CHAPTER 1 : RELATIONS AND FUNCTIONS

Ordered Pair:

A pair of elements listed in a specific order separated by comma and enclosing the pair in parenthesis is called an ordered pair.

For example, (a, b) is an ordered pair with a as the first element and b as the second element.

Cartesian Product or Cross Product of sets A and B:

The set of ordered pairs (a, b) such that a∈ A , b∈ B is called the cartesian product of A to B. The set of ordered pairs (b, a) such that a∈ A, b∈ B is called the cartesian product of B to A.

It is written as:

A x B = {(a, b): a∈ A , b∈ B }

A x B = {(b, a): a∈ A , b∈ B }

Number of elements in A x B:

If n(A) = p and n(B) = q then n(A x B) = pq

Relation from Set A to set B:

Let A and B be two non-empty sets, then a relation R from set A to set B is a subset of cartesian product A x B.

Relation on a Set:

Let A be a non-empty set. Then, a relation from A to A is called a relation on set A.

Domain, Range and Codomain of Relation:

Let R be a relation from set A to set B, then the set of all the first elements of the ordered pairs in R is called the domain and the set of all the second elements of the ordered pairs in R is called the range of R, i.e., Domain of R = {a: (a, b) ∈ R} and Range of R = {a: (a, b) ∈ R}. The set B is called the codomain of relation R.

Empty Relation:

A relation from set A to set B is said to be empty if no element of A is related to any element of B, and is denoted by ∅. An empty relation is a subset of A x B.

Universal Relation:

A relation from set A to set B is said to be universal if each element of A is related to every element of B. Universal relation U = A x B.

NOTE: Empty relation and Universal relation are said to be trivial relations.

Identity Relation:

A relation R on the set A is an identity relation if and only if R = {(a, a) for each a ∈ A}.

Types of Relations:

A relation on a non-empty set A is said to be

(i) Reflexive, if (a, a) ∈ R for all a ∈ A.

(ii) Symmetric, if (a, b) ∈ R implies (b, a) ∈ R, for all a, b∈ A.

(iii) Transitive, if (a, b) ∈ R and (b, c) ∈ R implies (a, c) ∈ R, for all a, b, c ∈ A.

Equivalence Relation:

A relation R on a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Equivalence Classes:

Let R be an equivalence relation on a set A. The set of all those elements of A, which are related to a, where a ∈ A, is said equivalence class determined by a and is denoted by [a].

Given an arbitrary relation R on an arbitrary set A, R divides A into mutually disjoint subsets A_i , called partitions or subdivisions of A, satisfying the conditions:

(i) All elements of A_i are related to each other, for each i.
(ii) No element of A_j is related to any element of A_i , for all $i \neq j$.
(iii) $A_i \cap A_j = \emptyset$, for all i, j.

Function (Mapping):

For any two non-empty sets A and B, a function f from A to B is a rule or mapping which associates each element of set A to a unique element in set B. It is denoted by f : A→B.

Domain, Codomain and Range of a Function:

Let f : A→B then elements of set A are called domain of f and the elements of set B

are called codomain of f. The set of all the images obtained in set B corresponding to each element belongs to A under f is called range.

Types of Functions:

One-one (or injective function): A function f : A→B is called a one-one or injective function, if distinct elements of A have distinct images in B.

i.e., for every $x_1, x_2 \in A$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$

Many-one function:

A function $f : A \to B$ is called a many-one function, if there exist at least two distinct elements in A, whose images are same in B.

Onto (or surjective function):

A function $f : A \to B$ is said to be onto or surjective function, if every element of B is the image of some elements of A under f.

i.e., for every $y \in B$ there exists an element $x \in A$ such that $f(x) = y$

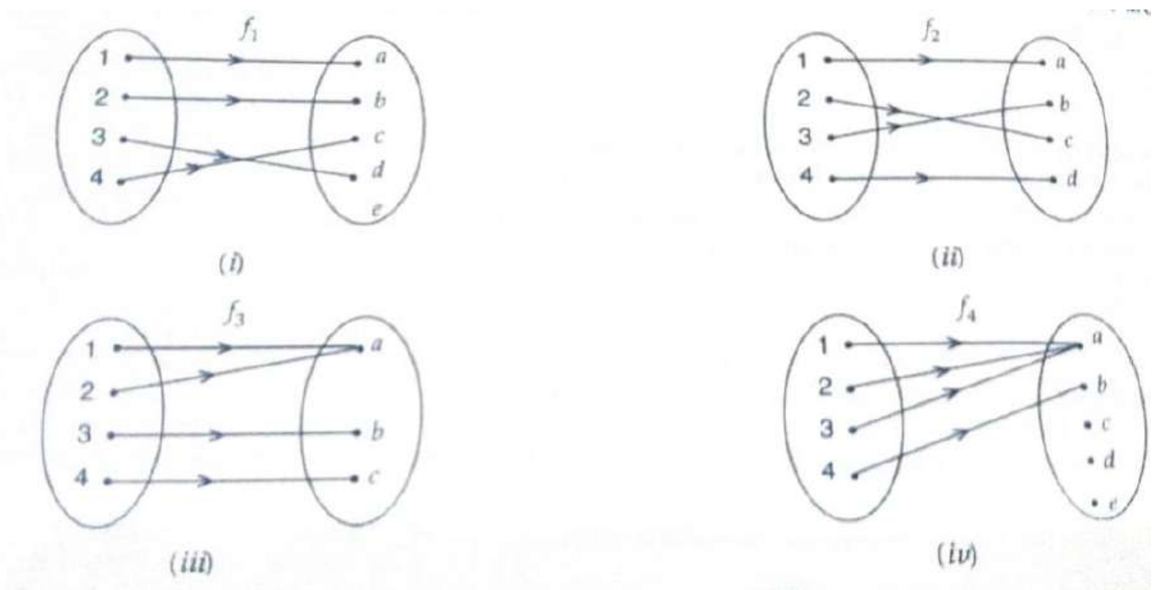
Into function:

A function $f : A \to B$ is an into function, if there exists an element in B which have no pre-image in A.

One-one and onto (or bijective function):

A function $f : A \to B$ is said to be one-one and onto (or bijective function), if f is both one-one and onto.

In the figures, the functions f_1 and f_2 are one-one and the functions f_3 and f_4 are many-one. The functions f_2 and f_3 are onto and the functions f_1 and f_4 are into.



Number of Relations from set A to set B:

If $n(A) = p$ and $n(B) = q$ then Number of Relations from A to B = 2^{pq}

Number of Reflexive Relations on a Set:

The number of reflexive relations on a set with the 'n' number of elements is given by $\mathbf{N = 2^{n(n-1)}}$

Number of Symmetric Relations on a Set:

Number of Symmetric relations for a set having 'n' number of elements is given as $\mathbf{N = 2^{n(n+1)/2}}$

Number of Functions:

If a set A has m elements and set B has n elements, then

The total number of functions from A to B = n^m

Number of Surjective Functions (Onto Functions):

If a set A has m elements and set B has n elements, then

The number of onto functions from A to B = $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - {}^nC_3(n-3)^m + \ldots - {}^nC_{n-1}$

Number of Injective Functions (One to One Functions):

If set A has m elements and set B has n elements, n≥m, then the number of injective functions or one to one function is given by n!/(n - m)!.

Number of Bijective functions:

If there is bijection between two sets A and B, then both sets will have the same number of elements. If n(A) = n(B) = m, then number of bijective functions = m!.

CHAPTER 2 : INVERSE TRIGONOMETRIC FUNCTIONS

CBSE SYLLABUS: - Definition, range, domain, principal value branch. Graphs of inverse trigonometric function.

Gist of topic: The domain of sine function is the set of all real numbers and range is the closed interval [–1, 1].

If we restrict its domain to $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$, then it becomes one-one and onto with range [– 1, 1].

Actually, sine function restricted to any of the intervals $\left[\frac{-3\pi}{2},\frac{-\pi}{2}\right],\left[\frac{-\pi}{2},\frac{\pi}{2}\right], or \left[\frac{\pi}{2}.\frac{3\pi}{2}\right]$........... etc., is one-one and its range is [–1, 1].

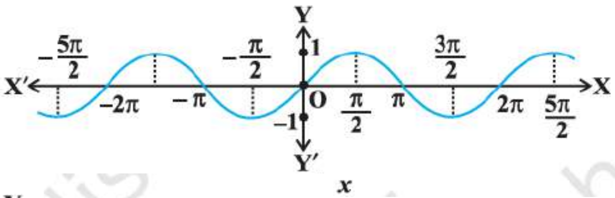
therefore, define the inverse of sine function in each of these intervals.

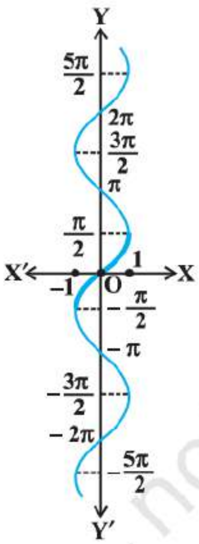
We denote the inverse of sine function by $\sin^{-1}$ (arc sine function).

