Duration: 1 hour   **Marks: 30**

SECTION A

Each carry 1 mark

1. Which of the following is not a homogeneous function of x and y?

(a) $x^2 + 2xy$ (b) $2x - y$ (c) $\cos^2\left(\frac{y}{x}\right) + \frac{y}{x}$ (d) $\sin x - \cos y$

2. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is

(a) $\frac{1}{x} + \frac{1}{y} = c$ (b) $\log x.\log y = c$ (c) $xy = c$ (d) $x + y = c$

3. The sum of the order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + 3x\left(\frac{d^2y}{dx^2}\right)^4 = \log x$

is

(a) 5   (b) 6   (c) 7   (d) 4

4. Assertion: The degree of the differential equation $\frac{d^3y}{dx^3} + 3\left(\frac{dy}{dx}\right) = x^2\log\left(\frac{d^2y}{dx^2}\right)$ is not defined.

Reason: If the differential equation is a polynomial in terms of its derivatives, then its degree is

defined.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of

Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of

Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Solve the differential equation $(y + 3x^2)\frac{dx}{dy} = x$

6. Find the general solution of the differential equation: $(xy - x^2)dy = y^2 dx$.

7. Find the general solution of the following differential equation: $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

SECTION C

Each carry 3 marks

8. Find the particular solution of the differential equation $\frac{dy}{dx} + y\tan x = 3x^2 + x^3\tan x, x \neq \frac{\pi}{2}$, given that $y = 0$ when $x = \frac{\pi}{3}$.

9. Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it.

SECTION D

Each carry 5 marks

10. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ given that $y = \frac{\pi}{2}$ when $x = 1$

11. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ given that $y = 1$ when $x = 0$.

SECTION E

12. Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administrated the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution of the differential equation $\frac{dy}{dx} = k(50 - y)$, where x denotes the number of weeks and y the number of children who have been given the drops.

Based on the above information answer the following questions:

(i) Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(50 - y)$?

(ii) Find the solution of the differential equation $\frac{dy}{dx} = k(50 - y)$.

(iii) Determine the value of C in the particular solution given that y(0) and k = 0,049.

UNIT TEST Answers

1. (d) sinx − cosy

2. (c) xy = c

3. (a) 5

4. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

5. $y - 3x^2 + Cx = 0.$

6. $\frac{y}{x} - \log\frac{y}{x} = \log|x| + c$

7. $e^y = \frac{x^3}{3} + e^x + C$

8. $y = x^3 - \frac{2\pi^3}{27}\cos x$

9. $-\frac{1}{2}\log|x^2 + xy + y^2| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = C$

10. $x^2\log(x) - y\sin y + \frac{\pi}{2} = 0$

11. $y = e^{\frac{x^2}{2y^2}}$

12. (i) Variable separable method   (ii) $-\log|50 - y| = kx + C$ (iii) $C = \log\frac{1}{50}$