

# 11

## Three Dimensional Geometry

### Short Answer Type Questions

**Q. 1** Find the position vector of a point  $A$  in space such that  $\vec{OA}$  is inclined at  $60^\circ$  to  $OX$  and at  $45^\circ$  to  $OY$  and  $|\vec{OA}| = 10$  units.

**Sol.** Since,  $\vec{OA}$  is inclined at  $60^\circ$  to  $OX$  and at  $45^\circ$  to  $OY$ . Let  $\vec{OA}$  makes angle  $\alpha$  with  $OZ$ .

$$\therefore \cos^2 60^\circ + \cos^2 45^\circ + \cos^2 \alpha = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 \alpha = 1 \quad [\because l^2 + m^2 + n^2 = 1]$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = 1 - \left(\frac{1}{2} + \frac{1}{4}\right)$$

$$\Rightarrow \cos^2 \alpha = 1 - \left(\frac{6}{8}\right)$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{4}$$

$$\Rightarrow \cos \alpha = \frac{1}{2} = \cos 60^\circ$$

$$\therefore \alpha = 60^\circ$$

$$\therefore \vec{OA} = |\vec{OA}| \left( \frac{1}{2} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} + \frac{1}{2} \hat{k} \right)$$

$$= 10 \left( \frac{1}{2} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} + \frac{1}{2} \hat{k} \right) \quad [\because |\vec{OA}| = 10]$$

$$= 5\hat{i} + 5\sqrt{2}\hat{j} + 5\hat{k}$$

**Q. 2** Find the vector equation of the line which is parallel to the vector  $3\hat{i} - 2\hat{j} + 6\hat{k}$  and which passes through the point  $(1, -2, 3)$ .

**Thinking Process**

Here, we use the formula  $\vec{r} = \vec{b} + \lambda \vec{a}$ , where  $\vec{r}$  is the equation of the line which passes through  $\vec{b}$  and parallel to  $\vec{a}$ .

**Sol.** Let  $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

So, vector equation of the line, which is parallel to the vector  $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$  and passes through the vector  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$  is  $\vec{r} = \vec{b} + \lambda \vec{a}$ .

$$\begin{aligned} \therefore \vec{r} &= (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k}) \\ \Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) &= \lambda(3\hat{i} - 2\hat{j} + 6\hat{k}) \\ \Rightarrow (x-1)\hat{i} + (y+2)\hat{j} + (z-3)\hat{k} &= \lambda(3\hat{i} - 2\hat{j} + 6\hat{k}) \end{aligned}$$

**Q. 3** Show that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-4}{5} = \frac{y-1}{2} = z$  intersect.

Also, find their point of intersection.

**Thinking Process**

If shortest distance between the lines is zero, then they intersect.

**Sol.** We have,  $x_1 = 1, y_1 = 2, z_1 = 3$  and  $a_1 = 2, b_1 = 3, c_1 = 4$

Also,  $x_2 = 4, y_2 = 1, z_2 = 0$  and  $a_2 = 5, b_2 = 2, c_2 = 1$

If two lines intersect, then shortest distance between them should be zero.

$\therefore$  Shortest distance between two given lines

$$\begin{aligned} & \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}} \\ &= \frac{\begin{vmatrix} 4-1 & 1-2 & 0-3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}}{\sqrt{(3 \cdot 1 - 2 \cdot 4)^2 + (4 \cdot 5 - 1 \cdot 2)^2 + (2 \cdot 2 - 5 \cdot 3)^2}} \\ &= \frac{\begin{vmatrix} 3 & -1 & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}}{\sqrt{25 + 324 + 121}} \\ &= \frac{3(3-8) + 1(2-20) - 3(4-15)}{\sqrt{470}} \\ &= \frac{-15 - 18 + 33}{\sqrt{470}} = \frac{0}{\sqrt{470}} = 0 \end{aligned}$$

Therefore, the given two lines are intersecting.

For finding their point of intersection for first line,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda + 2 \text{ and } z = 4\lambda + 3$$

Since, the lines are intersecting. So, let us put these values in the equation of another line.

$$\text{Thus, } \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3}{1}$$

$$\Rightarrow \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2} = \frac{4\lambda + 3}{1}$$

$$\Rightarrow \frac{2\lambda - 3}{5} = \frac{4\lambda + 3}{1}$$

$$\Rightarrow 2\lambda - 3 = 20\lambda + 15$$

$$\Rightarrow 18\lambda = -18 = -1$$

So, the required point of intersection is

$$x = 2(-1) + 1 = -1$$

$$y = 3(-1) + 2 = -1$$

$$z = 4(-1) + 3 = -1$$

Thus, the lines intersect at  $(-1, -1, -1)$ .

**Q. 4** Find the angle between the lines

$$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k}) \text{ and } \vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k}).$$

**Thinking Process**

We know that,  $\cos \theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| \cdot |\vec{b}_2|}$ , where,  $\theta$  is the angle between the lines  $\vec{a}_1 + \lambda \vec{b}_1$

and  $\vec{a}_2 + \mu \vec{b}_2$ .

**Sol.** We have,

$$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$$

and

$$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$$

where,

$$\vec{a}_1 = 3\hat{i} - 2\hat{j} + 6\hat{k}, \vec{b}_1 = 2\hat{i} + \hat{j} + 2\hat{k}$$

and

$$\vec{a}_2 = 2\hat{j} - 5\hat{k}, \vec{b}_2 = 6\hat{i} + 3\hat{j} + 2\hat{k}$$

If  $\theta$  is angle between the lines, then

$$\begin{aligned} \cos \theta &= \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| \cdot |\vec{b}_2|} \\ &= \frac{|(2\hat{i} + \hat{j} + 2\hat{k}) \cdot (6\hat{i} + 3\hat{j} + 2\hat{k})|}{|2\hat{i} + \hat{j} + 2\hat{k}| |6\hat{i} + 3\hat{j} + 2\hat{k}|} \\ &= \frac{|12 + 3 + 4|}{\sqrt{9} \sqrt{49}} = \frac{19}{21} \end{aligned}$$

$\therefore$

$$\theta = \cos^{-1} \frac{19}{21}$$

**Q. 5** Prove that the line through  $A(0, -1, -1)$  and  $B(4, 5, 1)$  intersects the line through  $C(3, 9, 4)$  and  $D(-4, 4, 4)$ .

**Sol.** We know that, the cartesian equation of a line that passes through two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Hence, the cartesian equation of line passes through  $A(0, -1, -1)$  and  $B(4, 5, 1)$  is

$$\begin{aligned} \frac{x - 0}{4 - 0} &= \frac{y + 1}{5 + 1} = \frac{z + 1}{1 + 1} \\ \Rightarrow \frac{x}{4} &= \frac{y + 1}{6} = \frac{z + 1}{2} \end{aligned} \quad \dots(i)$$

and cartesian equation of the line passes through  $C(3, 9, 4)$  and  $D(-4, 4, 4)$  is

$$\begin{aligned} \frac{x - 3}{-4 - 3} &= \frac{y - 9}{4 - 9} = \frac{z - 4}{4 - 4} \\ \Rightarrow \frac{x - 3}{-7} &= \frac{y - 9}{-5} = \frac{z - 4}{0} \end{aligned} \quad \dots(ii)$$

If the lines intersect, then shortest distance between both of them should be zero.

