# GRADE XII
# Question Bank (MATHEMATICS)
## Chapter-5 Continuity and Differentiability

**1 marks:**

1. If $f(x)=\begin{cases}3x-2, & 0<x\leq 1\\ 2x^2+ax, & 1<x<2\end{cases}$ is continuous for $x\in(0,2)$ then a is **[BOARD 2025]**

   a) -4 b) $-\frac{7}{2}$ c) -2 d) -1

2. If $f(x)=\begin{cases}\frac{1-\sin^3 x}{3\cos^2 x}, & x\neq\frac{\pi}{2}\\ k, & x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$ then the value of k is

   **[BOARD 2025]**

   a) $\frac{3}{2}$ b) $\frac{1}{6}$ c) $\frac{1}{2}$ d) 1

3. If $f(x)=\begin{cases}\frac{\sin^2 ax}{x^2}, & x\neq 0\\ 1, & x=0\end{cases}$ is continuous at $x=0$ then the value of a is

   **[BOARD 2025]**

   a) 1 b) -1 c) $\pm 1$ d) 0

4. If $f(x)=\begin{cases}\frac{\log(1+ax)+\log(1-bx)}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$ then the value of k is

   **[BOARD 2025]**

   a) a b) $a+b$ c) $a-b$ d) b

5. For what value of k the function $f(x)=\begin{cases}\frac{\sqrt{4+x}-2}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$?

   **[BOARD 2024]**

   a) 0 b) $\frac{1}{4}$ c) 1 d) 4

6. The value of the constant c that makes the function $f(x)=\begin{cases}x^2-c^2, & x<4\\ cx+20, & x\geq 4\end{cases}$ continuous for all real numbers, is **[BOARD 2024]**

   a) -2 b) -1 c) 0 d) 2

7. If $f(x)=\begin{cases}1, & x\leq 3\\ ax+b, & 3<x<5\\ 7, & 5\leq x\end{cases}$ is continuous in R, then the values of a and b are

   **[BOARD 2025]**

   a) $a=3,b=-8$ b) $a=3,b=8$ c) $a=-3,b=-8$ d) $a=-3,b=8$

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8. If $f(x) = \begin{cases} 3ax - b, & x > 1 \\ 11, & x = 1 \\ -5ax - 2b, & x < 1 \end{cases}$ is continuous at $x = 1$ then the values of a and b are

**[BOARD 2025]**

a) $a = 3, b = 5$ b) $a = 8, b = -1$ c) $a = 1, b = -8$ d) $a = -3, b = 5$

9. The function f is defined by $f(x) = \begin{cases} x, & x \leq 1 \\ 5, & x > 1 \end{cases}$ is not continuous at **[BOARD 2025]**

a) $x = 0$ b) $x = 1$ c) $x = 2$ d) $x = 5$

10. A function $f(x) = |1 - x + |x||$ is **[BOARD 2024]**

a) Discontinuous at $x = 1$ only
b) Discontinuous at $x = 0, 1$
c) discontinuous at $x = 0$ only
d) continuous everywhere

11. The function $f(x) = |x| + |x - 2|$ is **[BOARD 2024]**

a) $f(x)$ is continuous but not differentiable at $x = 0$ and $x = 2$
b) $f(x)$ is differentiable but not continuous at $x = 0$ and $x = 2$
c) $f(x)$ is continuous but not differentiable at $x = 0$ only
d) $f(x)$ is neither continuous nor differentiable at $x = 0$ and $x = 2$

12. If $f(x) = |x| + |x - 1|$ then which of the following is correct? **[BOARD 2025]**

a) $f(x)$ is both continuous and differentiable at $x = 0$ and $x = 1$
b) $f(x)$ is differentiable but not continuous at $x = 0$ and $x = 1$
c) $f(x)$ is continuous but not differentiable at $x = 0$ and $x = 1$
d) $f(x)$ is neither continuous nor differentiable at $x = 0$ and $x = 1$.

