Marking Scheme
Strictly Confidential
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Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE - 65/S/1(Comp.)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right **(✓)**while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out `with a note "Extra Question".**

10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME – 65/S/1

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A **(Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)**	
1.	If $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & -2 \end{bmatrix}$, then the value of $\lvert A \text{ adj } (A) \rvert$ is : (A) -1 (B) 1 (C) 2 (D) 3	
Ans	**(A) -1**	**1**
2.	For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of (4A) is : (A) 4B (B) B (C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$	
Ans	**(D) $\frac{1}{16}B$**	**1**
3.	If X, Y and XY are matrices of order 2×3, $m \times n$ and 2×5 respectively, then number of elements in matrix Y is : (A) 6 (B) 10 (C) 15 (D) 35	
Ans	**(C) 15**	**1**
4.	The number of discontinuities of the function f given by $f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ e^x, & \text{if } 0 \le x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$ is : (A) 0 (B) 1 (C) 2 (D) 3	
Ans	**(C) 2**	**1**

5.	Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$? (A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$ (C) -32 (D) -64	
Ans	**(C) -32**	**1**
6.	If $y = \log\sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is : (A) $\frac{1}{\pi}$ (B) π (C) $\frac{1}{2}$ (D) $\frac{1}{4}$	
Ans	**(A) $\frac{1}{\pi}$**	**1**
7.	If $x = 3\cos\theta$ and $y = 5\sin\theta$, then $\frac{dy}{dx}$ is equal to : (A) $-\frac{3}{5}\tan\theta$ (B) $-\frac{5}{3}\cot\theta$ (C) $-\frac{5}{3}\tan\theta$ (D) $-\frac{3}{5}\cot\theta$	
Ans	**(B) $-\frac{5}{3}\cot\theta$**	**1**
8.	For the function $f(x) = x^3$, $x = 0$ is a point of : (A) local maxima (B) local minima (C) non-differentiability (D) inflexion	
Ans	**(D) inflexion**	**1**

9.	The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$ (A) 0 (B) 1 (C) 2 (D) $\frac{3}{2}$	
Ans	**(C) 2**	**1**
10.	If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is : (A) $\frac{2\pi}{3}$ cm/s (B) π cm/s (C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s	
Ans	**(B) π cm/s**	**1**
11.	$\int_{-\pi/4}^{\pi/4} x^3 \cos^2 x \, dx$ is equal to : (A) 0 (B) -1 (C) 1 (D) 2	
Ans	**(A) 0**	**1**
12.	$\int \frac{x-3}{(x-1)^3} e^x \, dx$ is equal to : (A) $\frac{2e^x}{(x-1)^3} + C$ (B) $\frac{-2e^x}{(x-1)^2} + C$ (C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$	
Ans	**(D) $\frac{e^x}{(x-1)^2} + C$**	**1**

13.	The area (in sq. units) of the region bounded by the curve $y = x$, x-axis, $x = 0$ and $x = 2$ is : (A) $\frac{3}{2}$ (B) $\frac{1}{2}\log 2$ (C) 2 (D) 4	
Ans	**(C) 2**	**1**
14.	The number of arbitrary constants in the general solution of the differential equation $\frac{dy}{dx} + y = 0$ is : (A) 0 (B) 1 (C) 2 (D) 3	
Ans	**(B) 1**	**1**
15.	What is the value of $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$ for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$? (A) $\frac{3}{7}$ (B) $\frac{7}{3}$ (C) $\frac{4}{3}$ (D) $\frac{4}{7}$	
Ans	**(B) $\frac{7}{3}$**	**1**
16.	The direction ratios of the line $\frac{x-1}{3} = \frac{2-y}{1} = \frac{3z}{2}$ are : (A) 3, 1, 2 (B) 4, 3, 2 (C) 9, – 3, 2 (D) 9, 3, 2	
Ans	**(C) 9,–3,2**	**1**

17.	The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line $\vec{r} = 2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$ is : (A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$ (C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{2-z}{-2}$	
Ans	**(D)** $\frac{x-1}{1} = \frac{y+3}{1} = \frac{2-z}{-2}$	**1**
18.	If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \cdot \vec{b} > 0$ and $\lvert\vec{a} \cdot \vec{b}\rvert = \lvert\vec{a} \times \vec{b}\rvert$, then the angle between $\vec{a}$ and $\vec{b}$ is : (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$	
Ans	**(A)** $\frac{\pi}{4}$	**1**
	(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)	
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is equal to $\frac{\pi}{6}$. *Reason (R) :* The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).**	**1**

20.	*Assertion (A) :* If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$. *Reason (R) :* If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.	
Ans	**(C) Assertion (A) is true, but Reason (R) is false.**	**1**
	SECTION-B **(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)**	
21.	Find the value of $\cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \text{cosec}^{-1}(-2)$.	
Ans	$\cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \text{cosec}^{-1}(-2)$ $= \frac{\pi}{3} + \frac{\pi}{6} - \frac{\pi}{6}$ $= \frac{\pi}{3}$	$1\frac{1}{2}$ $\frac{1}{2}$
22.	(a) If $y = (\sin^{-1} x)^2$, then find $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$. **OR** (b) If $y^x = x^y$, then find $\frac{dy}{dx}$.	
Ans	**(a)** $\frac{dy}{dx} = \frac{2\sin^{-1} x}{\sqrt{1-x^2}} \Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$ **Differentiating with respect to 'x', we get** $(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = 4\frac{dy}{dx}$ $\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ **Or**	**1** $\frac{1}{2}$ $\frac{1}{2}$

	(b) $y^x = x^y \Rightarrow x\log y = y\log x$ **Differentiating with respect to 'x',** $\frac{x}{y}\frac{dy}{dx} + \log y = \frac{y}{x} + \frac{dy}{dx}\log x$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y - x\log y}{x - y\log x}\right)$	$\frac{1}{2}$ 1 $\frac{1}{2}$
23.	Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of f(x).	
Ans	$f'(x) = \frac{1-\log x}{x^2}$, $\therefore f'(x) = 0 \Rightarrow \log x = 1 \Rightarrow x = e$ $f''(x) = \frac{2x\log x - 3x}{x^4} \Rightarrow f''(e) = -\frac{1}{e^3} < 0$ **i.e.** $x = e$ **is a point of local maximum.**	1 1
24.	(a) Find : $\int \frac{x^3 - 1}{x^3 - x}\, dx$ **OR** (b) Evaluate : $\int_{-4}^{0} \lvert x + 2 \rvert\, dx$	
Ans	**(a)** $\int \frac{x^3-1}{x^3-x}dx = \int\left(1 + \frac{1}{x} - \frac{1}{x+1}\right)dx$ $= x + \log\lvert x\rvert - \log\lvert x+1\rvert + c$ **Or** **(b)** $\int_{-4}^{0} \lvert x+2\rvert dx = \int_{-4}^{-2} -(x+2)dx + \int_{-2}^{0}(x+2)dx$ $= -\frac{(x+2)^2}{2}\Big]_{-4}^{-2} + \frac{(x+2)^2}{2}\Big]_{-2}^{0} = 2 + 2 = 4$	1 1 1 1

25.	Find the angle between the lines $\frac{5-x}{-7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.	
Ans	**The direction ratios of the two lines are: $7,-5,1$ and $1,2,3$ respectively.** **$\therefore$ The angle between the two lines is given by:** $\theta=\cos^{-1}\left(\frac{7(1)-5(2)+1(3)}{\sqrt{49+25+1}\sqrt{1+4+9}}\right)$ $\Rightarrow \theta=\cos^{-1}0=\frac{\pi}{2}$	**½** **1** **½**
	SECTION-C **(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)**	
26.	(a) Find a matrix A such that $A\begin{bmatrix}4 & 0\\ -1 & -2\end{bmatrix}=\begin{bmatrix}17 & 10\\ 0 & -16\end{bmatrix}$. Also, find A^{-1}. **OR** (b) Given a square matrix A of order 3 such that $A^2=\begin{bmatrix}3 & -4 & 4\\ 0 & -1 & 0\\ -2 & 2 & -3\end{bmatrix}$, show that $A^3=A^{-1}$.	
Ans	**(a)** $A=\begin{bmatrix}17 & 10\\ 0 & -16\end{bmatrix}\begin{bmatrix}4 & 0\\ -1 & -2\end{bmatrix}^{-1}$ $=-\frac{1}{8}\begin{bmatrix}17 & 10\\ 0 & -16\end{bmatrix}\begin{bmatrix}-2 & 0\\ 1 & 4\end{bmatrix}$ $=\begin{bmatrix}3 & -5\\ 2 & 8\end{bmatrix}$ **Also,** $A^{-1}=\frac{1}{34}\begin{bmatrix}8 & 5\\ -2 & 3\end{bmatrix}$	**½** **1** **½** **1**

	Or **(b)** $A^3 = A^{-1} \Leftrightarrow A^4 = I$ $A^4 = A^2A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$	$1\frac{1}{2}$ $1\frac{1}{2}$
27.	(a) If $x \sin(a + y) - \sin y = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **OR** (b) Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$.	
Ans	**(a)** $x = \frac{\sin y}{\sin(a+y)} \Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$ $\Rightarrow \frac{dx}{dy} = \frac{\sin a}{\sin^2(a+y)}, \therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **Or** **(b)** Let $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $u = (\cos x)^x \Rightarrow \log u = x \log \cos x$ $\Rightarrow \frac{du}{dx} = (\cos x)^x [\log \cos x - x \tan x]$ $v = \cos^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = -\frac{1}{2\sqrt{x}\sqrt{1-x}}$ $\therefore \frac{dy}{dx} = (\cos x)^x [\log \cos x - x \tan x] - \frac{1}{2\sqrt{x}\sqrt{1-x}}$	2 1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$

