Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Supplementary Examination, 2025
SUBJECT- MATHEMATICS (041) (Q.P. CODE – 65/S/1)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	If the matrix $A = [a_{ij}]_{2\times2}$ is such that $a_{ij} = \begin{cases}1, & \text{if } i \neq j\\ 0, & \text{if } i = j\end{cases}$, then $A + A^2$ is equal to : (A) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (B) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ (C) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ (D) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	
Ans	(C) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$	1
Q2.	The domain of $f(x) = \cos^{-1}(2x)$ is : (A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	
Ans	(D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	1
Q3.	The value of the determinant $\begin{vmatrix}\cos 75^\circ & \sin 75^\circ\\ \sin 15^\circ & \cos 15^\circ\end{vmatrix}$ is : (A) 1 (B) zero (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$	
Ans	(B) zero	1

Q4.	For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to : (A) X (B) X^2 (C) I (D) O	
Ans	(A) X	1
Q5.	The cofactor of the element a_{32} in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is : (A) ± 5 (B) – 5 (C) 5 (D) 0	
Ans	(C) 5	1
Q6.	If A is an identity matrix of order n, then A (Adj A) is a/an : (A) identity matrix (B) row matrix (C) zero matrix (D) skew symmetric matrix	
Ans	(A) identity matrix	1
Q7.	If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is : (A) $\frac{3}{2}$ (B) $-\frac{2}{9}$ (C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$	
Ans	(B) $-\frac{2}{9}$	1

Q8.	The area bounded by the parabola $x^2 = y$ and the line y = 1 is : (A) $\frac{2}{3}$ sq unit (B) $\frac{1}{3}$ sq unit (C) $\frac{4}{3}$ sq units (D) 2 sq units	
Ans	(C) $\frac{4}{3}$ sq units	1
Q9.	If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is : (A) 1 sq unit (B) 2 sq units (C) 3 sq units (D) 4 sq units	
Ans	(B) 2 sq units	1
Q10.	$\int \frac{3 \cos \sqrt{x}}{\sqrt{x}} dx$ is equal to : (A) $-6 \sin \sqrt{x} + C$ (B) $-6 \cos \sqrt{x} + C$ (C) $6 \cos \sqrt{x} + C$ (D) $6 \sin \sqrt{x} + C$	
Ans	(D) $6 \sin \sqrt{x} + C$	1

Q11.	If $\frac{d}{dx}f(x) = 3x^2 - \frac{3}{x^4}$ such that f(1) = 0, then f(x) is : (A) $6x + \frac{12}{x^5}$ (B) $x^4 - \frac{1}{x^3} + 2$ (C) $x^3 + \frac{1}{x^3} - 2$ (D) $x^3 + \frac{1}{x^3} + 2$	
Ans	(C) $x^3 + \frac{1}{x^3} - 2$	1
Q12.	In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If Z = ax + by, where a, b > 0 is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be : (A) $a = 2b$ (B) $a = \frac{b}{2}$ (C) $a = 3b$ (D) $a = b$	
Ans	(B) $a = \frac{b}{2}$	1
Q13.	The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is : (A) 3 (B) 4 (C) 7 (D) 0	
Ans	(B) 4	1

Q14.	The general solution of the differential equation $\frac{dy}{dx} = 2x \, . e^{x^2 + y}$ is : (A) $e^{x^2 + y} = C$ (B) $e^{x^2} + e^{-y} = C$ (C) $e^{x^2} = e^{y} + C$ (D) $e^{x^2 - y} = C$	
Ans	(B) $e^{x^2} + e^{-y} = C$	1
Q15.	If ‘m’ and ‘n’ are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of (m + n) is : (A) 4 (B) 3 (C) 2 (D) 5	
Ans	(B) 3	1
Q16.	If $\mid \vec{a} \mid = 1$, $\mid \vec{b} \mid = 2$ and $\vec{a} \, . \, \vec{b} = 2$, then the value of $\mid \vec{a} + \vec{b} \mid$ is : (A) 9 (B) 3 (C) – 3 (D) 2	
Ans	(B) 3	1
Q17.	Two vectors $\vec{a}$ and $\vec{b}$ are such that $\mid \vec{a} \times \vec{b} \mid = \vec{a} \, . \, \vec{b}$. The angle between the two vectors is : (A) 30° (B) 60° (C) 45° (D) 90°	
Ans	(C) 45°	1

Q18.	A coin is tossed three times. The probability of getting at least two heads is : (A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$	
Ans	(A) $\frac{1}{2}$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

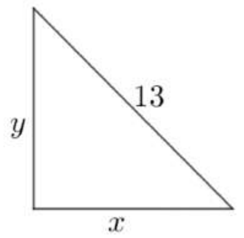
(D) Assertion (A) is false, but Reason (R) is true.

Q19.	Consider the function f : R $\longrightarrow$ R, defined as $f(x) = x^3$. *Assertion (A) :* f(x) is a one-one function. *Reason (R) :* f(x) is a one-one function, if co-domain = range.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1
Q20.	*Assertion (A) :* $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at x = 2. *Reason (R) :* The greatest integer function is not continuous at any integral value.	
Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1

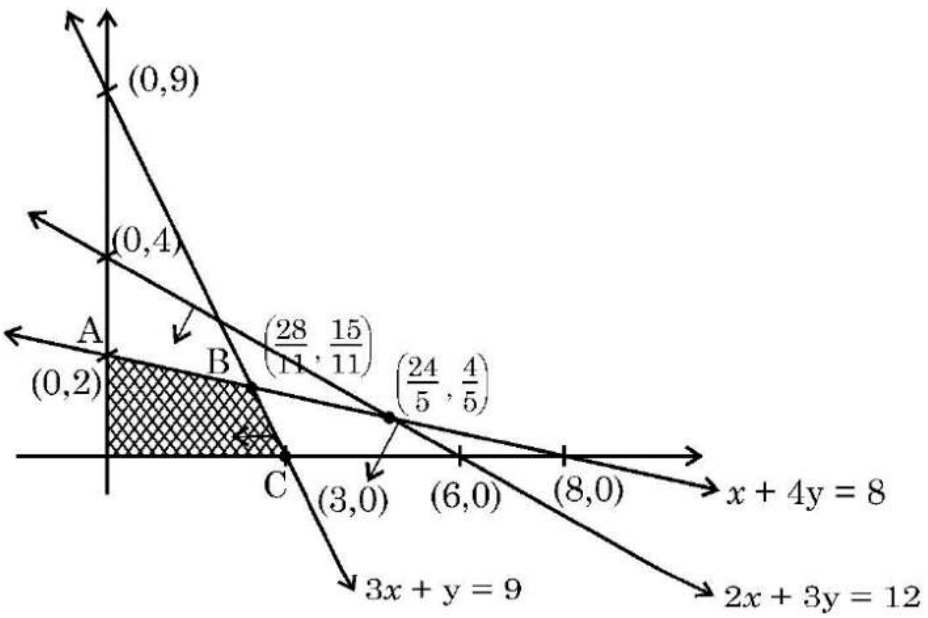
SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.	(a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. **OR** (b) Prove that : $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$	
Ans(a)	$cos^{-1}\left(-\frac{1}{2}\right) + 2\,sin^{-1}\left(\frac{1}{2}\right)$ $= \left(\pi - \frac{\pi}{3}\right) + 2\left(\frac{\pi}{6}\right)$ $= \pi$	 1 + ½ ½
	OR	
Ans(b)	Put $x = tan^2\,\theta \Rightarrow \theta = tan^{-1}\sqrt{x}$ RHS $= \frac{1}{2}cos^{-1}\left(\frac{1 - tan^2\,\theta}{1 + tan^2\,\theta}\right)$ $= \frac{1}{2}cos^{-1}(cos\,2\,\theta)$ $= \frac{1}{2}(2\theta)$ $= \theta = tan^{-1}\sqrt{x} =$ LHS	½ 1 ½
Q22.	If $e^y(x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$.	
Ans	$e^y(x+1) = 1 \Rightarrow e^y = \frac{1}{x+1}$ $\Rightarrow y = -\log(x+1)$ $\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$ $= -e^y \qquad \left[\because \frac{1}{x+1} = e^y\right]$	 ½ 1 ½
Q23.	A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?	

Ans	$x^2+y^2=169$ Differentiate both sides w.r.t. t $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$ $\Rightarrow 12(2)+5\left(\frac{dy}{dt}\right)=0[\because \text{ when } x=12m, y=5m]$ $\Rightarrow \frac{dy}{dt}=-\frac{24}{5}$ Hence, the height decreases at the rate of $\frac{24}{5}$ m/s 	½ 1 ½
Q24.	(a) Find the value of λ, if the points (–1, –1, 2), (2, 8, λ) and (3, 11, 6) are collinear. **OR** (b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $\vert\vec{a}\vert = 10$, $\vert\vec{b}\vert = 2$ and $\vec{a}\,.\,\vec{b} = 12$. Find the area of the parallelogram.	
Ans(a)	$A(-1,-1,2), B(2,8,\lambda), C(3,11,6)$ $\overrightarrow{AB}=3\hat{\imath}+9\hat{\jmath}+(\lambda-2)\hat{k}$ and $\overrightarrow{BC}=\hat{\imath}+3\hat{\jmath}+(6-\lambda)\hat{k}$ Since A, B and C are collinear, $\frac{3}{1}=\frac{9}{3}=\frac{\lambda-2}{6-\lambda}$ $\Rightarrow \lambda=5$	1 ½ ½
	OR	
Ans(b)	Let θ is the angle between $\vec{a}$ and $\vec{b}$. $\vec{a}.\vec{b}=12 \Rightarrow \vert\vec{a}\vert\vert\vec{b}\vert\cos\theta=12$ $\Rightarrow (10)(2)\cos\theta=12 \Rightarrow \cos\theta=\frac{3}{5}$ $\therefore \sin\theta=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$ Now, area of parallelogram $=\vert\vec{a}\times\vec{b}\vert=\vert\vec{a}\vert\vert\vec{b}\vert\sin\theta$ $=(10)(2)\left(\frac{4}{5}\right)=16$ $\therefore$ area of parallelogram $=16$	½ ½ ½ ½
Q25.	Find the angle between the lines $\vec{r}=(3+2\lambda)\hat{i}-(2-2\lambda)\hat{j}+(6+2\lambda)\hat{k}$ and $\vec{r}=(2\hat{j}-5\hat{k})+\mu(6\hat{i}+3\hat{j}+2\hat{k})$.	

