

Put, $\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{2ve^{v} - 1}{2e^{v}}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2ve^{v} - 1}{2e^{v}} - v = -\frac{1}{2e^{v}}$$

$$\Rightarrow 2e^{v}dv = -\frac{dx}{x}$$

Integrating to get, $2e^{v} = -\log|x| + C$

$$\Rightarrow 2e^{y/x} + \log|x| = C$$

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IV. Diffrentail equation contains

$\sin\frac{x}{y}$, $\cos\frac{x}{y}$, $\tan\frac{x}{y}$, $e^{\frac{x}{y}}$, $\log\frac{x}{y}$ terms : $\frac{x}{y} = v, x = vy$:

1.a

Find the particular solution of the differential equation

$2y\, e^{x/y}\, dx + (y - 2x\, e^{x/y})\, dy = 0$, given that $x = 0$ when $y = 1$.

Sol.

Given differential equation can be written as

$$\frac{dx}{dy} = \frac{x}{y} - \frac{1}{2e^{x/y}}$$

$$\text{Put } x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\therefore\ v + y\frac{dv}{dy} = v - \frac{1}{2e^{v}}$$

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$$\Rightarrow \int \frac{dy}{y} = -2\int e^{v} dv$$

$$\Rightarrow \log|y| = -2e^{v} + c = -2e^{x/y} + c$$

$$\text{when } x = 0,\ y = 1 \Rightarrow c = 2$$

$$\therefore\ \log|y| = 2(1 - e^{x/y})$$

1.b

Solve the differential equation $y \cdot e^{\frac{x}{y}} \cdot dx = (x\, e^{\frac{x}{y}} + y^2)dy$, $(y \neq 0)$.

Answer:

Given differential equation can be written as

$$\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y^2}{ye^{\frac{x}{y}}}$$

$$\text{Put } \frac{x}{y} = v \quad \Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\Rightarrow \quad v + y\frac{dv}{dy} = \frac{ve^v + y}{e^v} \qquad \Rightarrow y\frac{dv}{dy} = \frac{y}{e^v}$$

$$\therefore \int e^v\, dv = \int dy \qquad \Rightarrow \quad e^v = y + C$$

$\Rightarrow \quad e^{\frac{x}{y}} = y + C$, which is the required solution.

1.c 2024

Find the general solution of the differential equation :

$$\frac{dx}{dy} = \frac{e^{x/y}\left(\frac{x}{y} - 1\right)}{1 + e^{x/y}}.$$

Sol.

$$\text{Let } x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$$

Substituting in the given differential equation, we get

$$v + y\frac{dv}{dy} = \frac{e^v(v-1)}{e^v + 1} \Rightarrow y\frac{dv}{dy} = -\frac{(e^v + v)}{e^v + 1}$$

$$\Rightarrow \frac{e^v + 1}{e^v + v}\, dv = -\frac{dy}{y}$$

Integrating we get

$$\log\left|e^v + v\right| = -\log\left|y\right| + \log C$$

$$\Rightarrow e^{x/y} + \frac{x}{y} = \frac{C}{y} \text{ or } ye^{x/y} + x = C$$

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V . Linear differential Equation : $\frac{dy}{dx} + Py = Q$

a. Direct form : (term with single variable y)

1.a 2024

Find the particular solution of the differential equation

$$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2} ; y(0) = 5.$$

Sol.

Given differential equation is a linear order differential equation with:

$$P = -2x, \; Q = 3x^2 e^{x^2}$$

$$\text{Integrating Factor} = e^{\int -2x dx} = e^{-x^2}$$

The general solution is: $y \cdot e^{-x^2} = \int e^{-x^2} \cdot 3x^2 e^{x^2} dx + C \Rightarrow y \cdot e^{-x^2} = x^3 + C$

Putting $x = 0, y = 5$**, we get,** $C = 5$

$\therefore$ **The Particular solution is:** $y \cdot e^{-x^2} = x^3 + 5$ **or** $y = (x^3 + 5)e^{x^2}$

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2. a 2017

Find the general solution of the differential equation

$$\frac{dy}{dx} - y = \sin x.$$

Sol.