28. (a) Find :

$$\int \frac{dx}{\cos x \sqrt{\cos 2x}}$$

OR

(b) Find :

$$\int \frac{5x-3}{\sqrt{1+4x-2x^2}}\,dx$$

Ans

(a) $\int \frac{dx}{\cos x\sqrt{\cos 2x}} = \int \frac{dx}{\cos x\sqrt{\cos^2 x - \sin^2 x}}$ — ½

$= \int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$ — 1

$= \int \frac{1}{\sqrt{1-t^2}}\,dt$, using $\tan x = t, \sec^2 x \cdot dx = dt$ — ½

$= \sin^{-1} t + c = \sin^{-1}(\tan x) + c$ — 1

Or

(b) $\int \frac{5x-3}{\sqrt{1+4x-2x^2}}dx = -\frac{5}{4}\int \frac{-4x+4}{\sqrt{1+4x-2x^2}}dx + \sqrt{2}\int \frac{1}{\sqrt{\left(\left(\sqrt{\frac{3}{2}}\right)^2 - (x-1)^2\right)}}dx$ — 1½

$= -\frac{5}{2}\sqrt{1+4x-2x^2} + \sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}(x-1)\right) + c$ — 1½

29. Find the general solution of the differential equation

$y\,dx - x\,dy + (x \log x)\,dx = 0.$

Ans

The given differential equation can be written as, $\frac{dy}{dx} - \frac{1}{x}y = \log x$ — ½

$\therefore\ P = -\frac{1}{x},\ Q = \log x$ — ½

$\text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log \frac{1}{x}} = \frac{1}{x}$ — ½

	The solution of the differential equation is: $$y\cdot\frac{1}{x}=\int \log x\cdot\frac{1}{x}dx$$ $$\Rightarrow \frac{y}{x}=\frac{(\log x)^2}{2}+c,\ \text{or}\ y=\frac{1}{2}x\cdot(\log x)^2+cx$$	**1** **½**
30.	If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ represent the three sides of a triangle, then show that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.	
Ans	$\vec{a}+\vec{b}+\vec{c}=\vec{O}$ $\Rightarrow \vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{O}$ $\Rightarrow \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{O}$ $\Rightarrow \vec{a}\times\vec{b}=-\vec{a}\times\vec{c},\ \therefore \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$ **Similarly,** $\vec{b}\times\vec{c}=\vec{c}\times\vec{a},\ \therefore \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$	**1** **½** **1** **½**
31.	It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?	
Ans	**Let B : Student getting 'A' grade** **E_1 : Student having above 90% attendance** **E_2 : Student being irregular** $$P(E_1)=\frac{20}{100};P(E_2)=\frac{80}{100}\ P(B\mid E_1)=\frac{80}{100};P(B\mid E_2)=\frac{20}{100}$$ $$P(E_2\mid B)=\frac{P(E_2)P(B\mid E_2)}{P(E_1)P(B\mid E_1)+P(E_2)P(B\mid E_2)}$$ $$=\frac{0.8\times0.2}{0.2\times0.8+0.8\times0.2}=\frac{1}{2}$$	**½** **1½** **½+½**

SECTION-D
(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)

32. Check whether the relation S in the set of all real numbers ($\mathbb{R}$) defined by $S = \{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive.

Ans

Reflexive: Let $\frac{1}{2} \not\le \left(\frac{1}{2}\right)^3$, $\therefore$ **S is not reflexive.** — 1

Symmetric: $1 \le 2^3 \Rightarrow (1,2) \in S$ but $2 \not\le 1^3 \Rightarrow (2,1) \notin S$. $\therefore$ **S is not symmetric.** — 2

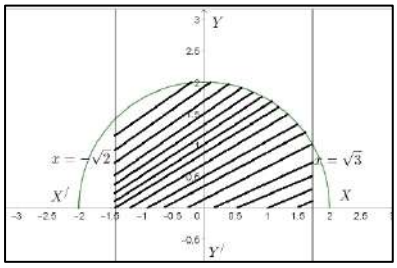
Transitive: $10 \le 6^3 \Rightarrow (10,6) \in S$ & $6 \le 2^3 \Rightarrow (6,2) \in S$ but $10 \not\le 2^3 \Rightarrow (10,2) \notin S$
$\therefore$ **S is not Transitive.** — 2

33. (a) Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

OR

(b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines $y = 1$ and $y = 3$ and the y-axis.

Ans



Correct Graph — 1

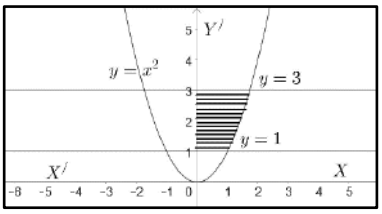
Area of the region bounded by the curve

$$= \int_{-\sqrt{2}}^{\sqrt{3}} \sqrt{4 - x^2}\,dx$$ — $1\frac{1}{2}$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_{-\sqrt{2}}^{\sqrt{3}}$$ — $1\frac{1}{2}$

$$= \frac{\sqrt{3}}{2} + 2\cdot\frac{\pi}{3} + 1 + 2\cdot\frac{\pi}{4} = \frac{\sqrt{3}}{2} + 1 + \frac{7\pi}{6}$$ — 1

Or

	Correct Graph  **Area of the region bounded by the curve** $= \int_1^3 \sqrt{y}\,dy$ $= \left[\frac{2y^{3/2}}{3}\right]_1^3 = \frac{2}{3}\left(3\sqrt{3}-1\right)$	**1** **2** **2**
34.	(a) Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **OR** (b) Find the point of intersection of the lines $\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and $\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$. Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.	
Ans	**(a)** $\vec{r} = \left(8\hat{i} - 9\hat{j} + 10\hat{k}\right) + \lambda\left(3\hat{i} - 16\hat{j} + 7\hat{k}\right)$ $\vec{r} = \left(15\hat{i} + 29\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 8\hat{j} - 5\hat{k}\right)$ **We have,** $\vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k}$, $\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$, $\vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}$, $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 7\hat{i} + 38\hat{j} - 5\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$ **Shortest Distance** $= \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right) \cdot \left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{98}{7} = 14$	**2** **½ + 1½** **1**

Or

If the given lines intersect then,

$$\hat{i}-\hat{j}+6\hat{k}+\lambda\left(3\hat{i}-\hat{k}\right)=-3\hat{j}+3\hat{k}+\mu\left(\hat{i}+2\hat{j}-\hat{k}\right)$$ **1**

Solving the equations, $1+3\lambda=\mu,-1=2\mu-3$ **, we get** $\lambda=0,\mu=1$ **2**

which do not satisfy the equation $,6-\lambda=3-\mu.$ **1**

$\therefore$ **The lines do not intersect, hence no point of intersection** **1**

35. Solve the following linear programming problem graphically :

Minimise Z = 6x + 7y

subject to constraints

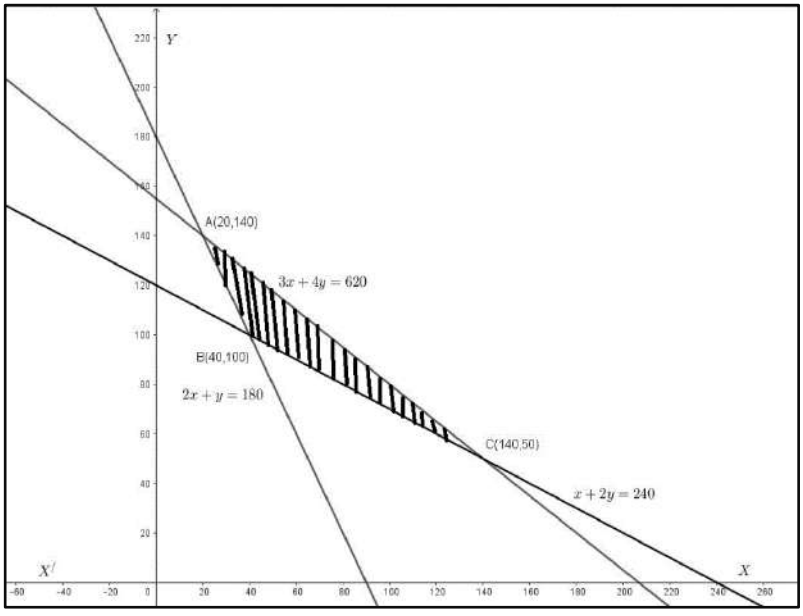
$x + 2y \geq 240$

$3x + 4y \leq 620$

$2x + y \geq 180$

$x, y \geq 0.$

Ans



Correct Graph $3\frac{1}{2}$

Corner Points	Value of $Z=6x+7y$
$A(20,140)$	$Z=1100$
$B(40,100)$	$Z=940$
$C(140,50)$	$Z=1190$