Ans	Given lines are: $\vec{r}=\left(3\hat{i}-2\hat{j}+6\hat{k}\right)+\lambda\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$ and $\vec{r}=\left(2\hat{j}-5\hat{k}\right)+\mu\left(6\hat{i}+3\hat{j}+2\hat{k}\right)$ Let θ be the angle between these two lines. $\cos\theta=\frac{2(6)+2(3)+2(2)}{\sqrt{4+4+4}\sqrt{36+9+4}}=\frac{22}{2\sqrt{3}\times 7}$ $\Rightarrow\cos\theta=\frac{11}{21}\sqrt{3}\Rightarrow\theta=\cos^{-1}\left(\frac{11}{21}\sqrt{3}\right)$	½ 1 ½
	SECTION C This section comprises short answer (SA) type questions of **3 marks each**.	
Q26.	Find the maximum slope of the curve $y=-x^3+3x^2+9x-30$.	
Ans	$y=-x^3+3x^2+9x-30$ Slope of the curve, $m=\frac{dy}{dx}=-3x^2+6x+9$ $\Rightarrow\frac{dm}{dx}=-6x+6$ For maximum/ minimum slope, put $\frac{dm}{dx}=0$ $\Rightarrow x=1$ As $\frac{d^2m}{dx^2}=-6<0\ \therefore m$ is maximum at $x=1$ Maximum slope $=-3(1)^2+6(1)+9=12$	 1 ½ ½ ½ ½
Q27.	(a) Find : $\int\sqrt{4x^2-4x+10}\ dx$ **OR** (b) Evaluate : $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}\ dx$	
Ans(a)	$I=\int\sqrt{4x^2-4x+10}\,dx$ $=\int\sqrt{(2x-1)^2+(3)^2}dx$ $=\frac{1}{2}\left[\left(\frac{2x-1}{2}\right)\sqrt{4x^2-4x+10}+\frac{9}{2}log\left\|(2x-1)+\sqrt{4x^2-4x+10}\right\|\right]+C$	 1 1+1
	OR	

Ans(b)	$I=\int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx \qquad ...(i)$	
	$=\int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$	
	$I=\int_0^\pi \frac{(\pi-x)\sin x}{1+\cos^2 x}dx \qquad ...(ii)$	½
	Adding (i) and (ii)	
	$2I=\int_0^\pi \frac{\pi\sin x}{1+\cos^2 x}dx$	½
	$\Rightarrow I=\frac{\pi}{2}\int_0^\pi \frac{\sin x}{1+\cos^2 x}dx$	
	Put $\cos x=t \Rightarrow -\sin x\,dx=dt$	½
	$I=-\frac{\pi}{2}\int_1^{-1}\frac{dt}{1+t^2}$	½
	$=\pi\left[\tan^{-1}t\right]_0^1=\frac{\pi^2}{4}$	1
Q28.	Solve the following LPP graphically : Maximize Z = 2x + 3y subject to the constraints $x + 4y \le 8$ $2x + 3y \le 12$ $3x + y \le 9$ $x \ge 0, y \ge 0.$	
Ans		For correct graph and shading 2

Corner Point	Value of $Z = 2x + 3y$
$O(0,0)$	0
$A(0,2)$	6
$B\left(\frac{28}{11}, \frac{15}{11}\right)$	$\frac{101}{11}$ Maximum
$C(3,0)$	6

For correct table 1

$Z_{\max} = \frac{101}{11}$ when $x = \frac{28}{11}, y = \frac{15}{11}$

Q29. (a) Find the general solution of the differential equation $(2x^2 + y)\,dx = x\,dy$.

OR

(b) For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that y = 0 when x = 1.

Ans(a)

$(2x^2 + y)\,dx = x\,dy$

$\Rightarrow \frac{dy}{dx} - \frac{1}{x}.y = 2x$ 1

$\text{I.F.} = e^{-\int \frac{1}{x} dx} = \frac{1}{x}$ 1

Solution is given by,

$y.\left(\frac{1}{x}\right) = \int 2x.\frac{1}{x}\,dx$ ½

$\Rightarrow \frac{y}{x} = 2x + C \text{ or } y = 2x^2 + Cx$ ½

OR

Ans(b)

$\frac{dy}{dx} = \frac{y}{x} - \cos ec\left(\frac{y}{x}\right)$

Put $\frac{y}{x} = v$ i.e. $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ½

The differential equation reduces to

$v + x\frac{dv}{dx} = v - \cos ec v$ ½

$\sin v\,dv = -\frac{1}{x}dx \Rightarrow \int \sin v\,dv = -\int \frac{dx}{x}$

$\Rightarrow -\cos v = -\log|x| - C$ 1

$\text{or}, \cos v = \log|x| + C'$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + C'$

$y = 0, x = 1 \text{ gives } C' = 1$ ½

$\therefore$ Solution is given by $\cos\left(\frac{y}{x}\right) = \log|x| + 1$ ½

Q30. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}\cdot\hat{b} = \hat{a}\cdot\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2\,(\hat{b} \times \hat{c})$.

Ans)

$\hat{a}.\hat{b} = 0 \Rightarrow \hat{a} \perp \hat{b}, \hat{a}.\hat{c} = 0 \Rightarrow \hat{a} \perp \hat{c}$ 1

$\Rightarrow \hat{a}$ is perpendicular to both $\hat{b}$ and $\hat{c} \Rightarrow \hat{a} \,||\, \left(\hat{b} \times \hat{c}\right)$

Let $\hat{a} = \lambda\left(\hat{b} \times \hat{c}\right)$ ½

$\Rightarrow |\hat{a}| = |\lambda|\left|\left(\hat{b} \times \hat{c}\right)\right| \Rightarrow |\hat{a}| = |\lambda||\hat{b}||\hat{c}|\sin\frac{\pi}{6}$ 1

$\Rightarrow |\lambda| = 2 \Rightarrow \lambda = \pm 2$ ½

$\therefore \hat{a} = \pm 2\left(\hat{b} \times \hat{c}\right)$

Q31. (a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

(b) The probability distribution of a random variable X is given below :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Find k, if E(X) = 2·94 and also find P(X ≤ 4).

Ans(a)

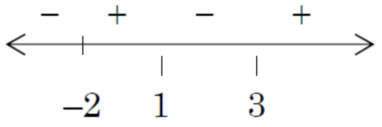
$P(\text{at most one of them will solve the problem})$

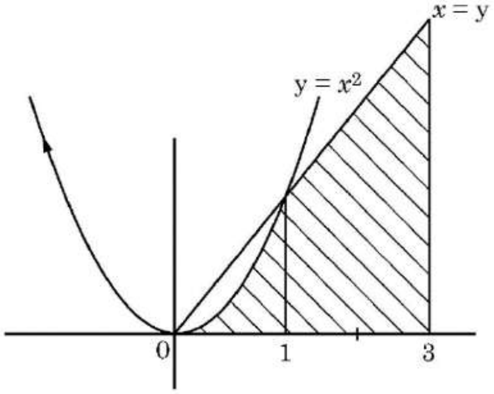
$= P(\text{none of them solves the problem}) + P(\text{only one of them solves the problem})$

$= \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times \frac{4}{5}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times \frac{4}{5} + \frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} \times \frac{4}{5} + \frac{1}{2} \times \frac{2}{3} \times \frac{2}{3} \times \frac{4}{5} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times \frac{1}{5}\right)$ 2½

$= \frac{19}{45}$ ½

OR		
Ans(b)	$E(X) = 2.94$ $\Rightarrow 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{5}\right) + 4\left(\frac{3}{25}\right) + 2k\left(\frac{1}{10}\right) + 3k\left(\frac{1}{25}\right) + 5k\left(\frac{1}{25}\right) = 2.94$ $\Rightarrow k = \frac{1.56}{0.52} \Rightarrow k = 3$ Now, $P(X \le 4) = P(X = 1) + P(X = 2) + P(X = 4)$ $= \frac{1}{2} + \frac{1}{5} + \frac{3}{25}$ $= \frac{41}{50}$	1 1 ½ ½
SECTION D This section comprises long answer (LA) type questions of **5 marks each**.		
Q32.	If A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations $3x + 4y + 7z = 14$; $2x - y + 3z = 4$; $x + 2y - 3z = 0$.	
Ans	$\|A\| = 3(-3) - 2(-26) + 1(19) = 62 \neq 0 \Rightarrow A^{-1}$ exists. cofactor Matrix $= \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$ $adjA = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ $A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ Given system of equations can be written as $A'.X = B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$ Now, $A'.X = B \Rightarrow X = (A')^{-1}.B$ $\Rightarrow X = (A^{-1})'.B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$ $= \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\Rightarrow x = 1, y = 1, z = 1$	½ 2 ½ ½ ½ 1

Q33.	(a) If $y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$, find $\frac{dy}{dx}$. **OR** (b) Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is : (i) strictly increasing. (ii) strictly decreasing.	
Ans(a)	$y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$	
	$\frac{d}{dx}(\cos x^2) = -2x\sin x^2$	1
	$\frac{d}{dx}(\cos^2 x) = 2\cos x\,(-\sin x) = -2\text{sinxcosx}$	1
	$\frac{d}{dx}(\cos^2(x^2)) = 2\cos(x^2)\,(-\sin(x^2))(2x) = -4x\sin x^2\cos x^2$	1
	$\frac{d}{dx}(\cos(x^x)) = -\sin(x^x)\,[x^x(1+\log x)]$	1½
	$\frac{dy}{dx} = -2x\sin x^2 - 2\text{sinxcosx} - 4x\sin x^2\cos x^2 - \sin(x^x)\,[x^x(1+\log x)]$	½
	OR	
Ans(b)	$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$	
	$\Rightarrow f'(x) = \frac{6}{5}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} = \frac{6}{5}(x^3 - 2x^2 - 5x + 6)$	2
	$= \frac{6}{5}(x-1)(x+2)(x-3)$  	1
	For strictly inc/dec, put $f'(x) = 0$	
	$\Rightarrow x = 1, -2, 3$	1
	(i) f is strictly increasing when $x \in (-2,1) \cup (3,\infty)$	
	(ii) f is strictly decreasing when $x \in (-\infty,-2) \cup (1,3)$ Note: Closed intervals are also acceptable.	1
Q34.	Using integration, find the area of the region $\{(x,y) : 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\}$.	