$\therefore$  Shortest distance between the lines

$$\begin{aligned} & \left| \begin{array}{ccc} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array} \right| \\ &= \frac{\left| \begin{array}{ccc} 3 - 0 & 9 + 1 & 4 + 1 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{array} \right|}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}} \\ &= \frac{\left| \begin{array}{ccc} 3 - 0 & 9 + 1 & 4 + 1 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{array} \right|}{\sqrt{(6 \cdot 0 + 10)^2 + (-14 - 0)^2 + (-20 + 42)^2}} \\ &= \frac{\left| \begin{array}{ccc} 3 & 10 & 5 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{array} \right|}{\sqrt{100 + 196 + 484}} \\ &= \frac{3(0 + 10) - 10(14) + 5(-20 + 42)}{\sqrt{780}} \\ &= \frac{30 - 140 + 110}{\sqrt{780}} = 0 \end{aligned}$$

So, the given lines intersect.

**Q. 6** Prove that the lines  $x = py + q, z = ry + s$  and  $x = p'y + q', z = r'y + s'$  are perpendicular, if  $pp' + rr' + 1 = 0$ .

**Sol.** We have,  $x = py + q \Rightarrow y = \frac{x - q}{p}$  ... (i)

and  $z = ry + s \Rightarrow y = \frac{z - s}{r}$  ... (ii)

$\Rightarrow \frac{x - q}{p} = \frac{y}{1} = \frac{z - s}{r}$  [using Eqs. (i) and (ii)] ... (iii)

Similarly,  $\frac{x - q'}{p'} = \frac{y}{1} = \frac{z - s'}{r'}$  ... (iv)

From Eqs. (iii) and (iv),

$$a_1 = p, b_1 = 1, c_1 = r$$

and

$$a_2 = p', b_2 = 1, c_2 = r'$$

If these given lines are perpendicular to each other, then

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$\Rightarrow$

$$pp' + 1 + rr' = 0$$

which is the required condition.

**Q. 7** Find the equation of a plane which bisects perpendicularly the line joining the points  $A(2, 3, 4)$  and  $B(4, 5, 8)$  at right angles.

**Sol.** Since, the equation of a plane is bisecting perpendicular the line joining the points  $A(2, 3, 4)$  and  $B(4, 5, 8)$  at right angles.

So, mid-point of  $AB$  is  $\left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2}\right)$  i.e.,  $(3, 4, 6)$ .

$$\text{Also, } \vec{N} = (4-2)\hat{i} + (5-3)\hat{j} + (8-4)\hat{k} = 2\hat{i} + 2\hat{j} + 4\hat{k}$$

So, the required equation of the plane is  $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$ .

$$\Rightarrow [(x-3)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}] \cdot (2\hat{i} + 2\hat{j} + 4\hat{k}) = 0 \quad [\because \vec{a} = 3\hat{i} + 4\hat{j} + 6\hat{k}]$$

$$\Rightarrow 2x - 6 + 2y - 8 + 4z - 24 = 0$$

$$\Rightarrow 2x + 2y + 4z = 38$$

$$\therefore x + y + 2z = 19$$

**Q. 8** Find the equation of a plane which is at a distance  $3\sqrt{3}$  units from origin and the normal to which is equally inclined to coordinate axis.

**Sol.** Since, normal to the plane is equally inclined to the coordinate axis.

$$\text{Therefore, } \cos \alpha = \cos \beta = \cos \gamma = \frac{1}{\sqrt{3}}$$

So, the normal is  $\vec{N} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$  and plane is at a distance of  $3\sqrt{3}$  units from origin.

The equation of plane is  $\vec{r} \cdot \hat{N} = 3\sqrt{3}$

$$\left[ \because \hat{N} = \frac{\vec{N}}{|\vec{N}|} \right]$$

[since, vector equation of the plane at a distance  $p$  from the origin is  $\vec{r} \cdot \hat{N} = p$ ]

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \frac{\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)}{1} = 3\sqrt{3}$$

$$\Rightarrow \frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 3\sqrt{3}$$

$$\therefore x + y + z = 3\sqrt{3} \cdot \sqrt{3} = 9$$

So, the required equation of plane is  $x + y + z = 9$ .

**Q. 9** If the line drawn from the point  $(-2, -1, -3)$  meets a plane at right angle at the point  $(1, -3, 3)$ , then find the equation of the plane.

**Sol.** Since, the line drawn from the point  $(-2, -1, -3)$  meets a plane at right angle at the point  $(1, -3, 3)$ . So, the plane passes through the point  $(1, -3, 3)$  and normal to plane is  $(-3\hat{i} + 2\hat{j} - 6\hat{k})$ .

$$\Rightarrow \vec{a} = \hat{i} - 3\hat{j} + 3\hat{k}$$

$$\text{and } \vec{N} = -3\hat{i} + 2\hat{j} - 6\hat{k}$$

So, the equation of required plane is  $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$$\Rightarrow [(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 3\hat{j} + 3\hat{k})] \cdot (-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$$

$$\Rightarrow [(x-1)\hat{i} + (y+3)\hat{j} + (z-3)\hat{k}] \cdot (-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$$

$$\Rightarrow -3x + 3 + 2y + 6 - 6z + 18 = 0$$

$$\Rightarrow -3x + 2y - 6z = -27$$

$$\therefore 3x - 2y + 6z - 27 = 0$$

**Q. 10** Find the equation of the plane through the points  $(2, 1, 0)$ ,  $(3, -2, -2)$  and  $(3, 1, 7)$ .

**Thinking Process**

Here, apply the equation of the plane passing through the points  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$

$$\text{and } (x_3, y_3, z_3) \text{ is given by } \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0.$$

**Sol.** We know that, the equation of a plane passing through three non-collinear points  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(-21+0) - (y-1)(7+2) + z(3) = 0$$

$$\Rightarrow -21x + 42 - 9y + 9 + 3z = 0$$

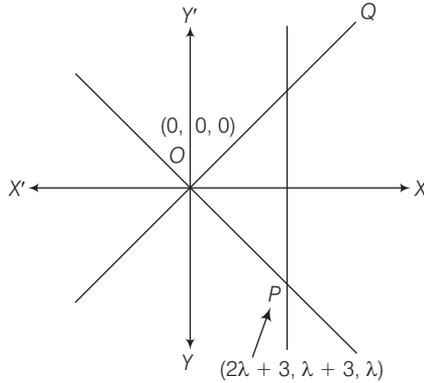
$$\Rightarrow -21x - 9y + 3z = -51$$

$$\therefore 7x + 3y - z = 17$$

So, the required equation of plane is  $7x + 3y - z = 17$ .

**Q. 11** Find the equations of the two lines through the origin which intersect the line  $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$  at angles of  $\frac{\pi}{3}$  each.

**Sol.** Given equation of the line is  $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} = \lambda$  ... (i)



So, DR's of the line are 2, 1, 1 and DC's of the given line are  $\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{16}}$ .

Also, the required lines make angle  $\frac{\pi}{3}$  with the given line.

From Eq. (i),

$$x = (2\lambda + 3), y = (\lambda + 3) \text{ and } z = \lambda$$

$$\therefore \cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\therefore \cos \frac{\pi}{3} = \frac{(4\lambda + 6) + (\lambda + 3) + (\lambda)}{\sqrt{6} \sqrt{(2\lambda + 3)^2 + (\lambda + 3)^2 + \lambda^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{6\lambda + 9}{\sqrt{6} \sqrt{4\lambda^2 + 9 + 12\lambda + \lambda^2 + 9 + 6\lambda + \lambda^2}}$$

$$\Rightarrow \frac{\sqrt{6}}{2} = \frac{6\lambda + 9}{\sqrt{6\lambda^2 + 18\lambda + 18}}$$

$$\Rightarrow 6\sqrt{\lambda^2 + 3\lambda + 3} = 2(6\lambda + 9)$$

$$\Rightarrow 36(\lambda^2 + 3\lambda + 3) = 36(4\lambda^2 + 9 + 12\lambda)$$

$$\Rightarrow \lambda^2 + 3\lambda + 3 = 4\lambda^2 + 9 + 12\lambda$$

$$\Rightarrow 3\lambda^2 + 9\lambda + 6 = 0$$

$$\Rightarrow \lambda^2 + 3\lambda + 2 = 0$$

$$\Rightarrow \lambda(\lambda + 2) + 1(\lambda + 2) = 0$$

$$\Rightarrow (\lambda + 1)(\lambda + 2) = 0$$

$$\therefore \lambda = -1, -2$$

So, the DC's are 1, 2, -1 and -1, 1, -2.