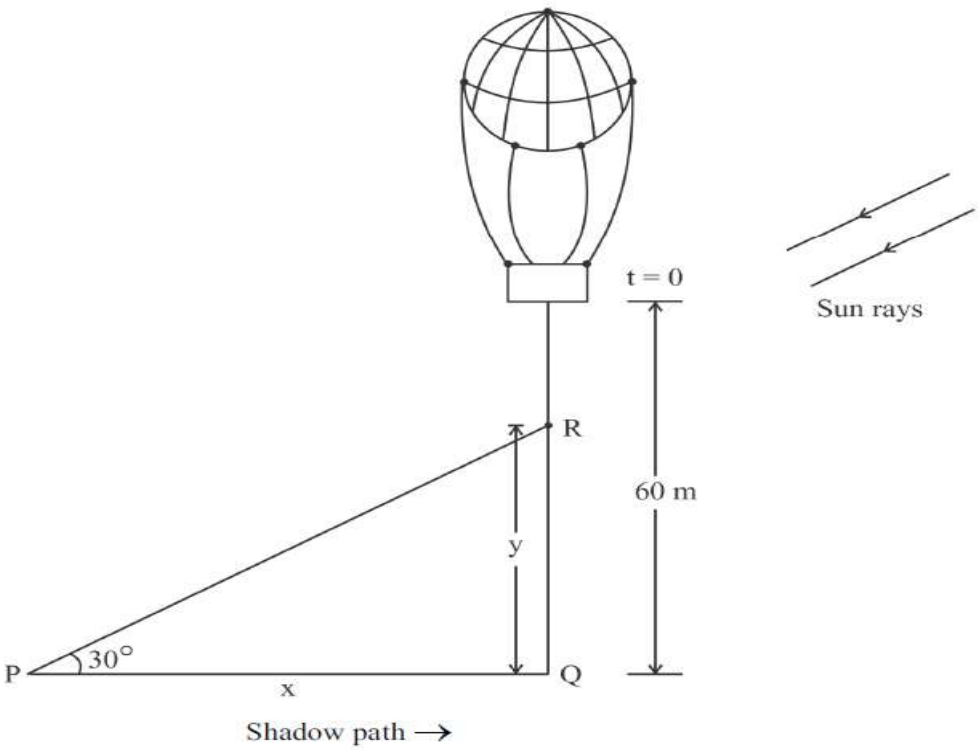
1

$\text{Min}(Z)=940$ **at** $x=40,y=100$ $\frac{1}{2}$

SECTION-E

(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)

36. A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9\ t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions :

(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.

(ii) After how much time will the sandbag be 35 metres above the ground ?

(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

Ans

(i) $\frac{y}{x} = \tan 30° \Rightarrow y = \frac{x}{\sqrt{3}}$ **or** $x = \sqrt{3}y$ — **1**

(ii) Using $y = 35m \Rightarrow 60 - 4.9t^2 = 35 \Rightarrow 4.9t^2 = 25 \Rightarrow t = \frac{5\sqrt{10}}{7}$ **seconds.** — **1**

(iii) (a) $x = \sqrt{3}y \Rightarrow x = 60\sqrt{3} - 4.9\sqrt{3}t^2 \Rightarrow \left.\frac{dx}{dt}\right]_{t=\frac{5\sqrt{10}}{7}} = -4.9\sqrt{3}(2t)\Big]_{t=\frac{5\sqrt{10}}{7}} = -7\sqrt{30}$ **m/s** — **2**

	Or **(iii) (b)** $\frac{dy}{dt} = -9.8t = -9.8 \times 2 = -19.6$ **m/s** **Height of the sandbag is decreasing at the rate of 19.6 m/s**	**2**
37.	A salesman receives a commission for each sale he makes together with a fixed daily income. The number of sales he makes in a day along with their probabilities are given in the table below :	

X :	0	1	2	3	4	5
P(X) :	0.42	3k	0.3	0.05	2k	0.03

His daily income Y (in ₹) is given by :

$$Y = 800X + 50$$

	On the basis of the above information, answer the following questions : (i) Find the value of k. (ii) Evaluate P(X ≥ 3). (iii) (a) Calculate the expected weekly income of the salesman assuming he works five days per week. **OR** (iii) (b) Calculate the expected weekly income of the salesman assuming he works only for three days of the week.	
Ans	**(i)** $0.42+3k+0.3+0.05+2k+0.03=1 \Rightarrow 5k=0.2 \Rightarrow k=\frac{1}{25}=0.04$ **(ii)** $P(X\geq 3)=0.05+2k+0.03=2k+0.08=0.08+0.08=0.16$ **(iii) (a) Expected sales per day =** $\sum x\cdot P(x)$ $=3k+0.6+0.15+8k+0.15=11k+0.9=1.34$ **Expected Daily income =** $Y=800(1.34)+50=$ ₹ 1122 **Expectedly sales income for five days per week = ₹ 5610** **Or** **(iii) (b) Expected sales per day =** $\sum x\cdot P(x)$ $=3k+0.6+0.15+8k+0.15=11k+0.9=1.34$ **Expected Daily income =** $Y=800(1.34)+50=$ ₹ 1122 **Expectedly sales income for three days of the week = ₹ 3366**	1 1 1 ½ ½ 1 ½ ½

38. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.

On the basis of the above information, answer the following questions :

(i) Formulate the linear equations in x and y to represent the given information.

(ii) Find the dimensions of the plot of land by matrix method.

Ans

(i) $(x-25)(y+25)=xy+625 \Rightarrow x-y=50$ — 1

$(x-20)(y+10)=xy-200 \Rightarrow x-2y=0$ — 1

(ii) The system of linear equations can be written in matrix form as

$$\begin{bmatrix}1 & -1\\ 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}50\\ 0\end{bmatrix}$$ — ½

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1 & -1\\ 1 & -2\end{bmatrix}^{-1}\begin{bmatrix}50\\ 0\end{bmatrix}=-\begin{bmatrix}-2 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}50\\ 0\end{bmatrix}=\begin{bmatrix}100\\ 50\end{bmatrix}$$ — 1

$x=100\text{m}, y=50\text{m}$ — ½

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE - 65/S/2(Comp.)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right **(✓)**while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out `with a note "Extra Question".**

10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME – 65/S/2

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A **(Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)**	
1.	If A and B are two square matrices of order 2 and $\|A\| = 2$ and $\|B\| = 5$, then $\|-3AB\|$ is : (A) -90 (B) -30 (C) 30 (D) 90	
Ans	**(D) 90**	**1**
2.	$\int \frac{x-3}{(x-1)^3} e^x \, dx$ is equal to : (A) $\frac{2e^x}{(x-1)^3} + C$ (B) $\frac{-2e^x}{(x-1)^2} + C$ (C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$	
Ans	**(D)** $\frac{e^x}{(x-1)^2} + C$	**1**
3.	The area (in sq. units) of the region bounded by the curve $y = x$, x-axis, $x = 0$ and $x = 2$ is : (A) $\frac{3}{2}$ (B) $\frac{1}{2}\log 2$ (C) 2 (D) 4	
Ans	**(C) 2**	**1**

4.	What is the value of $\dfrac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$ for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$? (A) $\frac{3}{7}$ (B) $\frac{7}{3}$ (C) $\frac{4}{3}$ (D) $\frac{4}{7}$	
Ans	**(B) $\frac{7}{3}$**	**1**
5.	If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \cdot \vec{b} > 0$ and $\lvert\vec{a} \cdot \vec{b}\rvert = \lvert\vec{a} \times \vec{b}\rvert$, then the angle between $\vec{a}$ and $\vec{b}$ is : (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$	
Ans	**(A) $\frac{\pi}{4}$**	**1**
6.	For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of (4A) is : (A) 4B (B) B (C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$	
Ans	**(D) $\frac{1}{16}$B**	**1**
7.	If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx}$ is equal to : (A) 2at (B) $\frac{1}{t}$ (C) $-\frac{1}{t^2}$ (D) $-\frac{1}{2at^3}$	

Ans	**(B) $\frac{1}{t}$**	**1**
8.	The function $f(x) = \|x\| - x$ where $x \in R$ is : (A) continuous and differentiable at $x = 0$ (B) continuous but not differentiable at $x = 0$ (C) not continuous but differentiable at $x = 0$ (D) neither continuous nor differentiable at $x = 0$	
Ans	**(B) continuous but not differentiable at $x = 0$**	**1**
9.	If X, Y and XY are matrices of order 2×3, $m \times n$ and 2×5 respectively, then number of elements in matrix Y is : (A) 6 (B) 10 (C) 15 (D) 35	
Ans	**(C) 15**	**1**
10.	The number of discontinuities of the function f given by $f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ e^x, & \text{if } 0 \le x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$ is : (A) 0 (B) 1 (C) 2 (D) 3	
Ans	**(C) 2**	**1**

11.	$\int e^{3\log x}\,(x^4+1)^{-1}\,dx$ is equal to : (A) $\frac{1}{4}\log(x^4+1)+C$ (B) $\frac{1}{4}\log\left(\frac{1}{x^4+1}\right)+C$ (C) $\frac{x^3}{x^4+1}+C$ (D) $\frac{e^x}{x^4+1}+C$	
Ans	**(A)** $\frac{1}{4}\log\left(x^4+1\right)+C$	**1**
12.	Let $y=f\left(\frac{1}{x}\right)$ and $f'(x)=x^3$. What is the value of $\frac{dy}{dx}$ at $x=\frac{1}{2}$? (A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$ (C) -32 (D) -64	
Ans	**(C) –32**	**1**
13.	If $y=\log\sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x=\frac{\pi^2}{16}$ is : (A) $\frac{1}{\pi}$ (B) π (C) $\frac{1}{2}$ (D) $\frac{1}{4}$	
Ans	**(A)** $\frac{1}{\pi}$	**1**

