Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
MATHEMATICS PAPER CODE 65/5/1

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9	<u>**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**</u>
10	<u>**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**</u>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

MATHEMATICS (Subject Code–041)

(PAPER CODE: 65/5/1)

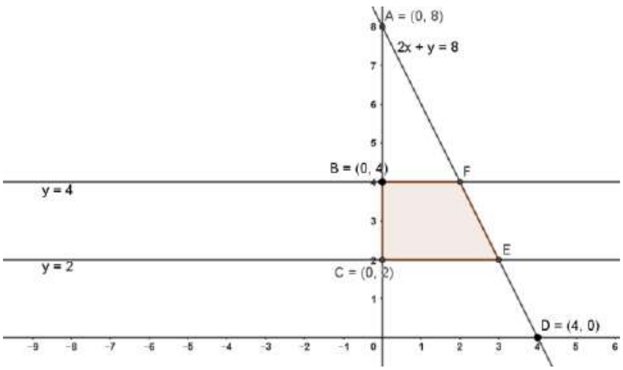
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	Let A = {3, 5}. Then number of reflexive relations on A is (a) 2 (b) 4 (c) 0 (d) 8	
Sol.	(b) 4	**1**
2.	$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$	
Sol.	(a) 1	**1**
3.	If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals (a) A (b) A + I (c) I – A (d) A – I	
Sol.	(c) I – A	**1**
4.	If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals (a) ± 1 (b) –1 (c) 1 (d) 2	
Sol.	(c) 1	**1**

5.	If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is (a) 1 (b) 2 (c) 3 (d) 4	
Sol.	(d) 4	**1**
6.	The derivative of x^{2x} w.r.t. x is (a) x^{2x-1} (b) $2x^{2x}\log x$ (c) $2x^{2x}(1+\log x)$ (d) $2x^{2x}(1-\log x)$	
Sol.	(c) $2x^{2x}(1 + \log x)$	**1**
7.	The function f(x) = [x], where [x] denotes the greatest integer less than or equal to x, is continuous at (a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$	
Sol.	(b) x = 1·5	**1**
8.	If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to (a) x (b) $-x$ (c) $16x$ (d) $-16x$	
Sol.	(d) $-16\,x$	**1**
9.	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is (a) $(-1, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -2)$ (d) $[-1, 1]$	
Sol.	(b) $(-2, -1)$	**1**
10.	$\int \frac{\sec x}{\sec x - \tan x}\,dx$ equals (a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$ (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$	
Sol.	(b) $\sec x + \tan x + c$	**1**

11.	$\int_{-1}^{1} \frac{\lvert x-2 \rvert}{x-2} \, dx, x \neq 2$ का मान है : (a) 1 (b) –1 (c) 2 (d) –2	
Sol.	(d) $\mathbf{-2}$	**1**
12.	The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is (a) 2 (b) 3 (c) 5 (d) 0	
Sol.	Due to error in the question, 1 mark should be awarded to each student who attempted the question	**1**
13.	Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if (a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ (c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$	
Sol.	(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$	**1**
14.	The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is (a) 1 (b) 5 (c) 7 (d) 12	
Sol.	(c) 7	**1**
15.	If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	
Sol.	(a) $\mathbf{0}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	**1**

16.	The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is (a) $0°$ (b) $30°$ (c) $45°$ (d) $90°$	
Sol.	(d) $\mathbf{90°}$	**1**
17.	If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to (a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$	
Sol.	(c) $\mathbf{\frac{7}{8}}$	**1**
18.	Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is (a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$	
Sol.	(c) $\mathbf{\frac{31}{32}}$	**1**
	Assertion – Reason Based Questions In the following questions **19 and 20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices : (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true, but (R) is not the correct explanation of (A). (c) (A) is true and (R) is false. (d) (A) is false, but (R) is true.	
19.	**Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$. **Reason (R) :** Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.	

Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
20.	**Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$ **Reason (R) :** $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$	
Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	Write the domain and range (principle value branch) of the following functions : $f(x) = \tan^{-1} x$	
Sol.	Domain = R ; Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	**1+1**
22(a).	If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.	
Sol.	Here RHD $= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 2$ LHD $= \lim_{h \to 0} \left[\frac{f(1-h) - f(1)}{-h}\right] = 1$ Since RHD $\neq$ LHD $\therefore$ f is not differentiable at x = 1.	$1\frac{1}{2}$ $\frac{1}{2}$
22(b).	Find the value(s) of 'λ', if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.	

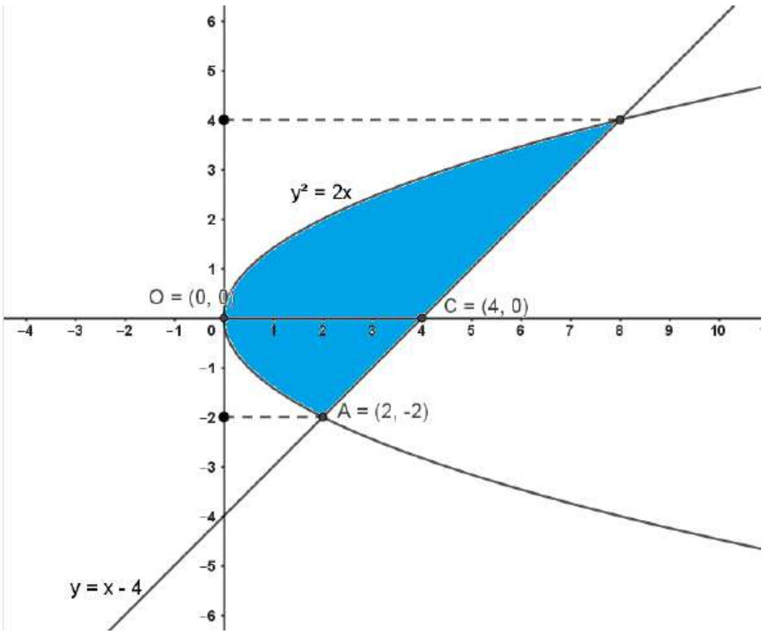
Sol.	$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{\sin^2 \lambda x}{x^2}\right) = \lim_{x \to 0} \left[\frac{\sin^2 \lambda x}{(\lambda x)^2} \cdot \lambda^2\right] = \lambda^2$ Since f(x) is continuous at x = 0 $\lim_{x \to 0} f(x) = f(0)$ $\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$	**1** **$\frac{1}{2}$** **$\frac{1}{2}$**
23.	Sketch the region bounded by the lines $2x + y = 8$, y = 2, y = 4 and the y-axis. Hence, obtain its area using integration.	
Sol.	 Required area = $\int_2^4 \frac{1}{2}\,(8 - y)dy$ $= \frac{1}{2}\left\|8y - \frac{y^2}{2}\right\|_2^4$ = 5	**$\frac{1}{2}$ for correct figure** **$\frac{1}{2}$** **$\frac{1}{2}$** **$\frac{1}{2}$**
24(a).	If the vectors $\vec{a}$ and $\vec{b}$ are such that $\|\vec{a}\| = 3$, $\|\vec{b}\| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.	

Sol.	Let θ be the angle between $\vec{a}$ and $\vec{b}$ Since $\vec{a} \times \vec{b}$ is a unit vector, we have $\|\vec{a} \times \vec{b}\| = 1$ $\Rightarrow \|\vec{a}\|\ \|\vec{b}\| \sin\theta = 1$ $\Rightarrow \sin\theta = \frac{1}{2}$, or $\theta = 30°$ (or $\frac{\pi}{6}$)	 **1** **1**
24(b).	Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.	
Sol.	Here $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$ $\Rightarrow \|\vec{a} \times \vec{b}\| = \sqrt{400 + 25 + 25} = \sqrt{450}$ **Area of parallelogram =** $\|\vec{a} \times \vec{b}\| = \sqrt{450} = 15\sqrt{2}$	 **$1\frac{1}{2}$** **$\frac{1}{2}$**
25.	Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.	
Sol.	The given line is $\frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}$, or $\frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$ So, the required vector equation of the line passing through (1,2,-1) is $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$	 **1** **$\frac{1}{2}$**

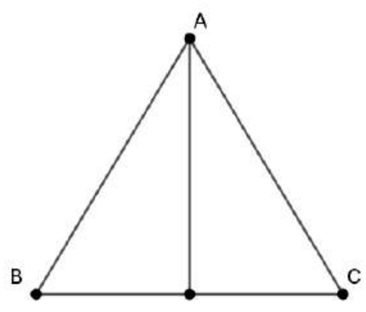
	Cartesian equation of the line is $$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$$	$\frac{1}{2}$
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	If A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.	
Sol.	Getting $A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$ Getting $A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$ $A^3 - 23A - 40I =$ $\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$ $= \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \mathbf{O}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$
27(a).	Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.	
Sol.	Let x = sin θ. Then $U = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$	