Also, both the required lines pass through origin.

So, the equations of required lines are  $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$  and  $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$ .

**Q. 12** Find the angle between the lines whose direction cosines are given by the equation  $l + m + n = 0$  and  $l^2 + m^2 - n^2 = 0$ .

**Sol.** Eliminating  $n$  from both the equations, we have

$$\begin{aligned} l^2 + m^2 - (l - m)^2 &= 0 \\ \Rightarrow l^2 + m^2 - l^2 - m^2 + 2ml &= 0 \quad \Rightarrow 2lm = 0 \\ \Rightarrow lm &= 0 \quad \Rightarrow (-m - n)m = 0 \quad [\because l = -m - n] \\ \Rightarrow (m + n)m &= 0 \\ \Rightarrow m = -n \quad \Rightarrow m &= 0 \\ \Rightarrow l = 0, l &= -n \end{aligned}$$

Thus, Dir's two lines are proportional to  $0, -n, n$  and  $-n, 0, n$  i.e.,  $0, -1, 1$  and  $-1, 0, 1$ .

So, the vector parallel to these given lines are  $\vec{a} = -\hat{j} + \hat{k}$  and  $\vec{b} = -\hat{i} + \hat{k}$

$$\begin{aligned} \text{Now,} \quad \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \Rightarrow \cos \theta = \frac{1}{2} \\ \therefore \theta &= \frac{\pi}{3} \quad \left[ \because \cos \frac{\pi}{3} = \frac{1}{2} \right] \end{aligned}$$

**Q. 13** If a variable line in two adjacent positions has direction cosines  $l, m, n$  and  $l + \delta l, m + \delta m, n + \delta n$ , then show that the small angle  $\delta \theta$  between the two positions is given by  $\delta \theta^2 = \delta l^2 + \delta m^2 + \delta n^2$ .

**Sol.** We have  $l, m, n$  and  $l + \delta l, m + \delta m, n + \delta n$  as direction cosines of a variable line in two different positions.

$$\begin{aligned} \therefore l^2 + m^2 + n^2 &= 1 \quad \dots (i) \\ \text{and} \quad (l + \delta l)^2 + (m + \delta m)^2 + (n + \delta n)^2 &= 1 \quad \dots (ii) \\ \Rightarrow l^2 + m^2 + n^2 + \delta l^2 + \delta m^2 + \delta n^2 + 2(l \delta l + m \delta m + n \delta n) &= 1 \\ \Rightarrow \delta l^2 + \delta m^2 + \delta n^2 + 2(l \delta l + m \delta m + n \delta n) &= -2(l \delta l + m \delta m + n \delta n) \quad [\because l^2 + m^2 + n^2 = 1] \\ \Rightarrow l \delta l + m \delta m + n \delta n &= \frac{-1}{2}(\delta l^2 + \delta m^2 + \delta n^2) \quad \dots (iii) \end{aligned}$$

Now,  $\vec{a}$  and  $\vec{b}$  are unit vectors along a line with direction cosines  $l, m, n$  and  $(l + \delta l), (m + \delta m), (n + \delta n)$ , respectively.

$$\begin{aligned} \therefore \vec{a} &= l\hat{i} + m\hat{j} + n\hat{k} \quad \text{and} \quad \vec{b} = (l + \delta l)\hat{i} + (m + \delta m)\hat{j} + (n + \delta n)\hat{k} \\ \Rightarrow \cos \delta \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \vec{a} \cdot \vec{b} \quad [\because |\vec{a}| = |\vec{b}| = 1] \\ \Rightarrow \cos \delta \theta &= l(l + \delta l) + m(m + \delta m) + n(n + \delta n) \\ &= (l^2 + m^2 + n^2) + (l \delta l + m \delta m + n \delta n) \\ &= 1 - \frac{1}{2}(\delta l^2 + \delta m^2 + \delta n^2) \quad \text{[using Eq. (iii)]} \\ \Rightarrow 2(1 - \cos \delta \theta) &= (\delta l^2 + \delta m^2 + \delta n^2) \\ \Rightarrow 2 \cdot 2 \sin^2 \frac{\delta \theta}{2} &= \delta l^2 + \delta m^2 + \delta n^2 \quad \left[ \because 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2} \right] \\ \Rightarrow 4 \left( \frac{\delta \theta}{2} \right)^2 &= \delta l^2 + \delta m^2 + \delta n^2 \quad \left[ \text{since, } \frac{\delta \theta}{2} \text{ is small, then } \sin \frac{\delta \theta}{2} = \frac{\delta \theta}{2} \right] \\ \therefore \delta \theta^2 &= \delta l^2 + \delta m^2 + \delta n^2 \end{aligned}$$

**Q. 14** If  $O$  is the origin and  $A$  is  $(a, b, c)$ , then find the direction cosines of the line  $OA$  and the equation of plane through  $A$  at right angle to  $OA$ .

**Sol.** Since, DC's of line  $OA$  are  $\frac{a}{\sqrt{a^2 + b^2 + c^2}}$ ,  $\frac{b}{\sqrt{a^2 + b^2 + c^2}}$  and  $\frac{c}{\sqrt{a^2 + b^2 + c^2}}$ .

Also, 
$$\vec{n} = \vec{OA} = \vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$$

The equation of plane passes through  $(a, b, c)$  and perpendicular to  $OA$  is given by

$$[\vec{r} - \vec{a}] \cdot \vec{n} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (a\hat{i} + b\hat{j} + c\hat{k}) = (a\hat{i} + b\hat{j} + c\hat{k}) \cdot (a\hat{i} + b\hat{j} + c\hat{k})$$

$$\Rightarrow ax + by + cz = a^2 + b^2 + c^2$$

**Q. 15** Two systems of rectangular axis have the same origin. If a plane cuts them at distances  $a, b, c$  and  $a', b', c'$ , respectively from the origin, then prove that  $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$ .

**Sol.** Consider  $OX, OY, OZ$  and  $ox, oy, oz$  are two system of rectangular axes. Let their corresponding equation of plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \dots(i)$$

and 
$$\frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1 \quad \dots(ii)$$

Also, the length of perpendicular from origin to Eqs. (i) and (ii) must be same.

$$\therefore \frac{\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{\frac{0}{a'} + \frac{0}{b'} + \frac{0}{c'} - 1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

$$\Rightarrow \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} = \sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$$

## Long Answer Type Questions

**Q. 16** Find the foot of perpendicular from the point  $(2, 3, -8)$  to the line  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ . Also, find the perpendicular distance from the given point to the line.

**Sol.** We have, equation of line as  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

$$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$$

$$\Rightarrow x = -2\lambda + 4, y = 6\lambda \text{ and } z = -3\lambda + 1$$

Let the coordinates of  $L$  be  $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$  and direction ratios of  $PL$  are proportional to  $(4 - 2\lambda - 2, 6\lambda - 3, 1 - 3\lambda + 8)$  i.e.,  $(2 - 2\lambda, 6\lambda - 3, 9 - 3\lambda)$ .