Given differential equation is $\frac{dy}{dx} - y = \sin x$

$\Rightarrow$ Integrating factor $= e^{-x}$

$\therefore$ Solution is: $y\ e^{-x} = \int \sin x\, e^{-x}\, dx + c$

$$I_1 = -\sin x e^{-x} + \int \cos x\, e^{-x} dx$$

$$= -\sin x\, e^{-x} + [-\cos x\, e^{-x} - \int + \sin x\, e^{-x} dx]$$

$$I_1 = \frac{1}{2}[-\sin x - \cos x] e^{-x}$$

$\therefore$ Solution is $y\ e^{-x} = \frac{1}{2}(-\sin x - \cos x)e^{-x} + c$

or $y = -\frac{1}{2}(\sin x + \cos x) + ce^{x}$ prepared by : BALAJI KANCHI

@kanchibalaji7
+91-8099454846
@ikbmaths7

2.b 2025

Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.

Sol.

The given differential equation can be written as:

$$\frac{dy}{dx} + 2y = \cos x\text{, Taking } P = 2, Q = \cos x$$

$$\text{Integrating factor is given by, } I = e^{\int 2dx} = e^{2x}$$

$$\therefore \text{The solution is, } y \cdot e^{2x} = \int e^{2x} \cos x dx$$

$$\text{Let, } I_1 = \int \cos x \cdot e^{2x} dx = \cos x \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x} \cos x}{2} + \frac{1}{2}\left[\sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx\right]$$

$$\Rightarrow I_1 = \frac{e^{2x} \cos x}{2} + \frac{e^{2x} \sin x}{4} - \frac{1}{4} I_1 \Rightarrow I_1 = \frac{e^{2x}}{5}(2\cos x + \sin x)$$

$\therefore$ The solution of the differential equation is

$$y \cdot e^{2x} = \frac{e^{2x}}{5}(2\cos x + \sin x) + C \Rightarrow y = \frac{1}{5}(2\cos x + \sin x) + Ce^{-2x}$$

2.c 2023

Find the general solution of the differential equation : $\frac{dy}{dx} - \frac{2y}{x} = \sin \frac{1}{x}$.

Sol.

Integrating factor $= e^{\int -\frac{2}{x} dx} = e^{-2 \log x} = \frac{1}{x^2}$

Solution is $y. \frac{1}{x^2} = \int \frac{1}{x^2} . \sin \frac{1}{x} \, dx + C$

Let $\frac{1}{x} = t, -\frac{1}{x^2} dx = dt$

$\therefore \int \frac{1}{x^2} . \sin \frac{1}{x} \, dx = -\int \sin t \, dt = \cos t = \cos \frac{1}{x}$

$\therefore y. \frac{1}{x^2} = \cos \frac{1}{x} + C$

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2.d

Solve the following differential equation :

$$\frac{dy}{dx} + y = \cos x - \sin x$$

Sol.

I.F. $= e^{x}$

Solution is $y.e^{x} = \int (\cos x - \sin x)\, e^{x}\, dx + C$

$$y.e^{x} = e^{x} \cos x + C \quad \text{or} \quad y = \cos x + Ce^{-x}$$

3.a

Find the particular solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.

Sol.

$$\frac{dy}{dx} + 2y \tan x = \sin x$$

$$\text{I.F.} = e^{\int 2\tan x dx} = e^{2\log \sec x} = \sec^2 x$$

$$\therefore y\sec^2 x = \int \sin x \sec^2 x\, dx = \int \sec x \tan x\, dx$$
$$= \sec x + c$$

$$y = 0, x = \frac{\pi}{3} \Rightarrow 0 = 2 + c \Rightarrow c = -2$$

$$\Rightarrow y \sec^2 x = \sec x - 2 \quad \text{Or} \quad y = \cos x - 2\cos^2 x$$

3.b

Find the particular solution of the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$, given that $y = 2$ when $x = \frac{\pi}{2}$.

Sol.

$$\text{I.F.} = e^{\int -3\cot x\, dx} = e^{-3\log(\sin x)} = \operatorname{cosec}^3 x$$

solution is given by:

$$y \cdot \operatorname{cosec}^3 x = \int \sin 2x \cdot \operatorname{cosec}^3 x\, dx + c$$

$$= 2\int \cot x \cdot \operatorname{cosec} x\, dx + c$$

$$= -2\operatorname{cosec} x + c$$

$$\therefore\ y = -2\sin^2 x + c\sin^3 x$$

$$\text{when } y = 2,\ x = \frac{\pi}{2} \Rightarrow c = 4$$

$$\therefore\ y = = -2\sin^2 x + 4\sin^3 x$$

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3.c 2023

Find the particular solution of the differential equation $\frac{dy}{dx} + \cot x \,.\, y = \cos^2 x$, given that when $x = \frac{\pi}{2}$, $y = 0$.

Sol.

