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Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Sr. Secondary School Supplementary Examination, July- 2023
MATHEMATICS PAPER CODE 65/C/1

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**

11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

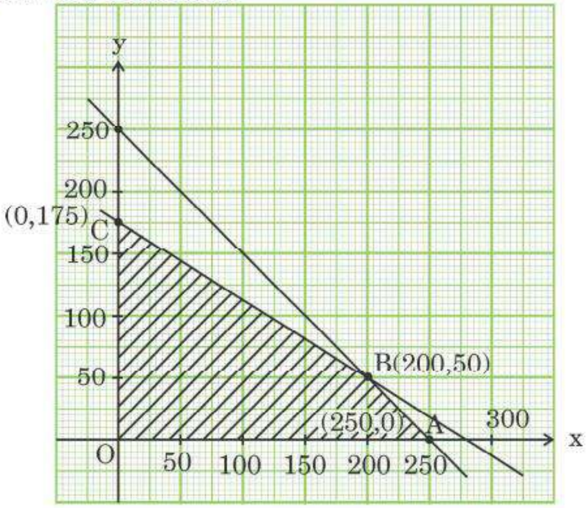
MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/C/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	If A is a square matrix of order 3 and $\|A\| = 6$, then the value of $\|\text{adj } A\|$ is : (a) 6 (b) 36 (c) 27 (d) 216	
Sol.	(b) 36	**1**
2.	The value of $\int_0^{\pi/6} \sin 3x \, dx$ is : (a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$	
Sol.	(d) $\frac{1}{3}$	**1**
3.	If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is : (a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$	
Sol.	(a) $\frac{2\pi}{3}$	**1**

4.	The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is : (a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$	
Sol.	(a) $\frac{1}{\sqrt{6}}$	**1**
5.	A family has 2 children and the elder child is a girl. The probability that both children are girls is : (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$	
Sol.	(c) $\frac{1}{2}$	**1**
6.	The vector equation of a line which passes through the point (2, – 4, 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is : (a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$ (c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$	
Sol.	(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$	**1**
7.	For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ? (a) ± 3 (b) -3 (c) ± 2 (d) 2	
Sol.	(c) ± 2	**1**
8.	The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is : (a) 5 (b) -5 (c) -11 (d) 11	
Sol.	(b) -5	**1**

9.	The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is : (a) 1 (b) 2 (c) -1 (d) 0	
Sol.	(d) 0	**1**
10.	If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is : (a) 1 (b) -2 (c) 2 (d) -1	
Sol.	(c) 2	**1**
11.	$\int \frac{\cos 2x}{\sin^2 x . \cos^2 x} dx$ is equal to (a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$ (c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$	
Sol.	(b) $-\cot x - \tan x + C$	**1**
12.	The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is (a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$	
Sol.	(a) $\frac{1}{x}$	**1**
13.	The point which lies in the half-plane $2x + y - 4 \leq 0$ is : (a) (0, 8) (b) (1, 1) (c) (5, 5) (d) (2, 2)	
Sol.	(b) (1, 1)	**1**
14.	If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :	

	(a) $\dfrac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$ (b) $\dfrac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$ (c) $\dfrac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$ (d) $\dfrac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$	
Sol.	(d) $\dfrac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$	**1**
15.	It is given that $X \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Then matrix X is : (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$	
Sol.	(c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	**1**
16.	If ABCD is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD}$ is : (a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$	
Sol.	(c) $2\overrightarrow{BC}$	**1**
17.	If $x = a \cos\theta + b \sin\theta$, $y = a \sin\theta - b \cos\theta$, then which one of the following is true ? (a) $y^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$ (b) $y^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$ (c) $y^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 0$ (d) $y^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - y = 0$	
Sol.	(a) $y^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$	**1**

18.	The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :  (a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$	
Sol.	(a) $2a = b$	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$. *Reason (R) :* Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.	
Sol.	(c) Assertion (A) is true, but Reason (R) is false.	**1**

20.	*Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus. *Reason (R) :* ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	If three non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, then show that $\vec{b} = \vec{c}$.	
Sol.	$\vec{a}.\vec{b}=\vec{a}.\vec{c}\Rightarrow\vec{a}.\left(\vec{b}-\vec{c}\right)=0$ $\Rightarrow\vec{a}=0 \text{ or } \vec{b}-\vec{c}=0 \text{ or } \vec{a}\perp\left(\vec{b}-\vec{c}\right)$ $\text{as } \vec{a}\neq 0\Rightarrow\vec{b}-\vec{c}=0 \text{ or } \vec{a}\perp\left(\vec{b}-\vec{c}\right) \quad ...(1)$ $\text{Again, } \vec{a}\times\vec{b}=\vec{a}\times\vec{c}\Rightarrow\vec{a}\times\left(\vec{b}-\vec{c}\right)=0$ $\Rightarrow\vec{a}=0 \text{ or } \vec{b}-\vec{c}=0 \text{ or } \vec{a}\parallel\left(\vec{b}-\vec{c}\right).$ $\text{as } \vec{a}\neq 0\Rightarrow\vec{b}-\vec{c}=0 \text{ or } \vec{a}\parallel\left(\vec{b}-\vec{c}\right) \quad ...(2)$ $\text{from } (1) \text{ and } (2), \vec{b}=\vec{c}$ $\quad$ ($\because \vec{a}$ can't be parallel and perpendicular to $\left(\vec{b}-\vec{c}\right)$ simultaneously.)	**1** ½ ½
22(a).	Simplify : $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$	

Sol.	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$	½
	$=\tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$	½
	$=\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$	½
	$=\frac{\pi}{4}+\frac{x}{2}$	½
	OR	
22(b).	Prove that the greatest integer function $f:\mathbb{R}\to\mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.	
Sol.	For not one-one: $1.1, 1.2 \in R$ (domain) now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow f$ is not one-one. For not onto: Let $\frac{1}{2} \in R$ (co-domain), but $[x]=\frac{1}{2}$ is not possible for x in domain. so, f is not onto.	**1** **1**
23.	Function f is defined as $f(x)=\begin{cases}2x+2, & \text{if } x<2\\ k, & \text{if } x=2\\ 3x, & \text{if } x>2\end{cases}$ Find the value of k for which the function f is continuous at x = 2.	
Sol.	As f is continuous at $x=2 \Rightarrow \lim_{x\to 2^+} f(x)=\lim_{x\to 2^-} f(x)=f(2)$ $\lim_{x\to 2^+} 3x=\lim_{x\to 2^-}(2x+2)=k$ $\Rightarrow k=6$	**1** **1**

24.	Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing.	
Sol.	$f'(x) = 4x^3 - 12x^2 + 8x = 4x(x-1)(x-2)$ $f'(x) = 0$ gives $x = 0, 1, 2$ for strictly increasing, $f'(x) > 0$ $x \in (0,1) \cup (2, \infty)$	**1** **1**
25(a).	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\|\vec{a}\| = 7$, $\|\vec{b}\| = 24$, $\|\vec{c}\| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.	
Sol.	$\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = (\vec{0})^2$ $\Rightarrow \|\vec{a}\|^2 + \|\vec{b}\|^2 + \|\vec{c}\|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow 49 + 576 + 625 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -625$	**1** **1**
	OR	
25(b).	If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.	
Sol.	d.c. are $\cos\alpha, \cos\beta, \cos\gamma$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$ $\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$	**1** **1**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	

26(a).	Evaluate : $$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$$	
Sol.	$I=\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx \quad ...(1)$ using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $I=\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$ $\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx \quad ...(2)$ adding (1) and (2) $2I=\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx=\pi\int_0^{\frac{\pi}{4}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ $\Rightarrow I=\frac{\pi}{2}\int_0^{\frac{\pi}{4}} \frac{\tan x\sec^2 x}{\left(\tan^2 x\right)^2+1}dx \quad (\because \text{dividing by} \cos^4 x)$ Putting $\tan^2 x=t$ gives $I=\frac{\pi}{4}\int_0^1 \frac{1}{t^2+1}dt$ $\Rightarrow I=\frac{\pi^2}{16}$	½ ½ **1** ½ ½
	OR	
26(b).	Evaluate : $$\int_1^3 (\lvert x-1\rvert+\lvert x-2\rvert)\,dx$$	

Sol.	$I=\int_1^3\left(\lvert x-1\rvert+\lvert x-2\rvert\right)dx$ $=\int_1^2\left[(x-1)-(x-2)\right]dx+\int_2^3\left[(x-1)+(x-2)\right]dx$ $=\int_1^2 1\,dx+\int_2^3(2x-3)\,dx$ $=\left[x\right]_1^2+\left[x^2-3x\right]_2^3$ $=1+2=3$	**1** ½ ½ **1**
27(a).	Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that y = 1 when x = 0.	
Sol.	$\frac{dy}{dx}=\frac{xy}{x^2+y^2}\quad ...(1)$ Put $\frac{y}{x}=v$ i.e. $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ Equation (1) gives $v+x\frac{dv}{dx}=\frac{v}{1+v^2}$ $\Rightarrow x\frac{dv}{dx}=-\frac{v^3}{1+v^2}$ $\Rightarrow\int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$ $\Rightarrow\frac{-1}{2v^2}+\log\lvert v\rvert=-\log\lvert x\rvert+\log c$ putting $v=\frac{y}{x}$ and simplifying gives $-\frac{x^2}{2y^2}=\log\left\lvert\frac{c}{y}\right\rvert$ now, $x=0$, $y=1$ gives $c=1$ required solution is : $\frac{x^2}{2y^2}=\log\lvert y\rvert$	½ **1** ½ ½ ½
	OR	

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27(b).	Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, given that y = 0 when x = 1.	
Sol.	Given diff. eqn. can be written as $\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{(1+x^2)^2}$ $\text{I.F.}=e^{\int\frac{2x}{1+x^2}dx}=1+x^2$ solution is given by: $y.(1+x^2)=\int\frac{1}{1+x^2}dx$ $\Rightarrow y.(1+x^2)=\tan^{-1}x+C$ Now $x=1, y=0$ gives $C=-\frac{\pi}{4}$ Reqired solution : $y.(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$	½ ½ ½ ½ ½ ½
28(a).	Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.	
Sol.	Let E_1: event of choosing bag A, E_2: event of choosing bag B, A: red ball is found here, $P(E_1)=P(E_2)=\frac{1}{2}; P(A\|E_1)=\frac{3}{5}, P(A\|E_2)=\frac{5}{9}$ $P(E_2\|A)=\frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)}$ $=\frac{\frac{5}{9}\times\frac{1}{2}}{\frac{3}{5}\times\frac{1}{2}+\frac{5}{9}\times\frac{1}{2}}=\frac{25}{52}$	½ **1** **1+ ½**
	OR	

28(b). Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

Sol. Let X be the random variable representing the number of persons who speak truth.

