

## UNIT TEST

Duration: 1 hour

Marks: 30

### SECTION A

Each carry 1 mark

1. If the matrix  $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$  then  $A^4 =$

- (a)  $4A$                       (b)  $8A$                       (c)  $16A$                       (d)  $6A$

2. If  $A$  is a symmetric matrix and  $B$  is a skew symmetric matrix such that  $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ ,

then  $AB$  is

- (a)  $\begin{bmatrix} 4 & -1 \\ -1 & -1 \end{bmatrix}$                       (b)  $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$                       (c)  $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$                       (d)  $\begin{bmatrix} -4 & 2 \\ 1 & 1 \end{bmatrix}$

3. If a matrix  $A$  is such that  $3A^3 + 2A^2 + 5A + I = 0$ , then what is  $A^{-1}$  equal to?

- (a)  $-(3A^2 + 2A + 5)$                       (b)  $(3A^2 + 2A + 5I)$   
(c)  $(3A^2 - 2A - 5I)$                       (d)  $-(3A^2 + 2A + 5I)$

4. Assertion (A): If  $A$  is a square matrix such that  $A^2 = I$ , then  $(I + A)^3 - 3A = I$

Reason (R):  $AI = IA = A$ , where  $I$  is the identity matrix.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).  
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).  
(c) Assertion (A) is true but Reason (R) is false.  
(d) Assertion (A) is false but Reason (R) is true.

### SECTION B

Each carry 2 marks

5. Construct a  $3 \times 3$  matrix  $A = [a_{ij}]$  such that  $a_{ij} = \begin{cases} 2^i; & i > j \\ i \cdot j; & i = j \\ 3^j; & i < j \end{cases}$

6. Find the values of  $x$  and  $y$ , if  $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  satisfies the equation  $A^2 + xA + yI = 0$

7. If  $A = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$  then prove that  $A^3 = \begin{pmatrix} \cos 3x & -\sin 3x \\ \sin 3x & \cos 3x \end{pmatrix}$

## SECTION C

**Each carry 3 marks**

8. If  $f(x) = \frac{1}{\sqrt{1-x^2}} \begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix}$ , prove that  $f(x).f(y) = f\left(\frac{x+y}{1+xy}\right)$ . Hence show that  $f(x).f(-x) = 1$

where  $|x| < 1$ .

9. Let  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$  and  $f(x) = x^2 - 4x + 7$ . Show that  $f(A) = 0$ . Use this result to find  $A^5$ .

## SECTION D

**Each carry 5 marks**

10. If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ , then show that  $A^3 - 23A - 40I = 0$ .

11. (i) If  $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , find  $k$  so that  $A^2 = 8A + kI$ .

(ii) If  $A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix}$  and  $I$  is an identity matrix of order 2, show that  $(2I - A)(10I - A) = 9I$ .

## SECTION E

12. Deepika, Yamini and Rachna were given the task of creating square matrix of order 2. Below are the matrices created by them namely  $A$ ,  $B$  and  $C$  respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

(i) Find  $A + B + C$

(ii) Find  $AC - BC$

(iii) Find the matrix  $(a+b)B$ ,  $a = 4$ ,  $b = -2$ .

## UNIT TEST Answers

1. (c) 16A

2. (b)  $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

3. (d)  $-(3A^2 + 2A + 5I)$

4. (d) Assertion (A) is false but Reason (R) is true.

5.  $\begin{pmatrix} 1 & 9 & 27 \\ 4 & 4 & 27 \\ 8 & 8 & 9 \end{pmatrix}$

6.  $x = -2$  and  $y = 0$

7. Proof

8. Proof

9.  $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

10. Proof

11. (i)  $-7$

(ii) Proof

12. (i)  $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

(ii)  $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$

(iii)  $\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$