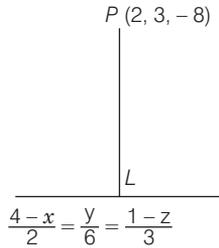
Also, direction ratios are proportional to  $-2, 6, -3$ . Since,  $PL$  is perpendicular to given line.

$$\therefore -2(2 - 2\lambda) + 6(6\lambda - 3) - 3(9 - 3\lambda) = 0$$

$$\Rightarrow -4 + 4\lambda + 36\lambda - 18 - 27 + 9\lambda = 0$$

$$\Rightarrow 49\lambda = 49 \Rightarrow \lambda = 1$$

So, the coordinates of  $L$  are  $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$  i.e.,  $(2, 6, -2)$ .



Also, length of  $PL = \sqrt{(2 - 2)^2 + (6 - 3)^2 + (-2 + 8)^2}$   
 $= \sqrt{0 + 9 + 36} = 3\sqrt{5}$  units

**Q. 17** Find the distance of a point  $(2, 4, -1)$  from the line

$$\frac{x + 5}{1} = \frac{y + 3}{4} = \frac{z - 6}{-9}$$

**Sol.** We have, equation of the line as  $\frac{x + 5}{1} = \frac{y + 3}{4} = \frac{z - 6}{-9} = \lambda$

$$\Rightarrow x = \lambda - 5, y = 4\lambda - 3, z = 6 - 9\lambda$$

Let the coordinates of  $L$  be  $(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$ , then Dr's of  $PL$  are  $(\lambda - 7, 4\lambda - 7, 7 - 9\lambda)$ .

Also, the direction ratios of given line are proportional to  $1, 4, -9$ .

Since,  $PL$  is perpendicular to the given line.

$$\therefore (\lambda - 7) \cdot 1 + (4\lambda - 7) \cdot 4 + (7 - 9\lambda) \cdot (-9) = 0$$

$$\Rightarrow \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$$

$$\Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1$$

So, the coordinates of  $L$  are  $(-4, 1, -3)$ .

$$\therefore \text{Required distance, } PL = \sqrt{(-4 - 2)^2 + (1 - 4)^2 + (-3 + 1)^2}$$

$$= \sqrt{36 + 9 + 4} = 7 \text{ units}$$

**Q. 18** Find the length and the foot of perpendicular from the point  $\left(1, \frac{3}{2}, 2\right)$  to the plane  $2x - 2y + 4z + 5 = 0$ .

**Sol.** Equation of the given plane is  $2x - 2y + 4z + 5 = 0$  ... (i)

$$\Rightarrow \vec{n} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

So, the equation of line through  $\left(1, \frac{3}{2}, 2\right)$  and parallel to  $\vec{n}$  is given by

$$\frac{x - 1}{2} = \frac{y - 3/2}{-2} = \frac{z - 2}{4} = \lambda$$

$$\Rightarrow x = 2\lambda + 1, y = -2\lambda + \frac{3}{2} \text{ and } z = 4\lambda + 2$$

If this point lies on the given plane, then

$$2(2\lambda + 1) - 2\left(-2\lambda + \frac{3}{2}\right) + 4(4\lambda + 2) + 5 = 0 \quad [\text{using Eq. (i)}]$$

$$\Rightarrow 4\lambda + 2 + 4\lambda - 3 + 16\lambda + 8 + 5 = 0$$

$$\Rightarrow 24\lambda = -12 \Rightarrow \lambda = \frac{-1}{2}$$

\(\therefore\) Required foot of perpendicular

$$= \left[ 2 \times \left(\frac{-1}{2}\right) + 1, -2 \times \left(\frac{-1}{2}\right) + \frac{3}{2}, 4 \times \left(\frac{-1}{2}\right) + 2 \right] \text{ i.e., } \left(0, \frac{5}{2}, 0\right)$$

$$\begin{aligned} \therefore \text{ Required length of perpendicular} &= \sqrt{(1-0)^2 + \left(\frac{3}{2} - \frac{5}{2}\right)^2 + (2-0)^2} \\ &= \sqrt{1+1+4} = \sqrt{6} \text{ units} \end{aligned}$$

**Q. 19** Find the equation of the line passing through the point (3, 0, 1) and parallel to the planes  $x + 2y = 0$  and  $3y - z = 0$ .

**Sol.** Equation of the two planes are  $x + 2y = 0$  and  $3y - z = 0$ .

Let  $\vec{n}_1$  and  $\vec{n}_2$  are the normals to the two planes, respectively.

$$\therefore \vec{n}_1 = \hat{i} + 2\hat{j} \quad \text{and} \quad \vec{n}_2 = 3\hat{j} - \hat{k}$$

Since, required line is parallel to the given two planes.

$$\begin{aligned} \text{Therefore, } \vec{b} = \vec{n}_1 \times \vec{n}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & 3 & -1 \end{vmatrix} \\ &= \hat{i}(-2) - \hat{j}(-1) + \hat{k}(3) \\ &= -2\hat{i} + \hat{j} + 3\hat{k} \end{aligned}$$

So, the equation of the lines through the point (3, 0, 1) and parallel to the given two planes are

$$\begin{aligned} (x-3)\hat{i} + (y-0)\hat{j} + (z-1)\hat{k} + \lambda(-2\hat{i} + \hat{j} + 3\hat{k}) \\ \Rightarrow (x-3)\hat{i} + y\hat{j} + (z-1)\hat{k} + \lambda(-2\hat{i} + \hat{j} + 3\hat{k}) \end{aligned}$$

**Q. 20** Find the equation of the plane through the points (2, 1, -1), (-1, 3, 4) and perpendicular to the plane  $x - 2y + 4z = 10$ .

**Sol.** The equation of the plane passing through (2, 1, -1) is

$$a(x-2) + b(y-1) + c(z+1) = 0 \quad \dots(i)$$

Since, this passes through (-1, 3, 4).

$$\therefore a(-1-2) + b(3-1) + c(4+1) = 0$$

$$\Rightarrow -3a + 2b + 5c = 0 \quad \dots(ii)$$

Since, the plane (i) is perpendicular to the plane  $x - 2y + 4z = 10$ .

$$\therefore 1 \cdot a - 2 \cdot b + 4 \cdot c = 0$$

$$\Rightarrow a - 2b + 4c = 0 \quad \dots(iii)$$

On solving Eqs. (ii) and (iii), we get

$$\frac{a}{8+10} = \frac{-b}{-17} = \frac{c}{4} = \lambda$$

$$\Rightarrow a = 18\lambda, b = 17\lambda, c = 4\lambda$$

From Eq. (i),

$$\begin{aligned} & 18\lambda(x-2) + 17\lambda(y-1) + 4\lambda(z+1) = 0 \\ \Rightarrow & 18x - 36 + 17y - 17 + 4z + 4 = 0 \\ \Rightarrow & 18x + 17y + 4z - 49 = 0 \\ \therefore & 18x + 17y + 4z = 49 \end{aligned}$$

**Q. 21** Find the shortest distance between the lines gives by

$$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$$

and  $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ .

**Sol.** We have,

$$\begin{aligned} \vec{r} &= (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k} \\ &= 8\hat{i} - 9\hat{j} + 10\hat{k} + 3\lambda\hat{i} - 16\lambda\hat{j} + 7\lambda\hat{k} \\ &= 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k}) \end{aligned}$$

$$\Rightarrow \vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k} \text{ and } \vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k} \quad \dots(i)$$

$$\text{Also } \vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

$$\Rightarrow \vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k} \text{ and } \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k} \quad \dots(ii)$$

Now, shortest distance between two lines is given by  $\left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$$\begin{aligned} \therefore \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} \\ &= \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48) \\ &= 24\hat{i} + 36\hat{j} + 72\hat{k} \end{aligned}$$

$$\begin{aligned} \text{Now, } |\vec{b}_1 \times \vec{b}_2| &= \sqrt{(24)^2 + (36)^2 + (72)^2} \\ &= 12\sqrt{2^2 + 3^2 + 6^2} = 84 \end{aligned}$$

$$\begin{aligned} \text{and } (\vec{a}_2 - \vec{a}_1) &= (15 - 8)\hat{i} + (29 + 9)\hat{j} + (5 - 10)\hat{k} \\ &= 7\hat{i} + 38\hat{j} - 5\hat{k} \end{aligned}$$

$$\begin{aligned} \therefore \text{Shortest distance} &= \left| \frac{(24\hat{i} + 36\hat{j} + 72\hat{k}) \cdot (7\hat{i} + 38\hat{j} - 5\hat{k})}{84} \right| \\ &= \left| \frac{168 + 1368 - 360}{84} \right| = \left| \frac{1176}{84} \right| = 14 \text{ units} \end{aligned}$$

**Q. 22** Find the equation of the plane which is perpendicular to the plane  $5x + 3y + 6z + 8 = 0$  and which contains the line of intersection of the planes  $x + 2y + 3z - 4 = 0$  and  $2x + y - z + 5 = 0$ .