X can takes the values $0, 1$ and 2. ½

$P(\text{speaking truth}) = \frac{20}{50}$, $P(\text{not speaking truth}) = \frac{30}{50}$ ½

$P(X=0) = \frac{30}{50} \times \frac{29}{49} = \frac{87}{245}$

$P(X=1) = 2 \times \frac{20}{50} \times \frac{30}{49} = \frac{120}{245}$ 1½

$P(X=2) = \frac{20}{50} \times \frac{19}{49} = \frac{38}{245}$

Probability Distribution Table is given by:

X	**0**	**1**	**2**
$P(X)$	$\frac{87}{245}$	$\frac{120}{245}$	$\frac{38}{245}$

½

29. Find :

$$\int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$$

Sol.

$$I = \int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$$

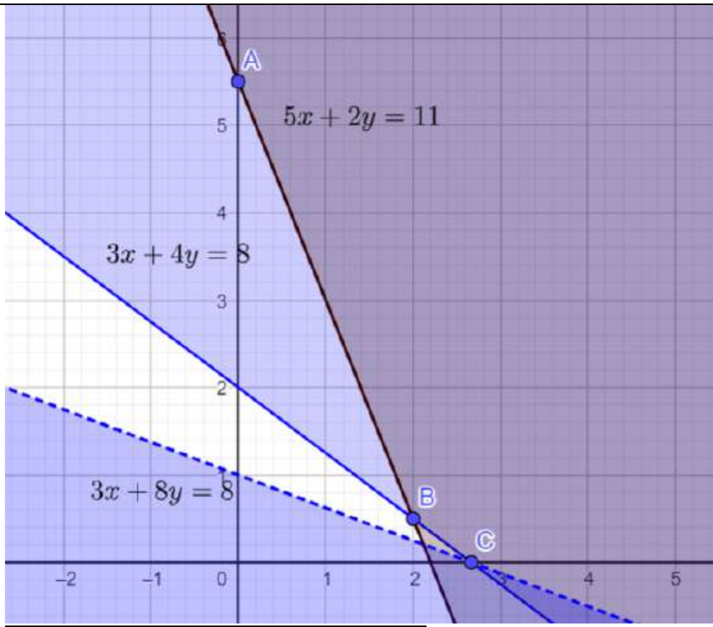
$$= \int \frac{\cos\theta}{\sqrt{\sin^2\theta - 3\sin\theta + 2}}\, d\theta$$ ½

Put $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$ ½

$$I = \int \frac{dt}{\sqrt{t^2 - 3t + 2}} = \int \frac{dt}{\sqrt{\left(t - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$ ½

$$= \log\left|\left(t - \frac{3}{2}\right) + \sqrt{t^2 - 3t + 2}\right| + C$$ 1

$$= \log\left|\left(\sin\theta - \frac{3}{2}\right) + \sqrt{\sin^2\theta - 3\sin\theta + 2}\right| + C$$ ½

30.	Solve the following Linear Programming Problem graphically : Minimise z = 3x + 8y subject to the constraints $3x + 4y \geq 8$ $5x + 2y \geq 11$ $x \geq 0, y \geq 0$	
Sol.		**Correct graph 1 mark**

Corner Point	$z = 3x + 8y$
$A\left(0, \frac{11}{2}\right)$	**44**
$B\left(2, \frac{1}{2}\right)$	**10**
$C\left(\frac{8}{3}, 0\right)$	**8**

1½

since $3x + 8y < 8$ do not have any point in common with the feasible region,

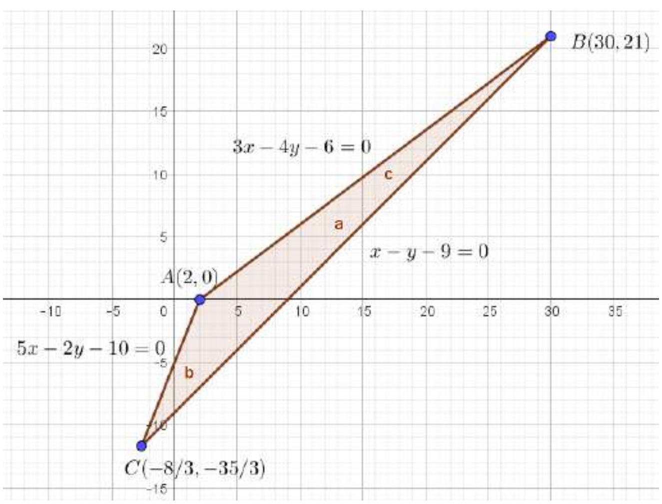
$z_{\min} = 8$ when $x = \frac{8}{3}, y = 0$ **½**

31.	Find : $\int \frac{2x^2 + 1}{x^2(x^2 + 4)} dx$	

Sol.	$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$ Let $\frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{2y+1}{y\left(y+4\right)}$, where $x^2=y$ Put $\frac{2y+1}{y\left(y+4\right)}=\frac{A}{y}+\frac{B}{y+4}$ $\Rightarrow 2y+1=A\left(y+4\right)+By$ $\Rightarrow A=\frac{1}{4},B=\frac{7}{4}$ $\therefore \frac{2y+1}{y\left(y+4\right)}=\frac{1}{4y}+\frac{7}{4\left(y+4\right)}=\frac{1}{4x^2}+\frac{7}{4\left(x^2+4\right)}$ $\Rightarrow I=\frac{1}{4}\int \frac{1}{x^2}dx+\frac{7}{4}\int \frac{1}{x^2+4}dx$ $=-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+C$	 ½ 1 ½ ½ ½
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32.	If matrix A = $\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations : $3x+2y+z=2000$ $4x+y+3z=2500$ $x+y+z=900$	

Sol.	$\|A\|=3(-2)-4(1)+1(5)=-5\neq 0\Rightarrow A^{-1}$ exists. $A_{11}=-2, A_{12}=-1, A_{13}=3$ $A_{21}=-1, A_{22}=2, A_{23}=-1$ $A_{31}=5, A_{32}=-5, A_{33}=-5$ $adjA=\begin{bmatrix}-2&-1&5\\-1&2&-5\\3&-1&-5\end{bmatrix}$ $A^{-1}=\frac{1}{\|A\|}adjA=-\frac{1}{5}\begin{bmatrix}-2&-1&5\\-1&2&-5\\3&-1&-5\end{bmatrix}$ Given system of equations can be written as $AX=B$, where $B=\begin{bmatrix}2000\\2500\\900\end{bmatrix}$ $X=A^{-1}B$ $=-\frac{1}{5}\begin{bmatrix}-2&-1&5\\-1&2&-5\\3&-1&-5\end{bmatrix}\begin{bmatrix}2000\\2500\\900\end{bmatrix}=-\frac{1}{5}\begin{bmatrix}-2000\\-1500\\-1000\end{bmatrix}=\begin{bmatrix}400\\300\\200\end{bmatrix}$ $\therefore x=400, y=300$ and $z=200$	**1** **1 ½** **1** ½ ½ ½
33(a).	Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.	

Sol.	line1: $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=\lambda$ ...(1) line 2: $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=\mu$ ...(2) General points on (1) and (2) are $(3\lambda-1, 5\lambda-3, 7\lambda-5)$ and $(\mu+2, 3\mu+4, 5\mu+6)$ for the lines to intersect, $3\lambda-1=\mu+2$ ...(3) $5\lambda-3=3\mu+4$ ...(4) $7\lambda-5=5\mu+6$ ...(5) solving (3) and (4) gives $\lambda=\frac{1}{2}$ and $\mu=-\frac{3}{2}$ clearly these values of λ and μ satisfies (5) $\Rightarrow$ given lines intersect. Point of intersection is $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$	**1** **1** **1** **1** **1**
	OR	
33(b).	Find the shortest distance between the pair of lines $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}; z=2$.	

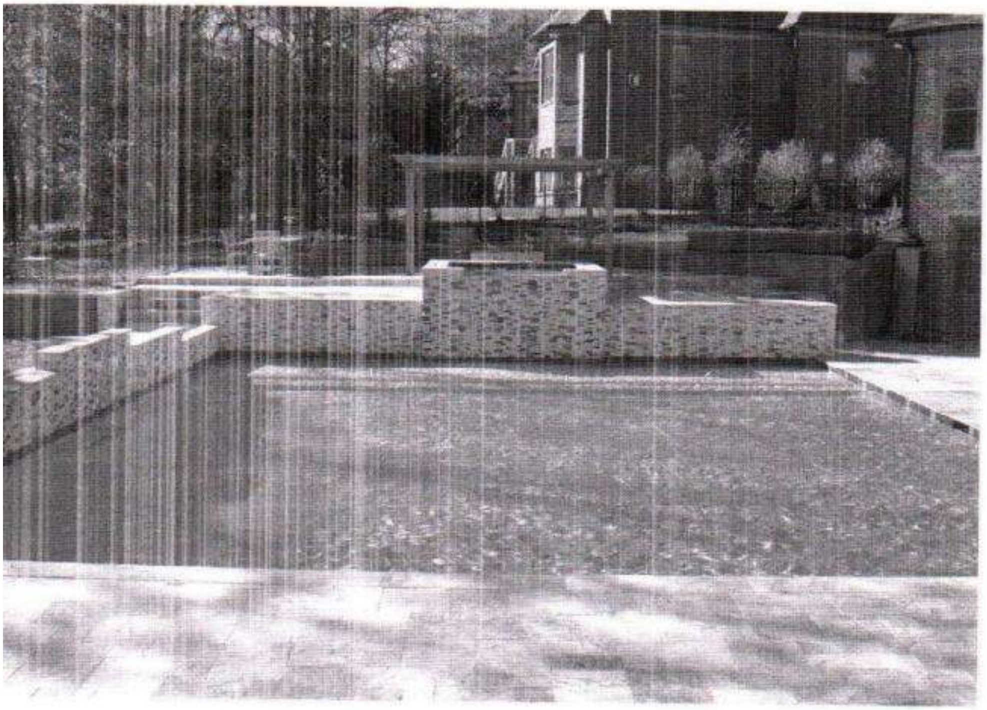
Sol.	Given lines are $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-0}{1}$ and $\frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$ In vector form, lines are $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+3\hat{j}+\hat{k})=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=(-\hat{i}+2\hat{j}+2\hat{k})+\mu(5\hat{i}+\hat{j})=\vec{a}_2+\lambda\vec{b}_2$ now, $\vec{a}_2-\vec{a}_1=-2\hat{i}+3\hat{j}+2\hat{k}$ $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 5 & 1 & 0\end{vmatrix}=-\hat{i}+5\hat{j}-13\hat{k}$ $\lvert\vec{b}_1\times\vec{b}_2\rvert=\sqrt{195}$ S.D.$=\left\lvert\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{\lvert\vec{b}_1\times\vec{b}_2\rvert}\right\rvert$ $=\left\lvert\frac{2+15-26}{\sqrt{195}}\right\rvert=\frac{9}{\sqrt{195}}$	1 1 ½ 1 ½ 1
34.	Find the area of the triangle ABC bounded by the lines represented by the equations 5x – 2y – 10 = 0, x – y – 9 = 0 and 3x – 4y – 6 = 0, using integration method.	
Sol.		**Correct figure 1 mark**

