CASE BASED QUESTIONS-Chapter 6 AOD

1	Mr.Suresh who is a popular businessman consulted a share market expert to know about the investment in a particular company. He predicted tha the trend of the company would be governed by the function $f(x) = \frac{x^3}{3} - 4x^2 + 15x + 8$ where x is the years of investment in the company.  i. In the first ten years when will he get growth in his investment? ii. There is going to be a lean patch in the investment. When is it going happen? When will the market pick up again?
2	Rain Water Harvesting pits are very essential to conserve water and use it furtheruse.An engineer was asked to design a cuboidal pit with a fixed volu of $256 m^3$ and with a square base  i. What will be the value of edge of base so that total surface minimum? ii. Find the height of tank and also the total surface area?
3	The soaring prices of tomatoes in our country in the recent part made the Trade Analysts to come up with an equation $$f(x) = 16x - \frac{1}{2}x^2$$ To find the cost of 1kg of tomatoes after x days.

	 After how many days will the cost of tomatoes be maximum? What will be the maximum cost? ii. When will the cost of tomatoes become Rs.64?
4	The Maths and crafts teachers of a school planned to assign a task to the students. A paper of area ‘k’ sq.uts was given to each one them and were asked to make a cylinder closed at one end and open at the other.  i. Find the value of r for which the cylinder has maximum volume. ii. Find the relation between r and h of the cylinder. Also find the maximum value.
5	A company was given contract to manufacture two varieties of bulbs A & B which will be sold at profits of Rs. 60 & Rs. 80 respectively. There was a condition that sum of squares of the number of bulbs of each type is a constant k.

	 i) What is the ratio of production of 2 bulbs for maximum profit? ii) What is the maximum profit if k = 100?
6	To honor the scientists associated with the success of chandrayaan-3, A school management decided to felicitate them. The students were asked to stand in a path governed by $y = x^2$. The scientists would be asked to move in a vehicle waving at the children along the path $y = x - 2$.  i) A student nearest to their path was given an opportunity to garland the dignitaries. Where on the curve does the student stand? ii) What is the distance of the student from the line?
7	To reduce global warming environmentalists and scientists came up with an innovative idea of developing a spherical bulb that would absorb harmful gases and thereby reduce global warming.But during the process of absorption the bulb would get inflated and it's radius would beincreasing at 1cm/sec.

	 i) Find the rate at which the volume increases when radius is 6 cm. ii) At an instant when volume was increasing at the rate of $400\pi cm^3/sec$ find the rate at which it's surface area is increasing?
8	An Aero plane is flying with a velocity of 300m/sec at an altitude of 1 km from the Earth's surface.  i) Find an expression for $\frac{dx}{dt}$ in terms of θ, where θ is the angle of elevation of the aeroplane from the bottom of the control tower and x is the horizontal distance between the aeroplane and the control tower. ii) Find the rate at which the angle of elevation of the Aeroplane changes from the control tower at an instant when the horizontal distance of the plane is 500m from it.
9	A cylindrical tank of fixed volume of $144\Pi\ m^3$ is to be constructed with an open top to throw all the garbage in an orphanage. The manager of the orphanage called a contractor for the construction ensure that a tank to dispose off biodegradable waste can be constructed at a minimum cost.

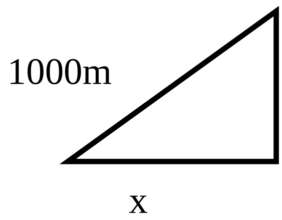
	 i) Find the cost of the least expensive tank that can be constructed if it costs Rs. 80 per sq. m for base and Rs. 120 per sq. m for walls ii) Find the radius and height as well.
10	An insect moves in a straight line to escape from a predator such that the distances travelled by the insect and its predator in time t (in seconds) are given by $s = t^3 - 7t^2 + 15t + 1$ and $s = t^4 + t^3 + 4t + 10$ respectively. i) What were their initial velocities? ii) After how many seconds will the predator be able to catch up the prey if initially it was behind it by 120m? 

ANSWERS

1	$f'(x) = x^2 - 8x + 15$ For critical points, $f'(x) = 0$ $x^2 - 8x + 15 = 0$ $x = 3, x = 5$ In (0, 3), $f'(x) > 0$ and in (3, 5) $f'(x) < 0$ By first derivative test f has maxima at $x = 3$

	ii) when x > 5, f ′ (x) > 0 f is Strictly decreasing in (3, 5) ---- lean patch
2	i) Given V = 256 m^3, $l^2 h = 256$ $h = \frac{256}{l^2}$ $S = l^2 + 4lh$ $S = l^2 + 4l \times \frac{256}{l^2} = f(l)$ $f'(l) = 2l - \frac{1024}{l^2}$ for critical points, f ′ (l) = 0 so we get l = 8m by second derivative test f has minima at l = 8m as f″(I) > 0 ii) $h = \frac{256}{l^2}$ so h = 4m S = f (8) = 192 m^2
3	$f(x)= 18x- \frac{1}{2}x^2$ f ‘(x)=18-x for critical points, f ‘(x)=0 ➡ x= 18 f ‘’(x)= -1<0 ➡ f ‘’(18)= -1<0 by second derivative test f has maxima at x= 18 maximum cost = $18 \times 18 – \frac{1}{2}x^2$ = Rs. 162 ii) $18x-\frac{1}{2}x^2= 64$ $36x- x^2 =128$ $X^2-36x+128= 0$ X= 4.
4	1. $2\pi rh + \pi r^2 = k$ $h= k-\pi r^2/2 \pi r$ $v = \pi r^2 h$ $= \pi r^2(k-\pi r^2/ 2\pi r)$ V= f(r)= ½(kr- πr^3) f ‘(r) = ½(k-3πr^2)