**Sol.** The equation of a plane through the line of intersection of the planes  $x + 2y + 3z - 4 = 0$  and  $2x + y - z + 5 = 0$  is

$$\begin{aligned} & (x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0 \\ \Rightarrow & x(1 + 2\lambda) + y(2 + \lambda) + z(-\lambda + 3) - 4 + 5\lambda = 0 \quad \dots(i) \end{aligned}$$

Also, this is perpendicular to the plane  $5x + 3y + 6z + 8 = 0$ .

$$\therefore 5(1 + 2\lambda) + 3(2 + \lambda) + 6(3 - \lambda) = 0 \quad [\because a_1 a_2 + b_1 b_2 + c_1 c_2 = 0]$$

$$\Rightarrow 5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$$

$$\therefore \lambda = -29/7$$

From Eq. (i),

$$x \left[ 1 + 2 \left( \frac{-29}{7} \right) \right] + y \left( 2 - \frac{29}{7} \right) + z \left( \frac{29}{7} + 3 \right) - 4 + 5 \left( \frac{-29}{7} \right) = 0$$

$$\Rightarrow x(7 - 58) + y(14 - 29) + z(29 + 21) - 28 - 145 = 0$$

$$\Rightarrow -51x - 15y + 50z - 173 = 0$$

So, the required equation of plane is  $51x + 15y - 50z + 173 = 0$ .

**Q. 23** If the plane  $ax + by = 0$  is rotated about its line of intersection with the plane  $z = 0$  through an angle  $\alpha$ , then prove that the equation of the plane in its new position is  $ax + by \pm (\sqrt{a^2 + b^2} \tan \alpha) z = 0$ .

**Sol.** Equation of the plane is  $ax + by = 0$  ... (i)

$\therefore$  Equation of the plane after new position is

$$\frac{ax \cos \alpha}{\sqrt{a^2 + b^2}} + \frac{by \cos \alpha}{\sqrt{b^2 + a^2}} \pm z \sin \alpha = 0$$

$$\Rightarrow \frac{ax}{\sqrt{a^2 + b^2}} + \frac{by}{\sqrt{b^2 + a^2}} \pm z \tan \alpha = 0 \quad [\text{on dividing by } \cos \alpha]$$

$$\Rightarrow ax + by \pm z \tan \alpha \sqrt{a^2 + b^2} = 0 \quad [\text{on multiplying with } \sqrt{a^2 + b^2}]$$

#### Alternate Method

Given, planes are  $ax + by = 0$  ... (i)

and  $z = 0$  ... (ii)

Therefore, the equation of any plane passing through the line of intersection of planes

(i) and (ii) may be taken as  $ax + by + k = 0$ . ... (iii)

Then, direction cosines of a normal to the plane (iii) are  $\frac{a}{\sqrt{a^2 + b^2 + k^2}}$ ,  $\frac{b}{\sqrt{a^2 + b^2 + k^2}}$ ,

$\frac{c}{\sqrt{a^2 + b^2 + k^2}}$  and direction cosines of the normal to the plane (i) are  $\frac{a}{\sqrt{a^2 + b^2}}$ ,  $\frac{b}{\sqrt{a^2 + b^2}}$ ,

0.

Since, the angle between the planes (i) and (ii) is  $\alpha$ ,

$$\therefore \cos \alpha = \frac{a \cdot a + b \cdot b + k \cdot 0}{\sqrt{a^2 + b^2 + k^2} \sqrt{a^2 + b^2}}$$

$$= \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + k^2}}$$

$$\Rightarrow k^2 \cos^2 \alpha = a^2 (1 - \cos^2 \alpha) + b^2 (1 - \cos^2 \alpha)$$

$$\Rightarrow k^2 = \frac{(a^2 + b^2) \sin^2 \alpha}{\cos^2 \alpha}$$

$$k = \pm \sqrt{a^2 + b^2} \tan \alpha$$

On putting this value in plane (iii), we get the equation of the plane as

$$ax + by + z \sqrt{a^2 + b^2} \tan \alpha = 0$$

**Q. 24** Find the equation of the plane through the intersection of the planes  $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$  and  $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$ , whose perpendicular distance from origin is unity.

**Sol.** We have,  $\vec{n}_1 = (\hat{i} + 3\hat{j}), d_1 = 6$  and  $\vec{n}_2 = (3\hat{i} - \hat{j} - 4\hat{k}), d_2 = 0$

Using the relation,  $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + d_2 \lambda$

$$\Rightarrow \vec{r} \cdot [(\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} - 4\hat{k})] = 6 + 0 \cdot \lambda$$

$$\Rightarrow \vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)] = 6 \quad \dots(i)$$

On dividing both sides by  $\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}$ , we get

$$\frac{\vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)]}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} = \frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}}$$

Since, the perpendicular distance from origin is unity.

$$\begin{aligned} \therefore & \frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} = 1 \\ \Rightarrow & (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2 = 36 \\ \Rightarrow & 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2 = 36 \\ \Rightarrow & 26\lambda^2 + 10 = 36 \\ \Rightarrow & \lambda^2 = 1 \\ \therefore & \lambda = \pm 1 \end{aligned}$$

Using Eq. (i), the required equation of plane is

$$\begin{aligned} & \vec{r} \cdot [(1 \pm 3)\hat{i} + (3 \mp 1)\hat{j} + (\mp 4)\hat{k}] = 6 \\ \Rightarrow & \vec{r} \cdot [(1 + 3)\hat{i} + (3 - 1)\hat{j} + (-4)\hat{k}] = 6 \\ \text{and} & \vec{r} \cdot [(1 - 3)\hat{i} + (3 + 1)\hat{j} + 4\hat{k}] = 6 \\ \Rightarrow & \vec{r} \cdot (4\hat{i} + 2\hat{j} - 4\hat{k}) = 6 \\ \text{and} & \vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) = 6 \\ \Rightarrow & 4x + 2y - 4z - 6 = 0 \\ \text{and} & -2x + 4y + 4z - 6 = 0 \end{aligned}$$

**Q. 25** Show that the points  $(\hat{i} - \hat{j} + 3\hat{k})$  and  $3(\hat{i} + \hat{j} + \hat{k})$  are equidistant from the plane  $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$  and lies on opposite side of it.

**Sol.** To show that these given points  $(\hat{i} - \hat{j} + 3\hat{k})$  and  $3(\hat{i} + \hat{j} + \hat{k})$  are equidistant from the plane  $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$ , we first find out the mid-point of the points which is  $2\hat{i} + \hat{j} + 3\hat{k}$ .

On substituting  $\vec{r}$  by the mid-point in plane, we get

$$\begin{aligned} \text{LHS} &= (2\hat{i} + \hat{j} + 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 \\ &= 10 + 2 - 21 + 9 = 0 \\ &= \text{RHS} \end{aligned}$$

Hence, the two points lie on opposite sides of the plane are equidistant from the plane.