	solving the given equations, the vertices of triangle are $A(2,0), B(30,21)$ and $C\left(-\frac{8}{3},-\frac{35}{3}\right)$	3
	$\text{ar}(\triangle ABC)=\frac{3}{4}\int_{2}^{30}(x-2)\,dx-\int_{9}^{30}(x-9)\,dx+\left\|\int_{-\frac{8}{3}}^{9}(x-9)\,dx\right\|-\left\|\frac{5}{2}\int_{-\frac{8}{3}}^{2}(x-2)\,dx\right\|$ $=\frac{3}{8}(x-2)^2\Big]_2^{30}-\frac{1}{2}(x-9)^2\Big]_9^{30}+\left\|\frac{1}{2}(x-9)^2\Big]_{-\frac{8}{3}}^{9}\right\|-\left\|\frac{5}{4}(x-2)^2\Big]_{-\frac{8}{3}}^{2}\right\|$ $=294-\frac{441}{2}+\frac{1225}{18}-\frac{245}{9}=\frac{343}{3}$	1
35(a).	**Show that the relation S in set $\mathbb{R}$ of real numbers defined by** **$S = \{(a, b) : a \le b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$** **is neither reflexive, nor symmetric, nor transitive.**	
Sol.	We have $S=\{(a, b : a \le b^3\}$ where $a, b \in R$. (i) Reflexive: we observe that, $\frac{1}{2}\le\left(\frac{1}{2}\right)^3$ is not true. $\therefore\ \left(\frac{1}{2},\frac{1}{2}\right)\notin S$. So, S is not reflexive. (ii) Symmetric: We observe that $1\le 3^3$ but $3\not\le 1^3$ i.e., (1, 3) $\in S$ but (3, 1) $\notin S$. So, S is not symmetric. (iii) Transitive: We observe that, $10\le 3^3$ and $3\le 2^3$ but $10\not\le 2^3$. i.e., (10, 3) $\in S$ and (3, 2) $\in S$ but (10, 2) $\notin S$. So, S is not transitive. $\therefore$ S is neither reflexive nor symmetric, not transitive.	1 ½ 1 ½ 2
	OR	

35(b).	Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].	
Sol.	$R=\{(a,b):\text{both } a \text{ and } b \text{ are either odd or even}\}$ for reflexive : Let $a \in A$ clearly both a and a are either odd or even $\therefore (a,a)\in R \Rightarrow R$ is reflexive. for symmetric : Let $a, b \in A$. Let $(a,b) \in R$ $\Rightarrow$ both a and b are either odd or even $\Rightarrow$ both b and a are either odd or even so, $(a,b)\in R \Rightarrow (b,a) \in R \Rightarrow R$ is symmetric. for transitive : Let $a,b,c \in A$. Let $(a,b)\in R, (b,c)\in R$ $\Rightarrow$ both a and b are either odd or even & both b and c are either odd or even $\Rightarrow$ both a and c are either odd or even so, $(a,b)\in R, (b,c) \in R \Rightarrow (a,c) \in R \Rightarrow R$ is transitive. equivalence class of $[1]=\{1,3,5,7\}$	**1** **1** **2** **1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36.	In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.  On the basis of the above information, answer the following questions : (i) Find the probability that the age of the selected student is a composite number. *1* (ii) Let X be the age of the selected student. What can be the value of X ? *1* (iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2* **OR** (iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*	
Sol.	$(i)\, P(\text{age of selected student is a composite number})$ $= P(\text{age is } 14, 15 \text{ or } 16) = \frac{6}{10} = \frac{3}{5}$ $(ii)\, X$ can be $14, 15, 16, 17, 19$	**1** **1**

	$(iii)(a)$ | X | **14** | **15** | **16** | **17** | **19** | | $P(X)$ | $\frac{1}{10}$ | $\frac{2}{10}$ | $\frac{3}{10}$ | $\frac{2}{10}$ | $\frac{2}{10}$ | $\text{mean} = \sum X.P(X)$ $= 14\left(\frac{1}{10}\right) + 15\left(\frac{2}{10}\right) + 16\left(\frac{3}{10}\right) + 17\left(\frac{2}{10}\right) + 19\left(\frac{2}{10}\right) = 16.4 \text{ years}$ **OR** $(iii)(b)\ A: \text{getting Prime number} = \{17, 19\}$ $B: \text{age is greater than } 15 \text{ years} = \{16, 17, 19\}$ $A \cap B = \{17, 19\}$ $P(A\|B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{3}$	**1** **1** **1** **1**

37.	A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.  Suppose the side of the square plot is x metres and depth is h metres. On the basis of the above information, answer the following questions : (i) Write cost C(h) as a function in terms of h. *1* (ii) Find critical point. *1* (iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2* **OR** (iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*	
Sol.		

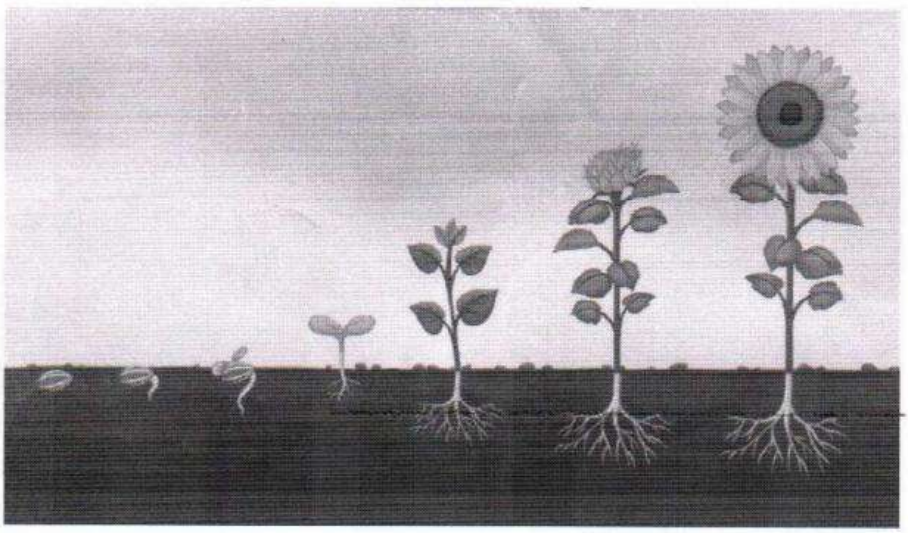
	(i) Capacity $=$ area $\times$ depth $= x^2h = 250 \Rightarrow x^2 = \frac{250}{h}$ $C\,(\text{cost}) = 500x^2 + 4000h^2$ $\Rightarrow C = 500\left(\frac{250}{h}\right) + 4000h^2 = \frac{125000}{h} + 4000h^2$ $(ii)\ \frac{dC}{dh} = -\frac{125000}{h^2} + 8000h$ $\frac{dC}{dh} = 0 \Rightarrow h = \frac{5}{2}\,m$ or $2.5\,m$ $(iii)(a)\ \frac{d^2C}{dh^2} = -125000\left(\frac{-2}{h^3}\right) + 8000 = \frac{250000}{h^3} + 8000$ $\left.\frac{d^2C}{dh^2}\right\vert_{h=2.5m} > 0 \Rightarrow$ Cost is minimum when $h = 2.5\,m$ Minimum cost $= C = \frac{125000}{\left(\frac{5}{2}\right)} + 4000\left(\frac{5}{2}\right)^2 = Rs.\,75,000$ OR $(iii)(b)$ we already have found above that $h = \frac{5}{2}\,m$ when $\frac{dC}{dh} = 0$ for the values of h less than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} < 0$ and, for the values of h more than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} > 0$ By first derivative test, there is a minimum at $h = \frac{5}{2}$ Now, $x^2 = \frac{250}{h} \Rightarrow x^2 = \frac{250}{\left(\frac{5}{2}\right)} = 100 \Rightarrow x = 10\,m$ also, $x = 4h$	 **1** **1** **1** **1** **1** ½ ½

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \quad 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? *2*

(ii) Using second derivative test, find the minimum value of the function. *2*

Sol. (i) $f'(x) = x^2 - 8x + 15 = (x-3)(x-5)$ **1**

$f'(x) = 0 \Rightarrow x = 3, 5$ are the critical points. **1**

(ii) Now $f''(x) = 2x - 8$

$f''(3) < 0$ and $f''(5) > 0$

so, mimimum value of $f(x)$ is at $x = 5$. **1**

min. value $= f(5) = \frac{5^3}{3} - 4(5)^2 + 15(5) + 2 = \frac{56}{3}$ **1**

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Sr. Secondary School Supplementary Examination, July- 2023
MATHEMATICS PAPER CODE 65/C/2

General Instructions: -

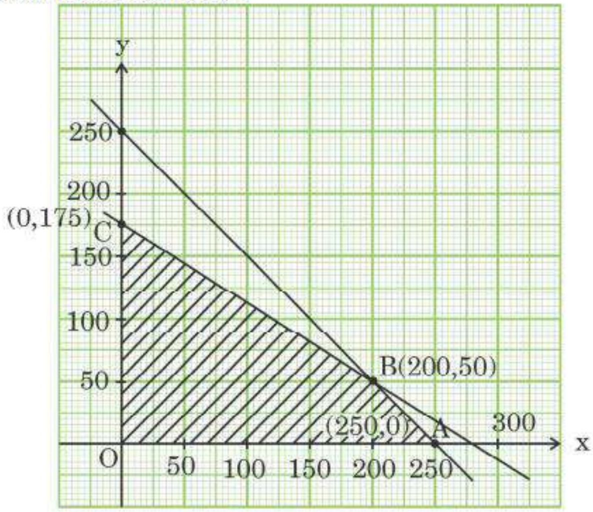
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**

11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/C/2)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each	
1.	Derivative of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$ is : (a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) $-\frac{1}{2}$	
Sol.	(b) $\frac{1}{2}$	1
2.	It is given that $X\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}=\begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}$. Then matrix X is : (a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\1 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}$	
Sol.	(c) $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$	1
3.	The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix}4 & 3 & 2\\2 & -1 & 0\\1 & 2 & 3\end{bmatrix}$ is : (a) 5 (b) -5 (c) -11 (d) 11	
Sol.	(b) -5	1

4.	Solution of the differential equation $(1 + y^2)(1 + \log x)\,dx + x\,dy = 0$ is : (a) $\tan^{-1} y + \log \lvert x \rvert + \frac{(\log \lvert x \rvert)^2}{2} = C$ (b) $\tan^{-1} y - \log \lvert x \rvert + \frac{(\log \lvert x \rvert)^2}{2} = C$ (c) $\tan^{-1} y - \log \lvert x \rvert - \frac{(\log \lvert x \rvert)^2}{2} = C$ (d) $\tan^{-1} y + \log \lvert x \rvert - \frac{(\log \lvert x \rvert)^2}{2} = C$	
Sol.	(a) $\tan^{-1} y + \log \lvert x \rvert + \frac{(\log \lvert x \rvert)^2}{2} = C$	1
5.	If ABCD is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD}$ is : (a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$	
Sol.	(c) $2\overrightarrow{BC}$	1
6.	If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then which one of the following is true ? (a) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (c) $y^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$	
Sol.	(a) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$	1