Integrating factor is $e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$

Solution is $y \sin x = \int \cos^2 x \sin x\, dx + C$

$\Rightarrow y \sin x = -\frac{\cos^3 x}{3} + C$

$x = \frac{\pi}{2}, y = 0 \Rightarrow C = 0$

$\therefore$ particular solution is $y \sin x = -\frac{\cos^3 x}{3}$ or $y = -\frac{\cos^3 x}{3}.\text{cosecx}$

prepared by : BALAJI KANCHI

3.d

Find a particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$, $(x \neq 0)$, given that $y = 0$ when $x = \frac{\pi}{2}$.

Ans.

$\frac{dy}{dx} + y.\cot x = 4x \operatorname{cosec} x$

$I.F. = e^{\int \cot x\, dx} = e^{\log(\sin x)} = \sin x$

Solution is given by :

$y.(\sin x) = \int (4x.\operatorname{cosec} x).\sin x\, dx$

$\Rightarrow y.\sin x = \int 4x\, dx$

$\Rightarrow y.\sin x = 2x^2 + C$

Now, $y = 0, x = \frac{\pi}{2}$ gives $C = -\frac{\pi^2}{2}$

Required solution is : $y.\sin x = 2x^2 - \frac{\pi^2}{2}$

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4.

Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \sec x = \tan x, \text{ where } x \in \left[0, \frac{\pi}{2}\right)$$

given that y = 1, when $x = \frac{\pi}{4}$.

Ans: $\text{I.F.} = e^{\int \sec x dx} = e^{\log|\sec x + \tan x|} = \sec x + \tan x$

$$\therefore \quad y \cdot (\sec x + \tan x) = \int \tan x(\sec x + \tan x) dx$$

$$= \sec x + \tan x - x + c$$

When $x = \frac{\pi}{4}, y = 1$ we get $c = \frac{\pi}{4}$

$$y(\sec x + \tan x) = \sec x + \tan x - x + \frac{\pi}{4}$$

5. 2023

Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$, given that $y(0) = 0$.

Sol.

Let (a) Integrating factor $= e^{\int \sec^2 x \, dx} = e^{\tan x}$

Solution is $ye^{\tan x} = \int \tan x \sec^2 x \, e^{\tan x} dx + C$

Let $\tan x = t \quad \sec^2 x \, dx = dt$

$\therefore \int e^{\tan x} \tan x \sec^2 x \, dx = \int e^t \, t \, dt = e^t (t - 1)$

$\therefore ye^{\tan x} = e^{\tan x} (\tan x - 1) + C$

$y(0) = 0$ gives $C = 1$

Particular solution is $ye^{\tan x} = e^{\tan x} (\tan x - 1) + 1$ or $y = \tan x - 1 + e^{-\tan x}$

b. Indirect form : (term with single variable y)

[write $\frac{dy}{dx}$ value followed by the term with single variable y]

6.a

Solve the differential equation :

$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$

Sol.

$(1+x^2)dy + 2xy\, dx = \cot x.dx.$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\cot x}{1+x^2}$$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$$

$$\therefore \text{ Solution is, } y \cdot (1+x^2) = \int \cot x\, dx = \log|\sin x| + c$$

$$\text{or } y = \frac{1}{1+x^2} \cdot \log|\sin x| + \frac{c}{1+x^2}$$

prepared by : BALAJI KANCHI

6.b 2022

Solve the following differential equation :

$$\left(y - \sin^2 x\right) dx + \tan x\, dy = 0$$

Sol.

$(y - \sin^2 x)\, dx + \tan x\, dy = 0$

$$\frac{dy}{dx} + \frac{y}{\tan x} = \frac{\sin^2 x}{\tan x}$$

$$\frac{dy}{dx} + (\cot x)\, y = \sin x \cos x$$

$$I.F. = e^{\int \cot x\, dx} = e^{\log(\sin x)} = \sin x$$

Solution is given by

$$y \sin x = \int \sin^2 x \cos x\, dx$$

prepared by : BALAJI KANCHI

$y\sin x = \int t^2 dt \qquad \because \sin x = t\,,\ \cos x\,dx = dt$

$y\sin x = \dfrac{t^3}{3} + C$

$y\sin x = \dfrac{\sin^3 x}{3} + C$

6.c

Solve the differential equation $(\tan^{-1} x - y)\,dx = (1 + x^2)\,dy$.

Sol.

Given differential equation can be written as

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x \Rightarrow \frac{dy}{dx} + \frac{1}{1+x^2}y = \frac{\tan^{-1} x}{1+x^2}$$

Integrating factor $= e^{\tan^{-1}x}$.