Thus, $\sin^{-1}$ is a function whose domain is [– 1, 1] and range could be any of the intervals $\left[\frac{-3\pi}{2},\frac{-\pi}{2}\right],\left[\frac{-\pi}{2},\frac{\pi}{2}\right], or \left[\frac{\pi}{2}.\frac{3\pi}{2}\right]$..........., and so on. Corresponding to each such interval,

we get a branch of the function $\sin^{-1}$. The branch with range , $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ is called the principal value branch, whereas other intervals as range give different branches of $\sin^{-1}$. When we refer to the function $\sin^{-1}$, we take it as the function whose domain is [–1, 1] and range is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$. We write

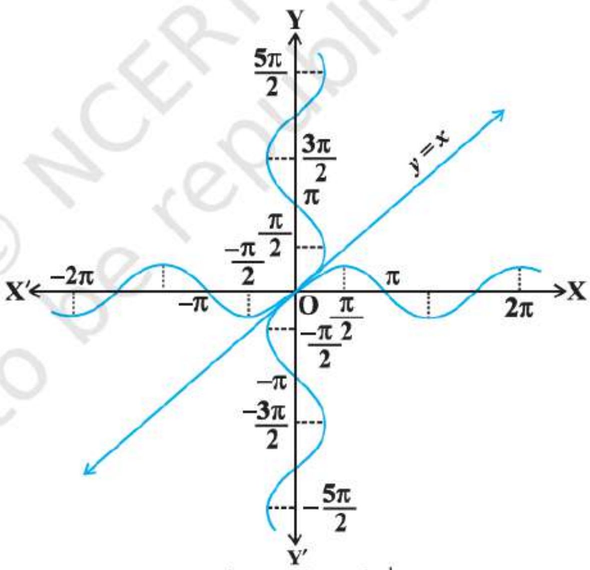
$\sin^{-1} : [-1, 1] \rightarrow \left[\frac{-\pi}{2},\frac{\pi}{2}\right]$





$y = \sin^{-1}x$

Fig 2.1 (ii)



$y = \sin x$ and $y = \sin^{-1}x$

Fig 2.1 (iii)

The cosine function is a function whose domain is the set of all real numbers and range is the set [–1, 1].

If we restrict the domain of cosine function to [0, π], then it becomes one-one and onto with range [–1, 1]. Actually, cosine function restricted to any of the intervals [– π, 0], [0,π], [π, 2π] etc., is bijective with range as [–1, 1].

We can, therefore, define the inverse of cosine function in each of these intervals. We denote the inverse of the cosine function by $\cos^{-1}$ (arc cosine function).

Thus, $\cos^{-1}$ is a function whose domain is [–1, 1] and range could be any of the intervals [–π, 0], [0, π], [π, 2π] etc. Corresponding to each such interval,

we get a branch of the function $\cos^{-1}$. The branch with range [0, π] is called the principal value branch of the function $\cos^{-1}$.

We write $\cos^{-1} : [-1, 1] \rightarrow [0, \pi]$

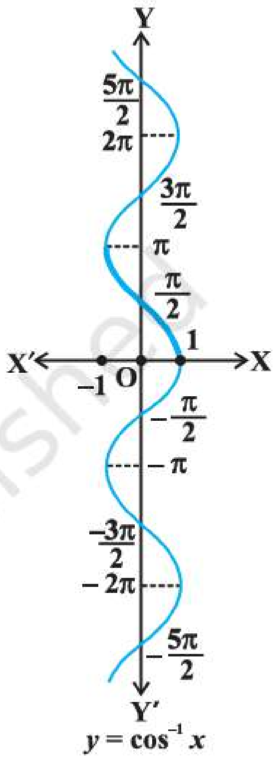


Fig 2.2 (ii)

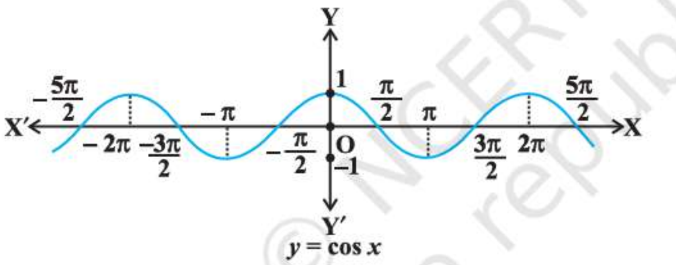


Fig 2.2 (i)

Trigonometric Functions	Domain	Range
$y = \sin x$	$(-\infty, \infty)$	$[-1, 1]$
$y = \cos x$	$(-\infty, \infty)$	$[-1, 1]$
$y = \tan x$	$x \in \mathbb{R} - \{\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\}$	$(-\infty, \infty)$
$y = \cot x$	$x \in \mathbb{R} - \{n\pi \mid n \in \mathbb{Z}\}$	$(-\infty, \infty)$
$y = \sec x$	$x \in \mathbb{R} - \{\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\}$	$(-\infty, -1] \cup [1, \infty)$
$y = \operatorname{cosec} x$	$x \in \mathbb{R} - \{n\pi \mid n \in \mathbb{Z}\}$	$(-\infty, -1] \cup [1, \infty)$

Inverse Trigonometric Functions	Domain	Range
$y = \sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	$(-\infty, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$y = \cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$
$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
$y = \operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$

IMPORTANT TRIGONOMETRIC RESULTS & SUBSTITUTIONS

** **Formulae for t-ratios of Allied Angles :**

All T-ratio changes in $\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$ while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$ $\quad$ $\sin\left(\frac{3\pi}{2} \pm \theta\right) = = \cos\theta$

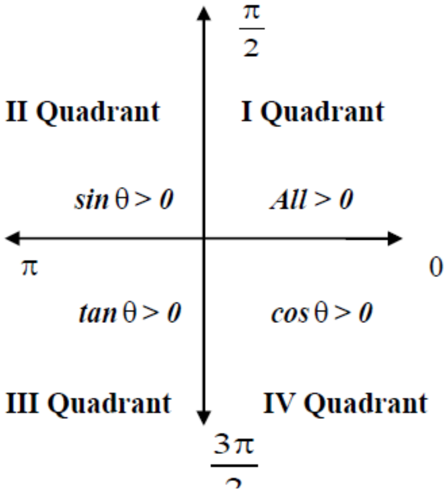
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$ $\quad$ $\cos\left(\frac{3\pi}{2} \pm \theta\right) = \pm\sin\theta$

$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$ $\quad$ $\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$

$\sin(\pi \pm \theta) = \mp\sin\theta$ $\quad$ $\sin(2\pi \pm \theta) = \pm\sin\theta$

$\cos(\pi \pm \theta) = = \cos\theta$ $\quad$ $\cos(2\pi \pm \theta) = \cos\theta$

$\tan(\pi \pm \theta) = \pm\tan\theta$ $\quad$ $\tan(2\pi \pm \theta) = \pm\tan\theta$



** **Sum and Difference formulae :**

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, $\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$,

$\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$, $\cot(A + B) = \frac{\cot A.\cot B - 1}{\cot B + \cot A}$ $\quad$ $\cot(A - B) = \frac{\cot A.\cot B + 1}{\cot B - \cot A}$

$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

** **Formulae for t-ratios of múltiple and sub-múltiple angles :**

$\sin 2A = 2\sin A \cos A = \frac{2\tan A}{1 + \tan^2 A}$.

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

Properties of Inverse trigonometric functions:

1. $\sin(\sin^{-1} x) = x, x \in [-1, 1]$ *and* $\sin^{-1}(\sin x) = x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
2. (i) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$, $x \geq 1$ *or* $x \leq -1$
   (ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$, $x \geq 1$ *or* $x \leq -1$
   (iii) $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$, $x > 0$
3. (i) $\sin^{-1}(-x) = -\sin^{-1} x, x \in [-1, 1]$
   (ii) $\tan^{-1}(-x) = -\tan^{-1} x$, $x \in R$
   (iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$ $|x| \geq 1$
4. (i) $\cos^{-1}(-x) = \pi - \cos^{-1} x, x \in [-1, 1]$
   (ii) $\sec^{-1}(-x) = \pi - \sec^{-1} x$ $|x| \geq 1$
   (iii) $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$
5. (i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $x \in [-1, 1]$
   (ii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$, $x \in R$
   (iii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $|x| \geq 1$

(i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$; $xy < 1$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; $xy > -1$

(i) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$; $|x| \leq 1$

(ii) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $x \geq 0$

(iii) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$; $-1 < x < 1$

(iv) $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$; $\frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

(v) $2\cos^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$; $\frac{-1}{\sqrt{2}} \leq x \leq 1$

(or)

$2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$; $0 \leq x \leq 1$

(i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

(ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

(iii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

(iv) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$

CHAPTER : MATRICES & DETERMINANTS

MATRIX: If mn elements can be arranged in the form of m row and n column in a rectangular array then this arrangement is called a matrix.

Order of a matrix: A matrix having m row and n column is called a matrix of $m \times n$ order.

Types of Matrices

(i) Column matrix: A matrix is said to be a column matrix if it has only one column e.g $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

(ii) Row matrix: A matrix is said to be a row matrix if it has only one row e.g..$[1 \quad 2 \quad 3]$

(iii) Square matrix :A matrix in which the number of rows are equal to the number of columns, is said to be a square matrix. Thus an m × n matrix is said to be a square matrix if m = n and is known as a square matrix of

order 'n' e.g.$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

(iv) Diagonal matrix: A square matrix B = $[b_{ij}]_{m \times m}$ is said to be a diagonal matrix if all its non diagonal elements are zero, that is a matrix B = [bij] m × m is said to be a diagonal matrix if bij = 0, when i ≠ j

$$\text{e.g.} \begin{bmatrix} a & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{bmatrix}$$

(v) Scalar matrix: A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix B = [bij] n × n is said to be a scalar matrix if bij = 0, when i ≠ j bij = k, when i = j, for some constant k.

$$\text{e.g.} \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

(vi) Identity matrix: A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an identity matrix.

$$\text{e.g.} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(vii) Zero matrix: A matrix is said to be zero matrix or null matrix if all its elements are zero. For example, [0], $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Addition and subtraction of matrices: Two matrices A and B can be added or subtracted if they are of the same order i.e. if A and B are two matrices of order $m \times n$ then A $\pm$ B is also a matrix of order m×n.

Multiplication of matrices: The product of two matrices A and B can be defined if the number of rows of B is equal to the number of columns of A i.e. if A be an $m \times n$ matrix and B be an $n \times p$ matrix then the product of matrices A and B is another matrix of order $m \times p$.

