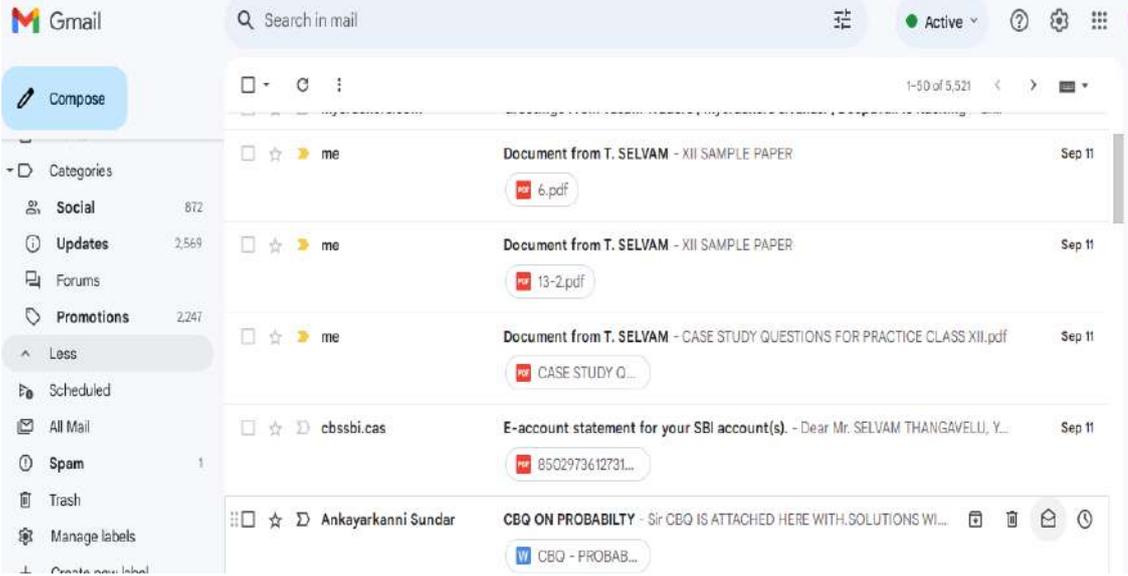# CASE BASED QUESTIONS
Chapter 12 Probability

1	In a bilateral hockey series of two matches between the countries India and Germany the probabilities of India Winning, Losing and Drawing are $\frac{1}{2}, \frac{1}{5}$ and $\frac{3}{10}$ resply. Each win, draw and loss gives the points 2, 1, 0 respectively to the team. Let X and Y denote total points scored by India and Germany after two games series. **Deleted** Based on the above information answer the following questions: (i) Write the probability distribution of X. (ii) Find the probability of India winning the series (iii) What is the probability of India drawing the series?
2.	A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. Based on the given information, answer the following questions. (i) Find the probability that A, B and C all will hit the target? (ii) What is the probability that any two of A, B and C will hit?
3.	Two persons A and B decided to play a game by using a die. They decided to throw the die alternately. They play with the condition that the person who gets 4 first will be the winner of the game. Based on the given information, answer the following questions. (i) A wins the game, if A starts the game first

	(ii) B wins the game, if A starts first.
4.	In a play zone, Alina is playing crane game. It has 12 blue soft toys, 8 red soft toys, 10 yellow soft toys and 5 green soft toys. Alina draws two soft toys one after the other without replacement. (i) What is the probability that the first soft toy is blue and the second is green. (ii). What is the probability that the first soft toy is green and the second is not yellow.
5.	Arun can detect spam e-mails in his inbox. It is found that the word "offer" occurs in 80% of the spam messages in his account. Also the word "offer" occurs in 10% of his desired e-mails. If 30% of the received e-mails are considered as spam. Based on the above information answer the following questions: (i) He received a new message which contains the word "offer", what is the probability that it is a spam mail? (ii) What is the probability of e-mails received with "offer" word?
6.	Husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$.

Based on the above information, answer the following questions.

(i) Both of them will be selected

(ii) Only one of them will be selected.

(iii) None of them will be selected. (OR)

At least one of them will be selected

7.	A fruit seller purchased Mango fruits from a farmer. The farmer told the vendor that he used chemical pesticide for 30% of the mango trees and organic pesticide for 70% of the mango trees during the flowering season to prevent the insects damaging the fruit. 80% of the flowers, which got chemical pesticide sprinkled, turned into fruits and 90% of the flowers, which got organic pesticide sprinkled, bore the fruits. Past experience shows that insects damage 20% of the fruits, sprinkled with chemical pesticide and only 10% of the fruits, with organic pesticide.

