

Reason (R): The number '2n' is composite for all natural numbers n.

16. **Assertion (A):** Let $A = \{x \in \mathbb{R}, -1 \leq x \leq 1\}$. If $f: A \rightarrow A$ be defined as $f(x) = x^2$ then f is not an onto function. **[BOARD 2025]**

Reason (R): If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

2 Marks:

1. Prove that the greatest integer function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = [x]$, is neither one-one nor onto. **[BOARD 2023]**
2. Let $f: A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$ where $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Discuss the bijectivity of the function. **[BOARD 2025]**
3. A function $f: A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$ then find the set B. **[BOARD 2023]**
4. Draw the graph of $\cos^{-1} x$ where $x \in [-1, 0]$. Also, write its range. **[BOARD 2023]**
5. Draw the graph of $\sin^{-1} x, x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write its range. **[BOARD 2023]**

3 Marks:

1. Let R be the relation defined over N, where N is set of natural numbers, defined as " mRn if and only if m is a multiple of n, $m, n \in \mathbb{N}$ ". find whether r is reflexive, symmetric and transitive or not. **[BOARD 2025]**
2. A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y): |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive. **[BOARD 2024]**
3. Let R be a relation on set of real numbers \mathbb{R} defined as $\{(x, y): x - y + \sqrt{3} \text{ is an irrational number}, x, y \in \mathbb{R}\}$, verify r for reflexive, symmetry and transitive. **[BOARD 2025]**
4. Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y): xy \text{ is a square of a natural number}, x, y \in \mathbb{N}\}$. Determine if the relation R is an equivalence relation. **[BOARD 2025]**
5. A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41, x, y \in \mathbb{N}$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not. **[BOARD 2025]**
6. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 4x^3 - 5, \forall x \in \mathbb{R}$ is one-one and onto. **[BOARD 2025]**

7. Show that the function $f: N \rightarrow N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is even} \\ n + 1, & \text{if } n \text{ is odd} \end{cases}$ is a bijection. **[BOARD 2025]**
8. Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y): x + y = 6, x \in A, y \in B\}$. **[BOARD 2025]**
- (i) Write all elements of R .
- (ii) Is R a function? Justify.
- (iii) Determine domain and range of R .
9. Prove that $f: N \rightarrow N$ defined as $f(x) = ax + b, a, b \in N$ is one-one but not onto. **[BOARD 2025]**
10. A function f is defined from $R \rightarrow R$ as $f(x) = ax + b$ such that $f(1) = 1$ and $f(2) = 3$. Find the function $f(x)$. Hence, check whether the function $f(x)$ is one-one and onto or not. **[BOARD 2024]**
11. If $f: R^+ \rightarrow R$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection, where R^+ is a set of all positive real numbers. **[BOARD 2025]**

5 Marks:

1. A relation R is defined on a set of real numbers \mathbb{R} as $R = \{(x, y): x, y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not. **[BOARD 2023]**
2. Check whether the relation S in the set of real numbers R defined by $S = \{(a, b): a - b + \sqrt{2} \text{ is an irrational number}\}$ is reflexive, symmetric or transitive. **[BOARD 2024]**
3. Show that the relation S in set \mathbb{R} of real numbers defined by $S = \{(a, b): a \leq b^3, a \in \mathbb{R}, b \in \mathbb{R}\}$ is neither reflexive, nor symmetric, nor transitive. **[BOARD 2023 & 2024]**
4. Check whether the relation S in the set \mathbb{R} of real numbers defined by $S = \{(a, b): a \leq b^2, a \in \mathbb{R}, b \in \mathbb{R}\}$ is reflexive, symmetric or transitive. Also, determine all $x \in R$ such that $(x, x) \in S$. **[BOARD 2024]**
5. Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b): \text{both } a \text{ and } b \text{ are either odd or even}\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class $[1]$. **[BOARD 2023]**