14.	A particular solution of the differential equation $x\frac{dy}{dx} + y = 0$, when $x = 1$ and $y = 1$, is : (A) $y = x$ (B) $y = e^x$ (C) $y = \frac{1}{x}$ (D) $y = \log x$	
Ans	**(C) $y = \frac{1}{x}$**	**1**
15.	The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$ (A) 0 (B) 1 (C) 2 (D) $\frac{3}{2}$	
Ans	**(C) 2**	**1**
16.	A vector makes equal angles with positive directions of x, y and z axes. The direction cosines of the vector are : (A) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (B) $\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (C) $\frac{-1}{2}, \frac{1}{2}, \frac{-1}{2}$ (D) $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$	
Ans	**(A) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$**	**1**

17.	The vector equation of the line passing through the points (0, 0, 2) and (3, – 2, 5) is : (A) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 5\hat{k})$ (B) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 5\hat{k})$ (C) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 3\hat{k})$ (D) $\vec{r} = 3\hat{i} - 2\hat{j} + 5\hat{k} + \lambda(2\hat{k})$	
Ans	**(C)** $\vec{r} = 2\hat{k} + \lambda\left(3\hat{i} - 2\hat{j} + 3\hat{k}\right)$	**1**
18.	If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is : (A) $\frac{2\pi}{3}$ cm/s (B) π cm/s (C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s	
Ans	**(B) π cm/s**	**1**
(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$. *Reason (R) :* If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.	
Ans	**(C) Assertion (A) is true, but Reason (R) is false.**	**1**

20.	*Assertion (A) :* $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is equal to $\frac{\pi}{6}$. *Reason (R) :* The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
	SECTION-B **(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)**	
21.	Evaluate : $\sin^{-1}\left(\sin\frac{13\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{3}\right) + \tan^{-1}(\sqrt{3})$	
Ans	$\sin^{-1}\left(\sin\frac{13\pi}{6}\right)+\cos^{-1}\left(\cos\frac{\pi}{3}\right)+\tan^{-1}\left(\sqrt{3}\right)$ $=\frac{\pi}{6}+\frac{\pi}{3}+\frac{\pi}{3}$ $=\frac{5\pi}{6}$	$1\frac{1}{2}$ $\frac{1}{2}$
22.	Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of f(x).	
Ans	$f'(x)=\frac{1-\log x}{x^2}$, $\therefore f'(x)=0 \Rightarrow \log x = 1 \Rightarrow x = e$ $f''(x)=\frac{2x\log x-3x}{x^4} \Rightarrow f''(e)=-\frac{1}{e^3}<0$ **i.e. $x = e$ is a point of local maximum.**	**1** **1**
23.	(a) Find : $\int \frac{x^3-1}{x^3-x}\,dx$ **OR** (b) Evaluate : $\int_{-4}^{0} \lvert x+2 \rvert\,dx$	

Ans	**(a)** $\int \frac{x^3-1}{x^3-x}dx = \int\left(1+\frac{1}{x}-\frac{1}{x+1}\right)dx$ $= x+\log\lvert x\rvert-\log\lvert x+1\rvert+c$ **Or** **(b)** $\int_{-4}^{0}\lvert x+2\rvert dx = \int_{-4}^{-2}-(x+2)dx+\int_{-2}^{0}(x+2)dx$ $= -\frac{(x+2)^2}{2}\Big]_{-4}^{-2}+\frac{(x+2)^2}{2}\Big]_{-2}^{0} = 2+2=4$	1 1 1 1
24.	(a) If $y=(\sin^{-1}x)^2$, then find $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$. **OR** (b) If $y^x = x^y$, then find $\frac{dy}{dx}$.	
Ans	**(a)** $\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}} \Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$ **Differentiating with respect to 'x', we get** $(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2 = 4\frac{dy}{dx}$ $\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$ **Or** **(b)** $y^x = x^y \Rightarrow x\log y = y\log x$ **Differentiating with respect to 'x',** $\frac{x}{y}\frac{dy}{dx}+\log y = \frac{y}{x}+\frac{dy}{dx}\log x$ $\Rightarrow \frac{dy}{dx}=\frac{y}{x}\left(\frac{y-x\log y}{x-y\log x}\right)$	1 ½ ½ ½ 1 ½
25.	Find the value of k so that the lines joining the points (1, – 1, 2) and (3, 4, k) is perpendicular to the line joining the points (0, 3, 2) and (3, 5, 6).	

Ans	**The direction ratios of the two lines are: $2,5,k-2$ and $3,2,4$ respectively.** **The lines are perpendicular $\therefore$, $2(3)+5(2)+4(k-2)=0 \Rightarrow k=-2$**	**1** **1**
	SECTION-C **(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)**	
26.	It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?	
Ans	**Let B : Student getting 'A' grade** **E_1 : Student having above 90% attendance** **E_2 : Student being irregular** $$P(E_1)=\frac{20}{100};P(E_2)=\frac{80}{100}\ P(B\mid E_1)=\frac{80}{100};P(B\mid E_2)=\frac{20}{100}$$ $$P(E_2\mid A)=\frac{P(E_2)P(B\mid E_2)}{P(E_1)P(B\mid E_1)+P(E_2)P(B\mid E_2)}$$ $$=\frac{0.8\times 0.2}{0.2\times 0.8+0.8\times 0.2}=\frac{1}{2}$$	**½** **1½** **½** **½**
27.	(a) If $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$, then find $\frac{dy}{dx}$. **OR** (b) If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$.	

Ans	**(a)** $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$	1
	$\Rightarrow y = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1} x$, $\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$	2
	Or	
	(b) $x^y = e^{x-y} \Rightarrow y \log x = x - y \Rightarrow y = \frac{x}{\log x + 1}$.	1
	$\Rightarrow \frac{dy}{dx} = \frac{\log x}{(\log x + 1)^2} = \frac{\log x}{\{\log (xe)\}^2}$	2
28.	(a) Find a matrix A such that $A\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}$. Also, find A^{-1}.	
	OR	
	(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$, show that $A^3 = A^{-1}$.	
Ans	**(a)** $A = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix}^{-1}$	½
	$= -\frac{1}{8}\begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}$	1
	$= \begin{bmatrix} 3 & -5 \\ 2 & 8 \end{bmatrix}$	½
	Also, $A^{-1} = \frac{1}{34}\begin{bmatrix} 8 & 5 \\ -2 & 3 \end{bmatrix}$	1
	Or	

	(b) $A^3 = A^{-1} \Leftrightarrow A^4 = I$ $A^4 = A^2A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$	$1\frac{1}{2}$ $1\frac{1}{2}$
29.	Find the particular solution of the differential equation : $x \cos y \, dy = (x e^x \log x + e^x) \, dx$ given that $y = \frac{\pi}{2}$ when $x = 1$.	
Ans	**Separating the variables and integrating:** $\int \cos y \, dy = \int e^x \left(\log x + \frac{1}{x} \right) dx$ $\Rightarrow \sin y = e^x \log x + c$ **Using** $y = \frac{\pi}{2}, x = 1$, **we get** $c = 1$ $\therefore$ **the particular solution is,** $\sin y = 1 + e^x \log x$	**1** **1** $\frac{1}{2}$ $\frac{1}{2}$
30.	Show that the vectors $3\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} - 8\hat{k}$ and $4\hat{i} - 2\hat{j} - 7\hat{k}$ form the vertices of a right triangle.	
Ans	**Let** $A(3\hat{i} + \hat{j} - 2\hat{k})$, $B(2\hat{i} - \hat{j} - 8\hat{k})$, **and** $C(4\hat{i} - 2\hat{j} - 7\hat{k})$, **then** $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ **and** $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{O}$ $\overrightarrow{BC} \cdot \overrightarrow{CA} = -2 - 3 + 5 = 0 \Rightarrow \overrightarrow{BC} \perp \overrightarrow{CA}$, $\therefore$ **ABC is a right triangle.**	**2** **1**

31. (a) Find :

$$\int \frac{dx}{\cos x \sqrt{\cos 2x}}$$

OR

(b) Find :

$$\int \frac{5x-3}{\sqrt{1+4x-2x^2}}\,dx$$

Ans

(a) $\int \frac{dx}{\cos x\sqrt{\cos 2x}} = \int \frac{dx}{\cos x\sqrt{\cos^2 x - \sin^2 x}}$ — ½

$= \int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$ — 1

$= \int \frac{1}{\sqrt{1-t^2}}\,dt$, using $\tan x = t, \sec^2 x \cdot dx = dt$ — ½

$= \sin^{-1} t + c = \sin^{-1}(\tan x) + c$ — 1

Or

(b) $\int \frac{5x-3}{\sqrt{1+4x-2x^2}}dx = -\frac{5}{4}\int \frac{-4x+4}{\sqrt{1+4x-2x^2}}dx + \sqrt{2}\int \frac{1}{\sqrt{\left(\left(\sqrt{\frac{3}{2}}\right)^2 - (x-1)^2\right)}}dx$ — 1½

$= -\frac{5}{2}\sqrt{1+4x-2x^2} + \sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}(x-1)\right) + c$ — 1½