$\therefore$ Solution is $y \cdot e^{\tan^{-1}x} = \int \tan^{-1} x \cdot e^{\tan^{-1} x} \dfrac{1}{1+x^2} dx$

$\Rightarrow \quad y \cdot e^{\tan^{-1}x} = e^{\tan^{-1}x} \cdot (\tan^{-1}x - 1) + c$

or $\quad y = (\tan^{-1} x - 1) + c \cdot e^{-\tan^{-1}x}$

6.d

Find the particular solution of the differential equation

$dy = \cos x\,(2 - y\,\text{cosec}\,x)\,dx$, given that $y = 2$ when $x = \frac{\pi}{2}$.

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} + y\cot x = 2\cos x$$

$$\text{I.F.} = e^{\int \cot x\,dx} = e^{\log \sin x} = \sin x$$

Solution is given by

$$y\sin x = \int 2\sin x\cos x\,dx = \int \sin 2x\,dx$$

$$= \frac{-\cos 2x}{2} + c$$

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when $x = \frac{\pi}{2}, y = 2, \Rightarrow c = \frac{3}{2}$

Solution is given by $y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$ or $y = \text{cosec}\,x + \sin x$

7.

Solve the differential equation :

$$\frac{dy}{dx} = -\left[\frac{x + y\cos x}{1 + \sin x}\right]$$

Sol.

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1 + \sin x} \cdot y = \frac{-x}{1 + \sin x};$$

$$\text{I.F.} = e^{\int \frac{\cos x}{1+\sin x} dx} = e^{\log(1+\sin x)} = 1 + \sin x$$

$\therefore$ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1 + \sin x} \times (1 + \sin x)\, dx + c$$

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1 + \sin x)} + \frac{c}{1 + \sin x}$$

8. 2024

Find the general solution of the differential equation

$$y\,dx - x\,dy + (x\log x)\,dx = 0.$$

Sol.

The given differential equation can be written as, $\frac{dy}{dx} - \frac{1}{x}y = \log x$

$$\therefore\ P = -\frac{1}{x},\ Q = \log x$$

$$\text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$$

The solution of the differential equation is:

$$y\cdot\frac{1}{x} = \int \log x\cdot\frac{1}{x}dx$$

$$\Rightarrow \frac{y}{x} = \frac{(\log x)^2}{2} + c,\ \text{or}\ y = \frac{1}{2}x\cdot(\log x)^2 + cx$$

prepared by : **BALAJI KANCHI**

@kanchibalaji7
+91-8099454846
@ikbmaths7

c.*Divide entire equation by the coefficient of* $\frac{dy}{dx}$:

[term with single variable y]

1.a

Find the particular solution of the differential equation $x\frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$.

Sol.

$$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x^2}{x}\right)e^x = xe^x$$

$$I \cdot F = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log x^{-1}} = \frac{1}{x}$$

Solution is $y \times \frac{1}{x} = \int e^x dx$

$$\frac{y}{x} = e^x + C$$

$$y = xe^x + Cx$$

$$C = -e$$

prepared by : BALAJI KANCHI


@kanchibalaji7
+91-8099454846
@ikbmaths7

1.b 2022

Find the particular solution of the differential equation $x\dfrac{dy}{dx} + y + \dfrac{1}{1+x^2} = 0$, given that $y(1) = 0$.

Ans.

Given differential equation can be written as $\dfrac{dy}{dx} + \dfrac{1}{x}y = \dfrac{-1}{x(1+x^2)}$

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

Solution is $y \cdot x = \int \dfrac{-1}{1+x^2}dx + C$

$$\Rightarrow xy = -\tan^{-1}x + C$$

Now $y(1) = 0 \Rightarrow C = \dfrac{\pi}{4}$

$\therefore$ Particular solution is $xy = \dfrac{\pi}{4} - \tan^{-1}x$ prepared by : BALAJI KANCHI

1.c

Find the particular solution of the differential equation

$$x\frac{dy}{dx} + 2y = x^2 \log x, \text{ given } y(1) = 1.$$

Sol.