6. A relation R on set $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ be defined as $R = \{(x, y): x + y \text{ is an integer divisible by } 2\}$. Show that R is an equivalence relation. Also, write the equivalence class [2]. **[BOARD 2024]**
7. A relation R on set $A = \{x: -10 \leq x \leq 10, x \in Z\}$ is defined as $R = \{(x, y): (x - y) \text{ is divisible by } 5\}$. Show that R is an equivalence relation. Also, write the equivalence class [5]. **[BOARD 2024]**
8. Prove that the relation R in the set of integers Z defined as $R = \{(a, b): 2 \text{ divides } (a + b)\}$ is an equivalence relation. Also, find [3]. **[BOARD 2024]**
9. If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b)R(c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation. **[BOARD 2023]**
10. A relation R is defined on $N \times N$ (where N is the set of natural numbers) as $(a, b)R(c, d) \Leftrightarrow a - c = b - d$. Show that R is an equivalence relation. **[BOARD 2024]**
11. A relation R is defined on $N \times N$ (where N is set of natural numbers) as $(a, b)R(c, d) \Leftrightarrow \frac{a}{c} = \frac{b}{d}$. Show that R is an equivalence relation. **[BOARD 2024]**
12. Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto. **[BOARD 2024]**
13. Let $A = R - \{3\}$ and $B = R - \{a\}$. Find the value of 'a' such that the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not. **[BOARD 2024]**
14. A function $f: [-4, 4] \rightarrow [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$. **[BOARD 2023]**
15. Show that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{5x-3}{4}$ is both one-one and onto. **[BOARD 2023]**
16. Prove that a function $f: [0, \infty) \rightarrow [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto. **[BOARD 2023]**
17. Check whether the function $f: \mathbb{R} \rightarrow \left[\frac{-1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is one-one and onto or not. **[BOARD 2023]**

18. Let $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not. **[BOARD 2023]**
19. Show that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f: \mathbb{R} \rightarrow A$ becomes an onto function. **[BOARD 2024]**
20. Show that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which $f(x) = 3$. **[BOARD 2024]**

Case Based Questions:

1. Let A be the set of 30 students of class XII in a school. Let $f: A \rightarrow N$, N is a set of natural numbers such that function $f(x) =$ Roll Number of student x .

[BOARD 2025]

Based on the above information, answer for the following:

- (i) Is f a bijective function? **1**
- (ii) Give reasons to support your answer to (i). **1**
- (iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where

$$R = \{(x, y): x, y \text{ are Roll numbers of students such that } y = 3x\}.$$

List the elements of R . Is the relation R reflexive, symmetric and transitive? Justify your answer. **2**

(OR)

- (b) Let R be a relation defined by

$$R = \{(x, y): x, y \text{ are Roll numbers of students such that } y = x^3\}$$

List the elements of R . Is R a function? Justify your answer. **2**

2. Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rails lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by $R =$

$\{(l_1, l_2): l_1 \text{ is parallel to } l_2\}$. **[BOARD 2024]**



Based on the above information, answer for the following questions:

- (i) Find whether the relation R is symmetric or not. **1**
- (ii) Find whether the relation R is transitive or not. **1**
- (iii) (a) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it. **2**

(OR)

- (b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$.
Check whether the relation S is symmetric and transitive. **2**

3. A class room teacher is keen to assess the learning of her students the concept of “relations” taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$: **[BOARD 2025]**

$$R_1 = \{(2, 3), (3, 2)\}$$

$$R_2 = \{(1, 2), (1, 3), (3, 2)\}$$

$$R_3 = \{(1, 2), (2, 1), (1, 1)\}$$

$$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$$

$$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$$

The students are asked to answer the following questions about the relations:

- (i) Identify the relations which is reflexive, symmetric but not transitive. **1**
- (ii) Identify the relations which is reflexive, transitive but not symmetric. **1**
- (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. **2**

(OR)

- (b) What pairs should be added to the relation R_2 to make it an equivalence relation? **2**

4. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as

$$R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}.$$

[BOARD 2025]



Based on the above information, answer for the following:

- (i) How many relations can be there from S to J? **1**
- (ii) A student identifies a function from S to J as
 $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$. Check if it is bijective. **1**
- (iii) (a) How many one-one functions can be there from set S to set J? **2**

(OR)

(b) Another student consider a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ is set S. write minimum order pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**

5. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$ where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer for the following questions:

- (i) How many relations are possible from B to G? **1**
- (ii) Among all the possible relations from B to G, how many functions can be formed from B to G? **1**
- (iii) (a) Let $R: B \rightarrow B$ be defined by $R = \{(x, y): x \text{ and } y \text{ are students of the same sex}\}$. Check if R is an equivalence relation. **2**

(OR)

(b) A function $f: B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer. **2**