13. If $f(x) = \{[x], x \in R\}$ is the greatest integer function then the correct statement is

**[BOARD 2025]**

a) $f(x)$ is continuous but not differentiable at $x = 2$
b) $f(x)$ is neither continuous nor differentiable at $x = 2$
c) $f(x)$ is both continuous and differentiable at $x = 2$
d) $f(x)$ is differentiable but not continuous at $x = 2$

14. Which of the following statements is true for the function $(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ ?

**[BOARD 2024]**

a) $f(x)$ is continuous and differentiable $\forall\, x \in \mathbb{R}$
b) $f(x)$ is continuous $\forall\, x \in \mathbb{R}$
c) $f(x)$ is continuous and differentiable $\forall\, x \in \mathbb{R} - \{0\}$
d) $f(x)$ is discontinuous at infinitely many points.

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15. The number of points where $f(x) = [x], 0 < x < 3$ ([.] denotes greatest integer function) is not differentiable is **[BOARD 2024]**

a) 1 b) 2 c) 3 d) 4

16. The greatest integer function defined by $f(x) = [x], 1 < x < 3$ is not differentiable at $x =$ **[BOARD 2024]**

a) 0 b) 1 c) 2 d) $\frac{3}{2}$

17. The function $f(x) = [x]$, where $[x]$ denotes greatest integer less than or equal to x, is continuous at **[BOARD 2023]**

a) $x = 1$ b) $x = 1.5$ c) $x = -2$ d) $x = 4$

18. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$ is **[BOARD 2024]**

a) 0 b) 1 c) 2 d) infinite

19. The number of discontinuities of the function f is given by

$f(x) = \begin{cases} x + 2, & x < 0 \\ e^x, & 0 \leq x \leq 1 \\ 2 - x, & x > 1 \end{cases}$ is **[BOARD 2024]**

a) 0 b) 1 c) 2 d) 3

20. If $f(x) = 2|x| + 3|\sin x| + 6$ then the right hand derivative of $f(x)$ at $x = 0$ is **[BOARD 2023]**

a) 6 b) 5 c) 3 d) 2

21. If $x = A\cos 4t + B\sin 4t$ then $\frac{d^2x}{dt^2}$ is equal to **[BOARD 2023]**

a) $x$ b) $-x$ c) $16x$ d) $-16x$

22. If $x = at, y = \frac{a}{t}$ then $\frac{dy}{dx}$ is **[BOARD 2024]**

a) $t^2$ b) $-t^2$ c) $\frac{1}{t^2}$ d) $-\frac{1}{t^2}$

23. If $x = at^2, y = 2at$ then $\frac{dy}{dx}$ is equal to **[BOARD 2024]**

a) $2at$ b) $\frac{1}{t}$ c) $-\frac{1}{t^2}$ d) $-\frac{1}{2at^3}$

24. If $x = 3\cos\theta$ and $y = 5\sin\theta$ then $\frac{dy}{dx}$ is equal to **[BOARD 2024]**

a) $-\frac{3}{5}\tan\theta$ b) $-\frac{5}{3}\cot\theta$ c) $-\frac{5}{3}\tan\theta$ d) $-\frac{3}{5}\cot\theta$

25. If $x = a\cos\theta + b\sin\theta, y = a\sin\theta - b\cos\theta$ then which of the following is true? **[BOARD 2023]**

a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

26. $\frac{d}{dx}[\cos(\log x + e^x)]$ at $x = 1$ is **[BOARD 2024]**

a) $-\sin e$ b) $\sin e$ c) $-(1+e)\sin e$ d) $(1+e)\sin e$

27. If $xe^y = 1$ then the value of $\frac{dy}{dx}$ at $x = 1$ is **[BOARD 2024]**

a) -1 b) 1 c) $-e$ d) $-\frac{1}{e}$

28. The derivative of $x^{2x}$ w.r.t x is **[BOARD 2023]**

a) $x^{2x-1}$ b) $2x^{2x}\log x$ c) $2x^{2x}(1+\log x)$ d) $2x^{2x}(1-\log x)$

29. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is **[BOARD 2024]**

a) 1 b) -1 c) $-2\sqrt{\pi}$ d) $2\sqrt{\pi}$

30. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is **[BOARD 2024]**

a) $\frac{x}{1+x^4}$ b) $\frac{2x}{1+x^4}$ c) $-\frac{2x}{1+x^4}$ d) $\frac{1}{1+x^4}$