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x} \cdot y = x \log x$$

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$$

Solution is given by

$$y \cdot x^2 = \int x^2 (x \log x) dx$$

$$\Rightarrow yx^2 = \int x^3 \log x \, dx$$

$$= \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx$$

$$= \frac{x^4}{4} \log x - \frac{1}{16} \cdot x^4 + C$$

$$\Rightarrow y = \frac{x^2}{4} \log x - \frac{1}{16} x^2 + \frac{C}{x^2}$$

when $x = 1$, $y = 1$, we get $C = \frac{17}{16}$

Required particular solution is

$$y = \frac{x^2}{4} \log x - \frac{1}{16} x^2 + \frac{17}{16x^2}$$

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1.d

Find the particular solution of the differential equation $x\frac{dy}{dx} + y = x \cos x + \sin x$; given $y\left(\frac{\pi}{2}\right) = 1$.

Sol.

The given differential equation can be written as

$$\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$$

The integrating factor, I.F. $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$\therefore$ **the solution is:**

$$y \cdot x = \int (x \cos x + \sin x) dx$$

$$= x \sin x - \int \sin x dx + \int \sin x dx$$

$$y \cdot x = x \sin x + C \Rightarrow y = \sin x + \frac{C}{x}$$

Put, $x = \frac{\pi}{2}, y = 1 \Rightarrow C = 0$

$\therefore$ **the particular solution is** $y = \sin x$

2.a

Find the particular solution of the differential equation $(1 + x^2)\dfrac{dy}{dx} + 2xy = \tan x$, given $y(0) = 1$.

Sol.

Given differential equation can be written as

$$\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\tan x}{1+x^2}$$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$$

Solution is given by

$$y \cdot (1 + x^2) = \int \frac{\tan x}{1+x^2} \cdot (1+x^2)\, dx$$

$$= \int \tan x \, dx$$

$$= \log |\sec x| + C$$

When $x = 0$, $y = 1$ gives $C = 1$

Required particular solution is $y \cdot (1 + x^2) = \log |\sec x| + 1$

or $y = \dfrac{\log |\sec x|}{1+x^2} + \dfrac{1}{1+x^2}$

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2.b 2023

Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, given that y = 0 when x = 1.

Sol.

Given diff. eqn. can be written as

$$\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{(1+x^2)^2}$$

$$\text{I.F.}=e^{\int\frac{2x}{1+x^2}dx}=1+x^2$$

solution is given by: $y.(1+x^2)=\int\frac{1}{1+x^2}dx$

$$\Rightarrow y.(1+x^2)=\tan^{-1}x+C$$

Now $x=1, y=0$ gives $C=-\frac{\pi}{4}$

Reqired solution : $y.(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$

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2.c 2023

Find the general solution of the differential equation :

$$(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$$

Sol.

Rewriting the given differential equation as:

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{\sqrt{x^2+4}}{1+x^2}$$

$$\textbf{Integrating factor}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$$

∴ solution of the differential equation is

$$y\left(1+x^2\right)=\int \frac{\sqrt{x^2+4}}{1+x^2}\cdot\left(1+x^2\right)dx+c$$

$$=\int \sqrt{x^2+4}\,dx+c$$

$$\therefore y\left(1+x^2\right)=\frac{x\sqrt{x^2+4}}{2}+2\log\left|x+\sqrt{x^2+4}\right|+c$$

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2.d 2025

Solve the differential equation $(1 + x^2)\,\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

Sol.

Given equation is $\frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{4x^2}{1+x^2}$

I.F. $= e^{\int \frac{2x}{1+x^2}dx} = 1+x^2$

Solution is given by,

$$y\cdot(1+x^2)=\int \frac{4x^2}{1+x^2}\cdot(1+x^2)\,dx=\int 4x^2\,dx$$

$$\Rightarrow\ y\cdot(1+x^2)=\frac{4x^3}{3}+c$$

when $x = 0,\ y = 0 \Rightarrow c = 0$

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$$y\cdot(1+x^2)=\frac{4x^3}{3}\quad \text{or}\quad y=\frac{4x^3}{3(1+x^2)}$$

2.e 2025

Solve the differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

Sol.

Given D.E. is $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$

Integrating factor is $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1 + x^2)$

Solution is $y(1 + x^2) = \int 4x^2 dx + C$

$$y(1 + x^2) = \frac{4x^3}{3} + C$$

$y(0) = 0$ gives $C = 0$, hence solution is $y(1 + x^2) = \frac{4x^3}{3}$

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4.

Find the particular solution of the differential equation

$(1 + \sin x)\dfrac{dy}{dx} = -x - y\cos x$, given $y(0) = 1$.