7.	The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :  (a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$	
Sol.	(a) $2a = b$	1
8.	A family has 2 children and the elder child is a girl. The probability that both children are girls is : (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$	
Sol.	(c) $\frac{1}{2}$	1
9.	If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is : (a) 1 (b) -2 (c) 2 (d) -1	
Sol.	(c) 2	1
10.	The vector equation of a line which passes through the point $(1, -2, 3)$ and is parallel to the vector $3\hat{i} - 2\hat{j} + 4\hat{k}$ is : (a) $\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$ (b) $\vec{r} = (-3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ (c) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$ (d) $\vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$	

Sol.	(c) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$	1
11.	If $\begin{bmatrix} 3 & 2 \\ 1 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \end{bmatrix}$, then x is : (a) $\frac{16}{3}$ (b) -3 (c) -4 (d) 4	
Sol.	(d) 4	1
12.	If A is a square matrix of order 3 and $\|A\| = 6$, then the value of $\|\text{adj } A\|$ is : (a) 6 (b) 36 (c) 27 (d) 216	
Sol.	(b) 36	1
13.	The value of $\int_0^{\pi/6} \sin 3x \, dx$ is : (a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$	
Sol.	(d) $\frac{1}{3}$	1
14.	If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is : (a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$	
Sol.	(a) $\frac{2\pi}{3}$	1

15.	$\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to (a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$ (c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$	
Sol.	(b) $-\cot x - \tan x + C$	1
16.	The point which lies in the half-plane $2x + y - 4 \leq 0$ is : (a) (0, 8) (b) (1, 1) (c) (5, 5) (d) (2, 2)	
Sol.	(b) (1, 1)	1
17.	Let P and Q be two points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ respectively. The position vector of a point which divides the join of P and Q externally in the ratio 3 : 2 is : (a) $4\vec{a} + 7\vec{b}$ (b) $\frac{8\vec{a} + 7\vec{b}}{5}$ (c) $4\vec{a} - 7\vec{b}$ (d) $\vec{a} + 4\vec{b}$	
Sol.	(a) $4\vec{a} + 7\vec{b}$	1
18.	The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is : (a) 1 (b) 2 (c) -1 (d) 0	
Sol.	(d) 0	1

	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$. *Reason (R) :* Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.	
Sol.	(c) Assertion (A) is true, but Reason (R) is false.	1
20.	*Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus. *Reason (R) :* ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1
	SECTION B This section comprises very short answer (VSA) type questions of 2 marks each.	
21.	Find the interval in which the function $x^3 - 12x^2 + 36x + 17$ is strictly increasing.	
Sol.	$f^{l}(x)=3x^2-24x+36=3(x-2)(x-6)$ f is strictly increasing, $f^{l}(x)>0$ $3(x-2)(x-6)>0 \Rightarrow x\in(-\infty,2)\cup(6,\infty)$	1 1
22.	Find the points at which the function $f(x) = \dfrac{4+x^2}{4x-x^3}$ is discontinuous.	

Sol.	$f(x)=\frac{4+x^2}{x(2-x)(2+x)}$ clearly f is not continuous when $x(2-x)(2+x)=0$ $\Rightarrow x=0,2,-2$	½ ½ 1
23(a).	**If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\|\vec{a}\| = 7$, $\|\vec{b}\| = 24$, $\|\vec{c}\| = 25$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.**	
Sol.	$\vec{a}+\vec{b}+\vec{c}=\vec{0}\Rightarrow\left(\vec{a}+\vec{b}+\vec{c}\right)^2=\left(\vec{0}\right)^2$ $\Rightarrow\|\vec{a}\|^2+\|\vec{b}\|^2+\|\vec{c}\|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$ $\Rightarrow 49+576+625+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$ $\Rightarrow\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-625$	**1** 1
	OR	
23(b).	**If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$.**	
Sol.	d.c. are $\cos\alpha, \cos\beta, \cos\gamma$ $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ $\Rightarrow\left(1-\sin^2\alpha\right)+\left(1-\sin^2\beta\right)+\left(1-\sin^2\gamma\right)=1$ $\Rightarrow\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$	1 1
24(a)	**Simplify :** $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$	
Sol.	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$	½

	$= \tan^{-1}\left(\dfrac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$ $= \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$ $= \frac{\pi}{4}+\frac{x}{2}$	½ ½ ½
24(b).	Prove that the greatest integer function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.	
Sol.	For not one-one: $1.1, 1.2 \in R$ (domain) now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow f$ is not one-one. For not onto: Let $\frac{1}{2} \in R$ (co-domain), but $[x] = \frac{1}{2}$ is not possible for x in domain. so, f is not onto.	1 1
25.	For the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, verify that the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$.	
Sol.	$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix} = \hat{i} + 11\hat{j} + 7\hat{k}$ $\vec{a}.\left(\vec{a}\times\vec{b}\right) = 1 - 22 + 21 = 0$ $\lvert\vec{a}\rvert\lvert\vec{a}\times\vec{b}\rvert\cos\theta = 0$ $\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$	1 ½ ½

	SECTION C This section comprises of Short Answer (SA) type questions of 3 marks each.	
26.	**Find :** $$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$$	
Sol.	$I=\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$ Let $e^x=t \Rightarrow e^x dx=dt$ $I=\int \frac{dt}{\sqrt{(3)^2-(t+2)^2}}$ $=\sin^{-1}\left(\frac{t+2}{3}\right)+C$ $=\sin^{-1}\left(\frac{e^x+2}{3}\right)+C$	1 1 ½ ½
27.	**Find :** $$\int \frac{2x^2+1}{x^2(x^2+4)}\,dx$$	

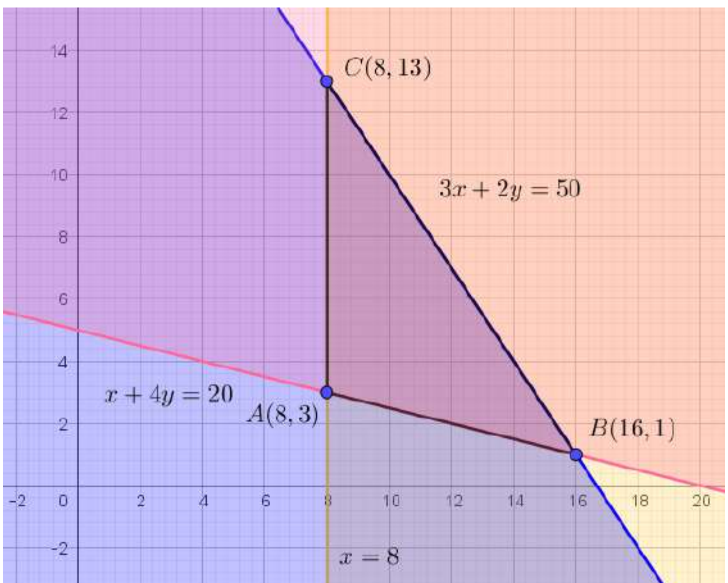
Sol.	$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$ Let $\frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{2y+1}{y\left(y+4\right)}$, where $x^2=y$ Put $\frac{2y+1}{y\left(y+4\right)}=\frac{A}{y}+\frac{B}{y+4}$ $\Rightarrow 2y+1=A\left(y+4\right)+By$ $\Rightarrow A=\frac{1}{4}, B=\frac{7}{4}$ $\therefore \frac{2y+1}{y\left(y+4\right)}=\frac{1}{4y}+\frac{7}{4\left(y+4\right)}=\frac{1}{4x^2}+\frac{7}{4\left(x^2+4\right)}$ $\Rightarrow I=\frac{1}{4}\int\frac{1}{x^2}dx+\frac{7}{4}\int\frac{1}{x^2+4}dx$ $=-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+C$	½ 1 ½ ½ ½
28(a).	Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.	
Sol.	Let E_1: event of choosing bag A, E_2: event of choosing bag B, A: red ball is found here, $P\left(E_1\right)=P\left(E_2\right)=\frac{1}{2}; P\left(A\mid E_1\right)=\frac{3}{5}, P\left(A\mid E_2\right)=\frac{5}{9}$ $P\left(E_2\mid A\right)=\frac{P\left(E_2\right)P\left(A\mid E_2\right)}{P\left(E_1\right)P\left(A\mid E_1\right)+P\left(E_2\right)P\left(A\mid E_2\right)}$ $=\frac{\frac{5}{9}\times\frac{1}{2}}{\frac{3}{5}\times\frac{1}{2}+\frac{5}{9}\times\frac{1}{2}}=\frac{25}{52}$	½ 1 1+ ½
	OR	

28(b).	Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.	
Sol.	Let X be the random variable representing the number of persons who speak truth. X can takes the values 0,1 and 2. $P(\text{speaking truth}) = \frac{20}{50}, P(\text{not speaking truth}) = \frac{30}{50}$ $P(X=0) = \frac{30}{50} \times \frac{29}{49} = \frac{87}{245}$ $P(X=1) = 2 \times \frac{20}{50} \times \frac{30}{49} = \frac{120}{245}$ $P(X=2) = \frac{20}{50} \times \frac{19}{49} = \frac{38}{245}$ Probability Distribution Table is given by: X: 0, 1, 2 $P(X)$: $\frac{87}{245}$, $\frac{120}{245}$, $\frac{38}{245}$	½ ½ 1½ ½
29(a)	Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.	

X	0	1	2
$P(X)$	$\frac{87}{245}$	$\frac{120}{245}$	$\frac{38}{245}$

Sol.	$\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ ...(1) Put $\frac{y}{x}=v$ i.e. $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ Equation (1) gives $v+x\frac{dv}{dx}=\frac{v}{1+v^2}$ $\Rightarrow x\frac{dv}{dx}=-\frac{v^3}{1+v^2}$ $\Rightarrow\int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$ $\Rightarrow\frac{-1}{2v^2}+\log\lvert v\rvert=-\log\lvert x\rvert+\log c$ putting $v=\frac{y}{x}$ and simplifying gives $-\frac{x^2}{2y^2}=\log\left\lvert\frac{c}{y}\right\rvert$ now, $x=0,\ y=1$ gives $c=1$ required solution is : $\frac{x^2}{2y^2}=\log\lvert y\rvert$	½ 1 ½ ½ ½
	OR	
29(b).	**Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, given that y = 0 when x = 1.**	
Sol.	Given diff. eqn. can be written as $\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{(1+x^2)^2}$ I.F. $=e^{\int\frac{2x}{1+x^2}dx}=1+x^2$ solution is given by: $y.(1+x^2)=\int\frac{1}{1+x^2}dx$ $\Rightarrow y.(1+x^2)=\tan^{-1}x+C$ Now $x=1,\ y=0$ gives $C=-\frac{\pi}{4}$ Reqired solution : $y.(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$	½ ½ ½ ½ ½ ½

30(a).	Evaluate : $$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$$	
Sol.	$I=\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx \quad ...(1)$ using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $I=\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$ $\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx \quad ...(2)$ adding (1) and (2) $2I=\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx=\pi\int_0^{\frac{\pi}{4}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ $\Rightarrow I=\frac{\pi}{2}\int_0^{\frac{\pi}{4}} \frac{\tan x\sec^2 x}{(\tan^2 x)^2+1}dx \quad (\because \text{dividing by} \cos^4 x)$ Putting $\tan^2 x=t$ gives $I=\frac{\pi}{4}\int_0^1 \frac{1}{t^2+1}dt$ $\Rightarrow I=\frac{\pi^2}{16}$	½ ½ 1 ½ ½
	OR	
30(b).	Evaluate : $$\int_1^3 (\lvert x-1\rvert + \lvert x-2\rvert)\, dx$$	

