UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. What is the domain of the function $\cos^{-1}(2x - 3)$?

(a) [–1, 1] (b) (1, 2) (c) (–1, 1) (d) [1, 2]

2. The principal value of $\left[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})\right]$ is

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

3. The domain of $\sin^{-1} x + \cos x$ is

(a) [–1, ∞) (b) (–1, 1) (c) [–1, 1] (d) (∞, 1]

4. Assertion: The range of the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1,1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason: The range of the principal value branch of $\sin^{-1} x$ is $[0, \pi]$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Evaluate: $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$

6. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

7. Find the principal value of $\cos^{-1}[\cos(-680°)]$.

SECTION C

Each carry 3 marks

8. Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$, where $-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form.

9. Solve: $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1} x$

SECTION D

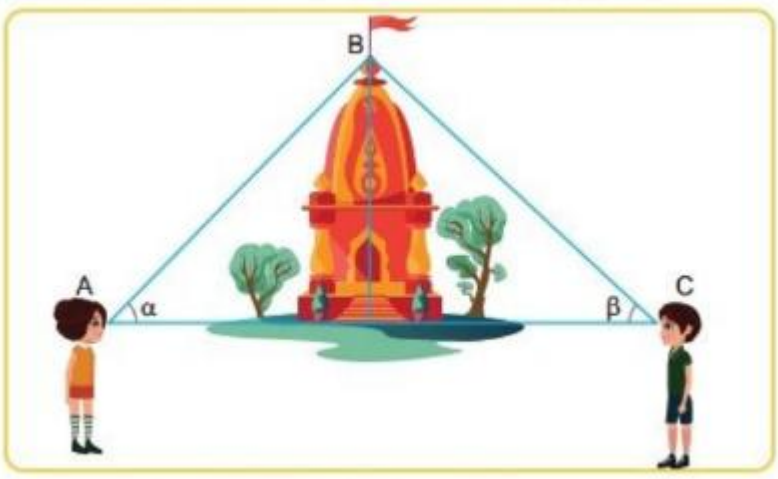
Each carry 5 marks

10. Express the following in simplest form: (i) $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$ (ii) $\tan^{-1}\left(\frac{a+bx}{a-bx}\right)$.

11. Express the following in simplest form: (i) $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ (ii) $\tan^{-1}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

SECTION E

12. Two men on either side of a temple 30 meters high observe its top at the angles of elevation α and β respectively (as shown in the figure). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters.



Based on the above information, answer the following questions:

(i) Find $\angle CAB = \alpha$ in terms of $\sin^{-1}$.

(ii) Find $\angle CAB = \alpha$ in terms of $\sin^{-1}$.

(iii) Find $\angle BCA = \beta$ in terms of $\tan^{-1}$.

UNIT TEST

1. (d) [1, 2]

2. (b) $-\frac{\pi}{2}$

3. (c) [–1, 1]

4. (c) Assertion (A) is true but Reason (R) is false.

5. $\frac{\pi}{3}$

6. $\frac{2\pi}{3}$

7. $40°$ or $\frac{2\pi}{9}$

8. $x + \frac{\pi}{4}$

9. $\frac{a+b}{1-ab}$

10. (i) $\tan^{-1}\left(\frac{a}{b}\right) - x$ (ii) $\frac{\pi}{4} + \tan^{-1}\frac{bx}{a}$

11. (i) $\sqrt{1-x^2}$ (ii) $\frac{1}{2}\cos^{-1}\sqrt{x}$

12. (i) $\sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (iii) $\tan^{-1}(\sqrt{3})$