

UNIT TEST

Duration: 1 hour

Marks: 30

SECTION A

Each carry 1 mark

1. The intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing in

- (a) $(-\infty, 2) \cup (2, \infty)$ (b) $(2, \infty)$ (c) $(-\infty, 2)$ (d) $(-\infty, 2] \cup (2, \infty)$

2. The maximum value of $\left(\frac{1}{x}\right)^x$ is

- (a) e (b) e^e (c) $e^{1/e}$ (d) $\left(\frac{1}{e}\right)^{1/e}$

3. The maximum value of $[x(x-1) + 1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is

- (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt[3]{\frac{1}{3}}$

4. Assertion: The ratio of change of area of a circle with respect to its radius r when $r = 6\text{cm}$ is $12\pi\text{cm}^2/\text{cm}$.

Reason: Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x -coordinate is 1, y -coordinate is changing twice as fast as x -coordinate. Find the value of a .

6. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is increasing or decreasing in the interval $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$.

7. Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals $(-3,0) \cup (0,3)$.

SECTION C

Each carry 3 marks

8. Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

9. Find the minimum value of $(ax + by)$, where $xy = c^2$.

SECTION D

Each carry 5 marks

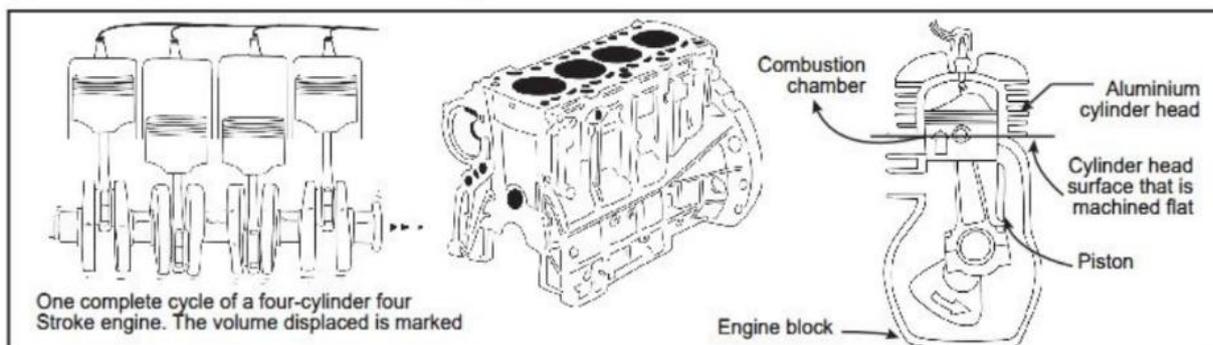
10. Show the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one third of the height of the cone and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

11. Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

SECTION E

12. Read the following passage and answer the following questions:

Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi\text{cm}^2$.

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of r .

(ii) Find $\frac{dV}{dr}$.

(iii) Find the radius of cylinder when its volume is maximum.

(Or)

For maximum volume, $h > r$. State true or false and justify.

Answers

1. (b) $(2, \infty)$

2. (c) $e^{1/e}$

3. (c) 1

4. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

5. $a = 2$.

6. $f(x)$ is increasing in $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

7. Proof

8. $f(x)$ is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ and strictly decreasing $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$.

9. $2c\sqrt{ab}$

10. Proof

11. $\frac{25}{2}$

12. (i) $\frac{\pi}{2} \times (75r - r^3)$

(ii) $\frac{3\pi}{2}(25 - r^2)$

(iii) $r = 5\text{cm}$ (Or) $h > r$ is false