Sol.	$I=\int_1^3 \left(\lvert x-1\rvert+\lvert x-2\rvert\right)dx$ $=\int_1^2 \left[(x-1)-(x-2)\right]dx+\int_2^3 \left[(x-1)+(x-2)\right]dx$ $=\int_1^2 1\,dx+\int_2^3 (2x-3)\,dx$ $=\left[x\right]_1^2+\left[x^2-3x\right]_2^3$ $=1+2=3$	1 ½ ½ 1
31.	**Solve the following Linear Programming Problem graphically:** **Maximise z = 10x + 15y** **subject to the constraints :** $3x + 2y \le 50$ $x + 4y \ge 20$ $x \ge 8,\ y \ge 0$	
Sol.		Correct Graph 1 Mark

Corner Point	$z=10x+15y$
$A(8,3)$	125
$B(16,1)$	175
$C(8,13)$	275

1 ½

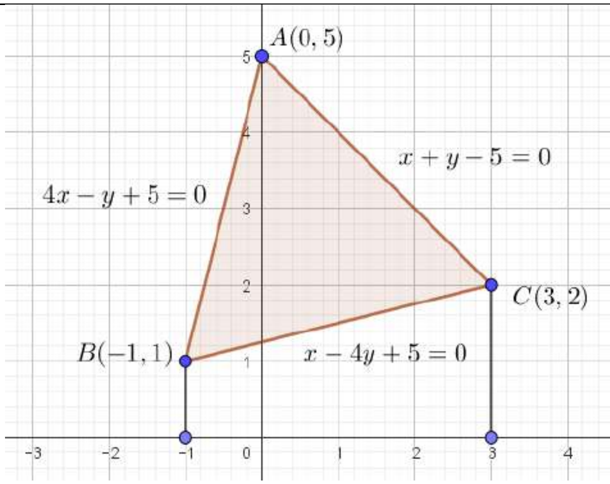
z_{max} is 275 when $x=8,\ y=13$ ½

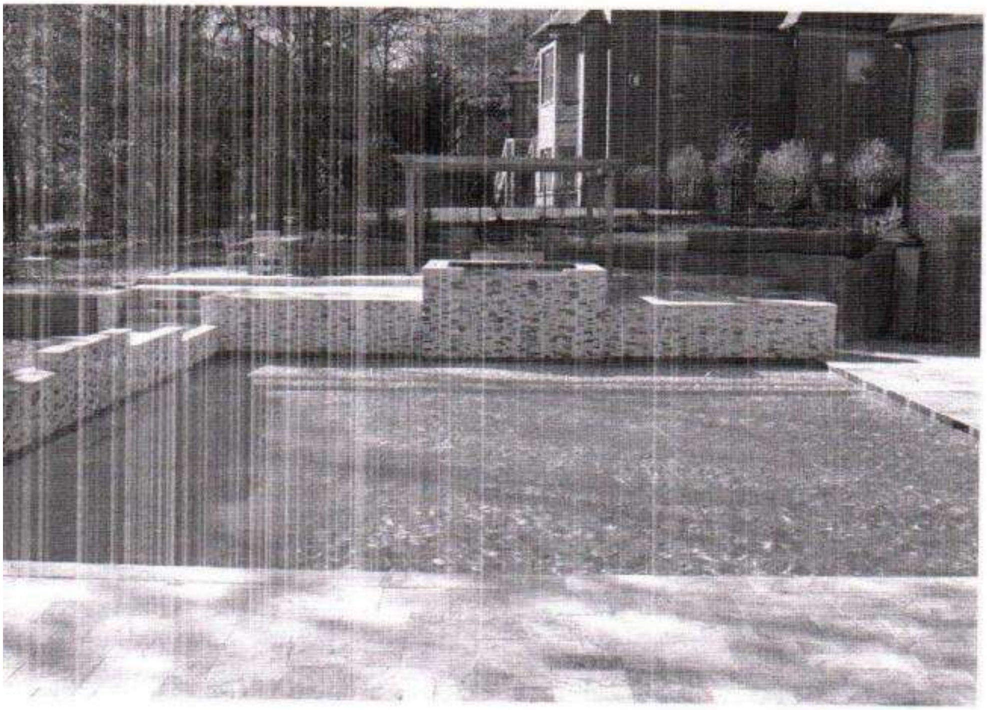
	SECTION D This section comprises of Long Answer (LA) type questions of 5 marks each.	
32(a).	Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.	
Sol.	line1: $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=\lambda$ ...(1) line 2: $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=\mu$ ...(2) General points on (1) and (2) are $(3\lambda-1, 5\lambda-3, 7\lambda-5)$ and $(\mu+2, 3\mu+4, 5\mu+6)$ for the lines to intersect, $3\lambda-1=\mu+2$ ...(3) $5\lambda-3=3\mu+4$ ...(4) $7\lambda-5=5\mu+6$ ...(5) solving (3) and (4) gives $\lambda=\frac{1}{2}$ and $\mu=-\frac{3}{2}$ clearly these values of λ and μ satisfies (5) $\Rightarrow$ given lines intersect. Point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$	1 1 1 1 1
	OR	
32(b).	Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$.	

Sol.	Given lines are $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-0}{1}$ and $\frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$ In vector form, lines are $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+3\hat{j}+\hat{k})=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=(-\hat{i}+2\hat{j}+2\hat{k})+\mu(5\hat{i}+\hat{j})=\vec{a}_2+\lambda\vec{b}_2$ now, $\vec{a}_2-\vec{a}_1=-2\hat{i}+3\hat{j}+2\hat{k}$ $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 5 & 1 & 0\end{vmatrix}=-\hat{i}+5\hat{j}-13\hat{k}$ $\lvert\vec{b}_1\times\vec{b}_2\rvert=\sqrt{195}$ S.D.$=\left\lvert\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{\lvert\vec{b}_1\times\vec{b}_2\rvert}\right\rvert$ $=\left\lvert\frac{2+15-26}{\sqrt{195}}\right\rvert=\frac{9}{\sqrt{195}}$	1 1 ½ 1 ½ 1
33(a).	**Show that the relation S in set $\mathbb{R}$ of real numbers defined by** **$S=\{(a, b) : a \le b^3,\ a\in\mathbb{R},\ b\in\mathbb{R}\}$** **is neither reflexive, nor symmetric, nor transitive.**	
Sol.	We have $S=\{(a, b : a\le b^3\}$ where $a, b\in R.$ (i) Reflexive: we observe that, $\frac{1}{2}\le\left(\frac{1}{2}\right)^3$ is not true. $\therefore\ \left(\frac{1}{2},\frac{1}{2}\right)\notin S.$ So, S is not reflexive. (ii) Symmetric: We observe that $1\le 3^3$ but $3\not\le 1^3$ i.e., (1, 3) $\in S$ but (3, 1) $\notin S$. So, S is not symmetric. (iii) Transitive: We observe that, $10\le 3^3$ and $3\le 2^3$ but $10\not\le 2^3$. i.e., (10, 3) $\in S$ and (3, 2) $\in S$ but (10, 2) $\notin S$. So, S is not transitive. $\therefore$ S is neither reflexive nor symmetric, not transitive.	1 ½ 1 ½ 2
	OR	

33(b).	Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].	
Sol.	$R=\{(a,b):\text{both } a \text{ and } b \text{ are either odd or even}\}$ for reflexive : Let $a \in A$ clearly both a and a are either odd or even $\therefore (a,a)\in R \Rightarrow R$ is reflexive. for symmetric : Let $a,b \in A$. Let $(a,b)\in R$ $\Rightarrow$ both a and b are either odd or even $\Rightarrow$ both b and a are either odd or even so, $(a,b)\in R \Rightarrow (b,a)\in R \Rightarrow R$ is symmetric. for transitive : Let $a,b,c\in A$. Let $(a,b)\in R, (b,c)\in R$ $\Rightarrow$ both a and b are either odd or even & both b and c are either odd or even $\Rightarrow$ both a and c are either odd or even so, $(a,b)\in R, (b,c)\in R \Rightarrow (a,c)\in R \Rightarrow R$ is transitive. equivalence class of $[1]=\{1,3,5,7\}$	1 1 2 1
34.	Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations : $x + y + z = 5000$ $6x + 7y + 8z = 35800$ $6x + 7y - 8z = 7000$	

Sol.	$\|A\| = (-56-56)-6(-8-7)+6(8-7) = -16 \neq 0 \Rightarrow A^{-1}$ exists. $A_{11} = -112, A_{12} = 96, A_{13} = 0$ $A_{21} = 15, A_{22} = -14, A_{23} = -1$ $A_{31} = 1, A_{32} = -2, A_{33} = 1$ $adjA = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ $A^{-1} = \frac{1}{\|A\|} adjA = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ Given system of equations can be written as $AX = B$, where $B = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$ $X = A^{-1}B$ $= -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$ $= -\frac{1}{16}\begin{bmatrix} -16000 \\ -35200 \\ -28800 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$ $\therefore x = 1000, y = 2200, z = 1800$	1 1 ½ 1 ½ ½ ½
35.	Using integration, find the area of the region bounded by the triangle ABC when its sides are given by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.	
Sol.		

	 solving the given equations, the vertices of triangle are $A(0,5), B(-1,1)$ and $C(3,2)$ $\text{ar}(\triangle ABC)=\int_{-1}^{0}(4x+5)\,dx+\int_{0}^{3}(-x+5)\,dx-\int_{-1}^{3}\frac{5+x}{4}\,dx$ $=\left[\frac{4x^2}{2}+5x\right]_{-1}^{0}+\left[\frac{-x^2}{2}+5x\right]_{0}^{3}-\left[\frac{5x}{4}+\frac{x^2}{8}\right]_{-1}^{3}$ $=\frac{15}{2}$	Correct figure 1 mark 1 2 ½ ½
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36.	A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.  Suppose the side of the square plot is x metres and depth is h metres. On the basis of the above information, answer the following questions : (i) Write cost C(h) as a function in terms of h. *1* (ii) Find critical point. *1* (iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2* **OR** (iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*	
Sol.		

	(i) Capacity = area × depth $= x^2h = 250 \Rightarrow x^2 = \frac{250}{h}$ $C\,(\text{cost}) = 500x^2 + 4000h^2$ $\Rightarrow C = 500\left(\frac{250}{h}\right) + 4000h^2 = \frac{125000}{h} + 4000h^2$ $(ii)\ \frac{dC}{dh} = -\frac{125000}{h^2} + 8000h$ $\frac{dC}{dh} = 0 \Rightarrow h = \frac{5}{2}\,m$ or $2.5\,m$ $(iii)(a)\ \frac{d^2C}{dh^2} = -125000\left(\frac{-2}{h^3}\right) + 8000 = \frac{250000}{h^3} + 8000$ $\left.\frac{d^2C}{dh^2}\right\|_{h=2.5m} > 0 \Rightarrow$ Cost is minimum when $h = 2.5\,m$ Minimum cost $= C = \frac{125000}{\left(\frac{5}{2}\right)} + 4000\left(\frac{5}{2}\right)^2 = Rs.\,75,000$ OR $(iii)(b)$ we already have found above that $h = \frac{5}{2}\,m$ when $\frac{dC}{dh} = 0$ for the values of h less than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} < 0$ and, for the values of h more than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} > 0$ By first derivative test, there is a minimum at $h = \frac{5}{2}$ Now, $x^2 = \frac{250}{h} \Rightarrow x^2 = \frac{250}{\left(\frac{5}{2}\right)} = 100 \Rightarrow x = 10\,m$ also, $x = 4h$	1 1 1 1 1 ½ ½

37. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

Sol.