Transpose of a Matrix: If A = [a_{ij}] be an $m \times n$ matrix, then the matrix obtained by interchanging. the rows and columns of A is called the *transpose* of A. Transpose of the matrix A is denoted by A' or A^T.

Properties of transpose of the Matrices: For any matrices A and B of suitable orders, we have

(i) $\left(A^T\right)^T = A$ (ii) $(KA)^T = KA^T$ (iii) $(A+B)^T = A^T + B^T$ (iv) $(AB)^T = B^T A^T$

Symmetric Matrix: A square matrix M is said to be symmetric if $A^T = A$

e.g. $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$, $\begin{bmatrix} x & y & z \\ y & u & v \\ z & v & w \end{bmatrix}$

Note: **there will be symmetry about the principal diagonal in Symmetric Matrix.**

Skew symmetric Matrix: A square matrix M is said to be skew symmetric if $A^T = -A$

e.g. $\begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix}$

Note: **All the principal diagonal element of a skew symmetric Matrix are zero.**

Determinant: For every Square Matrix we can associate a number which is called the Determinant of the square Matrix.

Determinant of a matrix of order one

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a.

Determinant of a matrix of order two

Let A= $\begin{bmatrix} a & b \\ x & y \end{bmatrix}$ be a Square Matrix of order2$\times$ 2 then the determinant of A is denoted by $|A|$ and defined by

$|A| = \begin{vmatrix} a & b \\ x & y \end{vmatrix}$ = ay - bx

Determinant of a matrix of order 3$\times$ 3: Let us consider the determinant of a square matrix of order 3$\times$ 3,

$|A| = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Expansion along first row $|A| = a(qz - yr) - b(pz - xr) + c(py - qx)$

We can expand the determinant with respect to any row or any column.

Area of a Triangle: area of a triangle whose vertices are (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3), can be written in the form of a determinant as

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Remarks: (i) Since area is a positive quantity, we always take the absolute value of the determinant

(ii) If area is given, use both positive and negative values of the determinant for calculation. (iii) The area of the triangle formed by three collinear points is zero.

Important properties of Determinant of Adjoint

***Minor** of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

* Minor of an element of a determinant of order $n (n \geq 2)$ is a determinant of order $n - 1$.

* **Cofactor** of an element a_{ij}, denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is minor of a_{ij}.

* If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

***Adjoint of matrix** :

If $A = [a_{ij}]$ be a square matrix then transpose of a matrix $[A_{ij}]$, where A_{ij} is the cofactor of A_{ij} element of matrix A, is called the adjoint of A.

Adjoint of A $= \text{Adj. } A = [A_{ij}]^T$.

If A be a square matrix of order n, then $|adjA| = |A|^{n-1}$.

If A be a square matrix of order n, then $|A\ adjA| = |A|^n$.

For any square matrix A, $A(\text{Adj.}A) = (\text{Adj.}A)A = |A|I$

If A be a square matrix of order n, then $|adj(adjA)| = |A|^{(n-1)^2}$

If A and B are square matrices of the same order, then adj(AB)=adjB.adj A

Important properties of Inverse of Matrix

A square matrix A is said to be non-singular if $|A| \neq 0$

If A and B are nonsingular matrices of the same order, then AB and BA are also nonsingular matrices of the same order

A square matrix A is invertible if and only if A is nonsingular matrix ($|A| \neq 0$)

$A^{-1} = \frac{1}{|A|}(adjA)$

If A is an invertible matrix, then $|A| \neq 0$ and $(A^{-1})^T = (A^T)^{-1}$

If A is a non-singular matrix $|(kA)^{-1}| = \frac{1}{k\ |A|}$

$(AB)^{-1} = B^{-1}A^{-1}$

***System of Linear Equations** :

$a_1x + b_1y + c_1z = d_1, \quad a_2x + b_2y + c_2z = d_2, \quad a_3x + b_3y + c_3z = d_3.$

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow AX = B \Rightarrow X = A^{-1}B \quad ; \quad \{|A| \neq 0\}.$$

***Criteria of Consistency**.

(i) If $|A| \neq 0$, then the system of equations is said to be consistent & has a unique solution.

(ii) If $|A| = 0$ and (adj. A)B = 0, then the system of equations is consistent and has infinitely many solutions.

(iii) If $|A| = 0$ and (adj. A)B $\neq 0$, then the system of equations is inconsistent and has no solution.

COUNTINUITY OF A FUNCTION

LEARNING OBJECTIVES/OUTCOMES

Understanding the concept of Continuity and differentiability and addressing the problems based on continuity
and derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of
implicit functions.
Learning the concept of exponential and logarithmic functions.
Skills to solve derivatives of logarithmic and exponential function. Logarithmic differentiation, derivative of
functions expressed in parametric forms. Second order derivatives

Knowledge of functions :

(i) Polynomial functions: e.g. f(x) = $x^2 + 2x + 5$
(ii) Modulus function : f(x) = $|x|$
(iii) Greatest Integer Function : f(x) = [x]
(iv) Signum function : The signum function, denoted sgn, is defined as follows: $sgn(x)$ = { 1, $x > 0$ −1, $x < 0$ 0, $x = 0$
(v) Trigonometric functions : sin x, cos x etc.
(vi) Inverse Trigonometric functions : sin- 1 x, cos- 1 x etc.
(vii) Logarithmic functions : f(x) = log x
(viii) Exponential functions : f(x) = e^x.

DEFINITAION OF COUNTINUITY

Continuity at a Point: A function f(x) is said to be continuous at a point x = a, if
Left hand limit of f(x) at (x = a) = Right hand limit of f(x) at (x = a) = Value of f(x) at (x = a)
i.e. if at x = a, LHL = RHL = f(a)
where, LHL = limx→a–f(x) and RHL = limx→a+f(x)
Note: To evaluate LHL of a function f(x) at (x = o), put x = a – h and to find RHL, put x = a + h.

Continuity in an Interval: A function y = f(x) is said to be continuous in an interval (a, b), where a < b if and only if f(x) is continuous at every point in that interval.

- **Every identity function is continuous.**
- **Every constant function is continuous.**
- **Every polynomial function is continuous.**
- **Every rational function is continuous.**
- **All trigonometric functions are continuous in their domain**..

Algebra of Continuous Functions

Suppose f and g are two real functions, continuous at real number c. Then,

- **f + g is continuous at x = c.**
- **f – g is continuous at x = c.**
- **f.g is continuous at x = c.**
- **cf is continuous, where c is any constant.**
- **(fog) is continuous at x = c, [provide g(c) ≠ 0]**

NOTE- Suppose f and g are two real valued functions such that (fog) is defined at c. If g is continuous at c and f is continuous at g (c), then (fog) is continuous at c.

- **If f is continuous, then |f| is also continuous.**

Standard Results of Limits

(i) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ (ii) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (iii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(iv) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ (v) $\lim_{x \to \infty} \frac{1}{x^p} = 0, p \in (0, \infty)$ (vi) $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

(vii) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$ (viii) $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$ (x) $\lim_{x \to 0} (1+x)^{1/x} = e$

(xi) $\lim_{x \to \infty} \frac{\sin x}{x} = 0$ (xii) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$

(xiii) $\lim_{x \to \infty} \sin x = \lim_{x \to \infty} \cos x =$ lies between -1 to 1.

Differentiability

1) The derivative of the function f at the point a in its domain is given by $f'(a) = \lim_{h \to 0} [f(a+h) - f(a)] / h$,
2) The function f is differentiable at the point a in its domain if f'(a) exists.
3) The function f is differentiable on the subset S of its domain if it differentiable at each point of S.

OR

Differentiability: A function f(x) is said to be differentiable at a point x = a, if
Left hand derivative at (x = a) = Right hand derivative at (x = a)
i.e. LHD at (x = a) = RHD (at x = a), where Right hand derivative, where

Right hand derivative, $Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Left hand derivative, $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

A function can fail to be differentiable at a point a if either $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ does not exist, or is infinite

Note :
(a) All continuous functions are differentiable. For instance, the closed-form function f(x) = |x| is continuous at every real number (including x = 0), but not differentiable at x = 0. (b) However, every differentiable function is continuous.

Note: Every differentiable function is continuous but every continuous function is not differentiable.

Differentiation: The process of finding a derivative of a function is called differentiation.

Rules of Differentiation:

Sum and Difference Rule: Let y = f(x) ± g(x).Then, by using sum and difference rule, it's derivative

is written as

$$\frac{dy}{dx} = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x).$$

Product Rule: Let y = f(x) g(x). Then, by using product rule, it's derivative is written as

$$\frac{dy}{dx} = \left[\frac{d}{dx}(f(x))\right] g(x) + \left[\frac{d}{dx}(g(x))\right] f(x).$$

Quotient Rule: Let y = f(x)g(x); g(x) ≠ 0, then by using quotient rule, it's derivative is written as

$$\frac{dy}{dx} = \frac{g(x) \times \frac{d}{dx}[f(x)] - f(x) \times \frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

Chain Rule: Let y = f(u) and u = f(x), then by using chain rule, we may write

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \text{ when } \frac{dy}{du} \text{ and } \frac{du}{dx} \text{ both exist.}$$

Rules of logarithmic function

$\log mn = \log m + \log n$

$\log\left(\frac{m}{n}\right) = \log m - \log n$

$\log(mn) = n \log mn$

Change of base rule, $\log_a b = \frac{\log_e b}{\log_e a}$

$\log e = 1, \log 1 = 0, e^{\log f(x)} = f(x)$

Differentiation of Functions in Parametric Form: A relation expressed between two variables x and y in the form x = f(t), y = g(t) is said to be parametric form with t as a parameter, when

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

(whenever dxdt≠0)

Note: dy/dx is expressed in terms of parameter only without directly involving the main variables x and y.