SECTION-D

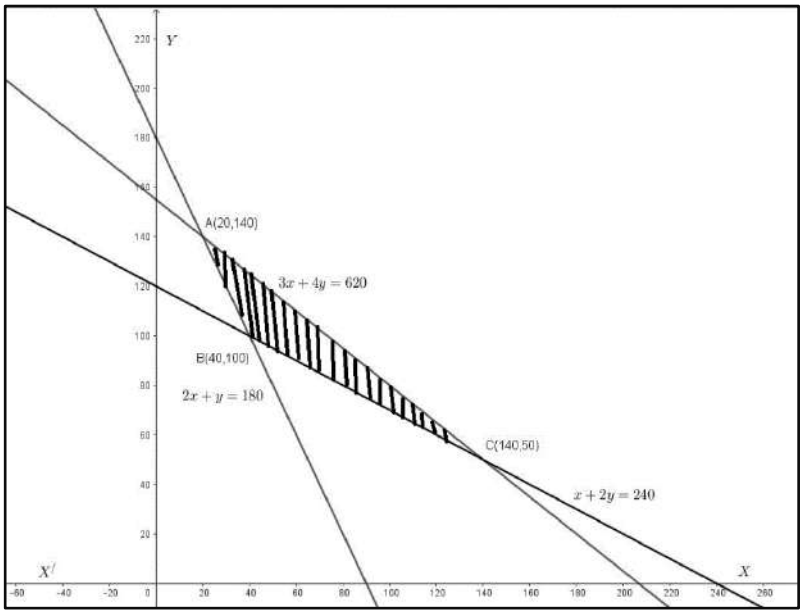
(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)

32. Check whether the relation S in the set $\mathbb{R}$ of real numbers, defined as $S = \{(a, b) : a \leq b^2\}$ is reflexive, symmetric or transitive. Also, determine all $x \in R$ such that $(x, x) \in S$.

Ans

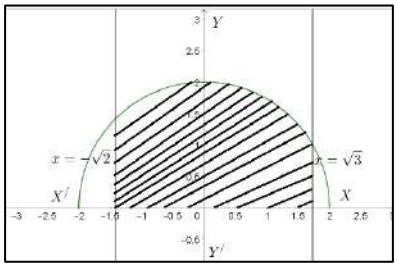
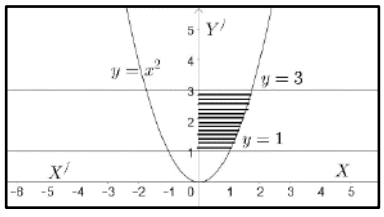
Reflexive: Let $\frac{1}{2} \not\leq \left(\frac{1}{2}\right)^2$, **$\therefore$ S is not reflexive.** — 1

Symmetric: $1 \leq 2^2 \Rightarrow (1,2) \in S$ **but** $2 \not\leq 1^2 \Rightarrow (2,1) \notin S$. **$\therefore$ S is not symmetric.** — 1½

	Transitive: $10 \le 6^2 \Rightarrow (10,6) \in S$ & $6 \le 3^2 \Rightarrow (6,3) \in S$ but $10 \not\le 3^2 \Rightarrow (10,3) \notin S$ $\therefore$ **S is not Transitive.** **Let** $(x,x) \in S \Leftrightarrow x \le x^2 \Leftrightarrow x \in R - (0,1)$	$1\frac{1}{2}$ **1**
33.	Solve the following linear programming problem graphically : Minimise Z = 6x + 7y subject to constraints $x + 2y \ge 240$ $3x + 4y \le 620$ $2x + y \ge 180$ $x, y \ge 0.$	
Ans	 **Correct Graph**	$3\frac{1}{2}$
		1
		$\frac{1}{2}$

Corner Points	Value of $Z = 6x + 7y$
$A(20,140)$	$Z = 1100$
$B(40,100)$	$Z = 940$
$C(140,50)$	$Z = 1190$

Min $(Z) = 940$ **at** $x = 40, y = 100$

34.	(a) Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis. **OR** (b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines $y = 1$ and $y = 3$ and the y-axis.	
Ans	 **Correct Graph**	**1**
	Area of the region bounded by the curve $= \int_{-\sqrt{2}}^{\sqrt{3}} \sqrt{4-x^2}\,dx$	**1½**
	$= \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_{-\sqrt{2}}^{\sqrt{3}}$	**1½**
	$= \frac{\sqrt{3}}{2} + 2\cdot\frac{\pi}{3} + 1 + 2\cdot\frac{\pi}{4} = \frac{\sqrt{3}}{2} + 1 + \frac{7\pi}{6}$	**1**
	Or	
	 **Correct Graph**	**1**
	Area of the region bounded by the curve $= \int_{1}^{3} \sqrt{y}\,dy$	**2**
	$= \left[\frac{2y^{3/2}}{3}\right]_{1}^{3} = \frac{2}{3}\left(3\sqrt{3} - 1\right)$	**2**

35. (a) Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

OR

(b) Find the point of intersection of the lines

$\vec{r}=\hat{i}-\hat{j}+6\hat{k}+\lambda(3\hat{i}-\hat{k})$, and

$\vec{r}=-3\hat{j}+3\hat{k}+\mu(\hat{i}+2\hat{j}-\hat{k})$.

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

Ans

(a) $\vec{r}=\left(8\hat{i}-9\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right)$

$\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$

We have, $\vec{a}_1=8\hat{i}-9\hat{j}+10\hat{k}$, $\vec{b}_1=3\hat{i}-16\hat{j}+7\hat{k}$, $\vec{a}_2=15\hat{i}+29\hat{j}+5\hat{k}$, $\vec{b}_2=3\hat{i}+8\hat{j}-5\hat{k}$ — 2

$\vec{a}_2-\vec{a}_1=7\hat{i}+38\hat{j}-5\hat{k}$, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-16&7\\3&8&-5\end{vmatrix}=24\hat{i}+36\hat{j}+72\hat{k}$ — ½+1½

Shortest Distance $=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|=\frac{98}{7}=14$ — 1

Or

If the given lines intersect then,

$$\hat{i}-\hat{j}+6\hat{k}+\lambda\left(3\hat{i}-\hat{k}\right)=-3\hat{j}+3\hat{k}+\mu\left(\hat{i}+2\hat{j}-\hat{k}\right)$$ — 1

Solving the equations, $1+3\lambda=\mu,-1=2\mu-3$, **we get** $\lambda=0,\mu=1$ — 2

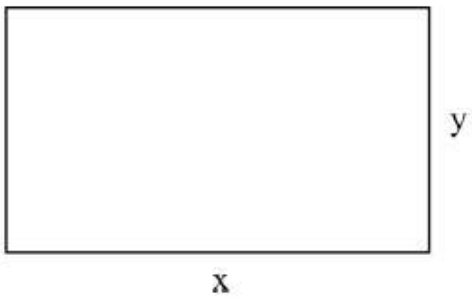
which do not satisfy the equation $,6-\lambda=3-\mu$. — 1

$\therefore$ The lines do not intersect, hence no point of intersection — 1

SECTION-E

(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)

36. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.



On the basis of the above information, answer the following questions :

(i) Formulate the linear equations in x and y to represent the given information.

(ii) Find the dimensions of the plot of land by matrix method.

Ans

(i) $(x-25)(y+25)=xy+625 \Rightarrow x-y=50$ — 1

$(x-20)(y+10)=xy-200 \Rightarrow x-2y=0$ — 1

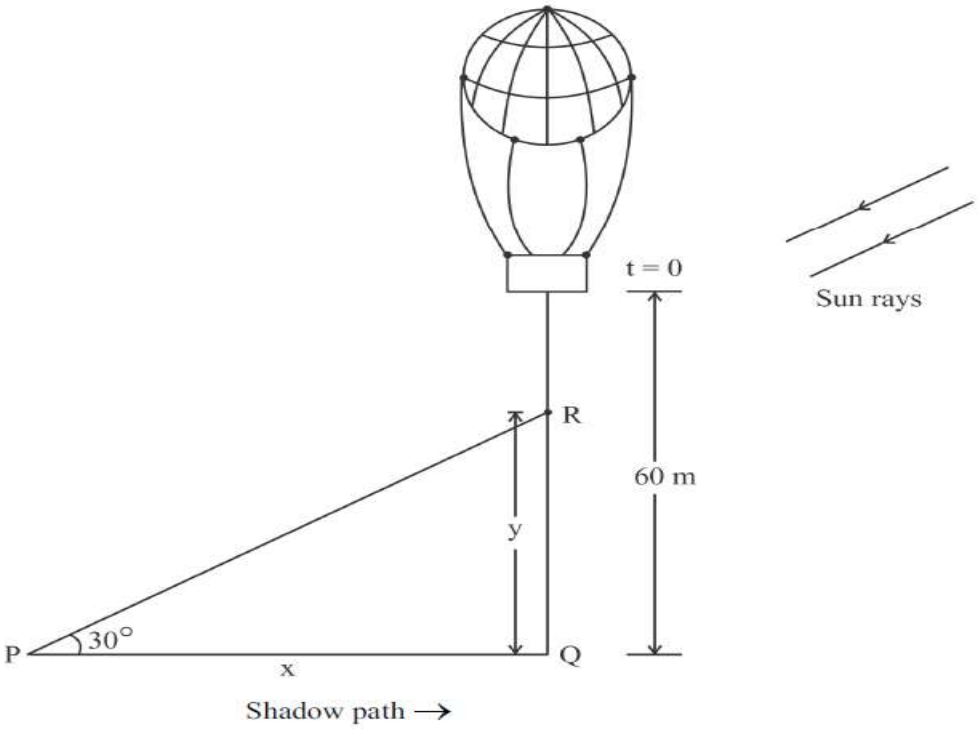
(ii) The system of linear equations can be written in matrix form as

$$\begin{bmatrix}1 & -1\\ 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}50\\ 0\end{bmatrix}$$ — ½

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1 & -1\\ 1 & -2\end{bmatrix}^{-1}\begin{bmatrix}50\\ 0\end{bmatrix}=-\begin{bmatrix}-2 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}50\\ 0\end{bmatrix}=\begin{bmatrix}100\\ 50\end{bmatrix}$$ — 1

$x=100\text{m}, \; y=50\text{m}$ — ½

37. A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9\ t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions :

(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.