	$= \sec^{-1}(\sec\theta) = \theta = \sin^{-1}x$ $\Rightarrow \frac{dU}{dx} = \frac{1}{\sqrt{1-x^2}}$ and $V = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$ $= \sin^{-1}[2\sin\theta\cos\theta] = 2\theta = 2\sin^{-1}x$ $\Rightarrow \frac{dV}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\Rightarrow \frac{dU}{dV} = \frac{dU/dx}{dV/dx} = \frac{1}{2}$ **Note: If the substitution is made as $x = \cos\theta$, answer will be $-\frac{1}{2}$**	**1** **1** **1**
27(b).	If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.	
Sol.	$y = \tan x + \sec x = \frac{\sin x + 1}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{\cos x(\cos x) + (\sin x + 1)\sin x}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1+\sin x}{1-\sin^2 x} = \frac{1}{1-\sin x}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-\sin x).0 - 1(0-\cos x)}{(1-\sin x)^2} = \frac{\cos x}{(1-\sin x)^2}$	 $1\frac{1}{2}$ $1\frac{1}{2}$
28(a).	Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx$	

Sol.	Let I = $\int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx = \int_0^{2\pi} \frac{1}{1 + e^{\sin(2\pi - x)}} dx$	**1**
	$= \int_0^{2\pi} \frac{1}{1 + e^{-\sin x}} dx = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + 1} dx$	**1**
	$\Rightarrow 2I = \int_0^{2\pi} \frac{e^{\sin x} + 1}{e^{\sin x} + 1} dx = \int_0^{2\pi} 1 \,.\, dx = 2\pi$	$\frac{1}{2}$
	$\Rightarrow$ **I = π**	$\frac{1}{2}$
28(b).	Find : $\int \frac{x^4}{(x-1)(x^2+1)} dx$	
Sol.	$I = \int \frac{x^4}{(x-1)(x^2+1)} dx = \int \left[x + 1 + \frac{1}{(x-1)(x^2+1)} \right] dx$	**1**
	$= \frac{x^2}{2} + x + \int \left[\frac{1}{2(x-1)} - \frac{1}{2} \frac{(x+1)}{(x^2+1)} \right] dx$ (Using partial fractions)	$\frac{1}{2}$+1
	$= \frac{x^2}{2} + x + \frac{1}{2} \log \lvert x - 1 \rvert - \frac{1}{4} \log \lvert x^2 + 1 \rvert - \frac{1}{2} \tan^{-1} x + C$	$\frac{1}{2}$
29.	Find the area of the following region using integration : $\{(x, y) : y^2 \le 2x \text{ and } y \ge x - 4\}$	

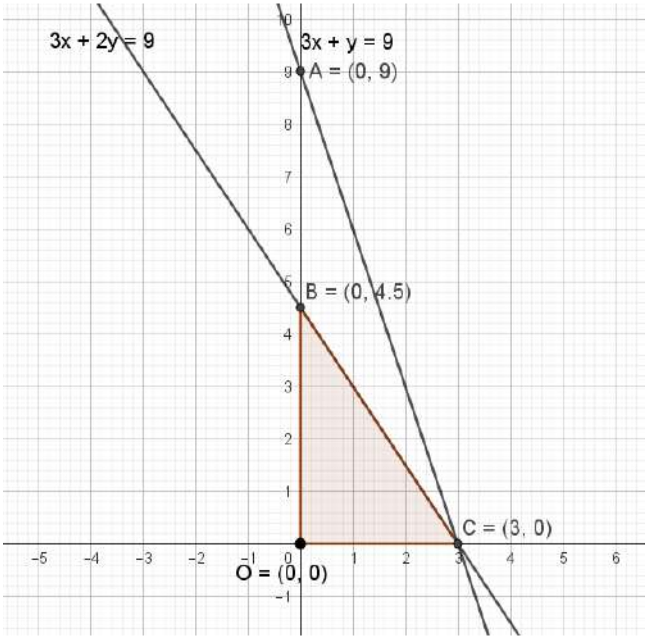
Sol.	 Solving $y^2 = 2x$ and $y = x - 4$, we get y = 4 or – 2 Required area = $\int_{-2}^{4} \left[(y + 4) - \frac{y^2}{2}\right] dy$ $= \left| \frac{y^2}{2} + 4y - \frac{1}{6}y^3 \right|_{-2}^{4}$ **= 18**	**1 mark for correct figure** $\frac{1}{2}$ **1** $\frac{1}{2}$
30(a).	Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.	
Sol.	General point on the given line is M (5λ – 3, 2λ + 1, 3λ – 4) Direction ratios of PM are $5\lambda - 3, 2\lambda - 1, 3\lambda - 7$	**1**

	If this point is the foot of the perpendicular from the point P (0, 2, 3), then PM is perpendicular to the line. Thus, $(5\lambda - 3).5 + (2\lambda - 1).2 + (3\lambda - 7).3 = 0$ $\Rightarrow \lambda = 1$ **Hence co-ordinates of M are (2, 3, – 1)**	1 1
30(b).	Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = 4$ and $\lvert\vec{c}\rvert = 2$.	
Sol.	$(\vec{a} + \vec{b} + \vec{c})^2 = 0$ $\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\mu) = 0$ $\Rightarrow \mu = -\frac{29}{2}$	1 1 1
31.	Find the distance between the lines : $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$	
Sol.	Here $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$ Here, $\vec{b_1}$ and $\vec{b_2}$ are parallel vectors.	 $\frac{1}{2}$

	$\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}, \quad \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$	$\frac{1}{2}$
	Thus, $(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$	**1**
	Distance between the lines = $\left\lvert \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}}{\lvert \vec{b} \rvert} \right\rvert$ $= \frac{\sqrt{81+196+16}}{\sqrt{4+9+36}}$ $= \frac{\sqrt{293}}{7}$ units.	**1**
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32(a).	The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.	
Sol.	In an equilateral triangle, median is same as altitude. Let 'h' denote the length of the median (or altitude) and 'x' be the side of Δ ABC.  Then, $h = \frac{\sqrt{3}}{2}x$ or $x = \frac{2h}{\sqrt{3}}$ ________ (i)	**1** **2**

	It is given that $\frac{dh}{dt} = 2\sqrt{3}$ So, by (i) we have	**1**
	$\frac{dx}{dt} = \frac{2}{\sqrt{3}}\frac{dh}{dt} \Rightarrow \frac{dx}{dt} = 4$	**1**
	Thus, the side of Δ ABC is increasing at the rate of 4 cm/sec.	
32(b).	Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.	
Sol.	Let the two numbers be x and y. Then, x + y = 5 or y = 5 – x	$\mathbf{\frac{1}{2}}$
	Let S denote the sum of the cubes of these numbers. Then	
	$S = x^3 + y^3 = x^3 + (5-x)^3$	**1**
	$\frac{dS}{dx} = 3x^2 - 3(5-x)^2 = 15(2x-5)$	**1**
	Now $\frac{dS}{dx} = 0$, gives $x = \frac{5}{2}$	$\mathbf{\frac{1}{2}}$
	Showing S is minimum at $x = \frac{5}{2}$	**1**
	So, the two numbers are $\frac{5}{2}$ and $\frac{5}{2}$	
	$\Rightarrow \boldsymbol{x^2 + y^2 = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}}$	**1**
33.	Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$	
Sol.	Let I = $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)\, dx$.	