Sol.

$$(1 + \sin x)\frac{dy}{dx} = -x - y\cos x$$

Given equation can be writte as

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x}\cdot y = \frac{-x}{1+\sin x}$$

$$\text{I.F.} = e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1 + \sin x$$

Solution is given by

$$y\cdot(1+\sin x) = \int \frac{-x}{(1+\sin x)}\cdot(1+\sin x)\,dx$$

$$= \int -x\,dx = -\frac{x^2}{2} + C$$

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Now, when $x = 0$, $y = 1 \Rightarrow C = 1$

$\therefore$ Required particular solution is $y\cdot(1+\sin x) = -\dfrac{x^2}{2} + 1$

d. $\frac{dy}{dx} + Py = Q$ form : after taking Reciprocal on both sides :

[single variable y term should come in numerator after writing $\frac{dy}{dx}$]

1.a

Find the particular solution of the differential equation $(y + 3x^2)\frac{dx}{dy} = x$, given that $y = 1$, when $x = 1$.

Ans.

Given differential equation can be written as

$$x\frac{dy}{dx} - y = 3x^2 \text{ or } \frac{dy}{dx} - \frac{1}{x}y = 3x$$

$$\text{I.F} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = -x^{-1} = \frac{1}{x}$$

$$\text{Solution is } y \cdot \frac{1}{x} = \int 3x\frac{1}{x}dx + C$$

$$\frac{y}{x} = 3x + C$$

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$x = 1,\ y = 1$ gives $C = -2$

Particular solution is $\frac{y}{x} = 3x - 2$ or $y = 3x^2 - 2x$

1.b

Solve the following differential equation

$$(y + 3x^2)\frac{dx}{dy} = x$$

Ans.

$$(y + 3x^2)\frac{dx}{dy} = x \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 3x$$

$$\text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\log(x)} = \frac{1}{x}$$

$$\text{Solution is } y \cdot \frac{1}{x} = \int 3x \cdot \frac{1}{x}dx \Rightarrow y = 3x^2 + cx$$

2. 2022

Find the particular solution of the differential equation $(2x^2 + y) \cdot \frac{dx}{dy} = x$; given that y = 2 when x = 1.

Sol.

Given diff. equation can be written as

$$x\frac{dy}{dx} - y = 2x^2 \quad or \quad \frac{dy}{dx} - \frac{1}{x}y = 2x$$

$$\text{IF} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$$

$$\therefore \text{ Solution is } y \cdot \frac{1}{x} = \int 2x \cdot \frac{1}{x} dx = 2x + C$$

$$\Rightarrow y = 2x^2 + Cx$$

$$\text{when } x = 1,\ y = 2 \Rightarrow 2 = 2 + C \Rightarrow C = 0$$

$$\Rightarrow \text{Particular Solution is } y = 2x^2$$

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3.

Find the general solution of the differential equation $(2x^2 + y)\, dx = x\, dy$.

sol.

$$(2x^2 + y)dx = x\,dy$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{x}.y = 2x$$

$$\text{I.F.} = e^{-\int \frac{1}{x}dx} = \frac{1}{x}$$

Solution is given by,

$$y.\left(\frac{1}{x}\right) = \int 2x.\frac{1}{x}dx$$

$$\Rightarrow \frac{y}{x} = 2x + C \quad or \quad y = 2x^2 + Cx$$

prepared by : BALAJI KANCHI

@kanchibalaji7
+91-8099454846
@ikbmaths7

VII. Single variable X : $\frac{dx}{dy} + Px = Q$ form :

[write $\frac{dx}{dy}$ value followed by the term with single variable x]

1.a 2024

Find the particular solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$, given that x = 1 when y = 0.

sol.

$$(\tan^{-1} y - x)dy = (1 + y^2)\, dx$$

$$\frac{dx}{dy} + \frac{1}{1+y^2}\, x = \frac{\tan^{-1} y}{1+y^2}$$

$$\text{I. F} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$x \times e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2}\, e^{\tan^{-1} y}\, dy$$

$$\Rightarrow x e^{\tan^{-1} y} = (\tan^{-1} y)\, e^{\tan^{-1} y} - e^{\tan^{-1} y} + C$$

OR

$$\Rightarrow x = \tan^{-1} y - 1 + C\, e^{-\tan^{-1} y}$$

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1.b

Find the particular solution of the differential equation

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0, \text{ given that } y = 0 \text{ when } x = 1.$$

Sol.