Second order Derivative: It is the derivative of the first order derivative.

$$\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Some Standard Derivatives

(i) $\frac{d}{dx}(\sin x) = \cos x$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(v) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

(vi) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(vii) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(viii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

(ix) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(x) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

(xi) $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

(xii) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

(xiii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(xiv) $\frac{d}{dx}(\text{constant}) = 0$

(xv) $\frac{d}{dx}(e^x) = e^x$

(xvi) $\frac{d}{dx}(\log_e x) = \frac{1}{x}, x > 0$

(xvii) $\frac{d}{dx}(a^x) = a^x \log_e a, a > 0$

Some Useful Substitutions for Finding Derivatives Expression

Expression	Substitution
(i) $a^2 + x^2$	$x = a\tan\theta$ or $x = a\cot\theta$
(ii) $a^2 - x^2$	$x = a\sin\theta$ or $x = a\cos\theta$
(iii) $x^2 - a^2$	$x = a\sec\theta$ or $x = a\text{cosec}\theta$
(iv) $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
(v) $\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2 = a^2\cos 2\theta$

Logarithmic Differentiation: Let $y = [f(x)]^{g(x)}$ ..(i)

So by taking log (to base e) we can write Eq. (i) as log y = g(x) log f(x). Then, by using chain rule

$$\frac{dy}{dx} = [f(x)]^{g(x)}\left[\frac{g(x)}{f(x)} f'(x) + g'(x)\log f(x)\right]$$

CHAPTER-6 Application of derivatives

- RATE OF CHANGE OF DERIVATIVES

Rate of Change of Quantities: Let y = f(x) be a function of x. Then, dy/dx represents the rate of change of y with respect to x.

If two variables x and y are varying with respect to another variable t, i.e. x = f(t) and y = g(t), then

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}, \text{ where } \frac{dx}{dt} \neq 0 \text{ (by chain rule)}$$

In other words, the rate of change of y with respect to x can be calculated using the rate of change of y and that of x both with respect to t.
Note: dy/dx is positive, if y increases as x increases and it is negative, if y decreases as x increases, dx

Marginal Cost: Marginal cost represents the instantaneous rate of change of the total cost at any level of output.
If C(x) represents the cost function for x units produced, then marginal cost (MC) is given by

$$\text{MC} = \frac{d}{dx}\{C(x)\}$$

Marginal Revenue: Marginal revenue represents the rate of change of total revenue with respect to the number of items sold at an instant.
If R(x) is the revenue function for x units sold, then marginal revenue (MR) is given by

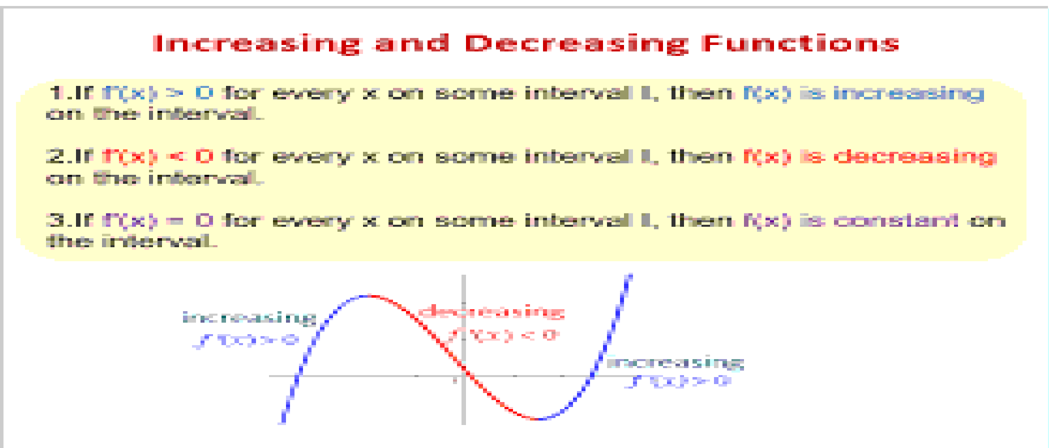
$$\text{MR} = \frac{d}{dx}\{R(x)\}$$

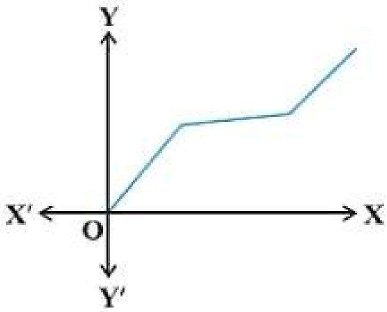
• INCREASING AND DECREASING FUCTION

Let I be an open interval contained in the domain of a real valued function f. Then, f is said to be

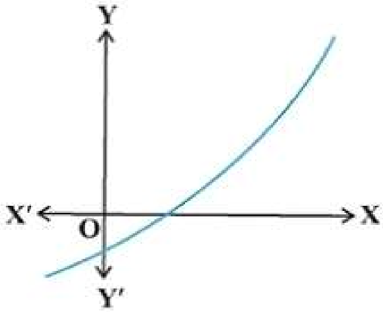
- Increasing on I, if $x_1 < x_2$ in I $\Rightarrow f(x_1) \leq f(x_2)$, $\forall\, x_1, x_2 \in$ I.
- Strictly increasing on I, if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$, $\forall\, x_1, x_2 \in$ I.
- Decreasing on I, if $x_1 < x_2$ in I $\Rightarrow f(x_1) \geq f(x_2)$, $\forall\, x_1, x_2 \in$ I.
- Strictly decreasing on I, if $x_1 < x_2$ in $f(x_1) > f(x_2)$, $\forall\, x_1, x_2 \in$ I.

Let x_0 be a point in the domain of definition of a real-valued function f, then f is said to be increasing, strictly increasing, decreasing or strictly decreasing at x_0, if there exists an open interval I containing x0 such that f is increasing, strictly increasing, decreasing or strictly decreasing, respectively in I.
Note: If for a given interval I ⊆ R, function f increase for some values in I and decrease for other values in I, then we say function is neither increasing nor decreasing.

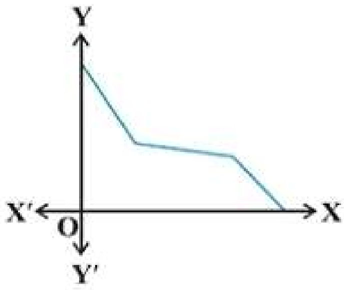




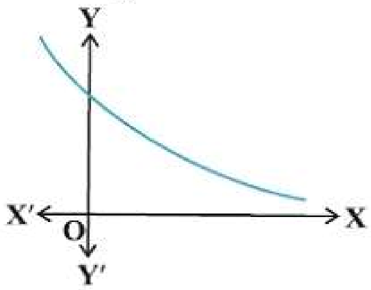
Increasing function
(i)



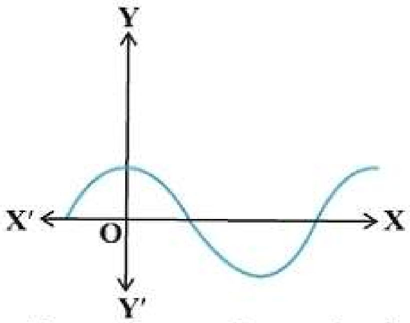
Strictly Increasing function
(ii)



Decreasing function
(iii)



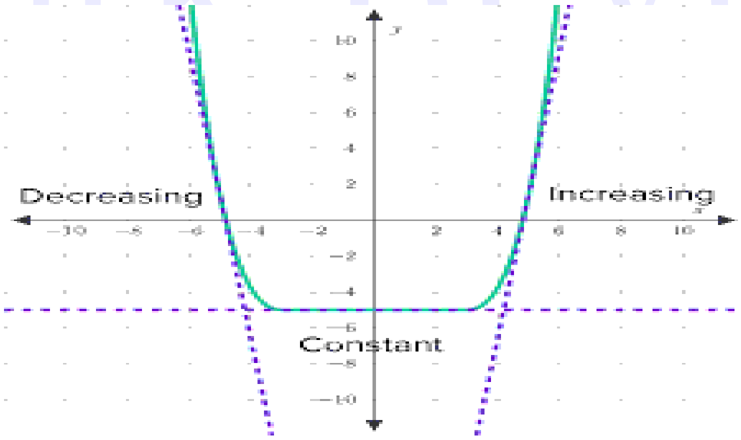
Strictly Decreasing function
(iv)

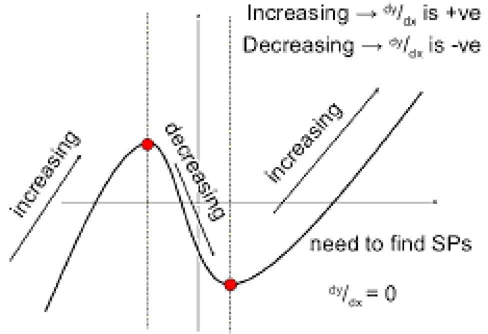


Neither Increasing nor Decreasing function
(v)

Note: If for a given interval I ⊆ R, function f increase for some values in I and decrease for other values in I, then we say function is neither increasing nor decreasing.

Let x_0 be a point in the domain of definition of a real-valued function f, then f is said to be increasing, strictly increasing, decreasing or strictly decreasing at x_0, if there exists an open interval I containing x0 such that f is increasing, strictly increasing, decreasing or strictly decreasing, respectively in I.
Note: If for a given interval I ⊆ R, function f increase for some values in I and decrease for other values in I, then we say function is neither increasing nor decreasing.