	$= \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1} (\sin x)\, dx$	$\frac{1}{2}$
	Put sin x = t so that cos x dx = dt	$\frac{1}{2}$
	Thus, $I = 2\int_0^1 t \tan^{-1} t\, dt$	$\frac{1}{2}$
	$= 2\left[\left\|\frac{t^2}{2} \tan^{-1} t\right\|_0^1 - \int_0^1 \frac{1}{1+t^2} \cdot \frac{t^2}{2}\, dt\right]$	1
	$= 2 \cdot \frac{1}{2} \cdot \frac{\pi}{4} - \int_0^1 \frac{t^2}{1+t^2}\, dt$	
	$= \frac{\pi}{4} - \int_0^1 \left[1 - \frac{1}{1+t^2}\right] dt$	1
	$= \frac{\pi}{4} - \| t \|_0^1 + \left\|\tan^{-1} t\right\|_0^1$	1
	$= \frac{\pi}{4} - 1 + \frac{\pi}{4}$	
	$= \frac{\pi}{2} - 1$	$\frac{1}{2}$
34.	Solve the following Linear Programming Problem graphically : Maximize : $P = 70x + 40y$ subject to : $3x + 2y \leq 9$, $3x + y \leq 9$, $x \geq 0,\ y \geq 0$	

Sol.		**3 for correct figure and shading**

Corner Points	Value of P
O (0,0)	0
B (0,4.5)	180
C (3,0)	210 → Max Value

$1\frac{1}{2}$

Maximum value of P =210 at $x = 3$ and y=0 **$\frac{1}{2}$**

35(a).	In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?	
Sol.	Let events A, B and E be defined as: A : Student knows the answer B : Student guesses the answer E : student answered correctly	**1**
	$P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$	$\frac{1}{2}$
	Here, $P\left(\frac{E}{A}\right) = 1$ and $P\left(\frac{E}{B}\right) = \frac{1}{3}$	**1**
	By Bayes' Theorem $P\left(\frac{A}{E}\right) = \frac{P(A) . P\left(\frac{E}{A}\right)}{P(A) . P\left(\frac{E}{A}\right) + P(B) . P\left(\frac{E}{B}\right)}$	

	$= \dfrac{\frac{3}{5}\times 1}{\left(\frac{3}{5}\times 1\right)+\left(\frac{2}{5}\times\frac{1}{3}\right)}$	$1\frac{1}{2}$
	$= \frac{9}{11}$	**1**
35(b).	A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.	
Sol.	Let X denote the prize value. Here X can take values of 8, 4 and 2. $P(X = 8) = \frac{2}{10}$, or $\frac{1}{5}$ $P(X = 4) = \frac{5}{10}$, or $\frac{1}{2}$ $P(X = 2) = \frac{3}{10}$	**1**
	(see table below)	**3**
	Hence, Mean value of X = $\sum X\,P(X) = \frac{8}{5} + 2 + \frac{6}{10}$	

X	8	4	2
P(X)	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{3}{10}$
XP(X)	$\frac{8}{5}$	$\frac{4}{2}$	$\frac{6}{10}$

	$= \frac{42}{10}$ or ₹ 4.20	**1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	
36.	An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.  Based on the above information, answer the following questions : (I) How many relations are possible from B to G ? (II) Among all the possible relations from B to G, how many functions can be formed from B to G ? (III) Let R : B → B be defined by R = $\{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation. **OR** (III) A function f : B → G be defined by f = $\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.	
Sol.	(I) Number of relations = $2^6 = 64$ (II) Number of possible functions = $2^3 = 8$ (III) R is an equivalence relation as it is reflexive, symmetric and transitive	**1** **1** **2**

	OR Since f is not one-one function $\therefore f$ is not bijective	 **1** **1**
37.	Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250. Based on the above information, answer the following questions : (I) Convert the given above situation into a matrix equation of the form AX = B. (II) Find $\|A\|$. (III) Find A^{-1}. **OR** (III) Determine $P = A^2 - 5A$.	
Sol.	(I) Matrix equation is AX = B, where $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$ where x is the number of pens bought, y the number of bags and z the number of instrument boxes. (II) $\|A\| = 5(4-6) - 3(8-3) + 1(4-1) = -22$ (III) $\text{adj}(A) = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	 **1** **1** **1**

	$\Rightarrow A^{-1} = \frac{1}{(-22)}\begin{bmatrix}-2 & -10 & 8\\ -5 & 19 & -13\\ 3 & -7 & -1\end{bmatrix}$	**1**
	OR	
	$P = A^2 - 5A = \begin{bmatrix}32 & 20 & 18\\ 15 & 13 & 17\\ 13 & 13 & 23\end{bmatrix} - \begin{bmatrix}25 & 15 & 5\\ 10 & 5 & 15\\ 5 & 10 & 20\end{bmatrix}$	**1+$\frac{1}{2}$**
	$= \begin{bmatrix}7 & 5 & 13\\ 5 & 8 & 2\\ 8 & 3 & 3\end{bmatrix}$	**$\frac{1}{2}$**
38.	An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero, whereas a function F(x, y) is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution y = vx and then separate the variables. Based on the above, answer the following questions : (I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$. (II) Solve the above equation to find its general solution.	
Sol.	(I) $(x^2 - y^2)dx + 2xydy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$	**1**

	$= \dfrac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$	
	$= g\left(\frac{y}{x}\right)$	1
	(II) $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dy}{dx}$	$\frac{1}{2}$
	$v + x\dfrac{dv}{dx} = \dfrac{v^2 - 1}{2v} - v = \dfrac{-1 - v^2}{2v}$	
	$\Rightarrow \displaystyle\int \dfrac{2v}{1 + v^2}\, dv = -\int \dfrac{dx}{x}$	1
	$\Rightarrow \log \lvert 1 + v^2 \rvert + \log \lvert x \rvert = \log C$	$\frac{1}{2}$
	or $x\left(1 + \dfrac{y^2}{x^2}\right) = C$	
	or $x^2 + y^2 = Cx$	

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
MATHEMATICS PAPER CODE 65/5/2

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

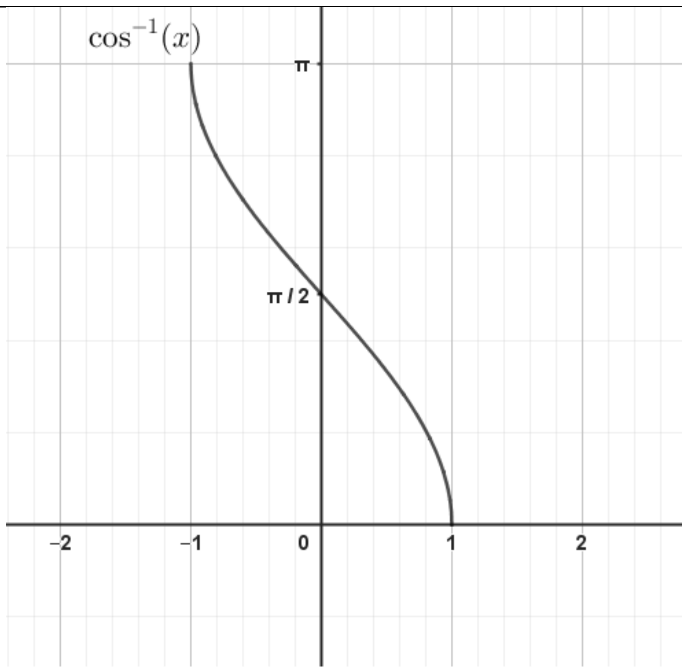
MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/5/2)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$	
Sol.	(a) 1	**1**
2.	Let A = {3, 5}. Then number of reflexive relations on A is (a) 2 (b) 4 (c) 0 (d) 8	
Sol.	(b) 4	**1**
3.	If $A=\begin{bmatrix}1&0\\2&1\end{bmatrix}$, $B=\begin{bmatrix}x&0\\1&1\end{bmatrix}$ and $A=B^2$, then x equals (a) ± 1 (b) -1 (c) 1 (d) 2	
Sol.	(c) 1	**1**
4.	If $A=[a_{ij}]$ is a square matrix of order 2 such that $a_{ij}=\begin{cases}1, & \text{when } i\neq j\\ 0, & \text{when } i=j\end{cases}$, then A^2 is (a) $\begin{bmatrix}1&0\\1&0\end{bmatrix}$ (b) $\begin{bmatrix}1&1\\0&0\end{bmatrix}$ (c) $\begin{bmatrix}1&1\\1&0\end{bmatrix}$ (d) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$	

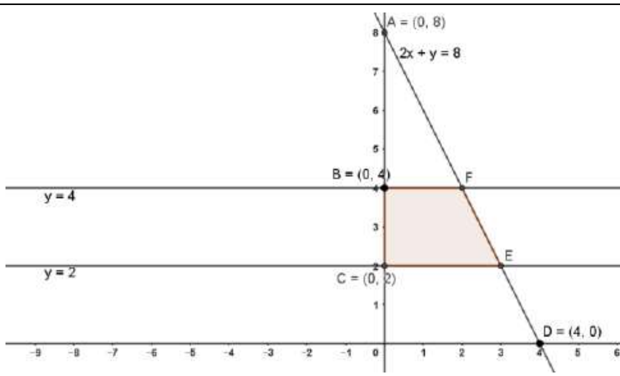
Sol.	(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	**1**
5.	The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is (a) 10 (b) 8 (c) 7 (d) –7	
Sol.	(d) $\mathbf{-7}$	**1**
6.	The function f(x) = [x], where [x] denotes the greatest integer less than or equal to x, is continuous at (a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$	
Sol.	(b) x = 1·5	**1**
7.	The derivative of x^{2x} w.r.t. x is (a) x^{2x-1} (b) $2x^{2x} \log x$ (c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$	
Sol.	(c) $2x^{2x}(1 + \log x)$	**1**
8.	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasin is (a) $(-1, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -2)$ (d) $[-1, 1]$	
Sol.	(b) $(\mathbf{-2, -1})$	**1**
9.	The function $f(x) = x \mid x \mid, x \in R$ is differentiable (a) only at $x = 0$ (b) only at $x = 1$ (c) in R (d) in R – {0}	
Sol.	(c) in R	**1**
10.	$\int \frac{\sec x}{\sec x - \tan x} dx$ equals (a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$ (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$	
Sol.	(b) $\mathbf{\sec x + \tan x + c}$	**1**

