UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. The value of k for which $f(x) = \begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$ (c) 11 (d) $\frac{11}{4}$

2. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$ (c) $\sec^2(\frac{y}{x})$ (d) - $\sec^2(\frac{y}{x})$

3. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x}\log x$ (c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

4. Assertion: $f(x) = \frac{x^2+5x+6}{x^2-4x+4}$ is continuous for R.

Reason: A polynomial function is everywhere continuous.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. If $y = \operatorname{cosec}(\cot\sqrt{x})$, then find $\frac{dy}{dx}$.

6. Differentiate the function y = $\log\sqrt{\frac{1-\cos x}{1+\cos x}}$ with respect to x.

7. If $y = \sin(2\sin^{-1}x)$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$.

SECTION C

Each carry 3 marks

8. If $y = x^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$

9. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, prove that $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$.

SECTION D

Each carry 5 marks

10. Differentiate the function $(\tan^{-1} x)^{\cot x} + (\cot^{-1} x)^{\tan x}$ with respect to x.

11. If $x = \sin t, y = \sin pt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

SECTION E

12. Example1: x – y – 6 = 0

Example 2: x + sinxy – y = 0.

When a relationship between x and y is expressed in a way that it is easy to solve for y and write y = f(x), we say that y is given as an explicit function of x. In the second case it is implicit that y is function of x and we say that the relationship of the second type above gives function implicitly. With the above information find the derivative of the following.

(i) y + siny = cosx

(ii) $x^2 + xy + y^2 = 5$.

(iii) $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

UNIT TEST

1. (d) $\frac{11}{4}$

2. (b) $\frac{y}{x}$

3. (c) $2x^{2x}(1 + \log x)$

4. (d) Assertion (A) is false but Reason (R) is true.

5. $\frac{\operatorname{cosec}(\cot\sqrt{x})\cot(\cot\sqrt{x})\times\operatorname{cosec}^2(\sqrt{x})}{2\sqrt{x}}$

6. cosecx

7. Proof

8. $x^{\cos x}\left(\frac{\cos x}{x} - \sin x\log x\right) + (\cos x)^{\sin x}[-\sin x\tan x + \cos x\log\cos x]$

9. Proof

10. $\frac{dy}{dx} = (\tan^{-1} x)^{\cot x}\left[\frac{\cot x}{\tan^{-1} x(1+x^2)} - \operatorname{cosec}^2\log(\tan^{-1} x)\right] + (\cot^{-1} x)^{\tan x}\left[\sec^2 x\log(\cot^{-1} x) - \frac{\tan x}{\cot^{-1} x(1+x^2)}\right]$

11. Proof

12. (i) $-\frac{\sin x}{1+\cos y}$ (ii) $-\frac{2x+y}{x+2y}$ (iii) $-\frac{2}{1+x^2}$