$(i)\, P(\text{age of selected student is a composite number})$

$= P(\text{age is } 14, 15 \text{ or } 16) = \frac{6}{10} = \frac{3}{5}$ 1

$(ii)\, X$ can be $14, 15, 16, 17, 19$ 1

$(iii)(a)$

	X	**14**	**15**	**16**	**17**	**19**
	$P(X)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$

1

$\text{mean} = \sum X.P(X)$

$$=14\left(\frac{1}{10}\right)+15\left(\frac{2}{10}\right)+16\left(\frac{3}{10}\right)+17\left(\frac{2}{10}\right)+19\left(\frac{2}{10}\right)=16.4\text{ years}$$ 1

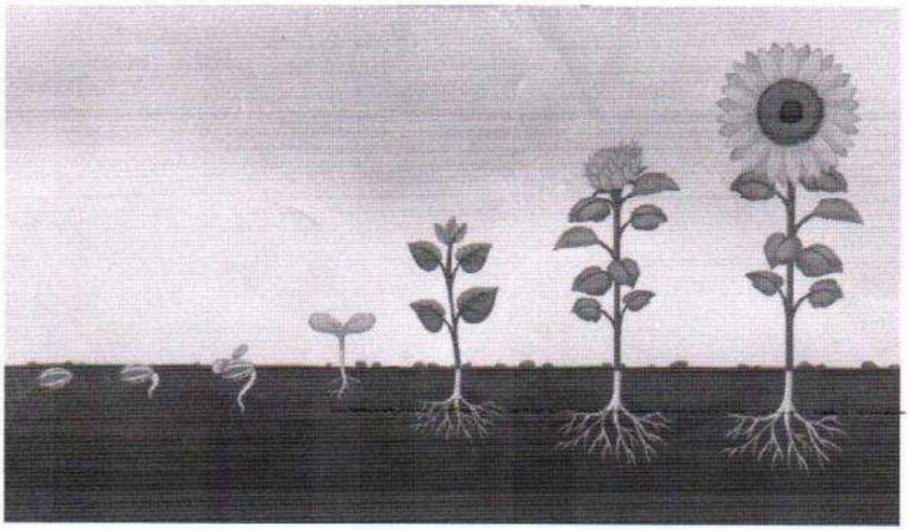
OR

$(iii)(b)\ A$: getting Prime number $=\{17,19\}$

B : age is greater than 15 years $=\{16,17,19\}$

$A\cap B=\{17,19\}$ 1

$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{2}{3}$ 1

38.	In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield. A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function $f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \ 0 \le x \le 10$ where x is the number of days the plant is exposed to sunlight.  On the basis of the above information, answer the following questions : (i) What are the critical points of the function f(x) ? *2* (ii) Using second derivative test, find the minimum value of the function. *2*	
Sol.	$f^{I}(x) = x^2 - 8x + 15 = (x-3)(x-5)$ $f^{I}(x) = 0 \Rightarrow x = 3, 5$ are the critical points. Now $f^{II}(x) = 2x - 8$ $f^{II}(3) < 0$ and $f^{II}(5) > 0$ so, mimimum value of $f(x)$ is at $x = 5$. min. value $= f(5) = \frac{5^3}{3} - 4(5)^2 + 15(5) + 2 = \frac{56}{3}$	1 1 1 1

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Sr. Secondary School Supplementary Examination, July- 2023
MATHEMATICS PAPER CODE 65/C/3

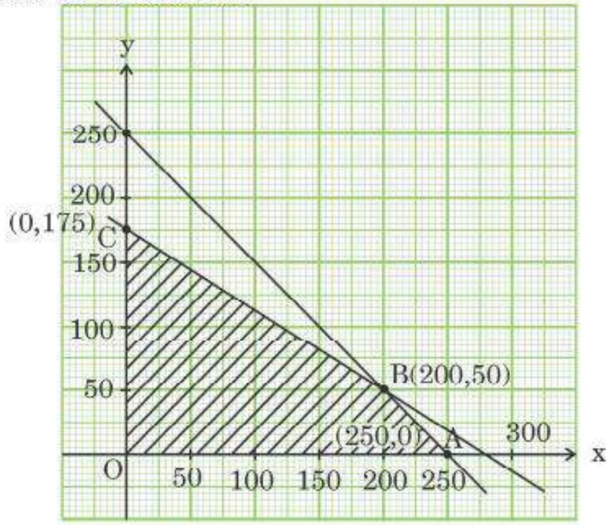
General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**

11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/C/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	Integrating factor of the differential equation $x\frac{dy}{dx} - 2y = 4x^2$ is : (a) x^2 (b) $-\frac{1}{x^2}$ (c) $\frac{1}{x^2}$ (d) $-x^2$	
Sol.	(c) $\frac{1}{x^2}$	**1**
2.	It is given that $X\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix} = \begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}$. Then matrix X is : (a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\1 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}$	
Sol.	(c) $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$	**1**
3.	For which value of x, are the determinants $\begin{vmatrix}2x & -3\\5 & x\end{vmatrix}$ and $\begin{vmatrix}10 & 1\\-3 & 2\end{vmatrix}$ equal ? (a) ± 3 (b) -3 (c) ± 2 (d) 2	
Sol.	(c) ± 2	**1**

4.	The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :  (a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$	
Sol.	(a) $2a = b$	**1**
5.	The angle between the lines $\frac{x+1}{1} = \frac{4-y}{-1} = \frac{z-5}{2}$ and $\frac{x+3}{-3} = \frac{y-2}{5} = \frac{z+5}{4}$ is : (a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$	
Sol.	(b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	**1**
6.	A fair die is rolled. Events E and F are E = {1, 3, 5} and F = {2, 3} respectively. Value of $P(E\|F)$ is : (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$	
Sol.	(d) $\frac{1}{2}$	**1**

7.	If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is : (a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$	
Sol.	(a) $\frac{2\pi}{3}$	**1**
8.	If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is : (a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$	
Sol.	(c) $2\vec{BC}$	**1**
9.	If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ? (a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$	
Sol.	(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$	**1**
10.	If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is : (a) 1 (b) -2 (c) 2 (d) -1	
Sol.	(c) 2	**1**
11.	The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is : (a) 1 (b) 2 (c) -1 (d) 0	
Sol.	(d) 0	**1**

12.	$\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to (a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$ (c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$	
Sol.	(b) $-\cot x - \tan x + C$	**1**
13.	If A is a square matrix of order 3 and $\lvert A \rvert = 6$, then the value of $\lvert adj\ A \rvert$ is : (a) 6 (b) 36 (c) 27 (d) 216	
Sol.	(b) 36	**1**
14.	In the matrix equation $\begin{bmatrix} x+y+z \\ x+z \\ y+2z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, the value of z is : (a) 1 (b) 2 (c) -1 (d) -2	
Sol.	(c) -1	**1**
15.	If $y = \log\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$, then $\frac{dy}{dx}$ is : (a) sec x (b) cosec x (c) tan x (d) sec x tan x	
Sol.	(a) sec x	**1**
16.	The point which lies in the half-plane $2x + y - 4 \leq 0$ is : (a) (0, 8) (b) (1, 1) (c) (5, 5) (d) (2, 2)	
Sol.	(b) (1, 1)	**1**

17.	The value of $\int_{0}^{\pi/6} \sin 3x \, dx$ is : (a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$	
Sol.	(d) $\frac{1}{3}$	**1**
18.	The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is : (a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$	
Sol.	(a) $\frac{1}{\sqrt{6}}$	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus. *Reason (R) :* ABCD is a rhombus if AB = BC = CD = DA, AC $\neq$ BD.	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
20.	*Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$. *Reason (R) :* Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.	

Sol.	(c) Assertion (A) is true, but Reason (R) is false.	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	Find a vector of magnitude 6, which is perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.	
Sol.	Let $\vec{c}=\vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$, and $\vec{d}=\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$ $\vec{c}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}=-2\hat{i}+4\hat{j}-2\hat{k}$ $\left\|\vec{c}\times\vec{d}\right\|=\sqrt{24}$ Required vector $=6\left(\frac{\vec{c}\times\vec{d}}{\left\|\vec{c}\times\vec{d}\right\|}\right)$ $=\frac{6}{\sqrt{24}}\left(-2\hat{i}+4\hat{j}-2\hat{k}\right)$ or $\sqrt{6}\left(-\hat{i}+2\hat{j}-\hat{k}\right)$	**1** **1**
22(a).	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\|\vec{a}\|=7$, $\|\vec{b}\|=24$, $\|\vec{c}\|=25$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.	
Sol.	$\vec{a}+\vec{b}+\vec{c}=\vec{0}\Rightarrow\left(\vec{a}+\vec{b}+\vec{c}\right)^2=\left(\vec{0}\right)^2$ $\Rightarrow\|\vec{a}\|^2+\|\vec{b}\|^2+\|\vec{c}\|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$ $\Rightarrow 49+576+625+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$ $\Rightarrow\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-625$	**1** **1**
	OR	
22(b).	If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$.	