31. The derivative of $2^x$ w.r.t. $3^x$ is **[BOARD 2024]**

a) $\left(\frac{3}{2}\right)^x\frac{\log 2}{\log 3}$ b) $\left(\frac{2}{3}\right)^x\frac{\log 3}{\log 2}$ c) $\left(\frac{2}{3}\right)^x\frac{\log 2}{\log 3}$ d) $\left(\frac{3}{2}\right)^x\frac{\log 3}{\log 2}$

32. The derivative of $5^x$ w.r.t. $e^x$ is **[BOARD 2024]**

a) $\left(\frac{5}{e}\right)^x\frac{1}{\log 5}$ b) $\left(\frac{e}{5}\right)^x\frac{1}{\log 5}$ c) $\left(\frac{5}{e}\right)^x\log 5$ d) $\left(\frac{e}{5}\right)^x\log 5$

33. The derivative of $e^{2x}$ w.r.t. $e^x$ is **[BOARD 2024]**

a) $e^x$ b) $2e^x$ c) $2e^{2x}$ d) $2e^{3x}$

34. The derivative of $x^2$ w.r.t. $x^3$ is **[BOARD 2024]**

a) $\frac{2}{3x}$ b) $\frac{3x}{2}$ c) $\frac{2x}{3}$ d) $6x^5$

35. The derivative of $e^{\sin^2 x}$ w.r.t. $\cos x$ is **[BOARD 2024]**

a) $\sin x\, e^{\sin^2 x}$

b) $-2\cos x\, e^{\sin^2 x}$

c) $\cos x\, e^{\sin^2 x}$

d) $-2\sin^2 x\cos x\, e^{\sin^2 x}$

36. If $f(x) = -2x^8$ then the correct statement is **[BOARD 2025]**

a) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$

b) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

c) $-f'\left(\frac{1}{2}\right) = f\left(\frac{1}{2}\right)$

d) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

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37. If $f(x) = x^x$ then $f'(e)$ is equal to **[BOARD 2024]**

a) 0 b) 2 c) $e^e$ d) $2e^e$

38. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant then $\frac{dy}{dx}$ is **[BOARD 2025]**

a) $\frac{x}{y}$ b) $-\frac{x}{y}$ c) $\frac{a}{x}$ d) $\frac{a}{y}$

39. If $y = \sin^2(x^3)$ then $\frac{dy}{dx}$ is equal to **[BOARD 2023]**

a) $2\sin x^3 \cos x^3$ b) $3x^3 \sin x^3 \cos x^3$ c) $6x^2 \sin x^3 \cos x^3$ d) $2x^2 \sin^2(x^3)$

40. If $y = \log\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$ then $\frac{dy}{dx}$ is equal to **[BOARD 2023]**

a) $\sec x$ b) $cosec\ x$ c) $\tan x$ d) $\sec x \tan x$

41. If $y = \log_{2x}\left(\sqrt{2x}\right)$ then $\frac{dy}{dx}$ is equal to **[BOARD 2025]**

a) 0 b) 1 c) $\frac{1}{x}$ d) $\frac{1}{\sqrt{2x}}$

42. If $y = log\sqrt{\sec\sqrt{x}}$ then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is **[BOARD 2024]**

a) $\frac{1}{\pi}$ b) $\frac{1}{2}$ c) $\pi$ d) $\frac{1}{4}$

43. If $y = \cos^{-1}(e^x)$ then $\frac{dy}{dx}$ is **[BOARD 2024]**

a) $\frac{1}{\sqrt{e^{-2x}+1}}$ b) $-\frac{1}{\sqrt{e^{-2x}+1}}$ c) $\frac{1}{\sqrt{e^{-2x}-1}}$ d) $-\frac{1}{\sqrt{e^{-2x}-1}}$

44. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is **[BOARD 2023]**

a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ c) $\log\left|sec\left(\frac{\pi}{4} - x\right)\right|$

b) $\sec^2\left(\frac{\pi}{4} - x\right)$ d) $-\log\left|sec\left(\frac{\pi}{4} - x\right)\right|$

45. If $y = \sin^{-1} x$ then $\frac{d^2y}{dx^2}$ is **[BOARD 2024]**

a) $\sec y$ b) $\sec y \tan y$ c) $\sec^2 y \tan y$ d) $\tan^y \sec y$

46. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$. **[BOARD 2024]**

a) $-\frac{1}{64}$ b) $-\frac{1}{32}$ c) $-32$ d) $-64$

47. If $e^{x^2y} = c$, then $\frac{dy}{dx}$ is **[BOARD 2024]**

a) $\frac{xe^{x^2y}}{2y}$ b) $\frac{-2y}{x}$ c) $\frac{2y}{x}$ d) $\frac{x}{2y}$

48. If $\sin(xy) = 1$ then $\frac{dy}{dx}$ is equal to **[BOARD 2024]**

a) $\frac{x}{y}$ b) $-\frac{x}{y}$ c) $\frac{y}{x}$ d) $-\frac{y}{x}$

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49. If $tan\left(\frac{x+y}{x-y}\right) = k$ then $\frac{dy}{dx}$ is equal to **[BOARD 2023]**

a) $-\frac{y}{x}$ b) $\frac{y}{x}$ c) $\sec^2\left(\frac{y}{x}\right)$ d) $-\sec^2\left(\frac{y}{x}\right)$

50. If $(\cos x)^y = (\cos y)^x$ then $\frac{dy}{dx}$ is equal to **[BOARD 2023]**

a) $\frac{y\tan x+\log(\cos y)}{x\tan y-\log(\cos x)}$ c) $\frac{y\tan x-\log(\cos y)}{x\tan y-\log(\cos x)}$

b) $\frac{x\tan y+\log(\cos x)}{y\tan x+\log(\cos y)}$ d) $\frac{y\tan x+\log(\cos y)}{x\tan y+\log(\cos x)}$

51. If $y = a\cos(\log x) + b\sin(\log x)$ then $x^2y_2 + xy_1$ is **[BOARD 2025]**

a) $\cot(\log x)$ b) y c) –y d) $\tan(\log x)$

**Options for Assertion and Reasoning Questions:**

a) Both assertion(A) and reason(R) are true and reason(R) is the correct explanation of assertion(A)

b) Both assertion(A) and reason(R) are true but reason(R) is not the correct explanation of assertion(A)

c) Assertion (A) is true but reason (R) is false

d) Assertion (A) is false but reason (R) is true

52. ***Assertion (A):*** $f(x) = \begin{cases} 3x-8, & x \le 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

***Reason (R):*** For a function f to be continuous at $x = a$,

$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$ **[BOARD 2025]**

53. ***Assertion (A):*** $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$. **[BOARD 2025]**

***Reason (R):*** When $x \to 0, \sin\frac{1}{x}$ is a finite value between -1 and 1.

**2 marks:**

1. Verify whether the function $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ or not.

**[BOARD 2024]**

2. Function f as defined as $f(x) = \begin{cases} 2x+2, & if\ x < 2 \\ k, & if\ x = 2 \\ 3x, & if\ x > 2 \end{cases}$ Find the value of k for which the function f is continuous at $x = 2$. **[BOARD 2023]**

3. Find the points at which the function $f(x) = \frac{4+x^2}{4x-x^3}$ is discontinuous.

**[BOARD 2023]**

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4. If $f(x) = \begin{cases} 2x-3, & -3 \le x \le -2 \\ x+1, & -2 < x \le 0 \end{cases}$ Check the differentiability of f(x) at $x = -2$.