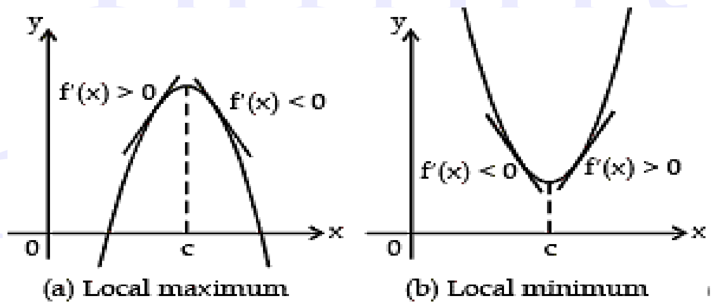


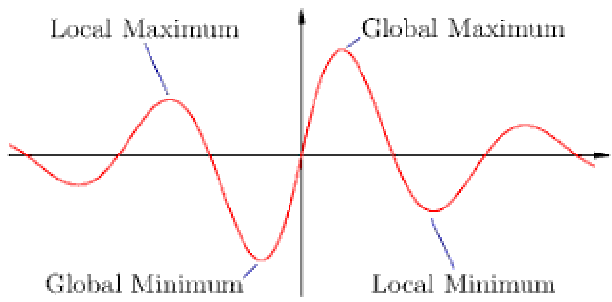
- **MAXIMA AND MINIMA**

Let f be continuous on [a, b] and differentiable on the open interval (a, b). Then,

Maximum and Minimum Value: Let f be a function defined on an interval I. Then,

- **f is said to have a maximum value in I, if there exists a point c in I such that f(c) > f(x), ∀ x ∈ I. The number f(c) is called the maximum value of f in I and the point c is called a point of a maximum value of f in I.**
  (ii) f is said to have a minimum value in I, if there exists a point c in I such that f(c) < f(x), ∀ x ∈ I. The number f(c) is called the minimum value of f in I and the point c is called a point of minimum value of f in I.
  (iii) f is said to have an extreme value in I, if there exists a point c in I such that f(c) is either a maximum value or a minimum value of f in I. The number f(c) is called an extreme value off in I and the point c is called an extreme point.





Local Maxima and Local Minima

(i) A function f(x) is said to have a local maximum value at point x = a, if there exists a neighbourhood (a – δ, a + δ) of a such that f(x) < f(a), ∀ x ∈ (a – δ, a + δ), x ≠ a. Here, f(a) is called the local maximum value of f(x) at the point x = a. (ii) A function f(x) is said to have a local

minimum value at point x = a, if there exists a neighbourhood (a – δ, a + δ) of a such that f(x) > f(a), ∀ x ∈ (a – δ, a + δ), x ≠ a. Here, f(a) is called the local minimum value of f(x) at x = a.

The points at which a function changes its nature from decreasing to increasing or vice-versa are called turning points.
Note:
(i) Through the graphs, we can even find the maximum/minimum value of a function at a point at which it is not even differentiable.
(ii) Every monotonic function assumes its maximum/minimum value at the endpoints of the domain of definition of the function.

Every continuous function on a closed interval has a maximum and a minimum value.

Let f be a function defined on an open interval I. Suppose cel is any point. If f has local maxima or local minima at x = c, then either f'(c) = 0 or f is not differentiable at c.

Critical Point: A point c in the domain of a function f at which either f'(c) = 0 or f is not differentiable, is called a critical point of f.

First Derivative Test: Let f be a function defined on an open interval I and f be continuous of a critical point c in I. Then,

- if f'(x) changes sign from positive to negative as x increases through c, then c is a point of local maxima.
- if f'(x) changes sign from negative to positive as x increases through c, then c is a point of local minima.
- if f'(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Such a point is called a point of inflection.

Second Derivative Test: Let f(x) be a function defined on an interval I and c ∈ I. Let f be twice differentiable at c. Then,
(i) x = c is a point of local maxima, if f'(c) = 0 and f''(c) < 0. (ii) x = c is a point of local minima, if f'(c) = 0 and f''(c) > 0.
(iii) the test fails, if f'(c) = 0 and f''(c) = 0.

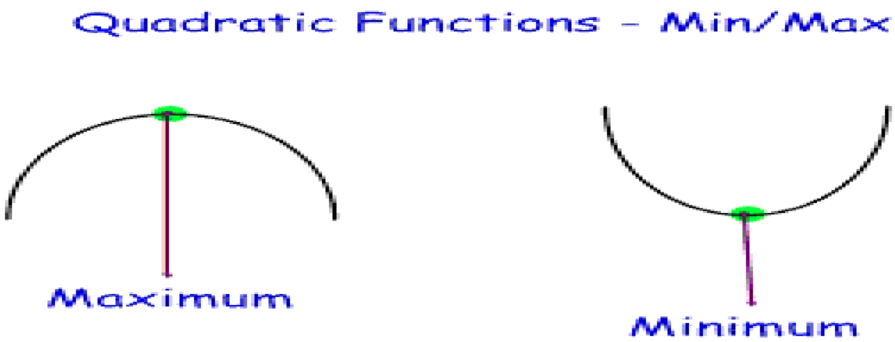
Note
(i) If the test fails, then we go back to the first derivative test and find whether a is a point of local maxima, local minima or a point of inflexion.
(ii) If we say that f is twice differentiable at o, then it means second order derivative exists at a.

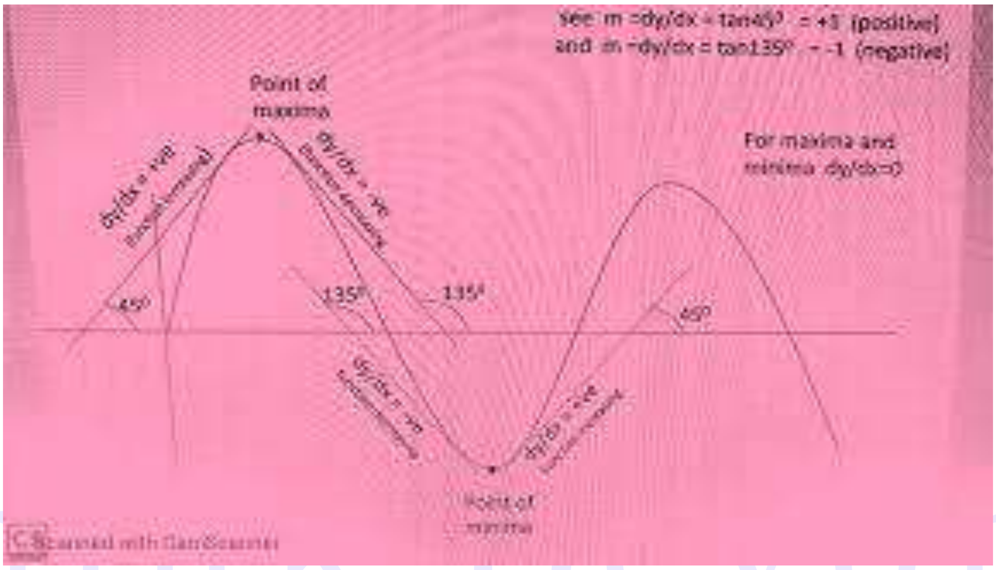
Absolute Maximum Value: Let f(x) be a function defined in its domain say Z ⊂ R. Then, f(x) is said to have the maximum value at a point a ∈ Z, if f(x) ≤ f(a), ∀ x ∈ Z.

Absolute Minimum Value: Let f(x) be a function defined in its domain say Z ⊂ R. Then, f(x) is said to have the minimum value at a point a ∈ Z, if f(x) ≥ f(a), ∀ x ∈ Z.

Note: Every continuous function defined in a closed interval has a maximum or a minimum value which lies either at the end points or at the solution of f'(x) = 0 or at the point, where the function is not differentiable.

Let f be a continuous function on an interval I = [a, b]. Then, f has the absolute maximum value and/attains it at least once in I. Also, f has the absolute minimum value and attains it at least once in





CHAPTER 7 :INTEGRALS

Basic Concepts:

1. **Antiderivative (or Primitive)** : A function $\emptyset(x)$ is said to be antiderivative or primitive of a function $f(x)$ if $\emptyset'(x) = f(x)$.

   For example : $\sin x$ is one of the antiderivative or primitive of $\cos x$, because

   $$\frac{d}{dx}(sinx) = \cos x$$
   $$\frac{d}{dx}(sinx + 1) = \cos x$$
   $$\frac{d}{dx}(sinx + 2) = \cos x$$
   $$\frac{d}{dx}(sinx + 3) = \cos x$$
   ……………………………
   ……………………………
   $$\frac{d}{dx}(sinx + C) = \cos x$$

   We conclude that a function has infinitely many antiderivatives.
   That is $\emptyset(x)$ be an antiderivative of $f(x)$, then $\emptyset(x) + C$ is also antiderivative of $f(x)$, where C is any constant.

2. **Indefinite Integrals** : If f(x) is a function then the family of all its antiderivatives is called Indefinite Integral of f(x). It is represented by:

   $\int f(x)dx$ (read as indefinite integral of f(x) with respect to x)

Derivatives	**Integrals**
$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$;	$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
Particularly, we note that	
$\frac{d}{dx}(x) = 1$;	$\int dx = x + C$
$\frac{d}{dx}(\sin x) = \cos x$;	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}(-\cos x) = \sin x$;	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$;	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$;	$\int \operatorname{cosec}^2 x\, dx = -\cot x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$;	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$;	$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$

Derivatives	**Integrals**
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
$\frac{d}{dx}(-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$;	$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
$\frac{d}{dx}(e^x) = e^x$;	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\log \lvert x \rvert) = \frac{1}{x}$;	$\int \frac{1}{x} dx = \log \lvert x \rvert + C$
$\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$;	$\int a^x dx = \frac{a^x}{\log a} + C$

3. Methods of Integration:

(i) Integration by substitution
(ii) Integration by Partial Fractions.
(iii) Integration by Parts

Integration by substitution : The given integral $\int f(x)\, dx$ can be transformed into another form by Changing the independent variable x to t by substituting $x = g(t)$.