Ans	 Required Area $= \int_0^1 x^2\,dx + \int_1^3 x\,dx$ $= \left.\frac{x^3}{3}\right]_0^1 + \left.\frac{x^2}{2}\right]_1^3$ $= \frac{1}{3} + 4 = \frac{13}{3}$	1 mark for correct figure 1+1 1 1
Q35.	(a) Find the shortest distance between the lines l_1 and l_2 given by : $l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$ **OR** (b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.	
Ans(a)	Given lines are : $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + 2\lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + 3\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ Clearly, the given lines are parallel. Here, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$ $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$ $\therefore \lvert(\vec{a}_2 - \vec{a}_1) \times \vec{b}\rvert = \sqrt{81 + 196 + 16} = \sqrt{293}$ Also, $\lvert\vec{b}\rvert = \sqrt{4 + 9 + 36} = 7$ S.D. $= \frac{\lvert(\vec{a}_2 - \vec{a}_1) \times \vec{b}\rvert}{\lvert\vec{b}\rvert}$ $= \frac{\sqrt{293}}{7}$	 1 ½ 1 ½ 1 ½ ½

OR

Ans(b) Let the given lines be

$l_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ and $l_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$

Any point on the line l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$ 1

Any point on the line l_2 is $(5\mu+4, 2\mu+1, \mu)$ 1

For the given lines to intersect, there must be a common point.

$$\left.\begin{array}{ll} \therefore 2\lambda+1=5\mu+4 \Rightarrow 2\lambda-5\mu=3 & ...(i) \\ 3\lambda+2=2\mu+1 \Rightarrow 3\lambda-2\mu=-1 & ...(ii) \\ 4\lambda+3=\mu \Rightarrow 4\lambda-\mu=-3 & ...(iii) \end{array}\right\}$$ 1

Solving (i) and (ii) gives, $\lambda = \mu = -1$ 1

We notice that $\lambda = \mu = -1$ also satisfies equation (iii)

$\therefore$ The given lines intersect.

Point of intersection is $(2(-1)+1, 3(-1)+2, 4(-1)+3)$ i.e. $(-1,-1,-1)$ 1

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.

Case Study – 1

A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively.

Based on the given information, answer the following questions :

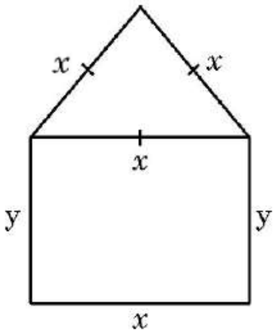
(i) If the perimeter of the window is 12 m, find the relation between x and y. *1*

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1*

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2*

OR

(iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*

Ans	$(i)\,\text{Perimeter}\,(P)=3x+2y=12$	1
	$(ii)\,\text{Area}\,(A)=xy+\frac{\sqrt{3}}{4}x^2$ $=x\left(\frac{12-3x}{2}\right)+\frac{\sqrt{3}}{4}x^2$ $=6x-\frac{3}{2}x^2+\frac{\sqrt{3}}{4}x^2$ 	1
	$(iii)(a)\,\frac{dA}{dx}=6-3x+\frac{\sqrt{3}}{2}x$	½
	For maximum light, $\frac{dA}{dx}=0$ $\Rightarrow 6-3x+\frac{\sqrt{3}}{2}x=0 \Rightarrow x=\frac{12}{6-\sqrt{3}}m$	½
	Also, $\frac{d^2A}{dx^2}=-3+\frac{\sqrt{3}}{2}<0 \therefore A$ is maximum when $x=\frac{12}{6-\sqrt{3}}m$	½
	Now, $y=\frac{12-3x}{2}=6-\frac{3}{2}\left(\frac{12}{6-\sqrt{3}}\right)=\frac{18-6\sqrt{3}}{6-\sqrt{3}}m$	½
	OR	
	$(iii)(b)\,xy+\frac{\sqrt{3}}{4}x^2=50$	½
	$\Rightarrow y=\frac{50}{x}-\frac{\sqrt{3}}{4}x$	1
	Now, $P=3x+2y$ $=3x+2\left(\frac{50}{x}-\frac{\sqrt{3}}{4}x\right)m$	½

Q37.

Case Study – 2

During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*

Ans

$(i)\ R_f = \{1, 4, 9, 16, ...\}$ i.e. set of perfect squares of natural numbers. 1

(ii) Let $x_1, x_2 \in N$ (domain)

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$ 1

$\Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$

$\therefore f$ is injective.

$(iii)(a)\ f(x) = x^2$

Let $x_1, x_2 \in \{1, 2, 3, 4, ...\}$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one. 1

As Co-domain = Range = $\{1, 4, 9, 16, ...\}$

$\therefore f$ is onto. 1

Since, f is one-one and onto, so f is bijective.

OR

$(iii)(b)\ f : R \rightarrow R, f(x) = x^2$

$-1, 1 \in R$ (domain)

As $f(-1) = f(1) = 1$ but $-1 \neq 1$ 1

$\therefore f$ is not injective.

Co-domain $= R$, but Range $= [0, \infty)$ 1

Since Co-domain $\neq$ Range, f is not surjective.

Q38.	**Case Study – 3**	
	Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed. Based on the above information, answer the following questions :	
	(i) What is the probability that the waste treatment plant is introduced ?	*2*
	(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ?	*2*
Ans	E_1 :Event that the first person is appointed.	
	E_2 :Event that the second person is appointed.	
	A:Event that the waste treatment plant is introducted.	
	Here, $\mathrm{P}(E_1)=0.5, P(E_2)=0.6$	
	$P(A\mid E_1)=0.7, P(A\mid E_2)=0.4$	
	$(i)\ P(\text{waste treatment plant is introducted})$	
	$=P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)$	
	$=(0.5)(0.7)+(0.6)(0.4)$	1
	$=0.35+0.24=0.59$	1
	$(ii)\ P(E_1\mid A)=\dfrac{P(E_1)P(A\mid E_1)}{P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)}$	
	$=\dfrac{(0.5)(0.7)}{(0.5)(0.7)+(0.6)(0.4)}$	1 ½
	$=\dfrac{0.35}{0.59}=\dfrac{35}{59}$	½
	Note: Full marks to be awarded, in case a student writes "Sum of probabilities of selecting first person and second person should not be greater than 1".	

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Supplementary Examination, 2025
SUBJECT- MATHEMATICS (041) (Q.P. CODE – 65/S/2)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC.”**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **“Extra Question”**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION – A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	The domain of $f(x) = \cos^{-1}(2x)$ is : (A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	
Ans	(D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	1
Q2.	If $[2x \;\; 3]\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$, then the value of x is : (A) zero (B) 3 (C) $2\sqrt{3}$ (D) $\pm 2\sqrt{3}$	
Ans	(D) $\pm 2\sqrt{3}$	1
Q3.	For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to : (A) X (B) X^2 (C) I (D) O	
Ans	(A) X	1
Q4.	The area of a triangle with vertices (3, 0), (0, k) and (– 3, 0) is 9 sq units. The value of k is : (A) 9 (B) – 9 (C) 3 (D) 6	
Ans	(C) 3	1

Q5.	The value of the determinant $\begin{vmatrix} \cos 75^{o} & \sin 75^{o} \\ \sin 15^{o} & \cos 15^{o} \end{vmatrix}$ is : (A) 1 (B) zero (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$	
Ans	(B) zero	1
Q6.	The derivative of $\sin^{-1}(2x^2-1)$ with respect to $\sin^{-1} x$ is (A) $\frac{2}{x}$ (B) 2 (C) $\frac{\sqrt{1-x^2}}{\sqrt{1-4x^2}}$ (D) $1-x^2$	
Ans	(B) 2	1
Q7.	If A is an identity matrix of order n, then A (Adj A) is a/an : (A) identity matrix (B) row matrix (C) zero matrix (D) skew symmetric matrix	
Ans	(A) identity matrix	1
Q8.	The area bounded by the curve $x = y^2$, y-axis and the lines y = 3 and y = 4 is : (A) $\frac{74}{3}$ sq units (B) $\frac{37}{3}$ sq units (C) 74 sq units (D) 37 sq units	
Ans	(B) $\frac{37}{3}$ sq units	1

Q9.	In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be : (A) $a = 2b$ (B) $a = \frac{b}{2}$ (C) $a = 3b$ (D) $a = b$	
Ans	(B) $a = \frac{b}{2}$	1
Q10.	If $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is : (A) $6x + \frac{12}{x^5}$ (B) $x^4 - \frac{1}{x^3} + 2$ (C) $x^3 + \frac{1}{x^3} - 2$ (D) $x^3 + \frac{1}{x^3} + 2$	
Ans	(C) $x^3 + \frac{1}{x^3} - 2$	1
Q11.	The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is : (A) 3 (B) 4 (C) 7 (D) 0	
Ans	(B) 4	1
Q12.	$\int \frac{\tan^2 \sqrt{x}}{\sqrt{x}} dx$ is equal to : (A) $\sec\sqrt{x} + C$ (B) $2\sqrt{x} \tan x - x + C$ (C) $2(\tan\sqrt{x} - \sqrt{x}) + C$ (D) $2\tan\sqrt{x} - x + C$	
Ans	(C) $2(\tan\sqrt{x} - \sqrt{x}) + C$	1