11.	The value of $\int_0^{\frac{\pi}{4}} (\sin 2x)\, dx$ is (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$	
Sol.	(c) $\frac{1}{2}$	**1**
12.	The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is (a) 2 (b) 3 (c) 5 (d) 0	
Sol.	Due to error in the question, 1 mark should be awarded to each student who attempted the question	**1**
13.	Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if (a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ (c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$	
Sol.	(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$	**1**
14.	A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is (a) $\frac{11}{15}$ (b) $\frac{11}{5\sqrt{3}}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5\sqrt{3}}$	
Sol.	(a) $\frac{11}{15}$	**1**
15.	The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is (a) $0°$ (b) $30°$ (c) $45°$ (d) $90°$	
Sol.	**(d) 90°**	**1**

16.	If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	
Sol.	(a) $\mathbf{0}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	**1**
17.	If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to (a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$	
Sol.	(c) $\frac{7}{8}$	**1**
18.	If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then P(B'/A) is (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{3}{4}$ (d) 1	
Sol.	(c) $\frac{3}{4}$	**1**
	Assertion – Reason Based Questions In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices : (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true, but (R) is not the correct explanation of (A). (c) (A) is true and (R) is false. (d) (A) is false, but (R) is true.	

19.	**Assertion (A) :** $\int_{2}^{8} \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$ **Reason (R) :** $\int_{a}^{b} f(x)\, dx = \int_{a}^{b} f(a+b-x)\, dx$	
Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
20.	**Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$. **Reason (R) :** Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.	
Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
	SECTION B This section comprises very short answer (VSA) type questions of **2 marks each.**	
21.	Draw the graph of the principal branch of the function $f(x) = \cos^{-1} x$.	
Sol.		**2**

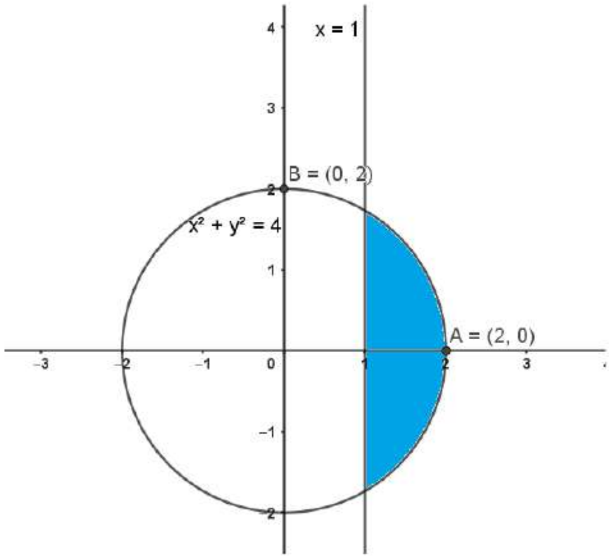
22(a).	If the vectors $\vec{a}$ and $\vec{b}$ are such that $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.	
Sol.	Let θ be the angle between $\vec{a}$ and $\vec{b}$ Since $\vec{a} \times \vec{b}$ is a unit vector, we have $\lvert\vec{a} \times \vec{b}\rvert = 1$ $\Rightarrow \lvert\vec{a}\rvert\ \lvert\vec{b}\rvert \sin\theta = 1$ $\Rightarrow \sin\theta = \frac{1}{2}$, or $\theta = 30°$ (or $\frac{\pi}{6}$)	 **1** **1**
22(b).	Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.	
Sol.	Here $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$ $\Rightarrow \lvert\vec{a} \times \vec{b}\rvert = \sqrt{400 + 25 + 25} = \sqrt{450}$ Area of parallelogram = $\lvert\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}\rvert = \sqrt{450} = 15\sqrt{2}$	 $\mathbf{1\frac{1}{2}}$ $\mathbf{\frac{1}{2}}$
23(a).	If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.	
Sol.	Here RHD $= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 2$ LHD $= \lim_{h \to 0} \left[\frac{f(1-h) - f(1)}{-h}\right] = 1$ Since RHD ≠ LHD	 $\mathbf{1\frac{1}{2}}$

	∴ f is not differentiable at x = 1.	$\frac{1}{2}$
23(b).	Find the value(s) of 'λ', if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.	
Sol.	$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{\sin^2 \lambda x}{x^2}\right) = \lim_{x \to 0} \left[\frac{\sin^2 \lambda x}{(\lambda x)^2} \cdot \lambda^2\right] = \lambda^2$ Since f(x) is continuous at x = 0 $\lim_{x \to 0} f(x) = f(0)$ $\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$	**1** $\frac{1}{2}$ $\frac{1}{2}$
24.	Sketch the region bounded by the lines 2x + y = 8, y = 2, y = 4 and the y-axis. Hence, obtain its area using integration.	
Sol.	 Required area = $\int_2^4 \frac{1}{2} (8 - y)dy$ $= \frac{1}{2} \left[8y - \frac{y^2}{2}\right]_2^4$ = 5	**$\frac{1}{2}$ for correct figure** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

25.	Find the angle between the following two lines : $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$; $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$	
Sol.	Let θ be the angle between the given lines. Then $\cos\theta = \left\|\frac{(3i + 2j + 6k).(i + 2j + 2k)}{\sqrt{9+4+36}\sqrt{1+4+4}}\right\| = \frac{19}{21}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$	$1\frac{1}{2}$ $\frac{1}{2}$
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	Using determinants, find the area of ΔPQR with vertices P(3, 1), Q(9, 3) and R(5, 7). Also, find the equation of line PQ using determinants.	
Sol.	Area = $\frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix}$ $= \frac{1}{2}\{3(-4) - 1(4) + 1(48)\} = 16$ sq. unit Equation of PQ is $\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$ **$-2x + 6y = 0$ or $x - 3y = 0$**	**1** $\frac{1}{2}$ **1** $\frac{1}{2}$
27(a).	Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.	
Sol.	Let x = sin θ. Then	

	$U = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$ $= \sec^{-1}(\sec\theta) = \theta = sin^{-1}x$ $\Rightarrow \frac{dU}{dx} = \frac{1}{\sqrt{1-x^2}}$ and $V = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$ $= \sin^{-1}[2\sin\theta\cos\theta] = 2\theta = 2\,sin^{-1}x$ $\Rightarrow \frac{dV}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\Rightarrow \frac{dU}{dV} = \frac{dU/dx}{dV/dx} = \frac{1}{2}$ **Note: If the substitution is made as $x = \cos\theta$, answer will be $-\frac{1}{2}$**	 **1** **1** **1**
27(b).	If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.	
Sol.	$y = \tan x + \sec x = \frac{\sin x + 1}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{\cos x(\cos x) + (\sin x + 1)\sin x}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1+\sin x}{1-\sin^2 x} = \frac{1}{1-\sin x}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-\sin x).0 - 1(0-\cos x)}{(1-\sin x)^2} = \frac{\cos x}{(1-\sin x)^2}$	 **$1\frac{1}{2}$** **$1\frac{1}{2}$**

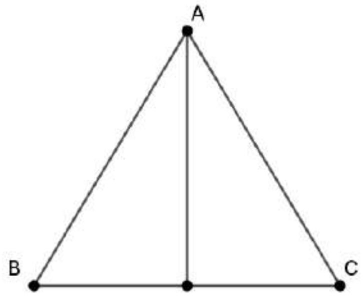
28(a).	Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x}\, dx$	
Sol.	$I = \int_{-\pi/4}^{\pi/4} \frac{\cos 2x}{1+\cos 2x}\, dx = 2\int_{0}^{\pi/4} \frac{\cos 2x}{1+\cos 2x}\, dx$ $= 2\int_0^{\pi/4}\left(1 - \frac{1}{1+\cos 2x}\right) dx$ $= 2\int_0^{\pi/4}\left(1 - \frac{1}{2\cos^2 x}\right) dx$ $= 2\int_0^{\pi/4}\left(1 - \frac{1}{2} sec^2 x\right) dx$ $= (2x - \tan x)\Big\|_0^{\pi/4}$ $= \left(\frac{\pi}{2} - 1\right)$	**1** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
28(b).	Find : $\int e^{x^2}(x^5 + 2x^3)\, dx$	
Sol.	Let $I = \int e^{x^2}(x^5 + 2x^3)\, dx$ Put $x^2 = t$ so that $2x\, dx = dt$ $\therefore I = \frac{1}{2}\int e^t(t^2 + 2t)\, dt$ $= \frac{1}{2} e^t t^2 + C$ **$= \frac{1}{2} e^{x^2}(x^4) + C$**	$\frac{1}{2}$ **1** **1** $\frac{1}{2}$