Consider $I = \int f(x)\, dx$
Put $x = g(t)$ so that
By differentiating find dx/dt and
We write $dx = g'(t)\, dt$

This change of variable formula is one of the important tools available to us in the name of integration by substitution. It is often important to guess what will be the useful substitution. Usually, we make a substitution for a function whose derivative also occurs in the integrand.

Integration by Partial Fractions :

A rational function is defined as the ratio of two polynomials in the form $\frac{P(x)}{Q(x)}$, where P(x) and Q(x) are polynomials in x and Q(x) ≠ 0. If the degree of P(x) is less than the degree of Q(x), then the rational function is called proper, otherwise, it is called improper. The improper rational functions can be reduced to the proper rational functions by long division process.

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ where $x^2 + bx + c$ cannot be factorised further	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$,

In the above table, A, B and C are real numbers to be determined suitably.

Integration by Parts :

If ***u*** and ***v*** are any two differentiable functions of a single variable x. Then,

$$\int \mathbf{u \cdot v\, dx = u \int v\, dx - \int \left[\frac{du}{dx} \cdot \int v\, dx\right] dx}$$

"The integral of the product of two functions = (first function) × (integral of the second function) – Integral of [(differential coefficient of the first function) × (integral of the second function)]"

(i) To integrate the product of two functions we choose the first function according to the word ***ILATE*** where ***I*** stands for inverse function
L stands for logarithmic function
A stands for algebraic function
T stands for trigonometric function and
E stands for exponential function

(ii) If integrand has only one function then 1 is taken to be the second function.

4. Integrals of Some Particular Functions:

1. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} tan^{-1}\frac{x}{a} + C$
2. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} log\left|\frac{x-a}{x+a}\right| + C$
3. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} log\left|\frac{a+x}{a-x}\right| + C$
4. $\int \frac{1}{\sqrt{a^2+x^2}} = log\left|x + \sqrt{a^2+x^2}\right|$ + C
5. $\int \frac{1}{\sqrt{x^2-a^2}} = log\left|x + \sqrt{x^2+a^2}\right|$ + C
6. $\int \frac{1}{\sqrt{a^2-x^2}} = sin^{-1}\frac{x}{a} + C$

5. Integrals of the type:

(i) $\int \frac{1}{ax^2+bx+c}$ dx Apply completing square method for $ax^2 + bx + c$ and use suitable formulae from above.

(ii) $\int \frac{1}{\sqrt{ax^2+bx+c}}$ dx Apply completing square method for $ax^2 + bx + c$ and use suitable formulae from above.

(iii) $\int \frac{px+q}{ax^2+bx+c}$ dx and $\int \frac{px+q}{\sqrt{ax^2+bx+c}}$ dx

Write px + q = $A\left(\frac{d}{dx}\{ax^2 + bx + c\}\right) + B$

px + q = A(2ax + b) + B

Find A and B by equating coefficients of like powers of x from both sides

Then express the given integrals as the sum of two integrals and apply substitution method in first part and in the second part completing square method for $ax^2 + bx + c$ and use suitable formulae from above.

6. Integrals of some more type:

(For the following integrals consider 1 as second function and evaluate)

(i) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

(ii) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

(iii) $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

7. **Integrals of the type**: $\int e^x[f(x) + f'(x)]\, dx = e^x f(x) + C$

Definite Integral

The definite integral has a unique value. A definite integral is denoted by $\int_a^b f(x)\, dx$ where *a* is called the lower limit of the integral and *b* is called the upper limit of the integral. The definite integral is introduced either as the limit of a sum or if it has an anti derivative F in the interval [*a*, *b*], then its value is the difference between the values of F at the end points, i.e., F(*b*) – F(*a*).

7.8 Fundamental Theorem of Calculus

7.8.1 *Area function*

We have defined $\int_a^b f(x)\, dx$ as the area of the region bounded by the curve $y = f(x)$, the ordinates $x = a$ and $x = b$ and x-axis. Let x be a given point in $[a, b]$. Then $\int_a^x f(x)\, dx$ represents the area of the shaded region

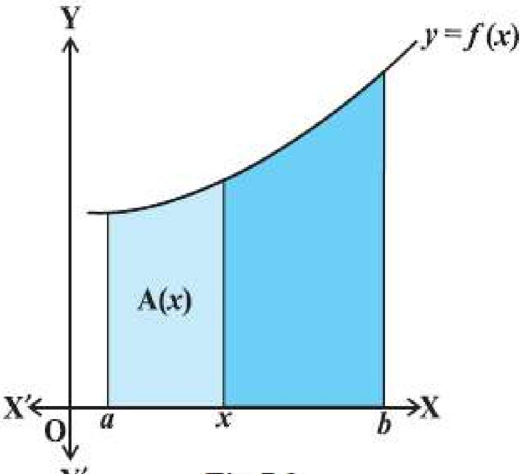


Fig 7.3

The area of this shaded region is a function of *x*. We denote this function of *x* by A(*x*). We call the function A(*x*) as *Area function* and is given by

$$A(x) = \int_a^b f(x)\, dx$$

$$A(x) = F(b) - F(a).$$

Properties of definite Integrals

P_0 : $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

P_1 : $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$. In particular, $\int_a^a f(x)\,dx = 0$

P_2 : $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

P_3 : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

P_4 : $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(Note that P_4 is a particular case of P_3)

P_5 : $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

P_6 : $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if $f(2a-x) = f(x)$ and

0 if $f(2a-x) = -f(x)$

P_7 : (i) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x)\,dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

Integration of trigonometric function

Working Rule (a) Express the given integrand as the algebraic sum of the functions of the following forms

(i) Sin $k\alpha$, (ii) cos $k\alpha$, (iii) tan $k\alpha$, (iv) cot $k\alpha$, (v) sec $k\alpha$, (vi) cosec $k\alpha$, (vii) $\sec^2 k\alpha$,

(viii) $\text{cosec}^2 k\alpha$, (ix) sec $k\alpha$ tan $k\alpha$ (x) cosec $k\alpha$ cot $k\alpha$

For this use the following formulae whichever applicable

(i) $\sin^2 x = \frac{1-\cos 2x}{2}$ **(ii)** $\cos^2 x = \frac{1+\cos 2x}{2}$

(iii) $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$ **(iv)** $\cos^3 x = \frac{3\cos x + \cos 3x}{4}$

(v) $\tan^2 x = \sec^2 x - 1$ **(vi)** $\cot^2 x = \text{cosec}^2 x - 1$

(Vii) $2\sin x \sin y = \cos(x-y) - \cos(x+y)$ **(vii)** $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

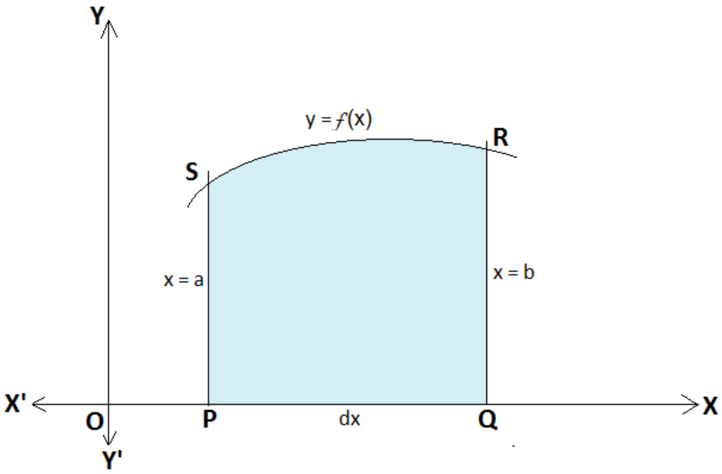
(ix) $2\sin x \cos y = \sin(x+y) + \sin(x-y)$ **(x)** $2\cos x \sin y = \sin(x+y) - \sin(x-y)$

CHAPTER 8 : APPLICATION OF INTEGRALS

INTRODUCTION

Area under Simple Curves

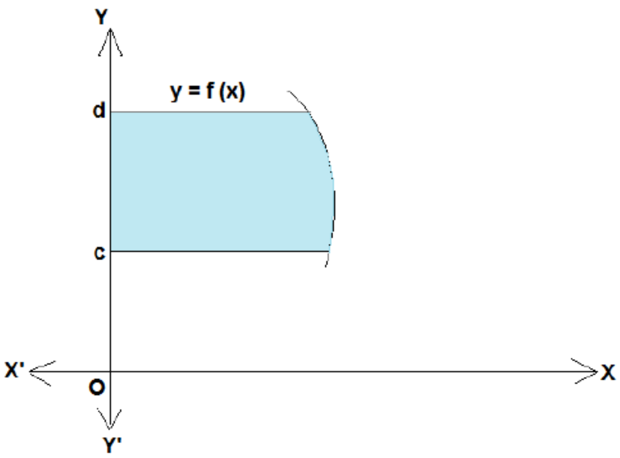
(i)



Area bounded by the curve y = f(x), the x-axis and between the ordinates at x = a and x = b is given by

Area = $\int_a^b y\,dx = \int_a^b f(x)\,dx$

(ii)



Area bounded by the curve y = f(x), the y axis and between abscissas at y = c and y = d is given by

Area = $\int_c^d x\,dx = \int_c^d g(y)dy$

Where $y = f(x) \Rightarrow x = g(y)$

Note: If area lies below x-axis or to left side of y-axis, then it is negative and in such a case we like its absolute value. (numerical value)

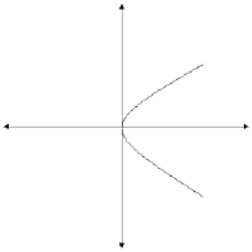
WORKING RULE

1. Draw the rough sketch of the given curve
2. Find whether the required area is included between two ordinate or two abscissa
3. (a) If the required area is included between two ordinates x = a and x= b then use the formula $\int_a^b y\,dx$