Q13.	A coin is tossed three times. The probability of getting at least two heads is : (A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$	
Ans	(A) $\frac{1}{2}$	1
Q14.	If $\lvert \vec{a} \rvert = 1$, $\lvert \vec{b} \rvert = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $\lvert \vec{a} + \vec{b} \rvert$ is : (A) 9 (B) 3 (C) – 3 (D) 2	
Ans	(B) 3	1
Q15.	If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is : (A) 1 sq unit (B) 2 sq units (C) 3 sq units (D) 4 sq units	
Ans	(B) 2 sq units	1
Q16.	The general solution of the differential equation $\frac{dy}{dx} = 2x \,.\, e^{x^2+y}$ is : (A) $e^{x^2+y} = C$ (B) $e^{x^2} + e^{-y} = C$ (C) $e^{x^2} = e^{y} + C$ (D) $e^{x^2-y} = C$	
Ans	(B) $e^{x^2} + e^{-y} = C$	1

Q17.	If ‘m’ and ‘n’ are the degree and order respectively of the differential equation $1+\left(\frac{dy}{dx}\right)^3=\frac{d^2y}{dx^2}$, then the value of (m + n) is : (A) 4 (B) 3 (C) 2 (D) 5	
Ans	(B) 3	1
Q18.	Two vectors $\vec{a}$ and $\vec{b}$ are such that $\lvert \vec{a}\times\vec{b} \rvert = \vec{a}\,.\,\vec{b}$. The angle between the two vectors is : (A) 30° (B) 60° (C) 45° (D) 90°	
Ans	(C) 45°	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

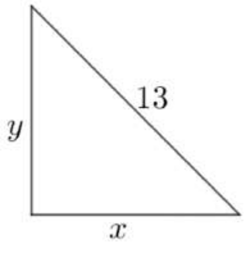
Q19.	Consider the function $f : R \longrightarrow R$, defined as $f(x) = x^3$. *Assertion (A) :* f(x) is a one-one function. *Reason (R) :* f(x) is a one-one function, if co-domain = range.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1

Q20.	*Assertion (A) :* f(x) = [x], $x \in \mathbb{R}$, the greatest integer function is not differentiable at x = 2. *Reason (R) :* The greatest integer function is not continuous at any integral value.	
Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.	For the curve $\sqrt{x} + \sqrt{y} = 1$, find the value of $\frac{dy}{dx}$ at the point $\left(\frac{1}{9}, \frac{1}{9}\right)$.	
Ans(a)	Differentiating both sides w.r.t. x, we get $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$ $\frac{dy}{dx}\ at \left(\frac{1}{9}, \frac{1}{9}\right) = -1$	1 ½ ½
Q22.	(a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. **OR** (b) Prove that : $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0, 1]$	
Ans (a)	$cos^{-1}\left(-\frac{1}{2}\right) + 2\,sin^{-1}\left(\frac{1}{2}\right)$ $= \left(\pi - \frac{\pi}{3}\right) + 2\left(\frac{\pi}{6}\right)$ $= \pi$ **OR**	1 + ½ ½
(b)	Put $x = tan^2\,\theta \Rightarrow \theta = tan^{-1}\sqrt{x}$ RHS $= \frac{1}{2}cos^{-1}\left(\frac{1 - tan^2\,\theta}{1 + tan^2\,\theta}\right)$ $= \frac{1}{2}cos^{-1}(cos\,2\,\theta)$ $= \frac{1}{2}(2\theta)$ $= \theta = tan^{-1}\sqrt{x}$ = LHS	½ 1 ½

Q23.	(a) Find the value of λ, if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear. **OR** (b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $\lvert\vec{a}\rvert = 10$, $\lvert\vec{b}\rvert = 2$ and $\vec{a}\,.\,\vec{b} = 12$. Find the area of the parallelogram.	
Ans (a)	$A(-1,-1,2), B(2,8,\lambda), C(3,11,6)$ $\overrightarrow{AB} = 3\hat{\imath} + 9\hat{\jmath} + (\lambda - 2)\hat{k}$ and $\overrightarrow{BC} = \hat{\imath} + 3\hat{\jmath} + (6 - \lambda)\hat{k}$ Since A, B and C are collinear, $\frac{3}{1} = \frac{9}{3} = \frac{\lambda - 2}{6 - \lambda}$ $\Rightarrow \lambda = 5$	1 ½ ½
	OR	
(b)	Let θ is the angle between $\vec{a}$ and $\vec{b}$. $\vec{a}.\vec{b} = 12 \Rightarrow \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos\theta = 12$ $\Rightarrow (10)(2)\cos\theta = 12 \Rightarrow \cos\theta = \frac{3}{5}$ $\therefore \sin\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$ Now, area of parallelogram $= \lvert\vec{a} \times \vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin\theta$ $= (10)(2)\left(\frac{4}{5}\right) = 16$ $\therefore$ area of parallelogram $= 16$	½ ½ ½ ½
Q24.	A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?	
Ans	$x^2 + y^2 = 169$ Differentiate both sides w.r.t. t $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\Rightarrow 12(2) + 5\left(\frac{dy}{dt}\right) = 0[\because$ when $x = 12m, y = 5m]$ $\Rightarrow \frac{dy}{dt} = -\frac{24}{5}$ Hence, the height decreases at the rate of $\frac{24}{5}$ m/s  	½ 1 ½

Q25.	Determine the vector equation of a line passing through the point (1, 2, – 3) and perpendicular to both the given lines $\frac{x-8}{3}=\frac{y+16}{-16}=\frac{x-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{-8}=\frac{z-5}{-5}$.	
	Let direction ratios of required line be a, b, c Therefore, 3a-16b + 7c = 0 3a – 8b – 5c = 0 Solving we get $\frac{a}{136}=\frac{b}{36}=\frac{c}{24}$ DRs are 136, 36, 24 or 34, 9, 6 Equation is $\frac{x-1}{136}=\frac{y-2}{36}=\frac{z+3}{24}$ or $\frac{x-1}{34}=\frac{y-2}{9}=\frac{z+3}{6}$ $\vec{r}=\hat{i}+2\hat{j}-3\hat{k}+\lambda(34\hat{i}+9\hat{j}+6\hat{k})$ Note: Due to typing error in the first equation, full marks may be awarded for any attempt	 ½ ½ ½ ½
SECTION C This section comprises short answer (SA) type questions of **3 marks each**.		
Q26.	(a) Evaluate : $I=\int_2^4(\lvert x-2\rvert+\lvert x-3\rvert+\lvert x-4\rvert)\,dx$ **OR** (b) Find : $\int\frac{dx}{(x+2)(x^2+1)}$	
Ans (a)	Let I = $\int_2^4(\lvert x-2\rvert+\lvert x-3\rvert+\lvert x-4\rvert)$ dx $=\int_2^3(x-2+3-x+4-x)$ dx + $\int_3^4(x-2+x-3+4-x)$ dx $=\int_2^3(5-x)$ dx + $\int_3^4(x-1)$ dx $=\left(-\frac{(5-x)^2}{2}\right)_2^3+\left(\frac{(x-1)^2}{2}\right)_3^4$ = 5 **OR**	 1 1 1
(b)	Let $\frac{1}{(x+2)(x^2+1)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}=\frac{1/5}{x+2}+\frac{-x}{5(x^2+1)}+\frac{2/5}{x^2+1}$ $I=\frac{1}{5}\log\lvert x+2\rvert-\frac{1}{10}\log(x^2+1)+\frac{2}{5}\tan^{-1}x+C$	½ + 1 1 ½
Q27.	Find the maximum slope of the curve $y=-x^3+3x^2+9x-30$.	

Ans(a)	$y=-x^3+3x^2+9x-30$	
	Slope of the curve, $m=\frac{dy}{dx}=-3x^2+6x+9$	1
	$\Rightarrow\frac{dm}{dx}=-6x+6$	½
	For maximum/ minimum slope, put $\frac{dm}{dx}=0$	
	$\Rightarrow x=1$	½
	As $\frac{d^2m}{dx^2}=-6<0\ \therefore m$ is maximum at $x=1$	½
	Maximum slope $=-3(1)^2+6(1)+9=12$	½
Q28.	(a) Find the general solution of the differential equation $x^2\frac{dy}{dx}=x^2+xy+y^2$. **OR** (b) Find the particular solution of the differential equation $\frac{dy}{dx}-3y\cot x=\sin 2x$, given that $y=2$ when $x=\frac{\pi}{2}$.	
Ans **(a)**	$\frac{dy}{dx}=\frac{y^2}{x^2}+\frac{y}{x}+1$	**½**
	Put $\frac{y}{x}=v$ i.e. $y=vx$	
	$\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$	**1**
	Equation becomes	
	$v+x\frac{dv}{dx}=v^2+v+1$	½
	$\frac{1}{v^2+1}dv=\frac{dx}{x}$	
	Integrating, we get	
	$\tan^{-1}v=\log\|x\|+C\ \Longrightarrow \tan^{-1}\frac{y}{x}=\log\|x\|+C$	**1**
	OR	
(b)		
	I.F. = $e^{\int -3\cot x dx}=\sin^{-3}x$	**1**
	Solution is	
	$y.\sin^{-3}x=\int \sin 2x.\sin^{-3}x\,dx+C$	**1**
	$y.\sin^{-3}x=2\int \text{cosec}x.\cot x\,dx+C$	
	$y.\sin^{-3}x=-2\text{cosec}x+C$	**½**
	putting $y=2$, $x=\frac{\pi}{2}$ gives $C=4$	

Particular solution is y. $\sin^{-3}x = -2\,cosecx + 4$ or $y = -2\sin^2 x + 4\sin^3 x$ ½

Q29.

If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2\,(\hat{b} \times \hat{c})$.