29.	Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.	
Sol.	 Required area $= 2\int_1^2 \sqrt{4-x^2}\,dx$ $= 2\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$ $= 2\left[\left\{0 + 2\left(\frac{\pi}{2}\right)\right\} - \left\{\frac{1}{2}\sqrt{3} + 2\cdot\frac{\pi}{6}\right\}\right]$ $= 2\left(\pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right)$ $= \left(\frac{4\pi}{3} - \sqrt{3}\right)$	**1 for correct figure** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

30.	Find the distance between the lines : $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$	
	Here $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$ Here, $\vec{b_1}$ and $\vec{b_2}$ are parallel vectors. $\vec{a_2} - \vec{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}, \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ Thus, $(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$ Distance between the lines = $\left\| \frac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{\|\vec{b}\|} \right\|$ $= \frac{\sqrt{81 + 196 + 16}}{\sqrt{4 + 9 + 36}}$ $= \frac{\sqrt{293}}{7}$ units.	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
31(a).	Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.	
Sol.	General point on the given line is M $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$	

	Direction ratios of PM are $5\lambda - 3, 2\lambda - 1, 3\lambda - 7$ If this point is the foot of the perpendicular from the point P (0, 2, 3), then PM is perpendicular to the line. Thus, $(5\lambda - 3).5 + (2\lambda - 1).2 + (3\lambda - 7).3 = 0$ $\Rightarrow \lambda = 1$ **Hence co-ordinates of M are (2, 3, – 1)**	**1** **1** **1**
31(b).	Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\lvert \vec{a} \rvert = 3$, $\lvert \vec{b} \rvert = 4$ and $\lvert \vec{c} \rvert = 2$.	
Sol.	$(\vec{a} + \vec{b} + \vec{c})^2 = 0$ $\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\mu) = 0$ $\mu = -\frac{29}{2}$	**1** **1** **1**
	SECTION D This section comprises of Long Answer (LA) type questions of **5 marks each.**	
32.	Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$	
Sol.	$I = \int_0^{\pi} \frac{x}{1+\sin x}\, dx = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\, dx$ $= \int_0^{\pi} \frac{\pi - x}{1+\sin x}\, dx$	**1**

	$= \int_0^{\pi} \frac{\pi}{1+\sin x}\,dx - \int_0^{\pi} \frac{x}{1+\sin x}\,dx$	
	$\Rightarrow 2I = \pi \int_0^{\pi} \frac{dx}{1+\sin x}$	**1**
	$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{dx}{1+\sin x} = \frac{\pi}{2} \int_0^{\pi} \frac{dx}{1+\cos(\pi/2 - x)}$	**1**
	$= \frac{\pi}{2} \int_0^{\pi} \frac{dx}{2\cos^2(\pi/4 - x/2)}$	
	$= \frac{\pi}{4} \int_0^{\pi} \sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right) dx$	**1**
	$= \frac{\pi}{4} \left| -2\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \right|_0^{\pi}$	
	$= -\frac{\pi}{2}\left[\tan\left(-\frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{4}\right)\right]$	
	$= \pi$	**1**
33(a).	The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.	
Sol.	In an equilateral triangle, median is same as altitude. Let ‘h’ denote the length of the median (or altitude) and ‘x’ be the side of Δ ABC.	**1**

	 Then, h = $\frac{\sqrt{3}}{2}$x or x = $\frac{2h}{\sqrt{3}}$ ________ (i) It is given that $\frac{dh}{dt} = 2\sqrt{3}$ So, by (i) we have $\frac{dx}{dt} = \frac{2}{\sqrt{3}}\frac{dh}{dt} \Rightarrow \frac{dx}{dt} = 4$ **Thus, the side of Δ ABC is increasing at the rate of 4 cm/sec.**	**2** **1** **1**
33(b).	Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.	
Sol.	Let the two numbers be x and y. Then, x + y = 5 or y = 5 – x Let S denote the sum of the cubes of these numbers. Then $S = x^3 + y^3 = x^3 + (5-x)^3$ $\frac{dS}{dx} = 3x^2 - 3(5-x)^2 = 15(2x-5)$ Now $\frac{dS}{dx}$ = 0, gives $x = \frac{5}{2}$ Showing S is minimum at $x = \frac{5}{2}$	$\frac{1}{2}$ **1** **1** $\frac{1}{2}$ **1**

	So, the two numbers are $\frac{5}{2}$ and $\frac{5}{2}$ $\Rightarrow x^2 + y^2 = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}$	**1**
34(a).	In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?	
Sol.	Let events A, B and E be defined as: A : Student knows the answer B : Student guesses the answer E : student answered correctly $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$ Here, $P\left(\frac{E}{A}\right) = 1$ and $P\left(\frac{E}{B}\right) = \frac{1}{3}$ By Bayes' Theorem	**1** **$\frac{1}{2}$** **1**

	$P\left(\frac{A}{E}\right) = \dfrac{P(A) \,.\, P\left(\frac{E}{A}\right)}{P(A) \,.\, P\left(\frac{E}{A}\right) + P(B) \,.\, P\left(\frac{E}{B}\right)}$ $= \dfrac{\frac{3}{5} \times 1}{\left(\frac{3}{5} \times 1\right) + \left(\frac{2}{5} \times \frac{1}{3}\right)}$ $= \frac{9}{11}$	**$1\frac{1}{2}$** **1**
34(b).	A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.	
Sol.	Let X denote the prize value. Here X can take values of 8, 4 and 2. $P(X = 8) = \frac{2}{10}$, or $\frac{1}{5}$ $P(X = 4) = \frac{5}{10}$, or $\frac{1}{2}$ $P(X = 2) = \frac{3}{10}$	**1** **3**

X	8	4	2
P(X)	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{3}{10}$

XP(X)	$\frac{8}{5}$	$\frac{4}{2}$	$\frac{6}{10}$

Hence, Mean value of X = $\frac{8}{5} + 2 + \frac{6}{10}$

$= \frac{42}{10}$ or ₹ 4.20 **1**

35. Solve the following Linear Programming Problem graphically :

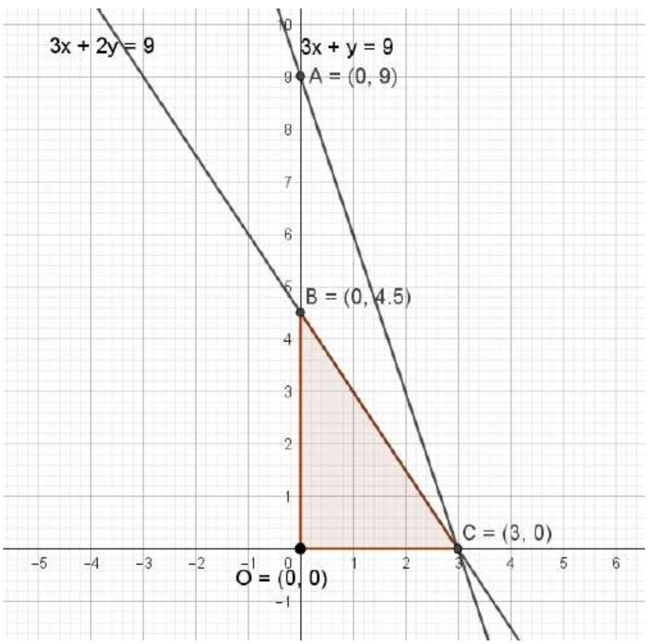
Maximize : $P = 70x + 40y$

subject to : $3x + 2y \leq 9$,

$3x + y \leq 9$,

$x \geq 0,\ y \geq 0$

Sol.



3 for correct figure and shading

Corner Points	Value of P
O (0,0)	0
B (0,4.5)	180
C (3,0)	210 → Max Value

$1\frac{1}{2}$

Maximum value of P =210 at $x = 3$ and y=0 $\frac{1}{2}$

SECTION E

This section comprises of 3 case-study based questions of 4 marks each.

36. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions :

(I) Convert the given above situation into a matrix equation of the form AX = B.

(II) Find |A|.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Sol.

(I) Matrix equation is AX = B, where

$$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \; B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$ **1**

	where x is the number of pens bought, y the number of bags and z the number of instrument boxes.	
	(II) $\lvert A \rvert = 5(4-6) - 3(8-3) + 1(4-1) = -22$	**1**
	(III) $\text{adj}(A) = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	**1**
	$\Rightarrow A^{-1} = \dfrac{1}{(-22)} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	**1**
	OR	
	$P = A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$	**1+$\frac{1}{2}$**
	$= \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$	**$\frac{1}{2}$**

37.	An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.  Based on the above information, answer the following questions : (I) How many relations are possible from B to G ? (II) Among all the possible relations from B to G, how many functions can be formed from B to G ? (III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation. **OR** (III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.	
Sol.	(I) Number of relations = $2^6 = 64$ (II) Number of possible functions = $2^3 = 8$ (III) R is an equivalence relation as it is reflexive, symmetric and transitive **OR** Since f is not one-one function	**1** **1** **2** **1**

	$\therefore f$ is not bijective	**1**
38.	An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables. Based on the above, answer the following questions : (I) Show that $(x^2 - y^2)\, dx + 2xy\, dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$. (II) Solve the above equation to find its general solution.	
Sol.	(I) $(x^2 - y^2)dx + 2xydy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ $= \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$ $= g\left(\frac{y}{x}\right)$ (II) $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$ $v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{-1 - v^2}{2v}$	**1** **1** $\mathbf{\frac{1}{2}}$

	$\Rightarrow \int \frac{2v}{1+v^2}\,dv = -\int \frac{dx}{x}$ $\log \lvert 1 + v^2 \rvert + \log \lvert x \rvert = \log C$ or $x\left(1+\frac{y^2}{x^2}\right) = C$ or $x^2 + y^2 = Cx$	**1** $\frac{1}{2}$

Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Senior School Certificate Examination, 2023** **MATHEMATICS PAPER CODE 65/5/3**
General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “**Guidelines for spot Evaluation**” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/5/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	Let R be a relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Then (a) $(8, 7) \in R$ (b) $(6, 8) \in R$ (c) $(3, 8) \in R$ (d) $(2, 4) \in R$	
Sol.	**(b) (6, 8)∈R**	**1**
2.	If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, where A^T is the transpose of the matrix A, then (a) $x = 0, y = 5$ (b) $x = y$ (c) $x + y = 5$ (d) $x = 5, y = 0$	
Sol.	**(b) $x = y$**	**1**
3.	$\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$	
Sol.	**(a) 1**	**1**
4.	If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals (a) A (b) A + I (c) I – A (d) A – I	
Sol.	**(c) I – A**	**1**

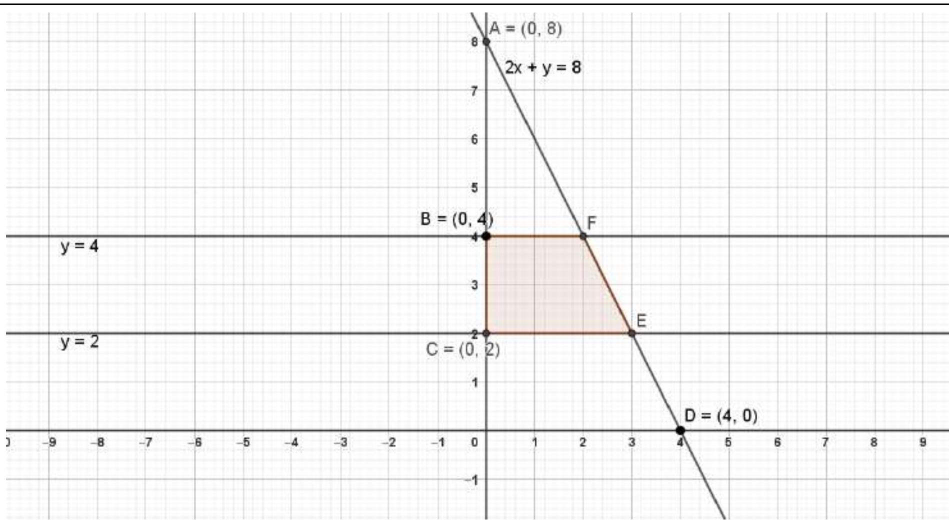
5.	If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is (a) 1 (b) 2 (c) 3 (d) 4	
Sol.	**(d) 4**	**1**
6.	If $f(x) = \lvert\cos x\rvert$, then $f\left(\frac{3\pi}{4}\right)$ is (a) 1 (b) -1 (c) $\frac{-1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$	
Sol.	**(d) $\frac{1}{\sqrt{2}}$**	**1**
7.	If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to (a) x (b) $-x$ (c) $16x$ (d) $-16x$	
Sol.	**(d) $-16\,x$**	**1**
8.	The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at (a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$	
Sol.	**(b) x = 1·5**	**1**
9.	The function $f(x) = x^3 + 3x$ is increasing in interval (a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) $\mathbb{R}$ (d) $(0, 1)$	
Sol.	**(c) R**	**1**
10.	$\int_{-1}^{1} \frac{\lvert x-2\rvert}{x-2}\,dx,\ x \neq 2$ का मान है : (a) 1 (b) -1 (c) 2 (d) -2	

Sol.	**(d) −2**	**1**
11.	$\int \frac{\sec x}{\sec x - \tan x}\, dx$ equals (a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$ (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$	
Sol.	**(b)** $\sec x + \tan x + c$	**1**
12.	The order and the degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$ respectively are : (a) $1, \frac{2}{3}$ (b) 3, 1 (c) 3, 3 (d) 1, 2	
Sol.	**(b) 3, 1**	**1**
13.	If $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$, then $\vec{a}$ is (a) $\hat{k}$ (b) $\hat{i}$ (c) $\hat{j}$ (d) $\hat{i} + \hat{j} + \hat{k}$	
Sol.	**(b)** $\hat{i}$	**1**
14.	Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is (a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$	
Sol.	**(c)** $\frac{31}{32}$	**1**
15.	If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to (a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$	
Sol.	**(c)** $\frac{7}{8}$	**1**

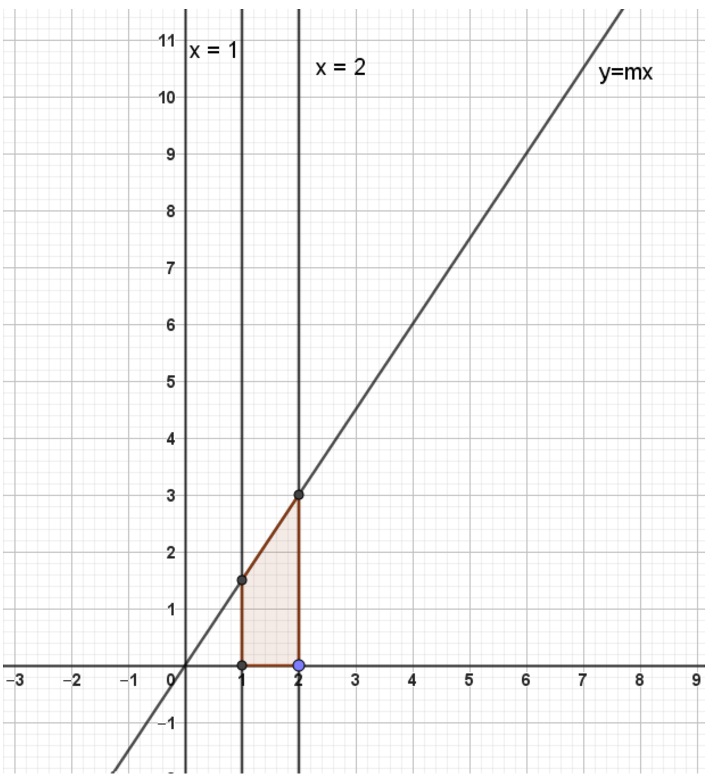
16.	The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is (a) 0° (b) 30° (c) 45° (d) 90°	
Sol.	**(d) 90°**	**1**
17.	If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	
Sol.	**(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$**	**1**
18.	The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is (a) 1 (b) 5 (c) 7 (d) 12	
Sol.	**(c) 7**	**1**
	Assertion – Reason Based Questions In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices : (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true, but (R) is not the correct explanation of (A). (c) (A) is true and (R) is false. (d) (A) is false, but (R) is true.	
19.	**Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx = 3$ **Reason (R) :** $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$	
Sol.	**(a) Both (A) and (R) are true and (R) is the correct explanation of (A)**	**1**