Based on the above information answer the following questions:

(i) A consumer purchased a fruit from that shop and found to be not affected by the insects. What is the probability that the tree had been sprinkled with organic pesticide?

(ii) What is the probability of selecting a fruit with insects inside the seed of it?

8.	A company has two plants to manufacture TVs. The first plant manufactures 70% of the TVs and the rest are manufactured by the second plant. 80% of the TVs manufactured by the first plant are rated of standard quality, while that of second plant

only 60% are of standard quality. One TV is selected at random.

 Based on the above information answer the following;

 (i) Find the probability that the selected TV is of standard quality.
 (ii) Find the probability that the TV is of standard quality, given that it was made by 1st plant.

9.

In a survey at Vande Bharat Train, IRCTC asked passengers to rate and review the food served in train. IRCTC asked 500 passengers selected at random to rate food according to price (low, medium ,or high) and food (1,2,3,or 4 stars). The results of this survey are presented in the two-way, or contingency, table below. The numbers in this table represent frequencies. For example, in the third row and fourth column, 30people rated the prices high and the food 4 stars.

Price/ rating	*	**	***	****	Total
Low	20	30	90	10	150
Medium	50	80	90	30	250
High	20	10	30	40	100
Total	90	120	210	80	500

 (1) Find the probability that the passenger rates the prices medium?

	(2) Find the probability that the passenger rates the food 2 stars.
	(3) Suppose the passenger selected rates the price high. What is the probability that he rates the restaurant 1 star?
	(4) Suppose the passenger selected does not rate the food 4 stars. . What is the probability that she rates the prices high?
10.	Two friends A and B had gone for a shopping, and they came across a beautiful antique piece and both want to buy it. They asked shop keeper for another piece but not available in shop. Both of them decided to go to coffee shop to have coffee and toss a PAIR OF COINS, whosoever gets the pair of heads first will buy the antique piece, both shook hands and sat down for their luck , Answer the following if A starts (1) What is the probability of getting pair of heads? (2) What is the probability of getting only one head in a throw? (3) What is the probability that A gets pair of heads in third throw and wins the game? (4) What is the probability that B wins the game if A starts? (5) What is the probability that A buys the antique piece if A starts ?
11.	During Examination, we need to reschedule on study hours and along with the study hours, we need quality revision of syllabus. In one such situation ,if X is a random variable which represents number of hours a student of class XII studied a particular subject per day, the probability distribution is given as, <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>>3</td></tr><tr><td>P(X)</td><td>0</td><td>K</td><td>3K</td><td>4K</td><td>0</td></tr></table> Deleted

(1) What is the value of K?

(2) The probability that less than two hours time is given to a subject per day is:

(3) What is the probability that two hours or three hours of time is given to a subject per day?

 (4) What is the probability that 3 hours or more than three hours of time is given to a subject per day?

| 12. | A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively. |

(1) When the doctor arrives late, what is the probability that he comes by metro?

 (2)When the doctor arrives late, what is the probability that he comes by cab?

(3) When the doctor arrives late, what is the probability that he comes by bike?

| 13. | Suman was doing a project on a school survey, on the average number of hours spent on study by students selected at random. At the end of survey, Suman prepared the following report related to the data. Let X denotes the average number of hours spent on study by the students. |

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X	0	1	2	3	4	>4
P(X)	0.2	K x	K x	K (6-x)	k(6-x)	0

(1) Find the value of k?

(2)What is the probability that the average study time is not more than 1 hour?

(3)What is the probability that the average study time is atleast 3 hours?

(4)What is the probability that the average study time is exactly 2 hours?

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14. Three persons A, B and C apply for manager post in a company ,the chances of selection is given by the ratio 1 : 2 : 4.The probability that if selected A , B and C can bring changes to improve profitability are 0.8, 0.5 and 0.3 respectively.

(1)If the changes does not take place Find The probability that C is selected as manager?

(2) What is the conditional probability that if change has taken place due to B?

ANSWERS

1.