Ans(a)

$\hat{a}.\hat{b}=0 \Rightarrow \hat{a}\perp\hat{b}, \hat{a}.\hat{c}=0 \Rightarrow \hat{a}\perp\hat{c}$ 1

$\Rightarrow \hat{a}$ is perpendicular to both $\hat{b}$ and $\hat{c} \Rightarrow \hat{a} \,||\, \left(\hat{b}\times\hat{c}\right)$

Let $\hat{a} = \lambda\left(\hat{b}\times\hat{c}\right)$ ½

$\Rightarrow |\hat{a}| = |\lambda|\left|\left(\hat{b}\times\hat{c}\right)\right| \Rightarrow |\hat{a}| = |\lambda||\hat{b}||\hat{c}|\sin\frac{\pi}{6}$ 1

$\Rightarrow |\lambda| = 2 \Rightarrow \lambda = \pm 2$ ½

$\therefore \hat{a} = \pm 2\left(\hat{b}\times\hat{c}\right)$

Q30.

(a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

(b) The probability distribution of a random variable X is given below :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Find k, if E(X) = 2·94 and also find P(X ≤ 4).

Ans)

$P\left(\text{at most one of them will solve the problem}\right)$

$= P\left(\text{none of them solves the problem}\right) + P\left(\text{only one of them solves the problem}\right)$

$=\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}\right)+\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}+\frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{4}{5}+\frac{1}{2}\times\frac{2}{3}\times\frac{2}{3}\times\frac{4}{5}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{5}\right)$ 2 ½

$=\frac{19}{45}$ ½

OR

$E(X) = 2.94$

$\Rightarrow 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{5}\right) + 4\left(\frac{3}{25}\right) + 2k\left(\frac{1}{10}\right) + 3k\left(\frac{1}{25}\right) + 5k\left(\frac{1}{25}\right) = 2.94$ 1

$\Rightarrow k = \frac{1.56}{0.52} \Rightarrow k = 3$ 1

Now, $P(X \le 4) = P(X = 1) + P(X = 2) + P(X = 4)$

$= \frac{1}{2} + \frac{1}{5} + \frac{3}{25}$ ½

$= \frac{41}{50}$ ½

Q31.

Solve the following LPP graphically :

Maximize Z = 2x + 3y

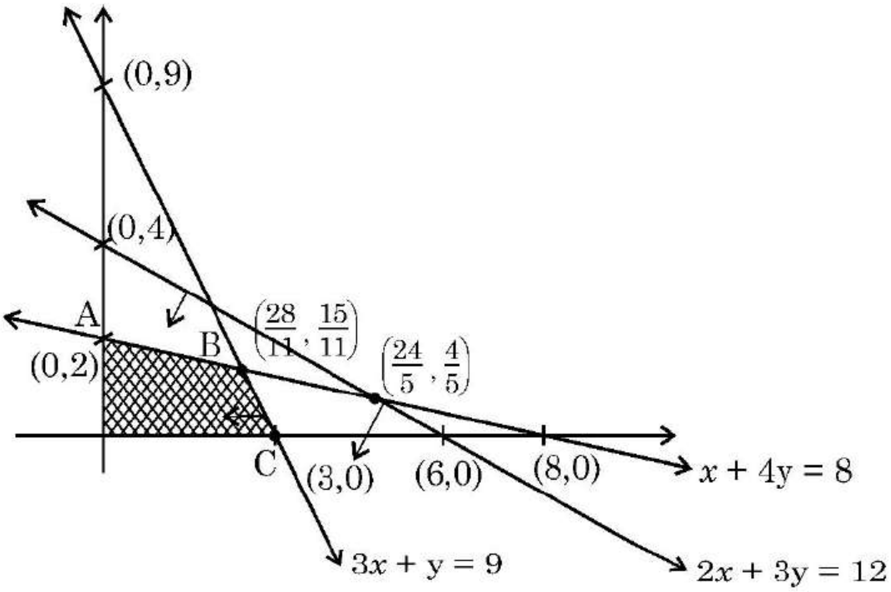
subject to the constraints $x + 4y \leq 8$

$2x + 3y \leq 12$

$3x + y \leq 9$

$x \geq 0, y \geq 0.$

Ans(a)



correct graph and shading
2

Corner Point	Value of $Z = 2x + 3y$
$O(0,0)$	0
$A(0,2)$	6
$B\left(\frac{28}{11}, \frac{15}{11}\right)$	$\frac{101}{11}$ Maximum
$C(3,0)$	6

For correct table
1

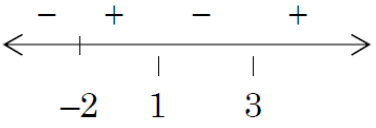
$$Z_{\max} = \frac{101}{11} \text{ when } x = \frac{28}{11}, y = \frac{15}{11}$$

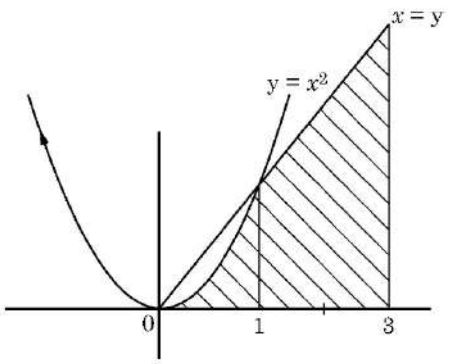
SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

Q32.	If $A = \begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations : $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$.	
Ans	**A** = $\begin{pmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{pmatrix}$ $\lvert A\rvert = -1 \neq 0$, hence A^{-1}exists	1
	adj(A) = $\begin{pmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{pmatrix}$	2
	$A^{-1} = \frac{\text{adjA}}{\lvert A\rvert} = \frac{1}{-1}\begin{pmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{pmatrix}$	½
	Given system in the form AX = B is $\begin{pmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}11\\ -5\\ -3\end{pmatrix}$ $X = A^{-1}B$ $= \frac{1}{-1}\begin{pmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{pmatrix}\begin{pmatrix}11\\ -5\\ -3\end{pmatrix}$	1
	$= \begin{pmatrix}1\\ 2\\ 3\end{pmatrix}$ $x = 1, y = 2, z = 3$	½
Q33.	(a) Find the shortest distance between the lines l_1 and l_2 given by : $l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$ **OR** (b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.	

Ans(a)	Given lines are : $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+2\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$	
	and $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+3\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$	
	Clearly, the given lines are parallel.	
	Here, $\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}$, $\vec{a}_2=\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$	1
	$\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$	½
	$\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 2 & 3 & 6\end{vmatrix}=9\hat{i}-14\hat{j}+4\hat{k}$	**1 ½**
	$\therefore\left\|\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}\right\|=\sqrt{81+196+16}=\sqrt{293}$	**1**
	Also, $\left\|\vec{b}\right\|=\sqrt{4+9+36}=7$	½
	S.D. $=\dfrac{\left\|\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}\right\|}{\left\|\vec{b}\right\|}$	
	$=\dfrac{\sqrt{293}}{7}$	½
OR		
Ans(b)	Let the given lines be	
	$l_1:\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}=\lambda$ and $l_2:\dfrac{x-4}{5}=\dfrac{y-1}{2}=\dfrac{z}{1}=\mu$	
	Any point on the line l_1 is $\left(2\lambda+1,3\lambda+2,4\lambda+3\right)$	**1**
	Any point on the line l_2 is $\left(5\mu+4,2\mu+1,\mu\right)$	**1**
	For the given lines to intersect, there must be a common point.	
	$\therefore 2\lambda+1=5\mu+4\Rightarrow 2\lambda-5\mu=3 \quad ...(i)$ $3\lambda+2=2\mu+1\Rightarrow 3\lambda-2\mu=-1 \quad ...(ii)$ $4\lambda+3=\mu\Rightarrow 4\lambda-\mu=-3 \quad ...(iii)$	**1**
	Solving (i) and (ii) gives, $\lambda=\mu=-1$	**1**
	We notice that $\lambda=\mu=-1$ also satisfies equation (iii)	
	$\therefore$ The given lines intersect.	
	Point of intersection is $\left(2(-1)+1,3(-1)+2,4(-1)+3\right)$ i.e. $\left(-1,-1,-1\right)$	**1**

Q34.	(a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$. **OR** (b) Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is : (i) strictly increasing. (ii) strictly decreasing.	
Ans **(a)**	$y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$	
	$\frac{d}{dx}(\cos x^2) = -2x \sin x^2$	1
	$\frac{d}{dx}(\cos^2 x) = 2\cos x\,(-\sin x) = -2\text{sinxcosx}$	1
	$\frac{d}{dx}(\cos^2(x^2)) = 2\cos(x^2)\,(-\sin(x^2))(2x) = -4x\sin x^2 \cos x^2$	1
	$\frac{d}{dx}(\cos(x^x)) = -\sin(x^x)\,[x^x(1+\log x)]$	1½
	$\therefore \frac{dy}{dx} = -2x\sin x^2 - 2\text{sinxcosx} - 4x\sin x^2\cos x^2 - \sin(x^x)\,[x^x(1+\log x)]$	½
	OR	
(b)	$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$	
	$\Rightarrow f'(x) = \frac{6}{5}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} = \frac{6}{5}\left(x^3 - 2x^2 - 5x + 6\right)$	2
	$= \frac{6}{5}(x-1)(x+2)(x-3)$  	1
	For strictly inc/dec, put $f'(x) = 0$	
	$\Rightarrow x = 1, -2, 3$	1
	(i) f is strictly increasing when $x \in (-2,1) \cup (3,\infty)$	1
	(ii) f is strictly decreasing when $x \in (-\infty,-2) \cup (1,3)$ Note: Closed intervals are also acceptable.	
Q35.	Using integration, find the area of the region $\left\{(x, y) : 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\right\}$.	

Ans(a)	 Required Area $=\int_0^1 x^2\,dx+\int_1^3 x\,dx$ $=\left.\frac{x^3}{3}\right]_0^1+\left.\frac{x^2}{2}\right]_1^3$ $=\frac{1}{3}+4=\frac{13}{3}$	1 mark for correct figure 1+1 1 1

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.

Case Study – 1

A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively.