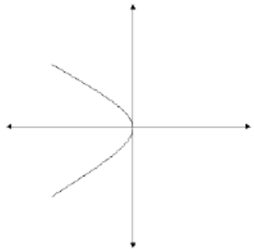
   (b) If the required area is included between two abscissa y = c and = d then use the Formula $\int_c^d x\,dy$

SOME IMPORTANT POINTS TO BE KEPT IN MIND FOR SKETCHING THE GRAPH

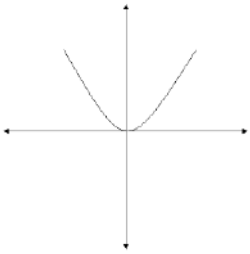
1. y^2 = 4ax is a parabola with vertex at origin, symmetric to X axis and right of origin



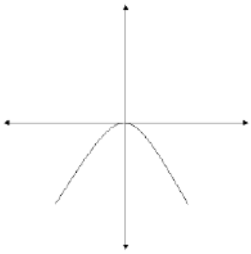
2. y^2 = - 4ax is a parabola with vertex at origin, symmetric to X axis and left of origin



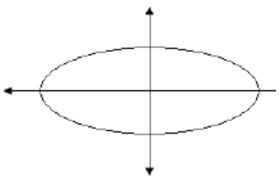
3. x^2 = 4ay is a parabola with vertex at origin, symmetricto y axis and above origin



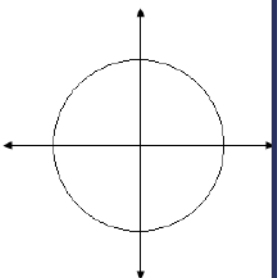
4. x^2 = - 4ay is a parabola with vertex at origin, symmetricto y axis and below origin



5. $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = 1 is an ellipse symmetric to both axis, Cut x axis at ($\pm$a,0) and y axis at (0,$\pm b$)

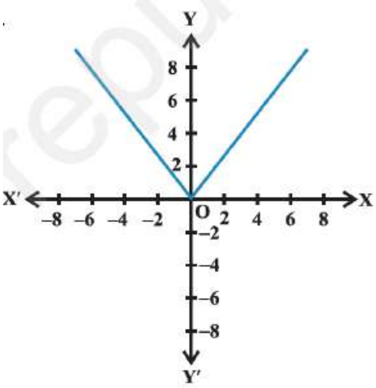


6. $x^2 + y^2 = r^2$ is a circle symmetric to both the axes With centre at origin and radius r



7. $(x-h)^2 + (y-k)^2 = r^2$ is a circle with centre at (h,k) and radius r.
8. ax +by + c = 0 representing a straight line

9. Graph of $y = |x|$



$f(x) = |x|$

–

CHAPTER 8 : Differential Equations

Basic concepts: $x^2 - 3x + 3 = 0$ …(1) $\sin x + \cos x = 0$ …(2) $x + 2y = 7$…….(3) $x + \frac{dy}{dx} + y = 0$….(4)

$\frac{d^2y}{dx^2} + y = 0$----(5) $\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$-----(6) Here are six equations.(1), (2), (3) are the equations are respectively quadratic, trigonometric , and linear equations. Whereas (4),(5) and (6) are differential equations. Actually, an equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called Differential Equation.

IMPORTANT : Sometimes differential involves derivatives with respect to more than one independent variables, such equations are called PARTIAL DIFFERENIAL EQUATIONS, else it is called ordinary differential equation. Here, we are to learn only about ordinary differential equation.

Order and Degree of differential equation.: The highest order derivative of the dependent variable with respect to independent variable involved in a differential equation is called its ORDER. The highest power of derivative forming polynomial equation in derivatives is called its degree.. e.g The degree of the differential equation. $\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$ is not defined. As it does not form polynomial equation in derivatives like y', y'', y''' etc.; The degree of $\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) - sin^2y = 0$ is Two.

General and Particular Solutions of Differential Equation: For differential equation $\frac{d^2y}{dx^2} + y = 0$…. (1)

The function $y = f(x) = a\sin(x + \text{b})$ is its General Solution as when its derivative and the value of y are substituted in L.H.S. of (1) R.H.S. are become equal to zero. While the function $y_1 = f_1(x) = 2\sin\left(x + \frac{\pi}{4}\right)$ is also the solution of the D. E. –(1) Here , the values arbitrary constant a and b are respectively 2 and $\frac{\pi}{4}$., $y_1 = f_1(x) = 2\sin\left(x + \frac{\pi}{4}\right)$ is called **particular solution**, while $y = f(x) = a\sin(x + \text{b})$ is called **General Solution. Note that the number of arbitrary constants in the general solution of a differential equation of order n is n. The number of arbitrary constants in the particular solution of a differential equation of any order is ZERO.**

Methods of solving First Order, First Degree Differential Equations:

There are three methods, (i) Variable Separable (ii) Homogeneous differential equation (iii) Linear differential equation.

(i) **Variable Separable :**

$\frac{dy}{dx} = \text{F}(x, y)$ ---(1) ⇒ *f(x) dx* = *f*(y)dy

Integrating both sides we get $\int f(x)dx = \int f(y)dy$ ⇒ F(x) = F(y) +C is the general solution of D.E. –(I)

(ii) Homogeneous differential equation:

To solve a homogeneous differential equation of the type $\frac{dy}{dx} = \text{F}(x, y) = g\left(\frac{y}{x}\right)$ ---(1)

We make the substitution $y = v.x$ ------(2)

Differentiating equation 2 w.r.t. x , we get $\frac{dy}{dx} = v + x\,\frac{dv}{dx}$ -------(3)

Substituting the value of $\frac{dy}{dx}$ from eqn (3) in eqn (1) we get $v + x\frac{dv}{dx} = g(v)$

Or.$x\frac{dv}{dx} = g(v) - v$ ------(4) separating variables in eqn(4) we get

$\frac{dv}{g(v)-v} = \frac{dx}{x}$ --------(5)

Integrating both sides of eqn(5), we get $\int \frac{dv}{g(v)-v} = \int \frac{1}{x}dx + C$ ------(6)

Eqn (6) gives general solution of the differential equation (1) when we replace v by $\frac{y}{x}$.

(iii) **Linear differential equation**

A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only , is known as first order linear differential equation also $\frac{dx}{dy} + Px = Q$, where P and Q are constants or functions of y only

To Solve . $\frac{dy}{dx} + Py = Q$ find Integrating factor (I.F.) $= e^{\int P\,dx}$

The general solution of $\frac{dy}{dx} + Py = Q$ is $y.(I.F) = \int Q.(I.F)\,dx + C$ (proceed)

And To Solve . $\frac{dx}{dy} + Px = Q$ find Integrating factor (I.F.) $= e^{\int P\,dy}$

The general solution of $\frac{dx}{dy} + Px = Q$ is $x.(I.F) = \int Q.(I.F)\,dy + C$ (proceed)

CHAPTER 10 : VECTORS ALGEBRA

SUMMARY

1. Position vector of a point P(x, y, z) is given as $\overrightarrow{OP}(\vec{r}) = x\hat{i} + y\hat{j} + z\hat{k}$, and its magnitude by $\sqrt{x^2 + y^2 + z^2}$.

2. The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
3. The magnitude(r), direction ratios $(a,\ b,\ c)$ and direction cosines $(l, m, n\ )$of any vector are related as: $l = \frac{a}{r}\ , m = \frac{b}{r}, n = \frac{c}{r}$
4. The vector sum of the three sides of a triangle taken in order is $\vec{0}$.
5. The vector sum of two co initial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
6. The multiplication of a given vector by a scalar α, changes the magnitude of the given vector by the multiple |α|, and keeps the direction same (or makes it opposite) according as the value of α is positive (or negative).
7. For a given vector $\vec{a}$, the vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the direction of $\vec{a}$.
8. The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio m : n

   (i) internally, is given by $\vec{R} = \frac{m\vec{b}+n\vec{a}}{m+n}$.

   (ii) externally, is given by $\vec{R} = \frac{m\vec{b}-n\vec{a}}{m-n}$.

   (iii) if R is the mid point of PQ, then $\vec{R} = \frac{\vec{b}+\vec{a}}{2}$.

9. The scalar product of two given vectors $\vec{a}$ and $\vec{b}$ having angle θ between them is defined as $\vec{a}.\vec{b}$ = $|\vec{a}||\vec{b}|\cos\theta$.