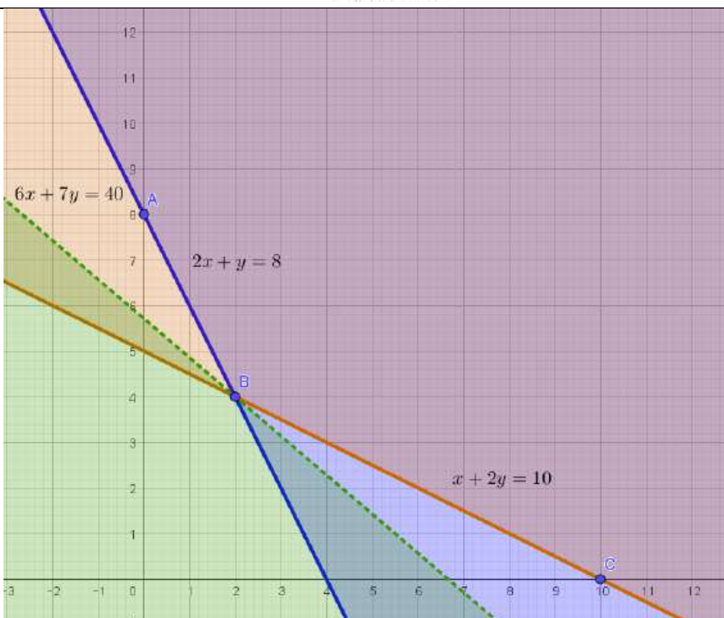
Sol.	d.c. are $\cos\alpha, \cos\beta, \cos\gamma$ $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ $\Rightarrow(1-\sin^2\alpha)+(1-\sin^2\beta)+(1-\sin^2\gamma)=1$ $\Rightarrow\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$	**1** **1**
23.	The function $f(x)=\begin{cases}3ax+b, & \text{if} \quad x>1\\ 11, & \text{if} \quad x=1\\ 5ax-2b, & \text{if} \quad x<1\end{cases}$ is continuous at x = 1. Find the values of a and b.	
Sol.	As f is continuous at $x=1\Rightarrow\lim_{x\to1^+}f(x)=\lim_{x\to1^-}f(x)=f(1)$ $\lim_{x\to1^+}(3ax+b)=\lim_{x\to1^-}(5ax-2b)=11$ $\Rightarrow 3a+b=11$ and $5a-2b=11$ solving, we get $a=3, b=2$	**1** ½ ½
24.	Find the interval in which the function $f(x)=x^3+\frac{1}{x^3}$, $x\neq0$ is decreasing.	
Sol.	$f'(x)=3x^2-\frac{3}{x^4}$ $f'(x)=0\Rightarrow\frac{3(x^6-1)}{x^4}=0\Rightarrow\frac{3(x^3-1)(x^3+1)}{x^4}=0$ $\Rightarrow x=-1,1 \quad (\because x\neq0)$ $\therefore f(x)$ is decreasing when $x\in[-1,1]-\{0\}$	½ **1** ½
25(a).	Simplify : $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$	

Sol.	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$	½
	$=\tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$	½
	$=\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$	½
	$=\frac{\pi}{4}+\frac{x}{2}$	½
	OR	
25(b).	Prove that the greatest integer function $f:\mathbb{R}\rightarrow\mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.	
Sol.	For not one-one: $1.1, 1.2 \in R$ (domain) now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow f$ is not one-one.	**1**
	For not onto: Let $\frac{1}{2} \in R$ (co-domain), but $[x] = \frac{1}{2}$ is not possible for x in domain. so, f is not onto.	**1**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26(a).	Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.	

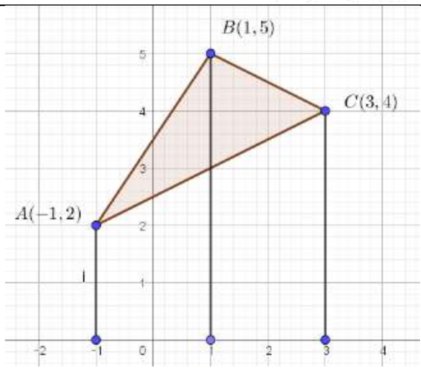
Sol.	Let E_1: event of choosing bag A, E_2: event of choosing bag B, A: red ball is found	½
	here, $P(E_1)=P(E_2)=\frac{1}{2}; P(A\|E_1)=\frac{3}{5}, P(A\|E_2)=\frac{5}{9}$	**1**
	$P(E_2\|A)=\frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)}$ $=\frac{\frac{5}{9}\times\frac{1}{2}}{\frac{3}{5}\times\frac{1}{2}+\frac{5}{9}\times\frac{1}{2}}=\frac{25}{52}$	**1+ ½**
	OR	
26(b).	Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.	
Sol.	Let X be the random variable representing the number of persons who speak truth. X can takes the values 0, 1 and 2.	½
	$P(\text{speaking truth})=\frac{20}{50}, P(\text{not speaking truth})=\frac{30}{50}$	½
	$P(X=0)=\frac{30}{50}\times\frac{29}{49}=\frac{87}{245}$ $P(X=1)=2\times\frac{20}{50}\times\frac{30}{49}=\frac{120}{245}$ $P(X=2)=\frac{20}{50}\times\frac{19}{49}=\frac{38}{245}$	**1½**
	Probability Distribution Table is given by:	
	X: **0**, **1**, **2** $P(X)$: $\frac{87}{245}$, $\frac{120}{245}$, $\frac{38}{245}$	½
27.	Find : $$\int \frac{\cos\theta}{\sqrt{3-3\sin\theta-\cos^2\theta}}\,d\theta$$	

Sol.	$I=\int \frac{\cos\theta}{\sqrt{3-3\sin\theta-\cos^2\theta}}d\theta$ $=\int \frac{\cos\theta}{\sqrt{\sin^2\theta-3\sin\theta+2}}d\theta$ Putting $\sin\theta=t$ gives $I=\int \frac{dt}{\sqrt{t^2-3t+2}}=\int \frac{dt}{\sqrt{\left(t-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$ $=\log\left\|\left(t-\frac{3}{2}\right)+\sqrt{t^2-3t+2}\right\|+C$ $=\log\left\|\left(\sin\theta-\frac{3}{2}\right)+\sqrt{\sin^2\theta-3\sin\theta+2}\right\|+C$	½ ½ ½ **1** ½
28(a).	Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x+\cos^4 x}\, dx$	
Sol.	$I=\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx \quad ...(1)$ using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $I=\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$ $\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx \quad ...(2)$ adding (1) and (2) $2I=\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx=\pi\int_0^{\frac{\pi}{4}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$	½ ½

	$\Rightarrow I=\frac{\pi}{2}\int_0^{\frac{\pi}{4}}\frac{\tan x\sec^2 x}{\left(\tan^2 x\right)^2+1}dx \quad (\because \text{dividing by} \cos^4 x)$	**1**
	Putting $\tan^2 x=t$ gives $I=\frac{\pi}{4}\int_0^1\frac{1}{t^2+1}dt$	½
	$\Rightarrow I=\frac{\pi^2}{16}$	½
	OR	
28(b).	Evaluate : $\int_1^3 (\lvert x-1\rvert+\lvert x-2\rvert)\,dx$	
Sol.	$I=\int_1^3\left(\lvert x-1\rvert+\lvert x-2\rvert\right)dx$	
	$=\int_1^2\left[(x-1)-(x-2)\right]dx+\int_2^3\left[(x-1)+(x-2)\right]dx$	**1**
	$=\int_1^2 1\,dx+\int_2^3(2x-3)\,dx$	½
	$=\left[x\right]_1^2+\left[x^2-3x\right]_2^3$	½
	$=1+2=3$	**1**
29.	Find : $\int \frac{x}{(x^2+1)(x-1)}dx$	

Sol.	$I=\int \frac{x}{(x^2+1)(x-1)}dx$ Let $\frac{x}{(x^2+1)(x-1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-1}$ $\Rightarrow x=(x-1)(Ax+B)+C(x^2+1)$ This gives $A=-\frac{1}{2}, B=\frac{1}{2}, C=\frac{1}{2}$ $\therefore I=-\frac{1}{2}\int \frac{x-1}{x^2+1}dx+\frac{1}{2}\int \frac{1}{x-1}dx$ $=-\frac{1}{2}\left\{\frac{1}{2}\int \frac{2x}{x^2+1}dx-\int \frac{1}{x^2+1}dx\right\}+\frac{1}{2}\int \frac{1}{x-1}dx$ $=-\frac{1}{4}\log(x^2+1)+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log\lvert x-1\rvert+C$	½ **1** ½ **1**
30.	Solve the following Linear Programming Problem graphically: Minimise z = 6x + 7y subject to the constraints $2x + y \geq 8$ $x + 2y \geq 10$ $x, y \geq 0$	
Sol.		**Correct graph 1 mark**

	<table><tr><td>**Corner Point**</td><td>$z=6x+7y$</td></tr><tr><td>$A(0,8)$</td><td>**56**</td></tr><tr><td>$B(2,4)$</td><td>**40**</td></tr><tr><td>$C(10,0)$</td><td>**60**</td></tr></table> since $6x+7y<40$ do not have any point in common with the feasible region, $z_{\min}=40$ when $x=2, y=4$	**1½** ½
31(a).	Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that y = 1 when x = 0.	
Sol.	$\frac{dy}{dx}=\frac{xy}{x^2+y^2}\quad ...(1)$ Put $\frac{y}{x}=v$ i.e. $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ Equation (1) gives $v+x\frac{dv}{dx}=\frac{v}{1+v^2}$ $\Rightarrow x\frac{dv}{dx}=-\frac{v^3}{1+v^2}$ $\Rightarrow \int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$ $\Rightarrow \frac{-1}{2v^2}+\log\lvert v\rvert=-\log\lvert x\rvert+\log c$ putting $v=\frac{y}{x}$ and simplifying gives $-\frac{x^2}{2y^2}=\log\left\lvert\frac{c}{y}\right\rvert$ now, $x=0, y=1$ gives $c=1$ required solution is : $\frac{x^2}{2y^2}=\log\lvert y\rvert$	½ 1 ½ ½ ½
	OR	
31(b).	Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, given that y = 0 when x = 1.	

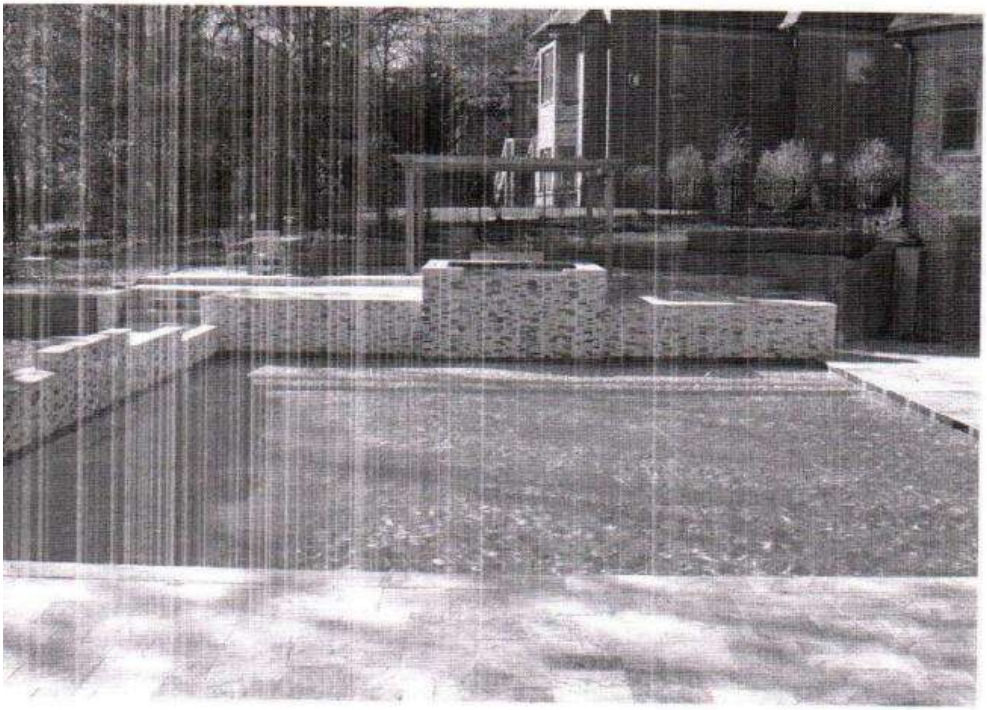
Sol.	Given diff. eqn. can be written as $\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{(1+x^2)^2}$ I.F. $=e^{\int\frac{2x}{1+x^2}dx}=1+x^2$ solution is given by: $y.(1+x^2)=\int\frac{1}{1+x^2}dx$ $\Rightarrow y.(1+x^2)=\tan^{-1}x+C$ Now $x=1, y=0$ gives $C=-\frac{\pi}{4}$ Reqired solution : $y.(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$	½ ½ ½ ½ ½ ½
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32.	Using integration, find the area of the region bounded by the triangle whose vertices are (– 1, 2), (1, 5) and (3, 4).	
Sol.	 $ar(ABC)=\int_{-1}^{1}y_{AB}\,dx+\int_{1}^{3}y_{BC}\,dx-\int_{-1}^{3}y_{AC}\,dx$ $=\int_{-1}^{1}\left(\frac{7+3x}{2}\right)dx+\int_{1}^{3}\left(\frac{11-x}{2}\right)dx-\int_{-1}^{3}\left(\frac{5+x}{2}\right)dx$ $=\frac{1}{2}\times\left(7x+\frac{3x^2}{2}\right)\Big]_{-1}^{1}+\frac{1}{2}\times\left(11x-\frac{x^2}{2}\right)\Big]_{1}^{3}-\frac{1}{2}\times\left(5x+\frac{x^2}{2}\right)\Big]_{-1}^{3}$ $=7+9-12=4$	**Correct figure 1 mark** **1** **1** **1** **1**