Given differential equation can be written as

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2}$$

$$\text{I.F.} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1} y}$$

Solution is given by

$$xe^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \times e^{\tan^{-1} y}\, dy = \int \frac{e^{2\tan^{-1} y}}{1+y^2}\, dy$$

$$\Rightarrow\ xe^{\tan^{-1} y} = \frac{e^{2\tan^{-1} y}}{2} + c$$

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when $x = 1,\ y = 0 \Rightarrow c = \frac{1}{2}$

$\therefore$ Solution is given by $xe^{\tan^{1} y} = \frac{1}{2}e^{2\tan^{-1} y} + \frac{1}{2}$ or $x = \frac{1}{2}(e^{\tan^{-1} y} + e^{-\tan^{-1} y})$

2.a 2024, 2017

Find the general solution of the differential equation

$$y\,dx - (x + 2y^2)\,dy = 0.$$

Sol.

Given differential equation can be written as

$$y\frac{dx}{dy} - x = 2y^2 \quad \text{or} \quad \frac{dx}{dy} - \frac{1}{y}\cdot x = 2y$$

Integrating factor is $e^{-\log y} = \dfrac{1}{y}$

$$\therefore \text{ Solution is } x\cdot\frac{1}{y} = \int 2\,dy = 2y + c$$

or $x = 2y^2 + cy.$ prepared by : BALAJI KANCHI

2.b

Find the particular solution of the differential equation :

$ye^y\,dx = (y^3 + 2xe^y)dy, y(0) = 1$

Ans. $y\cdot e^y dx = \left(y^3 + 2xe^y\right)dy \Rightarrow y\cdot e^y\dfrac{dy}{dx} = y^3 + 2xe^y$

$$\therefore \frac{dx}{dy} - \frac{2}{y}x = y^2\cdot e^{-y}$$

$$\text{I.F. (Integrating factor)} = e^{-2\int\frac{1}{y}dy} = e^{-2\log y} = e^{\log\frac{1}{y^2}} = \frac{1}{y^2}$$

$\therefore$ Solution is

$$x\cdot\frac{1}{y^2} = \int y^2\cdot e^{-y}\cdot\frac{1}{y^2}dy + c = \int e^{-y}dy + c$$

$$\Rightarrow \frac{x}{y^2} = -e^{-y} + c \quad \text{or} \quad x = -y^2e^{-y} + cy^2$$

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3. 2025

Solve the differential equation $(x - \sin y)\,dy + (\tan y)\,dx = 0$, given $y(0) = 0$.

Sol

The differential equation can be written as:

$$\frac{dx}{dy} + \cot y \cdot x = \cos y \text{, which is a linear order differential equation}$$

Here, $P = \cot y,\ Q = \cos y$ **, I.F. (Integrating Factor) =** $e^{\int \cot y\, dy} = e^{\log \sin y} = \sin y$

The solution is , $x(\sin y) = \int \cos y \cdot \sin y\ dy$

$$\Rightarrow x(\sin y) = \frac{(\sin y)^2}{2} + C \text{, For } x = 0, y = 0, C = 0.$$

$\therefore$ **The Particular solution is:** $x \sin y = \frac{\sin^2 y}{2}$ or $\sin y = 2x$ or $y = \sin^{-1} 2x$

4.

Solve the following differential equation :

$$\frac{dx}{dy} + x = (\tan y + \sec^2 y)$$

Sol.

I.F. $= e^{\int 1\, dy} = e^y$

Solution is

$$x \cdot e^y = \int e^y (\tan y + \sec^2 y) dy + C$$

$$x \cdot e^y = e^y \tan y + C \quad \text{or} \quad x = \tan y + ce^{-y}$$

VIII. Incomplete D.E – convert to D.E first (diiferentiate):

1. 2023

Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$$

Sol.

Given differential equation is

$2xy\frac{dy}{dx} + y^2 = 2y(1 + x^2)$

$\Rightarrow \frac{dy}{dx} + \frac{y}{2x} = \frac{1}{x} + x$

Integrating factor $= e^{\int \frac{1}{2x} dx} = e^{\log \sqrt{x}} = \sqrt{x}$

Solution is given by $y\sqrt{x} = \int \left(\frac{1}{\sqrt{x}} + x^{\frac{3}{2}}\right) dx$

$\Rightarrow y\sqrt{x} = 2\sqrt{x} + \frac{2x^{\frac{5}{2}}}{5} + C$,or $y = 2 + \frac{2x^2}{5} + \frac{C}{\sqrt{x}}$ prepared by : BALAJI KANCHI

@kanchibalaji7
+91-8099454846
@ikbmaths7

Case study :

1. 2023

An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

@kanchibalaji7
+91-8099454846
@ikbmaths7

Sol.