(ii) After how much time will the sandbag be 35 metres above the ground ?

(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

Ans

(i) $\frac{y}{x} = \tan 30° \Rightarrow y = \frac{x}{\sqrt{3}}$ **or** $x = \sqrt{3}y$ — **1**

(ii) Using $y = 35m \Rightarrow 60 - 4.9t^2 = 35 \Rightarrow 4.9t^2 = 25 \Rightarrow t = \frac{5\sqrt{10}}{7}$ **seconds.** — **1**

(iii) (a) $x = \sqrt{3}y \Rightarrow x = 60\sqrt{3} - 4.9\sqrt{3}t^2 \Rightarrow \left.\frac{dx}{dt}\right]_{t=\frac{5\sqrt{10}}{7}} = -4.9\sqrt{3}(2t)\Big]_{t=\frac{5\sqrt{10}}{7}} = -7\sqrt{30}$ m/s — **2**

Or

(iii) (b) $\frac{dy}{dt} = -9.8t = -9.8 \times 2 = -19.6$ m/s — **2**

Height of the sandbag is decreasing at the rate of 19.6 m/s

38. A salesman receives a commission for each sale he makes together with a fixed daily income. The number of sales he makes in a day along with their probabilities are given in the table below :

X :	0	1	2	3	4	5
P(X) :	0.42	3k	0.3	0.05	2k	0.03

His daily income Y (in ₹) is given by :

$$Y = 800X + 50$$

On the basis of the above information, answer the following questions :

(i) Find the value of k.

(ii) Evaluate $P(X \geq 3)$.

(iii) (a) Calculate the expected weekly income of the salesman assuming he works five days per week.

OR

(iii) (b) Calculate the expected weekly income of the salesman assuming he works only for three days of the week.

Ans	**(i)** $0.42+3k+0.3+0.05+2k+0.03=1 \Rightarrow 5k=0.2 \Rightarrow k=\frac{1}{25}=0.04$	1
	(ii) $P(X \geq 3)=0.05+2k+0.03=2k+0.08=0.08+0.08=0.16$	1
	(iii) (a) Expected sales per day = $\sum x \cdot P(x)$	
	$=3k+0.6+0.15+8k+0.15=11k+0.9=1.34$	1
	Expected Daily income = $Y=800(1.34)+50=$ ₹ 1122	½
	Expectedly sales income for five days per week = ₹ 5610	½
	Or	
	(iii) (b) Expected sales per day = $\sum x \cdot P(x)$	
	$=3k+0.6+0.15+8k+0.15=11k+0.9=1.34$	1
	Expected Daily income = $Y=800(1.34)+50=$ ₹ 1122	½
	Expectedly sales income for three days of the week = ₹ 3366	½

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE - 65/S/3(Comp.)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right **(✓)**while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out `with a note "Extra Question".**

10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME – 65/S/3

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A **(Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)**	
1.	If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$, then the value of $\det(A^{-1})$ is : (A) -1 (B) 1 (C) 0 (D) 2	
Ans	**(A) -1**	**1**
2.	The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$ (A) 0 (B) 1 (C) 2 (D) $\frac{3}{2}$	
Ans	**(C) 2**	**1**
3.	If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is : (A) $\frac{2\pi}{3}$ cm/s (B) π cm/s (C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s	
Ans	**(B) π cm/s**	**1**
4.	$\int \frac{x-3}{(x-1)^3} e^x \, dx$ is equal to : (A) $\frac{2e^x}{(x-1)^3} + C$ (B) $\frac{-2e^x}{(x-1)^2} + C$ (C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$	

Ans	**(D)** $\dfrac{e^x}{(x-1)^2}+C$	**1**
5.	The area (in sq. units) of the region bounded by the curve $y = x$, x-axis, $x = 0$ and $x = 2$ is : (A) $\frac{3}{2}$ (B) $\frac{1}{2}\log 2$ (C) 2 (D) 4	
Ans	**(C) 2**	**1**
6.	What is the value of $\dfrac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$ for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$? (A) $\frac{3}{7}$ (B) $\frac{7}{3}$ (C) $\frac{4}{3}$ (D) $\frac{4}{7}$	
Ans	**(B)** $\dfrac{7}{3}$	**1**
7.	If $f(x) = x^x$, then $f'(e)$ is equal to : (A) 0 (B) 2 (C) e^e (D) $2e^e$	
Ans	**(D)** $2e^e$	**1**
8.	The function f given by $f(x) = x^3 - 3x^2 + 3x$, $x \in \mathbb{R}$ is increasing on : (A) $[1, \infty)$ (B) $(1, \infty)$ (C) $(-\infty, \infty)$ (D) $(-\infty, 1)$	
Ans	**(C)** $(-\infty, \infty)$	**1**

9.	If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \cdot \vec{b} > 0$ and $\left\|\vec{a} \cdot \vec{b}\right\| = \left\|\vec{a} \times \vec{b}\right\|$, then the angle between $\vec{a}$ and $\vec{b}$ is : (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$	
Ans	**(A) $\frac{\pi}{4}$**	**1**
10.	For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of (4A) is : (A) 4B (B) B (C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$	
Ans	**(D) $\frac{1}{16}B$**	**1**
11.	$\int_{\pi/6}^{\pi/3} \log(\tan x)\, dx$ is equal to : (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 0 (D) $\frac{\pi}{12}$	
Ans	**(C) 0**	**1**
12.	If X, Y and XY are matrices of order 2×3, $m \times n$ and 2×5 respectively, then number of elements in matrix Y is : (A) 6 (B) 10 (C) 15 (D) 35	
Ans	**(C) 15**	**1**

13.	The number of discontinuities of the function f given by $f(x) = \begin{cases} x+2, & \text{if } x<0 \\ e^x, & \text{if } 0 \le x \le 1 \\ 2-x, & \text{if } x>1 \end{cases}$ is : (A) 0 (B) 1 (C) 2 (D) 3	
Ans	**(C) 2**	**1**
14.	The integrating factor of the differential equation $(x^2-1)\frac{dy}{dx}+2xy=\frac{1}{x^2-1}$ is : (A) e^{2x} (B) e^{x^2-1} (C) $\log(x^2-1)$ (D) x^2-1	
Ans	**(D) x^2-1**	**1**
15.	Let $y=f\left(\frac{1}{x}\right)$ and $f'(x)=x^3$. What is the value of $\frac{dy}{dx}$ at $x=\frac{1}{2}$? (A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$ (C) -32 (D) -64	
Ans	**(C) -32**	**1**
16.	The vectors $\vec{a}=2\hat{i}-4\hat{j}+\lambda\hat{k}$ and $\vec{b}=3\hat{i}-6\hat{j}+\hat{k}$ are collinear if value of λ is : (A) -30 (B) 30 (C) $\frac{3}{2}$ (D) $\frac{2}{3}$	
Ans	**(D) $\frac{2}{3}$**	**1**

17.	The vector equation of the line passing through the origin and perpendicular to the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ and $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is : (A) $\vec{r} = \lambda \hat{i}$ (B) $\vec{r} = \lambda \hat{j}$ (C) $\vec{r} = \lambda \hat{k}$ (D) $\vec{r} = \lambda (\hat{i} + \hat{k})$	
Ans	**(B) $\vec{r} = \lambda \hat{j}$**	**1**
18.	If $y = \log \sqrt{\sec \sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is : (A) $\frac{1}{\pi}$ (B) π (C) $\frac{1}{2}$ (D) $\frac{1}{4}$	
Ans	**(A) $\frac{1}{\pi}$**	**1**
(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$ is equal to $\frac{\pi}{6}$. *Reason (R) :* The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).**	**1**

20.	*Assertion (A) :* If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$. *Reason (R) :* If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.	
Ans	**(C) Assertion (A) is true, but Reason (R) is false.**	**1**
	SECTION-B **(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)**	
21.	Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.	
Ans	$\tan^{-1}\left(\tan\frac{3\pi}{5}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$ $=-\frac{2\pi}{5}+\frac{\pi}{6}-\frac{\pi}{6}$ $=-\frac{2\pi}{5}$	$1\frac{1}{2}$ $\frac{1}{2}$
22.	(a) Find : $\int \frac{x^3-1}{x^3-x}\,dx$ **OR** (b) Evaluate : $\int_{-4}^{0} \lvert x+2 \rvert\, dx$	
Ans	**(a)** $\int \frac{x^3-1}{x^3-x}dx = \int\left(1+\frac{1}{x}-\frac{1}{x+1}\right)dx$ $= x+\log\lvert x\rvert-\log\lvert x+1\rvert+c$ **Or**	**1** **1**