33.	If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations : $x + y + z = 6$ $x + 2z = 7$ $3x + y + z = 12$	
Sol.	$\|A\| = 1(-2) - 1(-5) + 1(1) = 4 \neq 0 \Rightarrow A^{-1}$ exists.	**1**
	$A_{11} = -2, A_{12} = 5, A_{13} = 1$ $A_{21} = 0, A_{22} = -2, A_{23} = 2$ $A_{31} = 2, A_{32} = -1, A_{33} = -1$ $adjA = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$	**1 ½**
	$A^{-1} = \frac{1}{\|A\|} adjA = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$	**1**
	Given system of equations can be written as $AX = B$, where $B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$	
	$X = A^{-1}B$	½
	$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$	½
	$\therefore x = 3, y = 1$ and $z = 2$	½
34(a).	Show that the relation S in set $\mathbb{R}$ of real numbers defined by $S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$ is neither reflexive, nor symmetric, nor transitive.	
Sol.	We have $S = \{(a, b : a \leq b^3\}$ where $a, b \in R.$ (i) Reflexive: we observe that, $\frac{1}{2} \leq \left(\frac{1}{2}\right)^3$ is not true.	

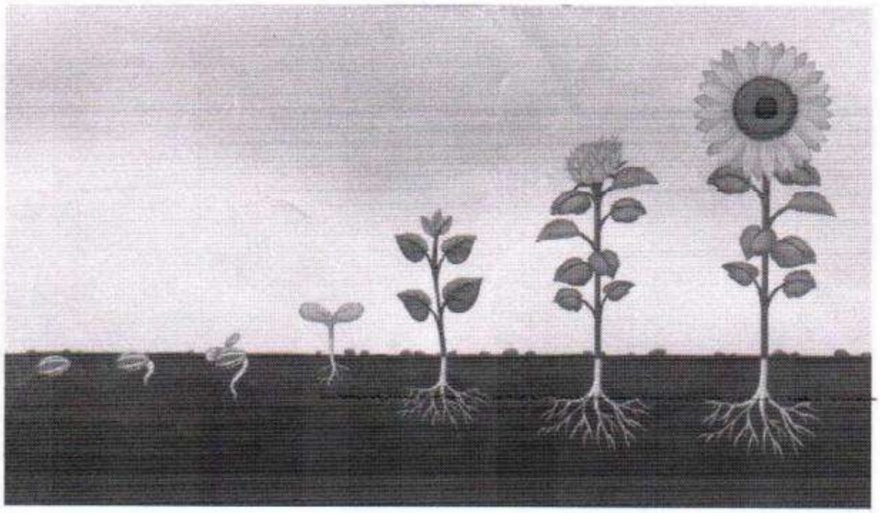
	$\therefore \left(\frac{1}{2},\frac{1}{2}\right) \notin S$. So, S is not reflexive. (ii) Symmetric: We observe that $1 \le 3^3$ but $3 \not\le 1^3$ i.e., (1, 3) $\in S$ but (3, 1) $\notin S$. So, S is not symmetric. (iii) Transitive: We observe that, $10 \le 3^3$ and $3 \le 2^3$ but $10 \not\le 2^3$. i.e., (10, 3) $\in S$ and (3, 2) $\in S$ but (10, 2) $\notin S$. So, S is not transitive. $\therefore$ S is neither reflexive nor symmetric, not transitive**.**	**1 ½** **1 ½** **2**
	OR	
34(b).	Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].	
Sol.	$R=\{(a,b):\text{both } a \text{ and } b \text{ are either odd or even}\}$ for reflexive : Let $a \in A$ clearly both a and a are either odd or even $\therefore (a,a) \in R \Rightarrow R$ is reflexive. for symmetric : Let $a,b \in A$. Let $(a,b) \in R$ $\Rightarrow$ both a and b are either odd or even $\Rightarrow$ both b and a are either odd or even so, $(a,\text{b}) \in R \Rightarrow (b,a) \in R \Rightarrow R$ is symmetric. for transitive : Let $a,b,c \in A$. Let $(a,b) \in R, (b,c) \in R$ $\Rightarrow$ both a and b are either odd or even & both b and c are either odd or even $\Rightarrow$ both a and c are either odd or even so, $(a,\text{b}) \in R, (b,c) \in R \Rightarrow (\text{a},\text{c}) \in R \Rightarrow R$ is transitive. equivalence class of $[1]=\{1,3,5,7\}$	**1** **1** **2** **1**
35(a).	Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.	

Sol.	line1: $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=\lambda$ ...(1) line 2: $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=\mu$ ...(2) General points on (1) and (2) are $(3\lambda-1, 5\lambda-3, 7\lambda-5)$ and $(\mu+2, 3\mu+4, 5\mu+6)$ for the lines to intersect, $3\lambda-1=\mu+2$ ...(3) $5\lambda-3=3\mu+4$ ...(4) $7\lambda-5=5\mu+6$ ...(5) solving (3) and (4) gives $\lambda=\frac{1}{2}$ and $\mu=-\frac{3}{2}$ clearly these values of λ and μ satisfies (5) $\Rightarrow$ given lines intersect. Point of intersection is $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$	**1** **1** **1** **1** **1**
	OR	
35(b).	Find the shortest distance between the pair of lines $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}$; $z=2$.	

Sol.	Given lines are $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-0}{1}$ and $\frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$ In vector form, lines are $\vec{r}=\left(\hat{i}-\hat{j}\right)+\lambda\left(2\hat{i}+3\hat{j}+\hat{k}\right)=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\left(-\hat{i}+2\hat{j}+2\hat{k}\right)+\mu\left(5\hat{i}+\hat{j}\right)=\vec{a}_2+\lambda\vec{b}_2$ now, $\vec{a}_2-\vec{a}_1=-2\hat{i}+3\hat{j}+2\hat{k}$ $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 5 & 1 & 0\end{vmatrix}=-\hat{i}+5\hat{j}-13\hat{k}$ $\left\|\vec{b}_1\times\vec{b}_2\right\|=\sqrt{195}$ S.D. $=\left\|\frac{\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)}{\left\|\vec{b}_1\times\vec{b}_2\right\|}\right\|$ $=\left\|\frac{2+15-26}{\sqrt{195}}\right\|=\frac{9}{\sqrt{195}}$	**1** **1** ½ **1** ½ **1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36.	A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.  Suppose the side of the square plot is x metres and depth is h metres. On the basis of the above information, answer the following questions : (i) Write cost C(h) as a function in terms of h. *1* (ii) Find critical point. *1* (iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2* **OR** (iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*	
Sol.		

	$(i)\text{Capacity} = \text{area}\times\text{depth} = x^2h = 250 \Rightarrow x^2 = \frac{250}{h}$ $C\,(\text{cost}) = 500x^2 + 4000h^2$ $\Rightarrow C = 500\left(\frac{250}{h}\right) + 4000h^2 = \frac{125000}{h} + 4000h^2$ $(ii)\frac{dC}{dh} = -\frac{125000}{h^2} + 8000h$ $\frac{dC}{dh} = 0 \Rightarrow h = \frac{5}{2}\,m \text{ or } 2.5\,m$ $(iii)(a)\frac{d^2C}{dh^2} = -125000\left(\frac{-2}{h^3}\right) + 8000 = \frac{250000}{h^3} + 8000$ $\left.\frac{d^2C}{dh^2}\right\|_{h=2.5m} > 0 \Rightarrow \text{Cost is minimum when } h = 2.5\,m$ $\text{Minimum cost} = C = \frac{125000}{\left(\frac{5}{2}\right)} + 4000\left(\frac{5}{2}\right)^2 = Rs.75,000$ OR $(iii)(b)$ we already have found above that $h = \frac{5}{2}\,m$ when $\frac{dC}{dh} = 0$ for the values of h less than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} < 0$ and, for the values of h more than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} > 0$ By first derivative test, there is a minimum at $h = \frac{5}{2}$ Now, $x^2 = \frac{250}{h} \Rightarrow x^2 = \frac{250}{\left(\frac{5}{2}\right)} = 100 \Rightarrow x = 10\,m$ also, $x = 4h$	**1** **1** **1** **1** **1** ½ ½

37.	In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield. A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function $f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2,\ 0 \le x \le 10$ where x is the number of days the plant is exposed to sunlight.  On the basis of the above information, answer the following questions : (i) What are the critical points of the function f(x) ? *2* (ii) Using second derivative test, find the minimum value of the function. *2*	
Sol.	$f^{I}(x) = x^2 - 8x + 15 = (x-3)(x-5)$ $f^{I}(x) = 0 \Rightarrow x = 3, 5$ are the critical points. Now $f^{II}(x) = 2x - 8$ $f^{II}(3) < 0$ and $f^{II}(5) > 0$ so, mimimum value of $f(x)$ is at $x = 5$. min. value $= f(5) = \frac{5^3}{3} - 4(5)^2 + 15(5) + 2 = \frac{56}{3}$	**1** **1** **1** **1**

38. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

Sol.

$(i)\ P(\text{age of selected student is a composite number})$

$= P(\text{age is } 14, 15 \text{ or } 16) = \frac{6}{10} = \frac{3}{5}$ **1**

$(ii)\ X$ can be 14, 15, 16, 17, 19 **1**

$(iii)(a)$

X	**14**	**15**	**16**	**17**	**19**	
$P(X)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	**1**

$\text{mean} = \sum X.P(X)$

$= 14\left(\frac{1}{10}\right) + 15\left(\frac{2}{10}\right) + 16\left(\frac{3}{10}\right) + 17\left(\frac{2}{10}\right) + 19\left(\frac{2}{10}\right) = 16.4 \text{ years}$ **1**

OR

$(iii)(b)\ A$: getting Prime number $= \{17, 19\}$

B : age is greater than 15 years $= \{16, 17, 19\}$

$A \cap B = \{17, 19\}$ **1**

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{3}$ **1**