Based on the given information, answer the following questions :

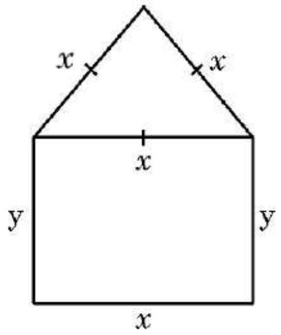
(i) If the perimeter of the window is 12 m, find the relation between x and y. *1*

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1*

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2*

OR

(iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*

Ans	$(i)\text{ Perimeter }(P)=3x+2y=12$ 	1
	$(ii)\text{ Area }(A)=xy+\frac{\sqrt{3}}{4}x^2$	
	$=x\left(\frac{12-3x}{2}\right)+\frac{\sqrt{3}}{4}x^2$	
	$=6x-\frac{3}{2}x^2+\frac{\sqrt{3}}{4}x^2$	1
	$(iii)(a)\,\frac{dA}{dx}=6-3x+\frac{\sqrt{3}}{2}x$	½
	For maximum light, $\frac{dA}{dx}=0$	
	$\Rightarrow 6-3x+\frac{\sqrt{3}}{2}x=0\Rightarrow x=\frac{12}{6-\sqrt{3}}m$	½
	Also, $\frac{d^2A}{dx^2}=-3+\frac{\sqrt{3}}{2}<0\therefore A$ is maximum when $x=\frac{12}{6-\sqrt{3}}m$	½
	Now, $y=\frac{12-3x}{2}=6-\frac{3}{2}\left(\frac{12}{6-\sqrt{3}}\right)=\frac{18-6\sqrt{3}}{6-\sqrt{3}}m$	½
	OR	
	$(iii)(b)\,xy+\frac{\sqrt{3}}{4}x^2=50$	½
	$\Rightarrow y=\frac{50}{x}-\frac{\sqrt{3}}{4}x$	1
	Now, $P=3x+2y$	
	$=3x+2\left(\frac{50}{x}-\frac{\sqrt{3}}{4}x\right)m$	½

Q37.	**Case Study – 2**	
	During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.	
	Based on the above information, answer the following questions :	
	(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ?	*1*
	(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not.	*1*
	(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective.	*2*
	OR	
	(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective.	*2*
Ans	$(i)\ R_f = \{1,4,9,16,...\}$ i.e. set of perfect squares of natural numbers.	1
	(ii) Let $x_1, x_2 \in N$ (domain)	
	Let $f(x_1) = f(x_2)$	
	$\Rightarrow x_1^2 = x_2^2$	
	$\Rightarrow x_1 = \pm x_2$	
	$\Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$	1
	$\therefore f$ is injective.	
	$(iii)(a)\ f(x) = x^2$	
	Let $x_1, x_2 \in \{1,2,3,4,...\}$	
	Let $f(x_1) = f(x_2)$	
	$\Rightarrow x_1^2 = x_2^2$	
	$\Rightarrow x_1 = x_2$	
	$\therefore f$ is one-one.	1
	As Co-domain = Range = $\{1,4,9,16,...\}$	
	$\therefore f$ is onto.	1
	Since, f is one-one and onto, so f is bijective.	
	OR	
	$(iii)(b)\ f : R \to R,\ f(x) = x^2$	
	$-1, 1 \in R$ (domain)	
	As $f(-1) = f(1) = 1$ but $-1 \neq 1$	
	$\therefore f$ is not injective.	1
	Co-domain $= R$, but Range $= [0, \infty)$	1
	Since Co-domain $\neq$ Range, f is not surjective.	

Q38.

Case Study – 3

Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed.

Based on the above information, answer the following questions :

(i) What is the probability that the waste treatment plant is introduced ? *2*

(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? *2*

Ans

E_1 : Event that the first person is appointed.
E_2 : Event that the second person is appointed.
A: Event that the waste treatment plant is introducted.
Here, $\mathrm{P}(E_1)=0.5, P(E_2)=0.6$
$P(A|E_1)=0.7, P(A|E_2)=0.4$

$(i)\ P(\text{waste treatment plant is introducted})$

$=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$

$=(0.5)(0.7)+(0.6)(0.4)$ 1

$=0.35+0.24=0.59$ 1

$$(ii)\ P(E_1|A)=\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{(0.5)(0.7)}{(0.5)(0.7)+(0.6)(0.4)}$$ 1 ½

$$=\frac{0.35}{0.59}=\frac{35}{59}$$ ½

Note: Full marks to be awarded, in case a student writes "Sum of probabilities of selecting first person and second person should not be greater than 1".

Marking Scheme
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Senior Secondary School Supplementary Examination, 2025
SUBJECT- MATHEMATICS (041) (Q.P. CODE – 65/S/3)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC.”**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **“Extra Question”**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	The principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is : (A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{2\pi}{3}$	
Ans	(C) $\frac{3\pi}{4}$	1
Q2.	The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is : (A) an identity matrix (B) a scalar matrix (C) a symmetric matrix (D) a skew-symmetric matrix	
Ans	(C) a symmetric matrix	1
Q3.	If $[3x \quad 5] . \begin{bmatrix} x \\ -15 \end{bmatrix} = 0$, then the value of x is : (A) zero (B) 5 (C) $2\sqrt{5}$ (D) ± 5	
Ans	(D) ± 5	1
Q4.	If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ and $\|3A\| = k\|A\|$, then the value of k is : (A) 3 (B) 9 (C) 6 (D) 27	
Ans	(B) 9	1

Q5.	The area of a triangle with vertices $(4, -1)$, $(1, k)$ and $(-2, -1)$ is 9 sq units. The value of k is : (A) 8 (B) -8 (C) 2 (D) 6	
Ans	(C) 2	1
Q6.	If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then the value of $\lvert \text{adj } A \rvert$ is : (A) 5 (B) 25 (C) -5 (D) -25	
Ans	(A) 5	1
Q7.	If $\frac{d}{dx}f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is : (A) $6x + \frac{12}{x^5}$ (B) $x^4 - \frac{1}{x^3} + 2$ (C) $x^3 + \frac{1}{x^3} - 2$ (D) $x^3 + \frac{1}{x^3} + 2$	
Ans	(C) $x^3 + \frac{1}{x^3} - 2$	1
Q8.	If $x = \sin t$ and $y = \cos t$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ is : (A) -1 (B) 2 (C) $2\sqrt{2}$ (D) $-2\sqrt{2}$	
Ans	(D) $-2\sqrt{2}$	1

Q9.	The area bounded by the line $y = x$, y-axis and the lines $y = 0$ and $y = 4$ is : (A) 16 sq units (B) 8 sq units (C) 4 sq units (D) 2 sq units	
Ans	(B) 8 sq units	1
Q10.	The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is : (A) 3 (B) 4 (C) 7 (D) 0	
Ans	(B) 4	1
Q11.	In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be : (A) $a = 2b$ (B) $a = \frac{b}{2}$ (C) $a = 3b$ (D) $a = b$	
Ans	(B) $a = \frac{b}{2}$	1
Q12.	If $\lvert \vec{a} \rvert = 1$, $\lvert \vec{b} \rvert = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $\lvert \vec{a} + \vec{b} \rvert$ is : (A) 9 (B) 3 (C) -3 (D) 2	
Ans	(B) 3	1

Q13.	$\int \frac{\cot\sqrt{x}}{\sqrt{x}}dx$ is equal to : (A) $\log \sin\sqrt{x} + C$ (B) $\frac{\log \sin\sqrt{x}}{2\sqrt{x}} + C$ (C) $2 \log \sin\sqrt{x} + C$ (D) $\frac{\log \sin\sqrt{x}}{\sqrt{x}} + C$	
Ans	(C) $2 \log \sin\sqrt{x} + C$	1
Q14.	If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is : (A) 1 sq unit (B) 2 sq units (C) 3 sq units (D) 4 sq units	
Ans	(B) 2 sq units	1
Q15.	If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of $(m + n)$ is : (A) 4 (B) 3 (C) 2 (D) 5	
Ans	(B) 3	1
Q16.	A coin is tossed three times. The probability of getting at least two heads is : (A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$	
Ans	(A) $\frac{1}{2}$	1

Q17.	Two vectors $\vec{a}$ and $\vec{b}$ are such that $\lvert \vec{a} \times \vec{b} \rvert = \vec{a} \cdot \vec{b}$. The angle between the two vectors is : (A) 30° (B) 60° (C) 45° (D) 90°	
Ans	(C) 45°	1
Q18.	The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is : (A) $\log x$ (B) $-\log x$ (C) $e^x - x$ (D) $\frac{e^x}{x}$	
Ans	(D) $\frac{e^x}{x}$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19.	*Assertion (A) :* $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at $x = 2$. *Reason (R) :* The greatest integer function is not continuous at any integral value.	
Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1
Q20.	Consider the function $f : R \longrightarrow R$, defined as $f(x) = x^3$. *Assertion (A) :* f(x) is a one-one function. *Reason (R) :* f(x) is a one-one function, if co-domain = range.	