	(b) $\int_{-4}^{0}\|x+2\|dx=\int_{-4}^{-2}-(x+2)dx+\int_{-2}^{0}(x+2)dx$	1
	$=-\frac{(x+2)^2}{2}\Big]_{-4}^{-2}+\frac{(x+2)^2}{2}\Big]_{-2}^{0}=2+2=4$	1
23.	(a) If $y=(\sin^{-1}x)^2$, then find $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$.	
	OR	
	(b) If $y^x=x^y$, then find $\frac{dy}{dx}$.	
Ans	(a) $\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}\Rightarrow(1-x^2)\left(\frac{dy}{dx}\right)^2=4y$	1
	Differentiating with respect to 'x', we get	
	$(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2=4\frac{dy}{dx}$	½
	$\Rightarrow(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$	½
	Or	
	(b) $y^x=x^y\Rightarrow x\log y=y\log x$	½
	Differentiating with respect to 'x',	
	$\frac{x}{y}\frac{dy}{dx}+\log y=\frac{y}{x}+\frac{dy}{dx}\log x$	1
	$\Rightarrow\frac{dy}{dx}=\frac{y}{x}\left(\frac{y-x\log y}{x-y\log x}\right)$	½
24.	Given that $f(x)=\frac{\log x}{x}$, find the point of local maximum of f(x).	
Ans	$f'(x)=\frac{1-\log x}{x^2}$, $\therefore f'(x)=0\Rightarrow\log x=1\Rightarrow x=e$	1

	$f''(x)=\frac{2x\log x-3x}{x^4}\Rightarrow f''(e)=-\frac{1}{e^3}<0$ **i.e.** $x=e$ **is a point of local maximum.**	**1**
25.	Find the Cartesian equation of the line passing through the origin, perpendicular to y-axis and making equal acute angles with x and z axes.	
Ans	**Let the line makes angle** α **with the x and z axes respectively.**	$\frac{1}{2}$
	$\therefore \cos^2\alpha+\cos^2\frac{\pi}{2}+\cos^2\alpha=1\Rightarrow\cos\alpha=\pm\frac{1}{\sqrt{2}}$	1
	$\frac{x}{\frac{1}{\sqrt{2}}}=\frac{y}{0}=\frac{z}{\frac{1}{\sqrt{2}}}$ **or** $\frac{x}{1}=\frac{y}{0}=\frac{z}{1}$	$\frac{1}{2}$
	SECTION-C **(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)**	
26.	(a) Find : $\int\frac{dx}{\cos x\sqrt{\cos 2x}}$ **OR** (b) Find : $\int\frac{5x-3}{\sqrt{1+4x-2x^2}}dx$	
Ans	**(a)** $\int\frac{dx}{\cos x\sqrt{\cos 2x}}=\int\frac{dx}{\cos x\sqrt{\cos^2 x-\sin^2 x}}$	$\frac{1}{2}$
	$=\int\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx$	1
	$=\int\frac{1}{\sqrt{1-t^2}}dt$, **using** $\tan x=t,\sec^2 x\cdot dx=dt$	$\frac{1}{2}$
	$=\sin^{-1}t+c=\sin^{-1}(\tan x)+c$	1
	Or	
	(b) $\int\frac{5x-3}{\sqrt{1+4x-2x^2}}dx=-\frac{5}{4}\int\frac{-4x+4}{\sqrt{1+4x-2x^2}}dx+\sqrt{2}\int\frac{1}{\sqrt{\left(\left(\sqrt{\frac{3}{2}}\right)^2-(x-1)^2\right)}}dx$	$1\frac{1}{2}$

	$= -\frac{5}{2}\sqrt{1+4x-2x^2} + \sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}(x-1)\right) + c$	$1\frac{1}{2}$
27.	(a) If $y = e^{a\cos^{-1}x}$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. **OR** (b) Find $\frac{dy}{dx}$, if $y = x^{\cos x} - 2^{\sin x}$.	
Ans	**(a)** $\frac{dy}{dx} = e^{a\cos^{-1}x}\frac{-a}{\sqrt{1-x^2}} = -\frac{ay}{\sqrt{1-x^2}}$, **squaring we get**	$1\frac{1}{2}$
	$\left(\frac{dy}{dx}\right)^2(1-x^2) = a^2y^2$, **differentiating again with respect to 'x'.**	
	$(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = 2a^2y\frac{dy}{dx} \Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$	$1\frac{1}{2}$
	Or	
	(b) Let $y = u - v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$, **where** $u = x^{\cos x}, v = 2^{\sin x}$	
	$\log u = \cos x \log x \Rightarrow \frac{du}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right]$, $\frac{dv}{dx} = 2^{\sin x}\cos x \log 2$	$1\frac{1}{2} + 1$
	$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] - 2^{\sin x}\cos x \log 2$	$\frac{1}{2}$
28.	It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?	
Ans	**Let B : Student getting 'A' grade** **E_1 : Student having above 90% attendance** **E_2 : Student being irregular** }	$\frac{1}{2}$

	$P(E_1)=\frac{20}{100}; P(E_2)=\frac{80}{100}\ P(B\mid E_1)=\frac{80}{100}; P(B\mid E_2)=\frac{20}{100}$ $P(E_2\mid B)=\frac{P(E_2)P(B\mid E_2)}{P(E_1)P(B\mid E_1)+P(E_2)P(B\mid E_2)}$ $=\frac{0.8\times 0.2}{0.2\times 0.8+0.8\times 0.2}=\frac{1}{2}$	1½ ½ + ½
29.	Find the general solution of the differential equation $\frac{dy}{dx}=xy\log x\log y$.	
Ans	**Separating the variables and integrating,** $\int\frac{dy}{y\log y}=\int x\log x\,dx$ $\Rightarrow \log\lvert\log y\rvert=\log x\left(\frac{x^2}{2}\right)-\int\frac{1}{x}\cdot\frac{x^2}{2}dx+c$ $\Rightarrow \log\lvert\log y\rvert=\frac{x^2}{2}\log x-\frac{x^2}{4}+c$	1 1½ ½
30.	Find all vectors of magnitude $8\sqrt{14}$ units that are perpendicular to the vectors $2\hat{i}-\hat{k}$ and $2\hat{j}+3\hat{k}$.	
Ans	**Vector parallel to the required vector is** $\begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 0 & 2 & 3 \end{vmatrix}=2\hat{i}-6\hat{j}+4\hat{k}$, $\lvert 2\hat{i}-6\hat{j}+4\hat{k}\rvert=2\sqrt{14}$ **The required vectors are** $=\pm 8\sqrt{14}\left(\frac{-2\hat{i}-6\hat{j}+4\hat{k}}{2\sqrt{14}}\right)=\pm\left(8\hat{i}+24\hat{j}-16\hat{k}\right)$	1½ + ½ 1

31. (a) Find a matrix A such that

$$A\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}.$$

Also, find A^{-1}.

OR

(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$,

show that $A^3 = A^{-1}$.

Ans

(a) $\mathbf{A = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix}^{-1}}$ — ½

$\mathbf{= -\frac{1}{8}\begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}}$ — 1

$\mathbf{= \begin{bmatrix} 3 & -5 \\ 2 & 8 \end{bmatrix}}$ — ½

Also, $\mathbf{A^{-1} = \frac{1}{34}\begin{bmatrix} 8 & 5 \\ -2 & 3 \end{bmatrix}}$ — 1

Or

(b) $\mathbf{A^3 = A^{-1} \Leftrightarrow A^4 = I}$ — 1½

$$\mathbf{A^4 = A^2A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I}$$ — 1½

SECTION-D
(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)

32. Prove that the relation R in the set of integers Z defined as

$R = \{(a, b) : 2 \text{ divides } (a + b)\}$ is an equivalence relation. Also, determine [3].

Ans

Reflexive: $a+a=2a$, which is divisible by 2, $\forall a \in Z \Rightarrow (a,a) \in R, \forall a \in Z$, $\therefore$ R is reflexive — 1

Symmetric: Let $(a,b) \in R \Rightarrow 2$ divides $(a+b) \Rightarrow 2$ divides $(b+a) \Rightarrow (b,a) \in R$ — $1\frac{1}{2}$

$\therefore$ R is symmetric.

Transitive: Let $(a,b),(b,c) \in R \Rightarrow 2$ divides $(a+b)$ & $(b+c)$ both — 1

$\Rightarrow a+b=2m, b+c=2n$

$\Rightarrow a+2b+c=2m+2n \therefore a+c=2(m+n-b)$

$\therefore 2$ divides $(a+c) \Rightarrow$ S is Transitive. — 1

$\therefore$ S is an equivalance relation.

$[3]=\{x : x \text{ is an odd integer}\}$ or $[3]=\{...,-1,0,1,3,5,7,...\}$ — $\frac{1}{2}$

33. (a) Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

OR

(b) Find the point of intersection of the lines

$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and

$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$.