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**Q. 26**  $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$  and  $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$  are two vectors. The position vectors of the points  $A$  and  $C$  are  $6\hat{i} + 7\hat{j} + 4\hat{k}$  and  $-9\hat{i} + 2\hat{k}$ , respectively. Find the position vector of a point  $P$  on the line  $AB$  and a point  $Q$  on the line  $CD$  such that  $\vec{PQ}$  is perpendicular to  $\vec{AB}$  and  $\vec{CD}$  both.

**Sol.** We have,  $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$  and  $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

Also, the position vectors of  $A$  and  $C$  are  $6\hat{i} + 7\hat{j} + 4\hat{k}$  and  $-9\hat{i} + 2\hat{k}$ , respectively. Since,  $\vec{PQ}$  is perpendicular to both  $\vec{AB}$  and  $\vec{CD}$ .

So,  $P$  and  $Q$  will be foot of perpendicular to both the lines through  $A$  and  $C$ .

Now, equation of the line through  $A$  and parallel to the vector  $\vec{AB}$  is,

$$\vec{r} = (6\hat{i} + 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$$

and the line through  $C$  and parallel to the vector  $\vec{CD}$  is given by

$$\vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k}) \quad \dots(i)$$

Let  $\vec{r} = (6\hat{i} + 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

and  $\vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k}) \quad \dots(ii)$

Let  $P(6 + 3\lambda, 7 - \lambda, 4 + \lambda)$  is any point on the first line and  $Q$  be any point on second line is given by  $(-3\mu, -9 + 2\mu, 2 + 4\mu)$ .

$$\begin{aligned} \therefore \vec{PQ} &= (-3\mu - 6 - 3\lambda)\hat{i} + (-9 + 2\mu - 7 + \lambda)\hat{j} + (2 + 4\mu - 4 - \lambda)\hat{k} \\ &= (-3\mu - 6 - 3\lambda)\hat{i} + (2\mu + \lambda - 16)\hat{j} + (4\mu - \lambda - 2)\hat{k} \end{aligned}$$

If  $\vec{PQ}$  is perpendicular to the first line, then

$$\begin{aligned} &3(-3\mu - 6 - 3\lambda) - (2\mu + \lambda - 16) + (4\mu - \lambda - 2) = 0 \\ \Rightarrow &-9\mu - 18 - 9\lambda - 2\mu - \lambda + 16 + 4\mu - \lambda - 2 = 0 \\ \Rightarrow &-7\mu - 11\lambda - 4 = 0 \quad \dots(iii) \end{aligned}$$

If  $\vec{PQ}$  is perpendicular to the second line, then

$$\begin{aligned} &-3(-3\mu - 6 - 3\lambda) + 2(2\mu + \lambda - 16) + 4(4\mu - \lambda - 2) = 0 \\ \Rightarrow &9\mu + 18 + 9\lambda + 4\mu + 2\lambda - 32 + 16\mu - 4\lambda - 8 = 0 \\ \Rightarrow &29\mu + 7\lambda - 22 = 0 \quad \dots(iv) \end{aligned}$$

On solving Eqs. (iii) and (iv), we get

$$\begin{aligned} &-49\mu - 77\lambda - 28 = 0 \\ \Rightarrow &319\mu + 77\lambda - 242 = 0 \\ \Rightarrow &270\mu - 270 = 0 \\ \Rightarrow &\mu = 1 \end{aligned}$$

Using  $\mu$  in Eq. (iii), we get

$$\begin{aligned} &-7(1) - 11\lambda - 4 = 0 \\ \Rightarrow &-7 - 11\lambda - 4 = 0 \\ \Rightarrow &-11 - 11\lambda = 0 \\ \Rightarrow &\lambda = -1 \end{aligned}$$

$$\begin{aligned} \therefore \vec{PQ} &= [-3(1) - 6 - 3(-1)]\hat{i} + [2(1) + (-1) - 16]\hat{j} + [4(1) - (-1) - 2]\hat{k} \\ &= -6\hat{i} - 15\hat{j} + 3\hat{k} \end{aligned}$$

**Q. 27** Show that the straight lines whose direction cosines are given by  $2l + 2m - n = 0$  and  $mn + nl + lm = 0$  are at right angles.

**Sol.** We have,  $2l + 2m - n = 0$  ... (i)  
and  $mn + nl + lm = 0$  ... (ii)

Eliminating  $m$  from the both equations, we get

$$m = \frac{n - 2l}{2} \quad \text{[from Eq. (i)]}$$

$$\begin{aligned} \Rightarrow & \left(\frac{n - 2l}{2}\right)n + nl + l\left(\frac{n - 2l}{2}\right) = 0 \\ \Rightarrow & \frac{n^2 - 2nl + 2nl + nl - 2l^2}{2} = 0 \\ \Rightarrow & n^2 + nl - 2l^2 = 0 \\ \Rightarrow & n^2 + 2nl - nl - 2l^2 = 0 \\ \Rightarrow & (n + 2l)(n - l) = 0 \\ \Rightarrow & n = -2l \text{ and } n = l \\ \therefore & m = \frac{-2l - 2l}{2}, m = \frac{l - 2l}{2} \\ \Rightarrow & m = -2l, m = \frac{-l}{2} \end{aligned}$$

Thus, the direction ratios of two lines are proportional to  $l, -2l, -2$  and  $l, \frac{-l}{2}, l$ .

$$\begin{aligned} \Rightarrow & 1, -2, -2 \text{ and } 1, \frac{-1}{2}, 1 \\ \Rightarrow & 1, -2, -2 \text{ and } 2, -1, 2 \end{aligned}$$

Also, the vectors parallel to these lines are  $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$  and  $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ , respectively.

$$\begin{aligned} \therefore \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{(\hat{i} - 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} - \hat{j} + 2\hat{k})}{3 \cdot 3} \\ &= \frac{2 + 2 - 4}{9} = 0 \end{aligned}$$

$$\therefore \theta = \frac{\pi}{2} \quad \left[ \because \cos \frac{\pi}{2} = 0 \right]$$

**Q. 28** If  $l_1, m_1, n_1, l_2, m_2, n_2$  and  $l_3, m_3, n_3$  are the direction cosines of three mutually perpendicular lines, then prove that the line whose direction cosines are proportional to  $l_1 + l_2 + l_3, m_1 + m_2 + m_3$  and  $n_1 + n_2 + n_3$  makes equal angles with them.

**Sol.** Let  $\vec{a} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$   
 $\vec{b} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$   
 $\vec{c} = l_3\hat{i} + m_3\hat{j} + n_3\hat{k}$   
 $\vec{d} = (l_1 + l_2 + l_3)\hat{i} + (m_1 + m_2 + m_3)\hat{j} + (n_1 + n_2 + n_3)\hat{k}$

Also, let  $\alpha, \beta$  and  $\gamma$  are the angles between  $\vec{a}$  and  $\vec{d}, \vec{b}$  and  $\vec{d}, \vec{c}$  and  $\vec{d}$ .

$$\begin{aligned} \therefore \cos \alpha &= \frac{l_1(l_1 + l_2 + l_3) + m_1(m_1 + m_2 + m_3) + n_1(n_1 + n_2 + n_3)}{l_1^2 + l_1 l_2 + l_1 l_3 + m_1^2 + m_1 m_2 + m_1 m_3 + n_1^2 + n_1 n_2 + n_1 n_3} \end{aligned}$$

$$= (l_1^2 + m_1^2 + n_1^2) + (l_1 l_2 + l_1 l_3 + m_1 m_2 + m_1 m_3 + n_1 n_2 + n_1 n_3)$$

$$= 1 + 0 = 1$$

$$[\because l_1^2 + m_1^2 + n_1^2 = 1 \text{ and } l_1 \perp l_2, l_1 \perp l_3, m_1 \perp m_2, m_1 \perp m_3, n_1 \perp n_2, n_1 \perp n_3]$$

$$\text{Similarly, } \cos \beta = l_2 (l_1 + l_2 + l_3) + m_2 (m_1 + m_2 + m_3) + n_2 (n_1 + n_2 + n_3)$$

$$= 1 + 0 \text{ and } \cos \gamma = 1 + 0$$

$$\Rightarrow \cos \alpha = \cos \beta = \cos \gamma$$

$$\Rightarrow \alpha = \beta = \gamma$$

So, the line whose direction cosines are proportional to  $l_1 + l_2 + l_3, m_1 + m_2 + m_3, n_1 + n_2 + n_3$  makes equal angles with the three mutually perpendicular lines whose direction cosines are  $l_1, m_1, n_1, l_2, m_2, n_2$  and  $l_3, m_3, n_3$  respectively.