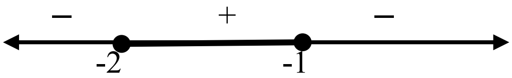
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

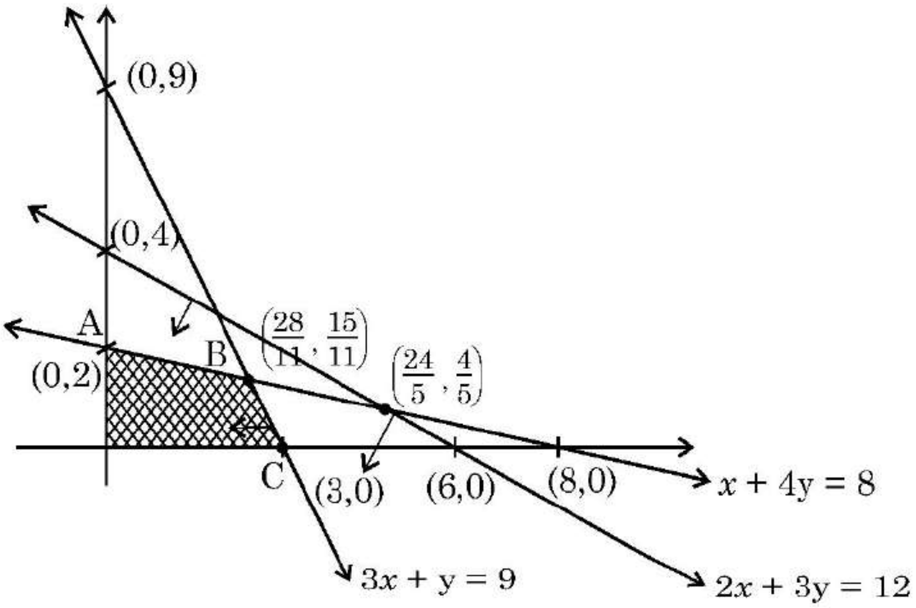
Q21.	If $e^y(x+1)=1$, prove that $\frac{dy}{dx}=-e^y$.	
Ans(a)	$e^y(x+1)=1\Rightarrow e^y=\frac{1}{x+1}$ $\Rightarrow y=-\log(x+1)$ $\Rightarrow \frac{dy}{dx}=-\frac{1}{x+1}$ $=-e^y \qquad \left[\because \frac{1}{x+1}=e^y\right]$	½ 1 ½
Q22.	(a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$. **OR** (b) Prove that : $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in[0,1]$	
Ans (a)	$cos^{-1}\left(-\frac{1}{2}\right)+2\,sin^{-1}\left(\frac{1}{2}\right)$ $=\left(\pi-\frac{\pi}{3}\right)+2\left(\frac{\pi}{6}\right)$ $=\pi$	1 + ½ ½
	OR	
(b)	Put $x=tan^2\,\theta\Rightarrow\theta=tan^{-1}\sqrt{x}$ RHS $=\frac{1}{2}cos^{-1}\left(\frac{1-tan^2\,\theta}{1+tan^2\,\theta}\right)$ $=\frac{1}{2}cos^{-1}(cos\,2\,\theta)$ $=\frac{1}{2}(2\theta)$ $=\theta=tan^{-1}\sqrt{x}=$ LHS	½ 1 ½
Q23.	A spherical balloon has a variable diameter $\frac{2}{3}(2t+1)$. Find the rate of change of its volume with respect to t.	
Ans	Clearly radius = $\frac{2t+1}{3}$	½

	$V = \frac{4\pi}{3}\frac{(2t+1)^3}{27}$ $\frac{dv}{dt} = \frac{8\pi}{27}(2t+1)^2$	½ 1
Q24.	Find the angle between the lines $\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ and $\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.	
Ans	Given lines are: $\vec{r} = \left(3\hat{i} - 2\hat{j} + 6\hat{k}\right) + \lambda\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{j} - 5\hat{k}\right) + \mu\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right)$ Let θ be the angle between these two lines. $\cos\theta = \frac{2(6)+2(3)+2(2)}{\sqrt{4+4+4}\sqrt{36+9+4}} = \frac{22}{2\sqrt{3}\times 7}$ $\Rightarrow \cos\theta = \frac{11}{21}\sqrt{3} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\sqrt{3}\right)$	½ 1 ½
Q25.	(a) Find the value of λ, if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear. **OR** (b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $\lvert\vec{a}\rvert = 10$, $\lvert\vec{b}\rvert = 2$ and $\vec{a}\cdot\vec{b} = 12$. Find the area of the parallelogram.	
(a)	$A(-1,-1,2), B(2,8,\lambda), C(3,11,6)$ $\overrightarrow{AB} = 3\hat{\imath} + 9\hat{\jmath} + (\lambda - 2)\hat{k}$ and $\overrightarrow{BC} = \hat{\imath} + 3\hat{\jmath} + (6 - \lambda)\hat{k}$ Since A, B and C are collinear, $\frac{3}{1} = \frac{9}{3} = \frac{\lambda - 2}{6 - \lambda}$ $\Rightarrow \lambda = 5$	1 ½ ½
	OR	
(b)	Let θ is the angle between $\vec{a}$ and $\vec{b}$. $\vec{a}.\vec{b} = 12 \Rightarrow \lvert\vec{a}\rvert\lvert\vec{b}\rvert\, cos\,\theta = 12$ $\Rightarrow (10)(2)\, cos\,\theta = 12 \Rightarrow cos\,\theta = \frac{3}{5}$ $\therefore sin\,\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$ Now, area of parallelogram $= \lvert\vec{a} \times \vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\, sin\,\theta$ $= (10)(2)\left(\frac{4}{5}\right) = 16$ $\therefore$ area of parallelogram $= 16$	½ ½ ½ ½

SECTION C This section comprises short answer (SA) type questions of **3 marks each**.		
Q26.	Find the intervals in which the function f given by $f(x) = -2x^3 - 9x^2 - 12x + 1$ is : (i) strictly increasing. (ii) strictly decreasing.	
Ans	$f'(x) = -6x^2 - 18x - 12$	1
	$f'(x) = 0$ gives $x = -2, -1$	1
	sign of $f'(x)$ 	
	f(x) is increasing in $(-2, -1)$	½
	f(x) is decreasing in $(-\infty, -2)$ *and* $(-1, \infty)$	½
Q27.	(a) Find : $\int \sqrt{4x^2 - 4x + 10}\ dx$ **OR** (b) Evaluate : $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$	
Ans(a)	$I = \int \sqrt{4x^2 - 4x + 10}\, dx$	
	$= \int \sqrt{(2x-1)^2 + (3)^2} dx$	1
	$= \frac{1}{2}\left[\left(\frac{2x-1}{2}\right)\sqrt{4x^2 - 4x + 10} + \frac{9}{2} log \left\lvert (2x-1) + \sqrt{4x^2 - 4x + 10} \right\rvert\right] + C$	1+1
OR		

Ans(b)	$I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx \quad ...(i)$ $=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$ $I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx \quad ...(ii)$ Adding (i) and (ii) $2I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx$ $\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$ Put $\cos x=t\Rightarrow -\sin x\,dx=dt$ $I=-\frac{\pi}{2}\int_1^{-1}\frac{dt}{1+t^2}$ $=\pi\left[\tan^{-1}t\right]_0^1=\frac{\pi^2}{4}$	½ ½ ½ ½ 1
Q28.	If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b}=\hat{a}.\hat{c}=0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a}=\pm 2(\hat{b}\times\hat{c})$.	
Ans	$\hat{a}.\hat{b}=0\Rightarrow\hat{a}\perp\hat{b},\hat{a}.\hat{c}=0\Rightarrow\hat{a}\perp\hat{c}$ $\Rightarrow\hat{a}$ is perpendicular to both $\hat{b}$ and $\hat{c}\Rightarrow\hat{a}\,\|\,(\hat{b}\times\hat{c})$ Let $\hat{a}=\lambda(\hat{b}\times\hat{c})$ $\Rightarrow\|\hat{a}\|=\|\lambda\|\|(\hat{b}\times\hat{c})\|\Rightarrow\|\hat{a}\|=\|\lambda\|\|\hat{b}\|\|\hat{c}\|\sin\frac{\pi}{6}$ $\Rightarrow\|\lambda\|=2\Rightarrow\lambda=\pm 2$ $\therefore\hat{a}=\pm 2(\hat{b}\times\hat{c})$	1 ½ 1 ½
Q29.	Solve the following LPP graphically : Maximize Z = 2x + 3y subject to the constraints $x+4y\le 8$ $2x+3y\le 12$ $3x+y\le 9$ $x\ge 0, y\ge 0.$	

Ans(a)



Correct graph with shading
2

Corner Point	Value of $Z = 2x + 3y$
$O(0,0)$	0
$A(0,2)$	6
$B\left(\frac{28}{11}, \frac{15}{11}\right)$	$\frac{101}{11}$ Maximum
$C(3,0)$	6

For correct table
1

$Z_{\max} = \frac{101}{11}$ when $x = \frac{28}{11}, y = \frac{15}{11}$

Q30.

(a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

(b) The probability distribution of a random variable X is given below :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Find k, if E(X) = 2·94 and also find P(X ≤ 4).

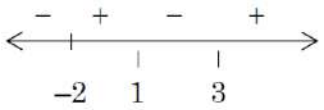
Ans)	$P\left(\text{at most one of them will solve the problem}\right)$ $=P\left(\text{none of them solves the problem}\right)+P\left(\text{only one of them solves the problem}\right)$ $=\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}\right)+\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}+\frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{4}{5}+\frac{1}{2}\times\frac{2}{3}\times\frac{2}{3}\times\frac{4}{5}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{5}\right)$ $=\frac{19}{45}$	2 ½ ½
	OR	
	$E(X) = 2.94$ $\Rightarrow 1\left(\frac{1}{2}\right)+2\left(\frac{1}{5}\right)+4\left(\frac{3}{25}\right)+2k\left(\frac{1}{10}\right)+3k\left(\frac{1}{25}\right)+5k\left(\frac{1}{25}\right)=2.94$ $\Rightarrow k=\frac{1.56}{0.52}\Rightarrow k=3$ Now, $P(X\leq 4)=P(X=1)+P(X=2)+P(X=4)$ $=\frac{1}{2}+\frac{1}{5}+\frac{3}{25}$ $=\frac{41}{50}$	1 1 ½ ½
Q31.	(a) Find the general solution of the differential equation $(2x^2 + y)\,dx = x\,dy$. **OR** (b) For the differential equation $\frac{dy}{dx}-\frac{y}{x}+\text{cosec}\left(\frac{y}{x}\right)=0$, find the particular solution, given that $y = 0$ when $x = 1$.	
Ans(a)	$\left(2x^2+y\right)dx=x\,dy$ $\Rightarrow\frac{dy}{dx}-\frac{1}{x}.y=2x$ $\text{I.F.}=e^{-\int\frac{1}{x}dx}=\frac{1}{x}$ Solution is given by, $y.\left(\frac{1}{x}\right)=\int 2x.\frac{1}{x}dx$ $\Rightarrow\frac{y}{x}=2x+C \text{ or } y=2x^2+Cx$	½ 1 ½ 1
	OR	