(i)

X	0	1	2	3	4
P(X)	$\frac{1}{5} \times \frac{1}{5}$	$2\left(\frac{3}{10} \times \frac{1}{5}\right)$	$2\left(\frac{1}{2} \times \frac{1}{5}\right) + \left(\frac{3}{10} \times \frac{3}{10}\right)$	$2\left(\frac{3}{10} \times \frac{1}{2}\right)$	$\left(\frac{1}{2} \times \frac{1}{2}\right)$

(ii) $P(India\ wins\ the\ series) = P(X = 3) + P(X = 4)$
$$= 2\left(\frac{3}{10} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$$
$$= \frac{11}{20}$$

(iii) $P(India\ draws\ the\ series) = P(X = 2)$ $$= 2\left(\frac{1}{2} \times \frac{1}{5}\right) + \left(\frac{3}{10} \times \frac{3}{10}\right)$$ $$= \frac{29}{100}$$	

2.	Answer : (i) $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$, $P(C) = \frac{2}{3}$ $$P(A \cap B \cap C) = P(A)P(B)P(C) = \frac{2}{5}$$ (ii) $P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C)$ $$P(A)\ P(B)\ P(\bar{C}) + P(A)P(\bar{B})P(C) + P(\bar{A})\ P(B)P(C) = \frac{13}{30}$$

3.	Answer: $$P(Win) = \frac{1}{6}\ ,\ P(lose) = \frac{5}{6}$$ (i) $P(A\ wins) = \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \cdots\cdots\cdots$ $$= \frac{1}{6} \times \frac{1}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$$ (ii). $P(B\ wins) = 1 - \frac{6}{11} = \frac{5}{11}$.

4.	Let B, R,Y and G denote the events that soft toy drawn is blue, red, yellow and green respectively.(without replacement)

B	R	Y	G	TOTAL
12	8	10	5	35

	(i) $\quad P(B \cap G) = P(B)P\left(\frac{G}{B}\right) = \frac{12}{35} \cdot \frac{5}{34} = \frac{6}{119}$ (ii) $\quad P(G \cap \bar{Y}) = P(G) \cdot P\left(\frac{\bar{Y}}{G}\right) = \frac{5}{35} \cdot \frac{24}{34} = \frac{12}{119}$

5.	(i)$E_1 \rightarrow desired\ mail;\ E_2 \rightarrow spam\ mail$ $A \rightarrow selected\ mail\ contains\ the\ word\ "offer"$ $$P(E_2/A) = \frac{P(A/E_2)\ P(E_2)}{P(A/E_1)P(E_1) + P(A/E_2)\ P(E_2)}$$ $$= \frac{\frac{80}{100} \times \frac{30}{100}}{\frac{80}{100} \times \frac{30}{100} + \frac{10}{100} \times \frac{70}{100}} = \frac{24}{31}$$

	(ii) $$(A) = P(A/E_1)P(E_1) + P(A/E_2)\,P(E_2)$$ $$= \frac{80}{100} \times \frac{30}{100} + \frac{10}{100} \times \frac{70}{100}$$ $$= \frac{31}{100}$$
6.	Answer : (i) $\quad \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$ (ii) $\quad \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} = \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$ (iii) $\quad \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$ (iv) $\quad$ 1 - P(both will not be selected) $= 1 - \frac{24}{35} = \frac{11}{35}$.
7.	(i) $E_1 \rightarrow Organic\ pesticide;\ E_2 \rightarrow Chemical\ pesticide$ $A \rightarrow not\ having\ insects\ in\ the\ seed\ of\ the\ fruit$ $$P(E_1/A) = \frac{P(A/E_1)\,P(E_1)}{P(A/E_1)P(E_1) + P(A/E_2)\,P(E_2)}$$ $$= \frac{\frac{90}{100} \times \frac{70}{100}}{\frac{90}{100} \times \frac{70}{100} + \frac{80}{100} \times \frac{30}{100}} = \frac{63}{87}$$ (ii) $Let\ B \rightarrow\ having\ insects\ in\ the\ seed\ of\ the\ fruit$ $$P(B) = P(B/E_1)P(E_1) + P(B/E_2)\,P(E_2)$$ $$= \frac{10}{100} \times \frac{70}{100} + \frac{20}{100} \times \frac{30}{100}$$ $$= \frac{13}{100}$$
8.	Answer : E_1 = TV produced by first plant E_2 = TV produced by second plant. A = Manufactured TV of standard quality. $$P(E_1) = \frac{7}{10}\ , P(E_2) = \frac{3}{10}\ ,\ P\left(\frac{A}{E_1}\right) = \frac{8}{10}\ , P\left(\frac{A}{E_2}\right) = \frac{6}{10}$$ (i) P(A) $= \frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{6}{10} = \frac{74}{100} = 0.74$ (ii) $P\left(\frac{E_1}{A}\right) = \dfrac{P(E_1)P(\frac{A}{E_1})}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P(\frac{A}{E_2})} = \frac{56}{74} = 0.756.$