**[BOARD 2025]**

5. Check whether the function $f(x) = x^2|x|$ is differentiable at $x = 0$ or not.

**[BOARD 2024]**

6. Check the differentiability of $f(x) = |\cos x|$ at $x = \frac{\pi}{2}$. **[BOARD 2024]**

7. Check the differentiability of the function $f(x) = |x - 5|$ at the point $x = 5$.

**[BOARD 2024]**

8. Check the differentiability of the function $f(x) = [x]$ at $x = -3$, where [.] denote greatest integer function. **[BOARD 2024]**

9. Check the differentiability of $f(x) = \begin{cases} x^2+1, & 0 \le x < 1 \\ 3-x, & 1 \le x \le 2 \end{cases}$ at $x = 1$. **[BOARD 2024]**

10. If $f(x) = \begin{cases} ax+b, & if\ 0 < x \le 1 \\ 2x^2 - x, & if\ 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2) then find the values of a and b. **[BOARD 2023]**

11. Show that the function $f(x) = |x|^3$ is differentiable at all points of its domain.

**[BOARD 2024]**

12. Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x. **[BOARD 2025]**

13. Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x. **[BOARD 2025]**

14. Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$. **[BOARD 2025]**

15. Differentiate $2^{\cos^2 x}$ w.r.t. $\cos^2 x$. **[BOARD 2025]**

16. If $\tan^{-1}(x^2 + y^2) = a^2$ then find $\frac{dy}{dx}$. **[BOARD 2025]**

17. If $y = x^{\frac{1}{x}}$ then find $\frac{dy}{dx}$ at $x = 1$. **[BOARD 2023]**

18. If $y = \cos^3(\sec^2 2t)$ find $\frac{dy}{dt}$. **[BOARD 2024]**

19. If $y = (\sin^{-1} x)^2$ then find $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$. **[BOARD 2024]**

20. If $y = 5\cos x - 3\sin x$ then prove that $\frac{d^2y}{dx^2} + y = 0$. **[BOARD 2025]**

21. If $y = cosec(\cot^{-1} x)$ then prove that $\sqrt{1 + x^2}\frac{dy}{dx} - x = 0$. **[BOARD 2024]**

22. If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$. **[BOARD 2024]**

23. If $y = \sqrt{\cos x + y}$ then prove that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$. **[BOARD 2024]**

24. If $y = \left(x + \sqrt{x^2 - 1}\right)^2$ then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$. **[BOARD 2023]**

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25. If $xy = e^{x-y}$ then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$. **[BOARD 2023]**

26. If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$. **[BOARD 2024]**

27. If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$. **[BOARD 2025]**

28. If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$. **[BOARD 2024]**

29. If $x^y = y^x$ then find $\frac{dy}{dx}$. **[BOARD 2024 & 2025]**

30. If $(x^2 + y^2)^2 = xy$ then find $\frac{dy}{dx}$. **[BOARD 2023]**

31. If $-2x^2 - 5xy + y^3 = 76$ then find $\frac{dy}{dx}$. **[BOARD 2025]**

32. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 1$ then find $\frac{dy}{dx}$ at the point $\left(\frac{1}{8}, \frac{1}{8}\right)$. **[BOARD 2024]**

33. If $f(x) = |\tan 2x|$ then find the value of $f'(x)$ at $x = \frac{\pi}{3}$. **[BOARD 2024]**

34. If $y = A \sin 2x + B \cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$ find the value of k. **[BOARD 2024]**

35. If $x = a \sin 2t$ , $y = a(\cos 2t + \log \tan t)$ then find $\frac{dy}{dx}$. **[BOARD 2023]**

36. If $x = \sqrt{a^{\tan^{-1} t}}, y = \sqrt{a^{\cot^{-1} t}}$ then show that $x \frac{dy}{dx} + y = 0$. **[BOARD 2023]**

**3 marks:**

1. Find k so that $f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$. **[BOARD 2025]**
2. Find the value of a and b so that the function is differentiable for all values of x:
   $f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \leq -1 \end{cases}$ **[BOARD 2024]**
3. Find the value of a and b so that the function f defined as
   $f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & x < 2 \\ a + b, & x = 2 \\ \frac{x-2}{|x-2|} + b, & x > 2 \end{cases}$ is a continuous function. **[BOARD 2024]**
4. Check the differentiability of the function $f(x) = x|x|$ at $x = 0$. **[BOARD 2025]**
5. If $y = (\tan x)^x$ then find $\frac{dy}{dx}$. **[BOARD 2024]**
6. Find $\frac{dy}{dx}$ if $5^x + 5^y = 5^{x+y}$. **[BOARD 2024]**
7. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$. **[BOARD 2024]**
8. Find $\frac{dy}{dx}$ if $y = x^{\cos x} - 2^{\sin x}$. **[BOARD 2024]**