Ans(b)	$\frac{dy}{dx}=\frac{y}{x}-\cos ec\left(\frac{y}{x}\right)$ Put $\frac{y}{x}=v$ i.e. $y=vx$ $\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ The differential equation reduces to $v+x\frac{dv}{dx}=v-\cos ecv$ $\sin v\,dv=-\frac{1}{x}dx\Rightarrow\int\sin v\,dv=-\int\frac{dx}{x}$ $\Rightarrow-\cos v=-\log\lvert x\rvert-C$ or, $\cos v=\log\lvert x\rvert+C'$ $\Rightarrow\cos\left(\frac{y}{x}\right)=\log\lvert x\rvert+C'$ $y=0, x=1$ gives $C'=1$ $\therefore$ Solution is given by $\cos\left(\frac{y}{x}\right)=\log\lvert x\rvert+1$	½ 1 1 ½

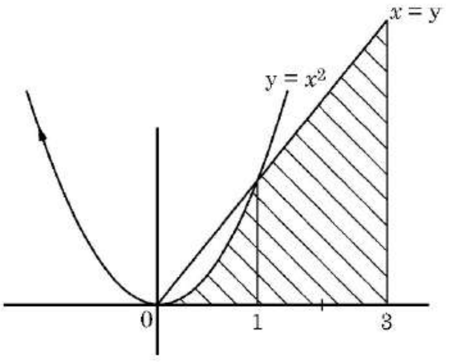
SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

Q32.	(a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$. OR (b) Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is : (i) strictly increasing. (ii) strictly decreasing.

Ans **(a)**	$y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$ $\frac{d}{dx}(\cos x^2) = -2x\sin x^2$ $\frac{d}{dx}(\cos^2 x) = 2\cos x(-\sin x) = -2\text{sinxcosx}$ $\frac{d}{dx}(\cos^2(x^2)) = 2\cos(x^2)\,(-\sin(x^2))(2x) = -4x\sin x^2\cos x^2$ $\frac{d}{dx}(\cos(x^x)) = -\sin(x^x)\,[x^x(1+\log x)]$ $\therefore \frac{dy}{dx} = -2x\sin x^2 - 2\text{sinxcosx} - 4x\sin x^2\cos x^2 - \sin(x^x)\,[x^x(1+\log x)]$	1 1 1 1½ ½
	OR	
(b)	$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ $\Rightarrow f'(x) = \frac{6}{5}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} = \frac{6}{5}(x^3 - 2x^2 - 5x + 6)$ $= \frac{6}{5}(x-1)(x+2)(x-3)$  For strictly inc/dec, put $f'(x) = 0$ $\Rightarrow x = 1, -2, 3$ (i) f is strictly increasing when $x \in (-2,1) \cup (3,\infty)$ (ii) f is strictly decreasing when $x \in (-\infty,-2) \cup (1,3)$ Note: Closed intervals are also acceptable.	2 1 1 1
Q33.	If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations : $y + 2z = 8,$ $x - y = -1,$ $2x + 3y + 4z = 20$	
Ans(a)	$BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$ Given system is $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 20 \\ 8 \end{bmatrix}$ i.e., BX = C say so $X = B^{-1}C$	2 1

	$=\frac{1}{6}AC$ $=\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} -1 \\ 20 \\ 8 \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	1 ½
	x =1 , y = 2, z = 3	½
Q34.	(a) Find the shortest distance between the lines l_1 and l_2 given by : $l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$ **OR** (b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.	
Ans (a)	Given lines are : $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+2\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$	
	and $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+3\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$	
	Clearly, the given lines are parallel.	
	Here, $\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}$, $\vec{a}_2=\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$	1
	$\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$	½
	$\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}=9\hat{i}-14\hat{j}+4\hat{k}$	1 ½
	$\therefore \left\lvert\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}\right\rvert=\sqrt{81+196+16}=\sqrt{293}$	1
	Also, $\lvert\vec{b}\rvert=\sqrt{4+9+36}=7$	½
	$\text{S.D.}=\frac{\left\lvert\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}\right\rvert}{\lvert\vec{b}\rvert}$	
	$=\frac{\sqrt{293}}{7}$	½
	OR	

(b)	Let the given lines be	
	$l_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ and $l_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$	
	Any point on the line l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$	**1**
	Any point on the line l_2 is $(5\mu+4, 2\mu+1, \mu)$	**1**
	For the given lines to intersect, there must be a common point.	
	$\therefore 2\lambda+1 = 5\mu+4 \Rightarrow 2\lambda-5\mu = 3 \quad ...(i)$	
	$3\lambda+2 = 2\mu+1 \Rightarrow 3\lambda-2\mu = -1 \quad ...(ii)$	**1**
	$4\lambda+3 = \mu \Rightarrow 4\lambda-\mu = -3 \quad ...(iii)$	
	Solving (i) and (ii) gives, $\lambda = \mu = -1$	**1**
	We notice that $\lambda = \mu = -1$ also satisfies equation (iii)	
	$\therefore$ The given lines intersect.	
	Point of intersection is $(2(-1)+1, 3(-1)+2, 4(-1)+3)$ i.e. $(-1,-1,-1)$	**1**
Q35.	Using integration, find the area of the region $\{(x, y) : 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\}$.	
Ans(a)		1 mark for correct figure
	Required Area	
	$= \int_0^1 x^2\,dx + \int_1^3 x\,dx$	1+1
	$= \left.\frac{x^3}{3}\right]_0^1 + \left.\frac{x^2}{2}\right]_1^3$	1
	$= \frac{1}{3} + 4 = \frac{13}{3}$	1

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.

Case Study – 1

A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively.

Based on the given information, answer the following questions :

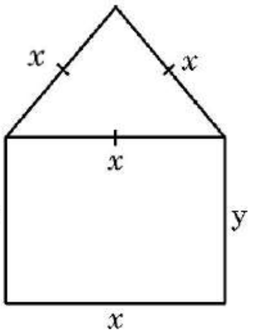
(i) If the perimeter of the window is 12 m, find the relation between x and y. *1*

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1*

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2*

OR

(iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*

Ans	$(i)\,\text{Perimeter}\,(P)=3x+2y=12$ 	1
	$(ii)\,\text{Area}\,(A)=xy+\frac{\sqrt{3}}{4}x^2$ $=x\left(\frac{12-3x}{2}\right)+\frac{\sqrt{3}}{4}x^2$ $=6x-\frac{3}{2}x^2+\frac{\sqrt{3}}{4}x^2$	1
	$(iii)(a)\,\frac{dA}{dx}=6-3x+\frac{\sqrt{3}}{2}x$	½
	For maximum light, $\frac{dA}{dx}=0$	
	$\Rightarrow 6-3x+\frac{\sqrt{3}}{2}x=0 \Rightarrow x=\frac{12}{6-\sqrt{3}}\,m$	½
	Also, $\frac{d^2A}{dx^2}=-3+\frac{\sqrt{3}}{2}<0 \therefore A$ is maximum when $x=\frac{12}{6-\sqrt{3}}\,m$	½
	Now, $y=\frac{12-3x}{2}=6-\frac{3}{2}\left(\frac{12}{6-\sqrt{3}}\right)=\frac{18-6\sqrt{3}}{6-\sqrt{3}}\,m$	½
	OR	
	$(iii)(b)\,xy+\frac{\sqrt{3}}{4}x^2=50$	½
	$\Rightarrow y=\frac{50}{x}-\frac{\sqrt{3}}{4}x$	1
	Now, $P=3x+2y$ $=3x+2\left(\frac{50}{x}-\frac{\sqrt{3}}{4}x\right)m$	½

Q37.

Case Study – 2

During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*

Ans

$(i)\ R_f = \{1,4,9,16,...\}$ i.e. set of perfect squares of natural numbers. 1

(ii) Let $x_1, x_2 \in N$ (domain)

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^{\ 2} = x_2^{\ 2}$

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$ 1

$\therefore f$ is injective.

$(iii)(a)\ f(x) = x^2$

Let $x_1, x_2 \in \{1,2,3,4,...\}$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^{\ 2} = x_2^{\ 2}$

$\Rightarrow x_1 = x_2$ 1

$\therefore f$ is one-one.

As Co-domain = Range = $\{1,4,9,16,...\}$

$\therefore f$ is onto. 1

Since, f is one-one and onto, so f is bijective.

OR

$(iii)(b)\ f : R \rightarrow R,\ f(x) = x^2$

$-1, 1 \in R$ (domain)

As $f(-1) = f(1) = 1$ but $-1 \neq 1$ 1

$\therefore f$ is not injective.

Co-domain = R, but Range = $[0, \infty)$ 1

Since Co-domain $\neq$ Range, f is not surjective.

Q38.	**Case Study – 3** Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed. Based on the above information, answer the following questions : (i) What is the probability that the waste treatment plant is introduced ? *2* (ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? *2*	
Ans	E_1 : Event that the first person is appointed. E_2 : Event that the second person is appointed. A: Event that the waste treatment plant is introducted. Here, $P(E_1)=0.5, P(E_2)=0.6$ $P(A\|E_1)=0.7, P(A\|E_2)=0.4$ $(i)\, P(\text{waste treatment plant is introducted})$ $=P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)$	
	$=(0.5)(0.7)+(0.6)(0.4)$	1
	$=0.35+0.24=0.59$	1
	$(ii)\, P(E_1\|A)=\dfrac{P(E_1)P(A\|E_1)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)}$	
	$=\dfrac{(0.5)(0.7)}{(0.5)(0.7)+(0.6)(0.4)}$	1 ½
	$=\dfrac{0.35}{0.59}=\dfrac{35}{59}$	½
	Note: Full marks to be awarded, in case a student writes "Sum of probabilities of selecting first person and second person should not be greater than 1".	