20.	**Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$. **Reason (R) :** Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.	
Sol.	**(a) Both (A) and (R) are true and (R) is the correct explanation of (A)**	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21(a).	(a) Find the value of k for which the function f given as $f(x) = \begin{cases} \frac{1 - \cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.	
Sol.	Here, $f(x) = \frac{1 - \cos x}{2x^2} = \frac{2\sin^2 \frac{x}{2}}{2x^2} = \left(\frac{\sin \frac{x}{2}}{2\frac{x}{2}}\right)^2$ $\Rightarrow \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{4}\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right) = \frac{1}{4}$ So, if f is continuous at x = 0, then $f(0) = \lim_{x \to 0} f(x)$ $\Rightarrow k = \frac{1}{4}$	**1** $\frac{1}{2}$ $\frac{1}{2}$

21(b).	If $x = a\cos t$ and $y = b\sin t$, then find $\frac{d^2y}{dx^2}$.	
Sol.	Given x = a cos t and y = b sin t, we have $\frac{dx}{dt} = -a\sin t$ and $\frac{dy}{dt} = b\cos t$ $\therefore \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{b\cos t}{-a\sin t} = -\frac{b}{a}\cot t$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(-\frac{b}{a}\cot t\right) \cdot \frac{dt}{dx}$ $= \frac{b}{a}\operatorname{cosec}^2 t \cdot \frac{1}{-a\sin t}$ $= -\frac{b}{a^2} \cdot \frac{1}{\sin^3 t}$ or $-\frac{b}{a^2}\operatorname{cosec}^3 t$	**1** $\mathbf{\frac{1}{2}}$ $\mathbf{\frac{1}{2}}$
22.	Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1} 1$.	
Sol.	$\tan^{-1}\left[2\cos\left(2 \cdot \frac{\pi}{6}\right)\right] + \frac{\pi}{4}$ $= \tan^{-1}\left[2 \times \frac{1}{2}\right] + \frac{\pi}{4} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$	**1** **1**
23.	Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.	

Sol.	The given line is $\frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}$, or $\frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$ So, the required vector equation of the line passing through (1,2,-1) is $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$ Cartesian equation of the line is $\boldsymbol{\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}}$	**1** $\frac{1}{2}$ $\frac{1}{2}$
24.	Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.	
Sol.	 Required area = $\int_2^4 \frac{1}{2}\,(8-y)dy$ $= \frac{1}{2}\left\|8y - \frac{y^2}{2}\right\|_2^4$ = 5	**$\frac{1}{2}$ for correct figure** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

25(a).	If the vectors $\vec{a}$ and $\vec{b}$ are such that $\|\vec{a}\| = 3$, $\|\vec{b}\| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.	
Sol.	Let θ be the angle between $\vec{a}$ and $\vec{b}$ Since $\vec{a} \times \vec{b}$ is a unit vector, we have $\|\vec{a} \times \vec{b}\| = 1$ $\Rightarrow \|\vec{a}\| \, \|\vec{b}\| \sin\theta = 1$ **$\Rightarrow \sin\theta = \frac{1}{2}$, or $\theta = 30°$ (or $\frac{\pi}{6}$)**	 **1** **1**
25(b).	Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.	
Sol.	Here $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$ $\Rightarrow \|\vec{a} \times \vec{b}\| = \sqrt{400 + 25 + 25} = \sqrt{450}$ Area of parallelogram = $\|\boldsymbol{\vec{a}} \times \boldsymbol{\vec{b}}\| = \sqrt{450} = 15\sqrt{2}$	 **$1\frac{1}{2}$** **$\frac{1}{2}$**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	Show that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.	

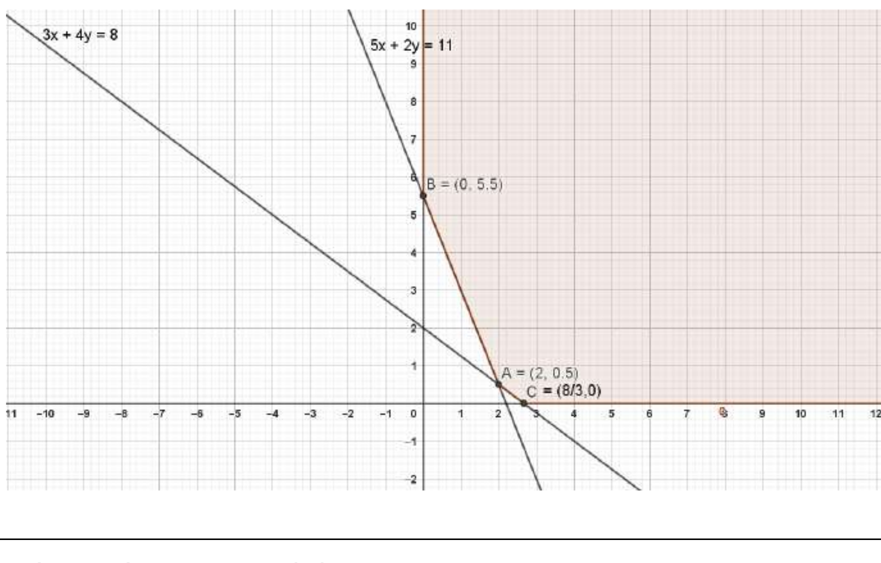
Sol.	$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ $= x(-x^2 - 1) - \sin\theta\,(-x\sin\theta - \cos\theta) + \cos\theta\,(-\sin\theta + x\cos\theta)$ $= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$ $= -x^3 - x + x$ $= -x^3$, independent of θ	$1\frac{1}{2}$ $1\frac{1}{2}$
27.	Using integration, find the area of the region bounded by $y = mx$ $(m > 0)$, $x = 1$, $x = 2$ and the x-axis.	
Sol.		**1 mark for correct figure**

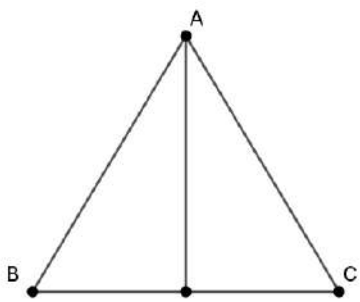
	Required area = $\int_1^2 (mx)dx$	**1**
	$= m\left[\frac{x^2}{2}\right]_1^2$	**$\frac{1}{2}$**
	$= \frac{3}{2}m$	**$\frac{1}{2}$**
28(a).	Find the coordinates of the foot of the perpendicular drawn from point (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.	
Sol.	A general point on the given line is M(3λ + 15, 8λ + 29, – 5λ + 5). DRs of $\overrightarrow{MP}$ are $(3\lambda + 10, 8\lambda + 22, -5\lambda + 2)$ This general point for some specific value of λ will be the foot of the perpendicular drawn from (5, 7, 3) on the given line if PM ⊥ line. i.e. if (3λ + 10) (3) + (8λ + 22) (8) + (– 5λ + 2) (– 5) = 0 ⇒ λ = – 2 Hence, M is (9, 13, 15) is the required foot of the perpendicular.	**1** **$\frac{1}{2}$** **$\frac{1}{2}$** **$\frac{1}{2}$** **$\frac{1}{2}$**
28(b).	If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.	

Sol.	$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ $\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$ $\therefore \perp$ vector, $\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$ $\vec{c} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ $\quad\quad |\vec{c}| = \sqrt{24} = 2\sqrt{6}$ Unit vector = $\frac{-2}{2\sqrt{6}}\hat{i} + \frac{4}{2\sqrt{6}}\hat{j} - \frac{2}{2\sqrt{6}}\hat{k}$ or $-\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
29.	Find the distance between the lines : $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$	
Sol.	Here $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b}_2 = 4\hat{i} + 6\hat{j} + 12\hat{k}$ Here, $\vec{b}_1$ and $\vec{b}_2$ are parallel vectors.	$\frac{1}{2}$

	$\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}, \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ Thus, $(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$ Distance between the lines = $\left\lvert \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}}{\lvert\vec{b}\rvert} \right\rvert$ $= \frac{\sqrt{81 + 196 + 16}}{\sqrt{4 + 9 + 36}}$ $= \frac{\sqrt{293}}{7}$ units.	$\frac{1}{2}$ 1 1
30(a).	Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.	
Sol.	Let x = sin θ. Then $U = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$ $= \sec^{-1}(\sec\theta) = \theta = \sin^{-1}x$ $\Rightarrow \frac{dU}{dx} = \frac{1}{\sqrt{1-x^2}}$ and $V = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$ $= \sin^{-1}[2\sin\theta\cos\theta] = 2\theta = 2\sin^{-1}x$ $\Rightarrow \frac{dV}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\Rightarrow \boldsymbol{\frac{dU}{dV} = \frac{dU/dx}{dV/dx} = \frac{1}{2}}$	 1 1 1