	$f'(r) = 0$ ➡ $r = \sqrt{k/3\pi}$ $f''(r) = -3\pi r$ $f''(\sqrt{k/3\pi}) = -3\pi\sqrt{k/3\pi} < 0$ by second derivative test, f has local max at $r = \sqrt{k/3\pi}$ ii) $h = (3\pi r^2 - \pi r^2)/ 2\pi r$ ➡ $h = 2\pi r^2/2\pi r$ **h= r**
5	1. let A & B variety bulbs be x & y respectively. $X^2+y^2 = k$ $Y=\sqrt{(k-x^2)}$ Profit= 60x + 80y $f(x) = 60x + 80\sqrt{(k-x^2)}$ $f'(x)= 0$ $25x^2 = 9k$ $X = \frac{3}{5}\sqrt{k}$ $f''(x) = -80(2k/ 2(k-x^2)^{3/2}) < 0$ f has max at $x = \frac{3}{5}\sqrt{k}$ $y = \frac{4}{5}\sqrt{k}$ ➡ x: y= 3:4 **ii)** profit= $60 \times \frac{3}{5} \times 10+80 \times \frac{4}{5} \times 10$ = R1000
6	1. let points on $y = x^2$ be $A=(t, t^2)$ $X – Y – 2 = 0$ distance = $I\ t – t^2 – 2\ I/ \sqrt{2}$ $f(t)= \lvert(t - t^2 - 2)\rvert/ \sqrt{2}$ $f'(t)= (2t-1)/ \sqrt{2}$ $f'(t)= 0$ t= ½ $f''(t)= 2/\sqrt{2}$ $f''(1/2) = \sqrt{2} > 0$ f has min at t= ½ ∴ A = (1/2 ,1/4) ii) distance = $((½)^2 – ½ + 2)/ \sqrt{2}$ $= \frac{7}{4\sqrt{2}}$ units.

7	$-\frac{dr}{dt}$= 1cm/sec i) $V = 4/3\pi r^3$ $dv/dt = 4\ \pi r^2 dr/dt$ $(dv/dt)_{r=6} = 4\ \pi\ 36 \times 1$ $= 144\ \pi\ cm^3/sec$ ii) Given $dv/dt = 400\ \pi$ $4\ \pi r^2 dr/dt = 400\ \pi$ r= 10 cm $s = 4\ \pi r^2$ $ds/dt = 8\ \pi r\ dr/dt$ $(ds/dt)_{r=10} = 8\ \pi \times 10 \times 1$ $= 80\ \pi\ cm^2/sec$
8	 $\frac{dx}{dt}$ = 300m/sec $\tan\theta = \frac{1000}{x}$ $x = 1000 \cot\theta$ ➡ $\frac{dx}{dt} = -1000\ cosec^2\theta\ \frac{d\theta}{dt}$ ii) when x = 500 $\tan\theta = 2$ $\cot\theta = ½$,$cosec\theta = \sqrt{(5/4)}$ $\therefore \frac{dx}{dt} = 1000 \times \frac{5}{4} \times \frac{d\theta}{dt}$ (when x = 500 m) $\frac{300}{-1250} = \frac{d\theta}{dt}$ $\frac{d\theta}{dt} = \frac{-6}{25} \frac{rad}{sec}$
9	$\Pi r^2 h = 144\Pi$ $h = \frac{144}{r^2}$ Area = $\Pi r^2 + 2\Pi rh$ Cost = $80 \times \Pi r^2 + 120 \times 2\Pi rh$ Substituting $h = \frac{144}{r^2}$

	Cost = f (r) $= 80 \times \Pi r^2 + \frac{34560\Pi}{r}$ $f'(r) = 160\,\Pi r - \frac{34560\Pi}{r^2}$ for critical points f ′ (r) = 0, we get r = 6m $f''(r) = 160\Pi + \frac{69120\Pi}{r^3} > 0$ for r = 6 so by second derivative test, cost is minimum at r = 6m minimum cost = f (6) = Rs. 8640Π
10	i) initial velocities: Prey ⟶ $\frac{ds}{dt} = 3t^2 - 14t + 15$ Initial velocity (t = 0) = 15m/sec Predator ⟶ $\frac{ds}{dt} = 4t^3 + 3t^2 + 4$ Initial velocity (t = 0) = 4m/sec iii) Let it catch the prey after t seconds, Difference in distances = 120m $(t^4 + t^3 + 4t + 10) - (t^3 - 7t^2 + 15t + 1) = 120$ $(t^4 + 7t^2 - 11t + 9 = 120$ Solving it we get t = 3 seconds