$$\text{(I)}\ (x^2 - y^2)dx + 2xydy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$= \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$$

$$= g\left(\frac{y}{x}\right)$$

$$\text{(II)}\ y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{-1 - v^2}{2v}$$

$$\Rightarrow \int \frac{2v}{1 + v^2}\,dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log|1 + v^2| + \log|x| = \log C$$

$$\text{or}\quad x\left(1 + \frac{y^2}{x^2}\right) = C$$

$$\text{or}\quad x^2 + y^2 = Cx$$

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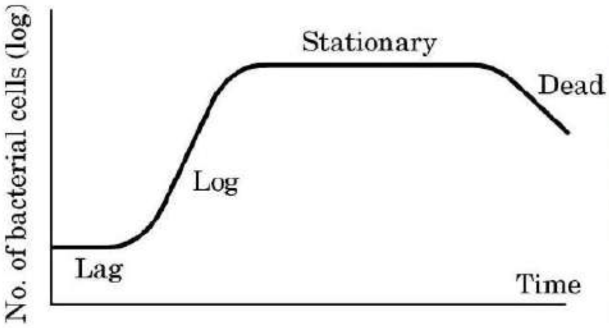
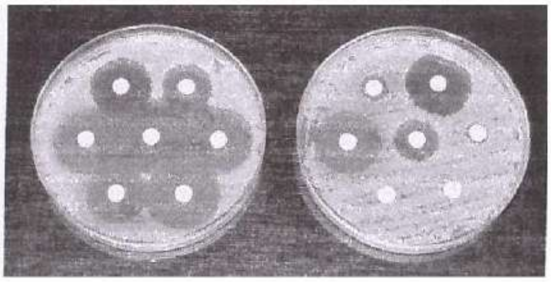
2.2024

A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt}$ = kP, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. 2

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. 2

Ans(i)

$$\frac{dP}{dt} = kP \Rightarrow \int \frac{dP}{P} = \int k\,dt$$

$$\Rightarrow \log P = kt + C \text{ or } P = e^{kt+C}$$

Ans(ii)

$$\log P = kt + C$$
$$\text{when } t = 0, P = 1000 \Rightarrow C = \log 1000$$
$$\text{when } t = 1, P = 2000 \Rightarrow \log 2000 = k + \log 1000$$
$$\Rightarrow k = \log 2$$

3. 2025

Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.

(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation.

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm.

(iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

OR

(iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

Sol.

(i) Order = 1, Degree = 1

(ii) Separating the variable and integrating, $\int dr = \frac{2k}{3}\int dt \Rightarrow r = \frac{2}{3}kt + C$

Putting $t = 0, r = 5$, we get $C = 5$

$$r = \frac{2}{3}kt + 5$$

(iii) (a) Putting $r = 3, t = 1$, $3 = \frac{2}{3}k(1) + 5 \Rightarrow k = -3$

$$r = -2t + 5\text{, For } r = 0,\ t = \frac{5}{2}\text{hrs or } 2.5 \text{ hrs}$$

OR

(iii) (b) Putting $r = 1, t = 1$, $1 = \frac{2}{3}k + 5 \Rightarrow k = -6$

$$\therefore\ r = -4t + 5\text{, For } r = 0,\ t = \frac{5}{4}\text{ hrs or } 1.25 \text{ hrs}$$

4. 2025

During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$,

where T(t) represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions :

(i) Find the expression for temperature of processor, T(t) given that $T(0) = 85°C$.

(ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0{\cdot}03$, $\log_e 4 = 1{\cdot}3863$.

Sol.

$$(i)\frac{dT}{dt}=-k(T-25)$$

$$\Rightarrow\frac{dT}{T-25}=-k\,dt$$

$$\Rightarrow\int\frac{dT}{T-25}=-k\int dt$$

$$\Rightarrow\log|T-25|=-kt+C \qquad ...(a)$$

When $t=0, T=85$

$$\Rightarrow\log 60=C$$

Using in equation (a), $\log|T-25|=-kt+\log 60 \qquad ...(b)$

(ii) When $k=0.03$, $\log|T-25|=-0.03t+\log 60$

$$\Rightarrow\log\left|\frac{T-25}{60}\right|=-0.03t$$

$$\Rightarrow T-25=60.e^{-0.03t}$$

When $T=40, t=t_1$

$$\Rightarrow\frac{15}{60}=e^{-0.03t_1}$$

$$\Rightarrow e^{-0.03t_1}=\frac{1}{4}\Rightarrow-0.03t_1=-\log 4$$

$$\Rightarrow t_1=\frac{\log 4}{0.03}=\frac{1.3863}{0.03}=46.21\,\text{m}$$