## Objective Type Questions

**Q. 29** Distance of the point  $(\alpha, \beta, \gamma)$  from Y-axis is

(a)  $\beta$

(b)  $|\beta|$

(c)  $|\beta| + |\gamma|$

(d)  $\sqrt{\alpha^2 + \gamma^2}$

**Sol. (d)** Required distance =  $\sqrt{(\alpha - 0)^2 + (\beta - \beta)^2 + (\gamma - 0)^2} = \sqrt{\alpha^2 + \gamma^2}$

**Q. 30** If the direction cosines of a line are  $k, k$  and  $k$ , then

(a)  $k > 0$

(b)  $0 < k < 1$

(c)  $k = 1$

(d)  $k = \frac{1}{\sqrt{3}}$  or  $-\frac{1}{\sqrt{3}}$

**Sol. (d)** Since, direction cosines of a line are  $k, k$  and  $k$ .

$$\therefore l = k, m = k \text{ and } n = k$$

$$\text{We know that, } l^2 + m^2 + n^2 = 1$$

$$\Rightarrow k^2 + k^2 + k^2 = 1$$

$$\Rightarrow k^2 = \frac{1}{3}$$

$$\therefore k = \pm \frac{1}{\sqrt{3}}$$

**Q. 31** The distance of the plane  $\vec{r} \left( \frac{2}{7} \hat{i} + \frac{3}{7} \hat{j} - \frac{6}{7} \hat{k} \right) = 1$  from the origin is

(a) 1

(b) 7

(c)  $\frac{1}{7}$

(d) None of these

**Sol. (a)** The distance of the plane  $\vec{r} \left( \frac{2}{7} \hat{i} + \frac{3}{7} \hat{j} - \frac{6}{7} \hat{k} \right) = 1$  from the origin is 1.

[since,  $\vec{r} \cdot \vec{n} = d$  is the form of above equation, where  $d$  represents the distance of plane from the origin i.e.,  $d = 1$ ]

**Q. 32** The sine of the angle between the straight line  $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

and the plane  $2x - 2y + z = 5$  is

- (a)  $\frac{10}{6\sqrt{5}}$       (b)  $\frac{4}{5\sqrt{2}}$       (c)  $\frac{2\sqrt{3}}{5}$       (d)  $\frac{\sqrt{2}}{10}$

**Sol. (d)** We have, the equation of line as

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

Now, the line passes through point (2, 3, 4) and having direction ratios (3, 4, 5).

Since, the line passes through point (2, 3, 4) and parallel to the vector  $(3\hat{i} + 4\hat{j} + 5\hat{k})$ .

$$\therefore \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Also, the cartesian form of the given plane is  $2x - 2y + z = 5$ .

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k}) = 5$$

$$\therefore \vec{n} = (2\hat{i} - 2\hat{j} + \hat{k})$$

$$\text{We know that, } \sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| \cdot |\vec{n}|} = \frac{|(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})|}{\sqrt{3^2 + 4^2 + 5^2} \cdot \sqrt{4 + 4 + 1}}$$

$$= \frac{|6 - 8 + 5|}{\sqrt{50} \cdot 3} = \frac{3}{15\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$\sin \theta = \frac{\sqrt{2}}{10}$$

**Q. 33** The reflection of the point  $(\alpha, \beta, \gamma)$  in the  $XY$ -plane is

- (a)  $(\alpha, \beta, 0)$       (b)  $(0, 0, \gamma)$       (c)  $(-\alpha, -\beta, \gamma)$       (d)  $(\alpha, \beta, -\gamma)$

**Sol. (d)** In  $XY$ -plane, the reflection of the point  $(\alpha, \beta, \gamma)$  is  $(\alpha, \beta, -\gamma)$ .

**Q. 34** The area of the quadrilateral  $ABCD$  where  $A(0, 4, 1)$ ,  $B(2, 3, -1)$ ,  $C(4, 5, 0)$ , and  $D(2, 6, 2)$  is equal to

- (a) 9 sq units      (b) 18 sq units  
(c) 27 sq units      (d) 81 sq units

**Sol. (a)** We have,  $\vec{AB} = (2-0)\hat{i} + (3-4)\hat{j} + (-1-1)\hat{k} = 2\hat{i} - \hat{j} - 2\hat{k}$

$$\vec{BC} = (4-2)\hat{i} + (5-3)\hat{j} + (0+1)\hat{k} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{CD} = (2-4)\hat{i} + (6-5)\hat{j} + (2-0)\hat{k} = -2\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{DA} = (0-2)\hat{i} + (4-6)\hat{j} + (1-2)\hat{k} = -2\hat{i} - 2\hat{j} - \hat{k}$$

$$\begin{aligned} \therefore \text{Area of quadrilateral } ABCD &= |\vec{AB} \times \vec{BC}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{vmatrix} \\ &= |\hat{i}(-1+4) - \hat{j}(2+4) + \hat{k}(4+2)| \\ &= |3\hat{i} - 6\hat{j} + 6\hat{k}| \\ &= \sqrt{9 + 36 + 36} = 9 \text{ sq units} \end{aligned}$$

**Q. 35** The locus represented by  $xy + yz = 0$  is

- (a) a pair of perpendicular lines
- (b) a pair of parallel lines
- (c) a pair of parallel planes
- (d) a pair of perpendicular planes

**Sol. (d)** We have,  $xy + yz = 0$   
 $\Rightarrow xy = -yz$   
 So, a pair of perpendicular planes.

**Q. 36** If the plane  $2x - 3y + 6z - 11 = 0$  makes an angle  $\sin^{-1} \alpha$  with  $X$ -axis, then the value of  $\alpha$  is

- (a)  $\frac{\sqrt{3}}{2}$
- (b)  $\frac{\sqrt{2}}{3}$
- (c)  $\frac{2}{7}$
- (d)  $\frac{3}{7}$

**Sol. (c)** Since,  $2x - 3y + 6z - 11 = 0$  makes an angle  $\sin^{-1} \alpha$  with  $X$ -axis.

$$\vec{b} = (2\hat{i} + 0\hat{j} + 6\hat{k}) \text{ and } \vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

We know that, 
$$\sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| \cdot |\vec{n}|}$$

$$= \frac{|(2\hat{i}) \cdot (2\hat{i} - 3\hat{j} + 6\hat{k})|}{\sqrt{4} \sqrt{4 + 9 + 36}} = \frac{2}{7}$$

## Fillers

**Q. 37** If a plane passes through the points  $(2, 0, 0)$ ,  $(0, 3, 0)$  and  $(0, 0, 4)$  the equation of plane is .....

**Sol.** We know that, equation of a the plane that cut the coordinate axes at  $(a, 0, 0)$ ,  $(0, b, 0)$  and  $(0, 0, c)$  is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ .

Hence, the equation of plane passes through the points  $(2, 0, 0)$ ,  $(0, 3, 0)$  and  $(0, 0, 4)$  is  $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ .

**Q. 38** The direction cosines of the vector  $(2\hat{i} + 2\hat{j} - \hat{k})$  are .....

**Sol.** Direction cosines of  $(2\hat{i} + 2\hat{j} - \hat{k})$  are  $\frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$  i.e.,  $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$ .

**Q. 39** The vector equation of the line  $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$  is .....

**Sol.** We have,  $\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$  and  $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$

So, the vector equation will be

$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) - (5\hat{i} - 4\hat{j} + 6\hat{k}) = \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

$$\Rightarrow (x-5)\hat{i} + (y+4)\hat{j} + (z-6)\hat{k} = \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

**Q. 40** The vector equation of the line through the points (3, 4, -7) and (1, -1, 6) is .....

**Sol.** We know that, vector equation of a line passes through two points is represented by  $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

Here,  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ,  $\vec{a} = 3\hat{i} + 4\hat{j} - 7\hat{k}$

and  $\vec{b} = \hat{i} - \hat{j} + 6\hat{k}$

$$\Rightarrow (\vec{b} - \vec{a}) = -2\hat{i} - 5\hat{j} + 13\hat{k}$$

So, the required equation is

$$x\hat{i} + y\hat{j} + z\hat{k} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$$

$$\Rightarrow (x-3)\hat{i} + (y-4)\hat{j} + (z+7)\hat{k} = \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$$

**Q. 41** The cartesian equation of the plane  $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$  is .....

**Sol.** We have,  $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$$

$$\Rightarrow x + y - z = 2$$

which is the required form

## True/False

**Q. 42** The unit vector normal to the plane  $x + 2y + 3z - 6 = 0$  is

$$\frac{1}{\sqrt{14}} \hat{i} + \frac{2}{\sqrt{14}} \hat{j} + \frac{3}{\sqrt{14}} \hat{k} .$$

**Sol.** True

We have,  $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\therefore \hat{n} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$$

**Q. 43** The intercepts made by the plane  $2x - 3y + 5z + 4 = 0$  on the coordinate axis are  $-2$ ,  $\frac{4}{3}$  and  $-\frac{4}{5}$ .

**Sol.** *True*

$$\begin{aligned} \text{We have, } & 2x - 3y + 5z + 4 = 0 \\ \Rightarrow & 2x - 3y + 5z = -4 \\ \Rightarrow & \frac{2x}{-4} - \frac{3y}{-4} + \frac{5z}{-4} = 1 \\ \Rightarrow & \frac{x}{-2} + \frac{y}{\frac{4}{3}} - \frac{z}{\frac{4}{5}} = 1 \\ \Rightarrow & \frac{x}{-2} + \frac{y}{\frac{4}{3}} + \frac{z}{\left(-\frac{4}{5}\right)} = 1 \end{aligned}$$

So, the intercepts are  $-2$ ,  $\frac{4}{3}$  and  $-\frac{4}{5}$ .

**Q. 44** The angle between the line  $\vec{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$  and the plane  $\vec{r} \cdot (3\hat{i} - 4\hat{j} - \hat{k}) + 5 = 0$  is  $\sin^{-1}\left(\frac{5}{2\sqrt{91}}\right)$ .

**Sol.** *False*

We have,  $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{n} = 3\hat{i} - 4\hat{j} - \hat{k}$

Let  $\theta$  is the angle between line and plane.

$$\begin{aligned} \text{Then, } \sin \theta &= \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| \cdot |\vec{n}|} = \frac{|(2\hat{i} - \hat{j} + \hat{k}) \cdot (3\hat{i} - 4\hat{j} - \hat{k})|}{\sqrt{6} \cdot \sqrt{26}} \\ &= \frac{|6 + 4 - 1|}{\sqrt{156}} = \frac{9}{2\sqrt{39}} \\ \therefore \theta &= \sin^{-1} \frac{9}{2\sqrt{39}} \end{aligned}$$

**Q. 45** The angle between the planes  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = 1$  and  $\vec{r} \cdot (\hat{i} - \hat{j}) = 4$  is  $\cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$ .

**Sol.** *False*

We know that, the angle between two planes is given by  $\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| |\vec{n}_2|}$

Here,  $\vec{n}_1 = (2\hat{i} - 3\hat{j} + \hat{k})$  and  $\vec{n}_2 = (\hat{i} - \hat{j})$

$$\begin{aligned} \therefore \cos \theta &= \frac{|(2\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j})|}{\sqrt{4 + 9 + 1} \cdot \sqrt{1 + 1}} \\ \Rightarrow \cos \theta &= \frac{|2 + 3|}{\sqrt{14} \cdot \sqrt{2}} = \frac{5}{2\sqrt{7}} \\ \therefore \theta &= \cos^{-1}\left(\frac{5}{2\sqrt{7}}\right) \end{aligned}$$

**Q. 46** The line  $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$  lies in the plane  
 $\vec{r} \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0$ .

**Sol.** *False*

We have,  $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) = \hat{i}(2 + \lambda) + \hat{j}(-3 - \lambda) + \hat{k}(-1 + 2\lambda)$$

Since,  $x = (2 + \lambda)$ ,  $y = (-3 - \lambda)$  and  $z = (-1 + 2\lambda)$  are coordinates of general point which should satisfy the equation of the given plane.

$$\therefore [(2 + \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (2\lambda - 1)\hat{k}] \cdot [\hat{i} + \hat{j} - \hat{k}] = 2$$

$$\Rightarrow (2 + \lambda) - 3 - \lambda - 2\lambda + 1 = 2$$

$$\Rightarrow -2\lambda = 2$$

$$\Rightarrow \lambda = -1$$

$$\therefore \vec{r} = (2 - 1)\hat{i} + (-3 + 1)\hat{j} + (-2 - 1)\hat{k} \\ = \hat{i} - 2\hat{j} - 3\hat{k}$$

Again, from the equation of the plane

$$\vec{r} \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0$$

$$\Rightarrow (\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0$$

$$\Rightarrow (3 - 2 + 3) + 2 = 0$$

$$\Rightarrow 6 \neq 0$$

which is not true.

So, the line  $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$  does not lie in a plane.

**Q. 47** The vector equation of the line  $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$  is

$$\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

**Sol.** *True*

We have,  $x = 5, y = -4, z = 6$

and  $a = 3, b = 7, c = 2$

$$\therefore \vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

**Q. 48** The equation of a line, which is parallel to  $2\hat{i} + \hat{j} + 3\hat{k}$  and which passes through the point  $(5, -2, 4)$  is  $\frac{x-5}{2} = \frac{y+2}{-1} = \frac{z-4}{3}$ .

**Sol.** *False*

Here,  $x_1 = 5, y_1 = -2, z_1 = 4$

and  $a = 2, b = 1, c = 3$

$$\Rightarrow \frac{x-5}{2} = \frac{y+2}{1} = \frac{z-4}{3}$$

**Q. 49** If the foot of perpendicular drawn from the origin to a plane is  $(5, -3, -2)$ , then the equation of plane is  $\vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 38$ .

**Sol.** True

Since, the required plane passes through the point  $P(5, -3, -2)$  and is perpendicular to  $\vec{OP}$ .

$$\therefore \vec{a} = 5\hat{i} - 3\hat{j} - 2\hat{k}$$

$$\text{and } \vec{n} = \vec{OP} = 5\hat{i} - 3\hat{j} - 2\hat{k}$$

Now, the equation of the plane is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = (5\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (5\hat{i} - 3\hat{j} - 2\hat{k})$$

$$\Rightarrow \vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 25 + 9 + 4$$

$$\Rightarrow \vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 38$$

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