   Also, when $\vec{a}.\vec{b}$ is given, the angle 'θ' between the vectors $\vec{a}$ and $\vec{b}$ may be determined by $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|\,|\vec{b}|}$.
10. The vector product is given as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$ such that $\vec{a}, \vec{b}, \hat{n}$ form right handed system of co-ordinate axes.
11. If we have two vectors $\vec{a}$ and $\vec{b}$, given in component form as

   $\vec{a} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$ and $\vec{b} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$

   and λ any scalar, then

   $\vec{a} + \vec{b} = (a_1 + a_2)\hat{\imath} + (b_1 + b_2)\hat{\jmath} + (c_1 + c_2)\hat{k}$

   $\lambda\vec{a} = \lambda a_1\hat{\imath} + \lambda b_1\hat{\jmath} + \lambda c_1\hat{k}$

   $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$

   and $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

CHAPTER 11 : 3D GEOMETRY

1. **Distance formula**: Distance between two points A(x_1, y_1, z_1) and B (x_2, y_2, z_2) is
   AB $=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
2. **Section formula**: Coordinates of a point P, which divides the line segment joining two given points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) in the ratio m : n
   (i) internally, are P $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$,
   (ii) the coordinates of a point Q divides the line segment joining two given points in the ratio m : n; externally are Q $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$
   (i) the coordinates of mid-point are R$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}, \frac{z_2+z_1}{2}\right)$
3. **Direction cosines of a line :**
   (i) The direction of a line OP is determined by the angles α, β, γ which makes with OX, OY,OZ respectively. These angles are called the direction angles and their cosines are called the direction cosines.
   (ii) Direction cosines of a line are denoted by l, m, n; l = cos α, m = cos $\beta, n = \cos\gamma$
   (iii) Sum of the squares of direction cosines of a line is always 1.
   $l^2 + m^2 + n^2 = 1$ i.e $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
4. **Direction ratio of a line :**
   (i) Numbers proportional to the direction cosines of a line are called direction ratios of a line. If a, b, and c are, direction ratios of a line, then $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$.
   (ii) If a, b, c are, direction ratios of a line, then the direction cosines are
   $\pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$
   (iii) Direction ratio of a line AB passing through the points A(x_1, y_1, z_1) and B (x_2, y_2, z_2) are $x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$
5. **STRAIGHT LINE:**
   (i) Vector equation of a Line passing through a point $\vec{a}$ and along the direction $\vec{\boldsymbol{b}}$, : $\vec{r} = \vec{a} + \mu\vec{b}$,
   (ii) Cartesian equation of a Line: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$. Where ($x_1$, y_1, z_1) is the given point and its direction ratios are a, b, c.
6. (i) Vector equation of a Line passing through two points, with position vectors $\vec{a}$ *and* $\vec{b}$
   $\vec{r} = \vec{a} + \boldsymbol{\mu}(\vec{\boldsymbol{b}} - \vec{a})$
   (ii) Cartesian equation of a Line: $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$, two points are ($x_1$,$y_1$) and ($x_2$,$y_2$).

7. **ANGLE** between two lines (i) Vector equations: $\vec{r} = \vec{a_1} + \boldsymbol{\lambda}\vec{b_1}$ and $\vec{r} = \vec{a_2} + \boldsymbol{\mu}\vec{b_2}$,

   (ii) Cartesian equations: If lines are $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$, $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

$$\cos\theta = \frac{\vec{b1}.\ \vec{b2}}{|\vec{b1}|.|\vec{b2}|}$$

(iii) If two lines are perpendicular, then $\vec{b}_1.\ \vec{b}_2 = 0$, i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(iv) If two lines are parallel, then $\vec{b}_1 = t\,\vec{b}_2$, where t is a scalar. OR $\vec{b}_1 \times \vec{b}_2 = 0$, OR $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(v) If θ *is the* angle between two lines with direction cosines, l_1, m_1, n_1 and l_2, m_2, n_2 then

(a) $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$ (b) if the lines are parallel, then $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

(c) If the lines are perpendicular, then $l_1l_2 + m_1m_2 + n_1n_2 = 0$

8 Shortest distance between two skew- lines:

(i) Vector equations: $\vec{r} = \vec{a_1} + \boldsymbol{\lambda}\vec{b_1}$, and : $\vec{r} = \vec{a_2} + \boldsymbol{\mu}\vec{b_2}$,

$$SD = \left|\frac{(\vec{a2}-\vec{a1}).(\vec{b1}\times\vec{b2})}{|\vec{b1}\times\vec{b2}|}\right|.$$

If shortest distance is zero, then lines intersect and line intersects in space if they are coplanar. Hence if above lines are coplanar

If $(\vec{a2} - \vec{a1}).(\vec{b1} \times \vec{b2}) = 0$

(ii) Cartesian equations: $\frac{x-x1}{a1} = \frac{y-y1}{b1} = \frac{z-z1}{c1}, \frac{x-x2}{a2} = \frac{y-y2}{b2} = \frac{z-z2}{c2}$

$$SD = \left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

If shortest distance is zero, then lines intersect and line intersects in space if they are coplanar. Hence if above lines are coplanar

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

9.Shortest distance between two parallel lines:

If two lines are parallel, then they are coplanar. Let the lines be: $\vec{r} = \vec{a_1} + \boldsymbol{\lambda}\vec{b}$, and:

$\vec{r} = \vec{a_2} + \boldsymbol{\mu}\vec{b}$, $SD = \left|\frac{\vec{b}\times(\vec{a2}-\vec{a1})}{|\vec{b}|}\right|$

CHAPTER 12 - LINEAR PROGRAMMING PROBLEMS

LINEAR PROGRAMMING PROBLEM(LPP)

A linear programming problem deals with the optimization (maximization/ minimization) of a linear function of two variables (say x and y) which is known as objective function subject to :

i) The variables x and y are non-negative
ii) The variables x and y satisfy a set of linear inequalities which are called linear constraints.

OBJECTIVE FUNCTION

A linear function z = ax + by where a and b are constants which has to be maximized or minimized is called a linear objective function.

DECISION VARIABLES

In the objective function z = ax + by, x and y are called decision variables.

CONSTRAINTS

The linear inequalities or restrictions on the variables of an LPP are called constraints. The conditions $x \geq 0$,$y \geq 0$ are called non-negative constraints.

FEASIBLE REGION

The common region determined by all the constraints including non-negative constraints $x \geq 0$,$y \geq 0$ of an LPP is called the feasible region for the problem.

FEASIBLE SOLUTIONS

Points within and on the boundary of the feasible region for an LPP represent feasible solutions.

INFEASIBLE SOLUTIONS

Any point outside feasible region is called infeasible solutions.

OPTIMAL(FEASIBLE) SOLUTION

Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

THEOREM-1

Let R be the feasible region (convex polygon) for an LPP and $Z=ax + by$ be the objective function. When Z has an optimal value (maximum or minimum) where x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point(vertex) of the feasible region.

THEOREM-2

Let R be the feasible region for a LPP and $Z = ax + by$ be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occur at a corner point of R.

If the feasible region R is unbounded, then a maximum or a minimum value of the objective function may or may not exist. If it exists, it must occur at a corner point of R.

CORNER POINT METHOD FOR SOLVING A LPP

The method comprises of the following steps:

1) Find the feasible region of the LPP and determine its corner points(vertices) either by inspection or by solving the two equations of the lines intersecting at the point.
2) Evaluate the objective function $Z = ax + by$ at each corner point.
   Let M and m respectively denote the largest and the smallest values of Z.

3).i) When the feasible region is bounded , M and m are respectively the maximum and minimum values of Z.

ii) In case the feasible region is unbounded

a) M is maximum value of Z , if the open half plane determined by $ax+by > M$ has no point in common with the feasible region. Otherwise Z has no maximum value.
b) Similarly , m is minimum value of Z , if the open half plane determined by $ax+by < m$ has no common point with the feasible region. Otherwise Z has no minimum value.

MULTIPLE OPTIMAL POINTS

If two corner points of the feasible region are optimal solutions of the same type i.e. both produce the same maximum or minimum , then any point on the line segment joining these two points is also an optimal solution of the same type.

CHAPTER 13 : PROBABILITY

Random Experiment:

An experiment whose outcomes can't be predicted is called a random experiment.

Trial:

Performing an event is known as trial.

Event:

The possible outcomes of a trial are called events.

Equally likely Events:

The events are said to be equally likely if there is no reason to expect any one in preference to any other.

Exhaustive Events:

The events E_1 , E_2 ,E_3 ,…………,E_n of a sample space S are called Exhaustive events if

$E_1 \cup E_2 \cup E_3 \cup \ldots\ldots\ldots \cup E_n = S$

Mutually Exclusive Events:

Two or more events are said to be mutually exclusive if they cannot happen simultaneously in a trial.

Favourable Events:

The cases which ensure the occurrence of the events are called favourable events.

Sample Space(S):

The set of all possible outcomes of an experiment is called sample space.

Probability of occurrences of an event:

Probability of occurrences of an event A, denoted by P (A), is defined as:

$$P(A)= \frac{Number\ \ of\ outcomes\ in\ favour\ of\ A}{Total\ number\ \ of\ possible\ outcomes} = \frac{n(A)}{n(S)}$$

THEOREMS:

(i) In a random experiment, if S be the sample space and A an event, then:

(a) $0 \le P(A) \le 1$ (b) $P(\emptyset) = 0$ (c) $P(S) = 1$

(ii) If A and B are mutually exclusive events, then

(a) $P(A \cap B) = 0$ (b) $P(A) + P(B) = 1$ (c) $P(A \cup B) = P(A) + P(B)$

(iii) For any two events A and B $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(iv) For each event A, P(not A)= 1 - P(A), where 'not A' is the complementary event of A.

MORE DEFINITIONS

Compound Event:

The simultaneous happening of two or more events is called a compound event if they occur in connection with each other.

Conditional Probability:

Let A and B be two events associated with the same sample space S, then Probability of occurrences of event A given that B occurs is

$$P(A/B) = \frac{P(A\cap B)}{P(B)}, \; P(B) \neq 0$$

Independent Events:

Two events are said to be independent if the occurrence of one does not depend upon the occurrence of the other.

$P(A\cap B) = P(A) . P(B)$ when A and B are independent events.

MULTIPLICATION THEOREM:

Let A and B be two events associated with the same sample space. Then

$P(A\cap B) = P(B) . P(A / B)$, where $P(B) \neq 0$

THEOREM ON TOTAL PROBABILITY:

If $E_1, E_2, E_3, \ldots, E_n$ are mutually exclusive and exhaustive events(partition) of sample space S and A is any event associated with S, then

$$P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2) + P(E_3).P(A|E_3) + \ldots P(E_n).P(A|E_n)$$

BAYES' THEOREM:

If $E_1, E_2, E_3, \ldots, E_n$ are mutually exclusive and exhaustive events(partition) of sample space S and A is any event associated with S, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)+\cdots\ldots+P(E_n)P(A/E_n)}$$