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

Ans

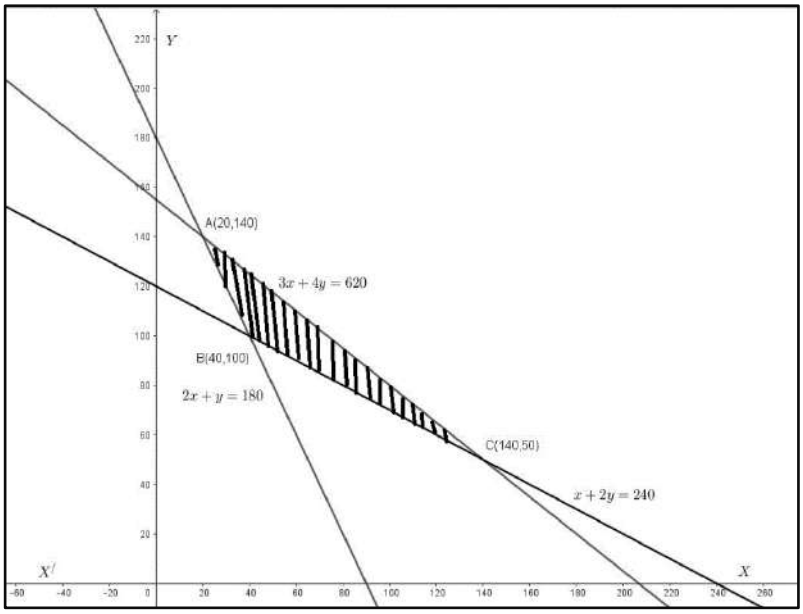
(a) $\vec{r} = (8\hat{i} - 9\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

We have, $\vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k}$, $\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$, $\vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}$, $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$ — 2

	$\vec{a}_2-\vec{a}_1=7\hat{i}+38\hat{j}-5\hat{k}$, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-16&7\\3&8&-5\end{vmatrix}=24\hat{i}+36\hat{j}+72\hat{k}$	$\frac{1}{2}+1\frac{1}{2}$
	Shortest Distance $=\left\lvert\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{\lvert\vec{b}_1\times\vec{b}_2\rvert}\right\rvert=\frac{98}{7}=14$	1
	Or	
	If the given lines intersect then, $\hat{i}-\hat{j}+6\hat{k}+\lambda(3\hat{i}-\hat{k})=-3\hat{j}+3\hat{k}+\mu(\hat{i}+2\hat{j}-\hat{k})$	1
	Solving the equations, $1+3\lambda=\mu,-1=2\mu-3$**, we get** $\lambda=0,\mu=1$	2
	which do not satisfy the equation $,6-\lambda=3-\mu$**.**	1
	$\therefore$ The lines do not intersect, hence no point of intersection	1
34.	Solve the following linear programming problem graphically : Minimise Z = 6x + 7y subject to constraints $x + 2y \geq 240$ $3x + 4y \leq 620$ $2x + y \geq 180$ $x, y \geq 0.$	

Ans



Correct Graph — $3\frac{1}{2}$

Corner Points	Value of $Z = 6x + 7y$
$A(20,140)$	$Z = 1100$
$B(40,100)$	$Z = 940$
$C(140,50)$	$Z = 1190$

1

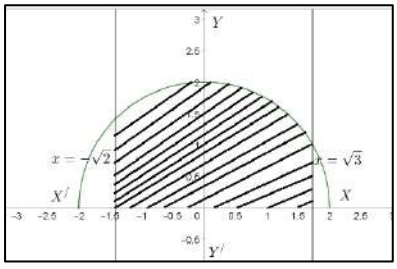
$\text{Min}(Z) = 940$ at $x = 40, y = 100$ — $\frac{1}{2}$

35. (a) Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

OR

(b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines $y = 1$ and $y = 3$ and the y-axis.

Ans



Correct Graph — 1

Area of the region bounded by the curve

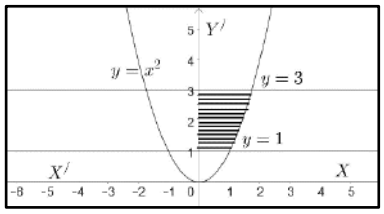
$$= \int_{-\sqrt{2}}^{\sqrt{3}} \sqrt{4 - x^2}\,dx$$ — $1\frac{1}{2}$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_{-\sqrt{2}}^{\sqrt{3}}$$ — $1\frac{1}{2}$

$$= \frac{\sqrt{3}}{2} + 2 \cdot \frac{\pi}{3} + 1 + 2 \cdot \frac{\pi}{4} = \frac{\sqrt{3}}{2} + 1 + \frac{7\pi}{6}$$ 1

Or

Correct Graph **1**



Area of the region bounded by the curve

$$= \int_1^3 \sqrt{y}\,dy$$ **2**

$$= \left[\frac{2y^{3/2}}{3}\right]_1^3 = \frac{2}{3}\left(3\sqrt{3} - 1\right)$$ **2**

SECTION-E

(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)

36. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.

On the basis of the above information, answer the following questions :

(i) Formulate the linear equations in x and y to represent the given information.

(ii) Find the dimensions of the plot of land by matrix method.

Ans

(i) $(x-25)(y+25)=xy+625 \Rightarrow x-y=50$ — **1**

$(x-20)(y+10)=xy-200 \Rightarrow x-2y=0$ — **1**

(ii) The system of linear equations can be written in matrix form as

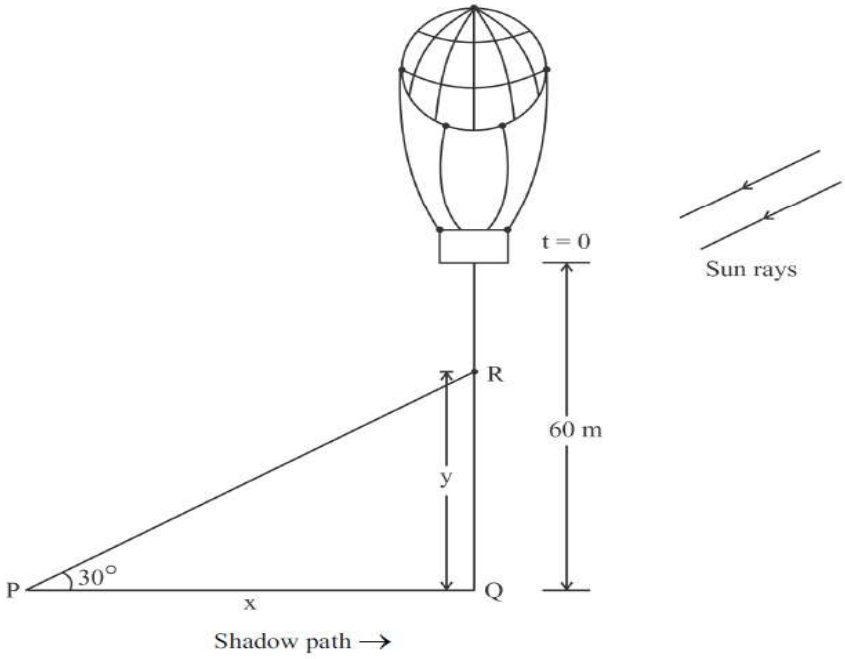
$$\begin{bmatrix}1 & -1\\ 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}50\\ 0\end{bmatrix}$$ — **½**

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1 & -1\\ 1 & -2\end{bmatrix}^{-1}\begin{bmatrix}50\\ 0\end{bmatrix}=-\begin{bmatrix}-2 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}50\\ 0\end{bmatrix}=\begin{bmatrix}100\\ 50\end{bmatrix}$$ — **1**

$$x=100\text{m},\ y=50\text{m}$$ — **½**

37.

A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9\ t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions :

(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.

(ii) After how much time will the sandbag be 35 metres above the ground ?

(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

Ans

(i) $\frac{y}{x} = \tan 30° \Rightarrow y = \frac{x}{\sqrt{3}}$ **or** $x = \sqrt{3}y$ — **1**

(ii) Using $y = 35m \Rightarrow 60 - 4.9t^2 = 35 \Rightarrow 4.9t^2 = 25 \Rightarrow t = \frac{5\sqrt{10}}{7}$ **seconds.** — **1**

(iii) (a) $x = \sqrt{3}y \Rightarrow x = 60\sqrt{3} - 4.9\sqrt{3}t^2 \Rightarrow \frac{dx}{dt}\Big]_{t=\frac{5\sqrt{10}}{7}} = -4.9\sqrt{3}(2t)\Big]_{t=\frac{5\sqrt{10}}{7}} = -7\sqrt{30}$ **m/s** — **2**

Or

(iii) (b) $\frac{dy}{dt} = -9.8t = -9.8 \times 2 = -19.6$ **m/s** — **2**

Height of the sandbag is decreasing at the rate of 19.6 m/s

38. A salesman receives a commission for each sale he makes together with a fixed daily income. The number of sales he makes in a day along with their probabilities are given in the table below :

X :	0	1	2	3	4	5
P(X) :	0.42	3k	0.3	0.05	2k	0.03

His daily income Y (in ₹) is given by :

$$Y = 800X + 50$$

	On the basis of the above information, answer the following questions : (i) Find the value of k. (ii) Evaluate P(X ≥ 3). (iii) (a) Calculate the expected weekly income of the salesman assuming he works five days per week. **OR** (iii) (b) Calculate the expected weekly income of the salesman assuming he works only for three days of the week.	
Ans	**(i)** $0.42+3k+0.3+0.05+2k+0.03=1 \Rightarrow 5k=0.2 \Rightarrow k=\frac{1}{25}=0.04$ **(ii)** $P(X \geq 3)=0.05+2k+0.03=2k+0.08=0.08+0.08=0.16$ **(iii) (a) Expected sales per day =** $\sum X \cdot P(X)$ $=3k+0.6+0.15+8k+0.15=11k+0.9=1.34$ **Expected Daily income =** $Y=800(1.34)+50=$ ₹ 1122 **Expectedly sales income for five days per week = ₹ 5610** **Or** **(iii) (b) Expected sales per day =** $\sum X \cdot P(X)$ $=3k+0.6+0.15+8k+0.15=11k+0.9=1.34$ **Expected Daily income =** $Y=800(1.34)+50=$ ₹ 1122 **Expectedly sales income for three days of the week = ₹ 3366**	**1** **1** **1** **½** **½** **1** **½** **½**