9.	Number of passengers =500
	
	    (1) P (Rating Medium) = 250/500 =1/2
	    (2) P(Rating 2 stars) = 12/500
	    (3) P (Rating one star/selecting the price high) = 20/100
	P (price high /does not rating 4 stars) =60/420

10.	(1) S = { HH,HT,TH,TT } P(getting two heads) = ¼ (2) P (getting only one head in a throw) =1/2 (3) P (A getting pair of heads in third throw) = $\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$ = $\frac{9}{64}$ (4) Ans : $\frac{3}{7}$ (5)Ans : $\frac{4}{7}$
11.	(1) We know $\therefore 0 + K + 3K + 4K + 0 = 1$ 8K=1, = $\frac{1}{8}$. (2) P (X < 2) = P (X =0) + P (X =1) =1K = $\frac{1}{8}$ (3) P (X =2) + P (X =3) = 3K + 4K = 7K = $\frac{7}{8}$ (4)P (X=3) + P(X =4) = 4K + 0 = $\frac{4}{8}$

12.	Let E be the event of coming late.
	A be the event of coming by cab.
	B be the event of coming by metro.
	C be the event of coming by bike.
	D be the event of coming by other transport.
	 P (A) = 3/10 P (B) = 2/10 P (C) = 1/10 P (D) = 4/10
	P (E/A) = 25/100 P (E/B) = 3/10 P (E/C) = 35/100
	P (E/D) = 1/10
	
	    (1) P (E /B) = $\dfrac{P(B)P(\frac{E}{B})}{P(A)P\left(\frac{E}{A}\right)+P(B)P\left(\frac{E}{B}\right)+P(C)P\left(\frac{E}{C}\right)+P(D)P\left(\frac{E}{D}\right)}$
	
	    = $\left(\dfrac{6/100}{\left(\frac{15}{200}+\frac{6}{100}+\frac{7}{200}+\frac{4}{100}\right)}\right)$ = $\dfrac{6\times2}{42}$ = 2/7
	    (2) ANSWER : $\frac{5}{14}$
	    (3) ANSWER : $\frac{1}{6}$

13.						

P(X)	0.2	k	2k	3k	2k	0

(1) Using $\sum P(x) = 1$, 8k = 1 -0.2

8k = 0.8 , k = 1/10

(2) Probability of study time NOT more than one ; P(X=0) + P(X =1)

        2/10 + 1/10 = 3/10

(3) P (X=3) + P(X =4) + P (X >4) = $\frac{5}{10}$

(4) P(X =2) = $\frac{1}{5}$

(5) P(X =1) + P(X=2) + P (X=3) + P(X =4) + P (X >4) ==1 – P(X=0)

    = 1- 0.2 =0.8

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14. Let the events be described as below :

A: No change takes place

E1: Person A gets appointed

E2: Person B gets appointed

E3: Person C gets appointed.

The chances of selection of A, B and C are in the ratio 1: 2: 4 .

Hence, P(E1)=1/7 ,P(E2)=2/7 ,P(E3)=4/7

Probabilities of A,B and C introducing changes to improve profits of company are 0.8,0.5 and 0.3 respectively. Hence probability of no changes on appointment of A,B and C are 0.2,0.5 and 0.7 respectively.

Hence ,P(A|E1)=0.2= 2/10

P(A|E2)=0.5= 5/10

P(A|E3)=0.7= 7/10

(1)∵the required probability is

$$P \; (E_3/ A) = \left(\frac{P(E_3)P\left(\frac{A}{E_3}\right)}{\sum_1^3\left(P \; (E_i)\times P \left(\frac{A}{E_i}\right)\right)} \right) = \frac{\frac{28}{70}}{\left(\frac{2}{70} + \frac{10}{70} + \frac{28}{70}\right)} \qquad =28/40$$

    = 7/10

∴ if no change takes place, the probability that it is due to appointment of C is 7/10

(2) E : Change takes place

$$P \left(\frac{E_2}{E} \right) \;\; = \frac{P(E_2)P\left(\frac{E}{E_3}\right)}{\sum_1^3\left(P \; (E_i)\times P \left(\frac{E}{E_i}\right)\right)} = \left(\frac{\frac{2}{7} \times 5/10}{\left(\frac{8}{70} + \frac{10}{70} + \frac{12}{70}\right)} \right) = 10/30 \;\; =1/3$$