	Note: If the substitution is made as $x = \cos\theta$, answer will be $-\frac{1}{2}$	
30(b).	If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.	
Sol.	$y = \tan x + \sec x = \frac{\sin x + 1}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{\cos x\,(\cos x) + (\sin x + 1)\sin x}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1 + \sin x}{1 - \sin^2 x} = \frac{1}{1 - \sin x}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-\sin x)\,.\,0 - 1\,(0 - \cos x)}{(1-\sin x)^2} = \frac{\cos x}{(1-\sin x)^2}$	$1\frac{1}{2}$ $1\frac{1}{2}$
31(a).	Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$	
Sol.	Let $I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi - x)}}\, dx$ $= \int_0^{2\pi} \frac{1}{1+e^{-\sin x}} dx = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1} dx$ $\Rightarrow 2I = \int_0^{2\pi} \frac{e^{\sin x}+1}{e^{\sin x}+1}\, dx = \int_0^{2\pi} 1\,.\,dx = 2\pi$ $\Rightarrow$ **$I = \pi$**	1 1 $\frac{1}{2}$ $\frac{1}{2}$
31(b).	Find : $\int \frac{x^4}{(x-1)(x^2+1)}\, dx$	

Sol.	$I = \int \frac{x^4}{(x-1)(x^2+1)} dx = \int \left[x + 1 + \frac{1}{(x-1)(x^2+1)} \right] dx$ $= \frac{x^2}{2} + x + \int \left[\frac{1}{2(x-1)} - \frac{1}{2} \frac{(x+1)}{(x^2+1)} \right] dx$ (Using partial fractions) $= \frac{x^2}{2} + x + \frac{1}{2} \log \lvert x-1 \rvert - \frac{1}{4} \log \lvert x^2+1 \rvert - \frac{1}{2} \tan^{-1} x + C$	**1** $\frac{1}{2}$**+1** $\frac{1}{2}$
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32.	Solve the following Linear Programming Problem graphically : Minimise : $Z = 60x + 80y$ subject to constraints : $3x + 4y \geq 8$ $5x + 2y \geq 11$ $x, y \geq 0$	
Sol.		**3 Marks for correct graph**

	$Z = 60x + 80y$ $Z_B = 0 + 440 = 440$ $Z_A = 120 + 40 = 160$ $Z_C = 160$ Minimum Z = 160 at all points of the line AC	**1** **1**
33(a).	The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.	
Sol.	In an equilateral triangle, median is same as altitude. Let 'h' denote the length of the median (or altitude) and 'x' be the side of Δ ABC.  Then, $h = \frac{\sqrt{3}}{2}x$ or $x = \frac{2h}{\sqrt{3}}$ ________ (i) It is given that $\frac{dh}{dt} = 2\sqrt{3}$ So, by (i) we have	**1** **2** **1**

	$\frac{dx}{dt} = \frac{2}{\sqrt{3}}\frac{dh}{dt} \Rightarrow \frac{dx}{dt} = 4$ **Thus, the side of Δ ABC is increasing at the rate of 4 cm/sec.**	**1**
33(b).	Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.	
Sol.	Let the two numbers be x and y. Then, x + y = 5 or y = 5 – x Let S denote the sum of the cubes of these numbers. Then $S = x^3 + y^3 = x^3 + (5-x)^3$ $\frac{dS}{dx} = 3x^2 - 3(5-x)^2 = 15(2x-5)$ Now $\frac{dS}{dx} = 0$, gives $x = \frac{5}{2}$ Showing S is minimum at $x = \frac{5}{2}$ So, the two numbers are $\frac{5}{2}$ and $\frac{5}{2}$ $\Rightarrow \boldsymbol{x^2 + y^2 = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}}$	$\frac{1}{2}$ **1** **1** $\frac{1}{2}$ **1** **1**
34.	Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$	
Sol.	Let I = $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)\, dx$. $= \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1}(\sin x)\, dx$	$\frac{1}{2}$

	Put sin x = t so that cos x dx = dt	$\frac{1}{2}$
	Thus, $I = 2\int_0^1 t \tan^{-1} t \, dt$	$\frac{1}{2}$
	$= 2\left[\left\|\frac{t^2}{2}\tan^{-1} t\right\|_0^1 - \int_0^1 \frac{1}{1+t^2} \cdot \frac{t^2}{2}\, dt\right]$	1
	$= 2 \cdot \frac{1}{2} \cdot \frac{\pi}{4} - \int_0^1 \frac{t^2}{1+t^2}\, dt$	
	$= \frac{\pi}{4} - \int_0^1 \left[1 - \frac{1}{1+t^2}\right] dt$	1
	$= \frac{\pi}{4} - \| t \|_0^1 + \left\|\tan^{-1} t\right\|_0^1$	1
	$= \frac{\pi}{4} - 1 + \frac{\pi}{4}$	
	$= \frac{\pi}{2} - 1$	$\frac{1}{2}$
35(a).	In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?	
Sol.		

	Let events A, B and E be defined as: A : Student knows the answer B : Student guesses the answer E : student answered correctly	1
	$P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$	$\frac{1}{2}$
	Here, $P\left(\frac{E}{A}\right) = 1$ and $P\left(\frac{E}{B}\right) = \frac{1}{3}$	1
	By Bayes' Theorem $P\left(\frac{A}{E}\right) = \frac{P(A).P\left(\frac{E}{A}\right)}{P(A).P\left(\frac{E}{A}\right)+P(B).P\left(\frac{E}{B}\right)}$	
	$= \frac{\frac{3}{5}\times 1}{\left(\frac{3}{5}\times 1\right)+\left(\frac{2}{5}\times\frac{1}{3}\right)}$	$1\frac{1}{2}$
	$= \frac{9}{11}$	1
35(b).	A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.	

Sol.		
	Let X denote the prize value. Here X can take values of 8, 4 and 2. $P(X = 8) = \frac{2}{10}$, or $\frac{1}{5}$ $P(X = 4) = \frac{5}{10}$, or $\frac{1}{2}$ $P(X = 2) = \frac{3}{10}$	**1**
	<table><tr><td>X</td><td>8</td><td>4</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{1}{5}$</td><td>$\frac{1}{2}$</td><td>$\frac{3}{10}$</td></tr><tr><td>XP(X)</td><td>$\frac{8}{5}$</td><td>$\frac{4}{2}$</td><td>$\frac{6}{10}$</td></tr></table>	**3**
	Hence, Mean value of X = $\sum X\, P(X) = \frac{8}{5} + 2 + \frac{6}{10}$ $= \frac{42}{10}$ or ₹ 4.20	**1**

36.

An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective. Justify your answer.

Sol.

(I) Number of relations = $2^6 = 64$ **1**

(II) Number of possible functions = $2^3 = 8$ **1**

(III) R is an equivalence relation as it is reflexive, symmetric and transitive **2**

OR

Since f is not one-one function **1**

$\therefore f$ is not bijective **1**

37.	Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250. Based on the above information, answer the following questions : (I) Convert the given above situation into a matrix equation of the form AX = B. (II) Find $\|A\|$. (III) Find A^{-1}. **OR** (III) Determine $P = A^2 - 5A$.	
Sol.	(I) Matrix equation is AX = B, where $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$	**1**
	where x is the number of pens bought, y the number of bags and z the number of instrument boxes.	
	(II) $\|A\| = 5(4-6) - 3(8-3) + 1(4-1) = -22$	**1**
	(III) $\text{adj}(A) = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	**1**
	$\Rightarrow A^{-1} = \frac{1}{(-22)} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	**1**
	OR	

	$P = A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$ $= \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$	**$1+\frac{1}{2}$** **$\frac{1}{2}$**
38.	An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables. Based on the above, answer the following questions : (I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$. (II) Solve the above equation to find its general solution.	
Sol.	(I) $(x^2 - y^2)dx + 2xydy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ $= \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$ $= g\left(\frac{y}{x}\right)$	**1** **1**

	(II) $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$	$\frac{1}{2}$
	$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{-1 - v^2}{2v}$	
	$\Rightarrow \int \frac{2v}{1+v^2}\,dv = -\int \frac{dx}{x}$	1
	$\Rightarrow \log\left\|1 + v^2\right\| + \log\left\|x\right\| = \log C$	
	or $x\left(1 + \frac{y^2}{x^2}\right) = C$	$\frac{1}{2}$
	or $x^2 + y^2 = Cx$	