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9. Given that $y = (\sin x)^x . x^{\sin x} + a^x$ find $\frac{dy}{dx}$. **[BOARD 2024]**

10. Given that $x^y + y^x = a^b$, where a and b are positive constants, find $\frac{dy}{dx}$. **[BOARD 2024]**

11. Given $y = (\cos x)^x + \cos^{-1}\sqrt{x}$, find $\frac{dy}{dx}$. **[BOARD 2024]**

12. If $y = \sin(\tan^{-1} e^x)$ then find $\frac{dy}{dx}$ at $x = 0$. **[BOARD 2024]**

13. Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x. **[BOARD 2025]**

14. Differentiate $\log(x^x + cosec^2 x)$ with respect to x. **[BOARD 2025]**

15. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$. **[BOARD 2023]**

16. Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. **[BOARD 2025]**

17. Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$. **[BOARD 2025]**

18. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$. **[BOARD 2025]**

19. If $x\cos(p+y) + \cos p \sin(p+y) = 0$, prove that $\cos p \frac{dy}{dx} = -\cos^2(p+y)$, where p is a constant. **[BOARD 2024]**

20. If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(x+1)^2}$. **[BOARD 2025]**

21. If $x^{30}y^{20} = (x+y)^{50}$ prove that $\frac{dy}{dx} = \frac{y}{x}$. **[BOARD 2024]**

22. If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$ then find $\frac{dy}{dx}$. **[BOARD 2024]**

23. If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$ prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$. **[BOARD 2024]**

24. If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$. **[BOARD 2024]**

25. If $x = a\sin^3\theta, y = b\cos^3\theta$ then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$. **[BOARD 2024]**

26. If $y = \tan x + \sec x$ then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$. **[BOARD 2023]**

27. If $y = (\log x)^2$ prove that $x^2y'' + xy' = 2$. **[BOARD 2024]**

28. If $y = (\tan^{-1} x)^2$ then show that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$. **[BOARD 2024]**

29. If $y = e^{a\cos^{-1}x}$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. **[BOARD 2024]**

30. If $x\sin(a+y) - \sin y = 0$ then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$. **[BOARD 2024]**

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31. Show that $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$. **[BOARD 2024]**

32. Show that the derivative of $\tan^{-1}(\sec x + \tan x), \left[-\frac{\pi}{2} < x < \frac{\pi}{2}\right]$ with respect to x is equal to $\frac{1}{2}$. **[BOARD 2025]**

**5 marks:**

1. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **[BOARD 2025]**
2. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$ then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$. **[BOARD 2025]**
3. Differentiate $\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}\left(2x\sqrt{1-x^2}\right), x \in \left(\frac{1}{\sqrt{2}}, 1\right)$. **[BOARD 2025]**
4. Find $\frac{dy}{dx}$, if $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$. **[BOARD 2025]**

**Case Based Questions:**

1. Let $f(x)$ be a real valued function. Then its **[BOARD 2023]**
   - Left Hand Derivative (L.H.D): $Lf'(a) = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$
   - Right Hand Derivative (R.H.D): $Rf'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

   Also, a function $f(x)$ is said to be differentiable at $x = a$ if its L.H.D and R.H.D at $x = a$ exist and both are equal.

   Foe the function $f(x) = \begin{cases} |x-3|, & if\ x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & if\ x < 1 \end{cases}$

   Answer for the following:

   (i) What is R.H.D of $f(x)$ at $x = 1$? **1**

   (ii) What is L.H.D of $f(x)$ at $x = 1$? **1**

   (iii) (a) Check if the function $f(x)$ is differentiable at $x = 1$. **2**

   **(OR)**

   (b) Find $f'(2)$and $f'(-1)$. **2**

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