

GRADE XII
Question Bank (MATHEMATICS)
Chapter-6 Applications of Derivatives

1 marks:

1. The point of inflection of a function $f(x)$ is the point where **[BOARD 2024]**
 - a) $f'(x) = 0$ and $f'(x)$ changes its sign from positive to negative from left to right of that point.
 - b) $f'(x) = 0$ and $f'(x)$ changes its sign from negative to positive from left to right of that point.
 - c) $f'(x) = 0$ and $f'(x)$ doesn't change its sign from left to right of that point.
 - d) $f'(x) = 0$.
2. The function $f(x) = kx - \sin x$ is strictly increasing for **[BOARD 2024]**
 - a) $k > 1$
 - b) $k < 1$
 - c) $k > -1$
 - d) $k < -1$
3. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are **[BOARD 2025]**
 - a) $1 < \lambda < \sqrt{2}$
 - b) $\lambda \geq 1$
 - c) $\lambda \geq \sqrt{2}$
 - d) $\lambda < 1$
4. If $f(x) = a(x - \cos x)$ is strictly decreasing in \mathbb{R} , then 'a' belongs to **[BOARD 2023]**
 - a) $\{0\}$
 - b) $(0, \infty)$
 - c) $(-\infty, 0)$
 - d) $(-\infty, \infty)$
5. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is **[BOARD 2023]**
 - a) $(-1, \infty)$
 - b) $(-2, -1)$
 - c) $(-\infty, -2)$
 - d) $[-1, 1]$
6. The function $f(x) = x^3 + 3x$ is increasing in interval **[BOARD 2023]**
 - a) $(-\infty, 0)$
 - b) $(0, \infty)$
 - c) \mathbb{R}
 - d) $(0, 1)$
7. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval **[BOARD 2025]**
 - a) $(0, 2)$
 - b) $(-\infty, 2]$
 - c) $[1, 2]$
 - d) $[2, \infty)$
8. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is **[BOARD 2024]**
 - a) Strictly decreasing on \mathbb{R}
 - b) Strictly increasing on \mathbb{R}
 - c) Neither strictly increasing nor strictly decreasing on \mathbb{R}
 - d) Strictly decreasing on $(-\infty, 0)$
9. $f(x) = x^x$ has a critical point at **[BOARD 2025]**
 - a) $x = e$
 - b) $x = e^{-1}$
 - c) $x = 0$
 - d) $x = 1$

10. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to **[BOARD 2024]**
 a) 2 b) 1 c) 0 d) -2
11. Let $f(x) = |x|, x \in R$ then, which of the following statements is incorrect? **[BOARD 2025]**
 a) f has a minimum value at $x = 0$ c) f is continuous at $x = 0$
 b) f has no maximum value in R d) f is differentiable at $x = 0$
12. For the function $f(x) = x^3, x = 0$ is a point of **[COMPARTMENT 2024]**
 a) Local maxima c) local minima
 b) Non-differentiability d) inflection
13. If $f(x) = 2x + \cos x$ then $f(x)$ **[BOARD 2025]**
 a) has a maxima at $x = \pi$ c) has a minima at $x = \pi$
 b) is an increasing function d) is a decreasing function
14. If $f: R \rightarrow R$ is defined as $f(x) = 2x - \sin x$, then f is **[BOARD 2025]**
 a) A decreasing function c) an increasing function
 b) Maximum at $x = \frac{\pi}{2}$ d) maximum at $x = 0$
15. The absolute maximum value of the function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is **[BOARD 2025]**
 a) 0 b) 2 c) 4 d) 5
16. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at **[BOARD 2025]**
 a) $(1, -10)$ b) $(1, 10)$ c) $(10, 1)$ d) $(-10, 1)$
17. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is **[COMPARTMENT 2024]**
 a) $\frac{2\pi}{3}$ cm/s b) π cm/s c) $\frac{4\pi}{3}$ cm/s d) 2π cm/s
18. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is **[BOARD 2024]**
 a) -60 unit/sec b) 60 units/sec c) -70 units/sec d) -140 units/sec
19. If the sides of a square are decreasing at the rate of 1.5 cm/s, the rate of decrease of its perimeter is **[BOARD 2024]**
 a) 1.5 cm/s b) 6 cm/s c) 3 cm/s d) 2.25 cm/s
20. The rate of change of surface area of a sphere with respect to its radius 'r', when $r = 4$ cm, is **[BOARD 2024]**
 a) 64π cm²/cm b) 48π cm²/cm c) 32π cm²/cm d) 16π cm²/cm

21. A spherical ball has a variable diameter $\frac{5}{2}(3x + 1)$. The rate of change of its volume w.r.t. 'x' when $x = 1$, is **[BOARD 2025]**
 a) 225π b) 300π c) 375π d) 125π
22. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of $100\pi \text{ cm}^3/\text{s}$. The rate, at which the height of the sugar inside the tank is increasing, is **[BOARD 2025]**
 a) 0.1 cm/s b) 0.5 cm/s c) 1 cm/s d) 1.1 cm/s
23. Edge of a variable cube increases at the rate of 5 cm/s . The rate at which the surface area of the cube increases when the edge is 2 cm long is **[BOARD 2025]**
 a) $24 \text{ cm}^2/\text{s}$ b) $120 \text{ cm}^2/\text{s}$ c) $12 \text{ cm}^2/\text{s}$ d) $5 \text{ cm}^2/\text{s}$

2 marks:

- Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing. **[COMPARTMENT 2023]**
- Find the intervals in which the function $f(x) = x^4 - 4x^3 + 10$, is strictly decreasing. **[BOARD 2024]**
- Find the intervals in which the function $f(x) = x^3 - 12x^2 + 36x + 17$, is strictly increasing. **[COMPARTMENT 2023]**
- Find the interval in which the function $f(x) = 2x^3 - 3x$ is strictly increasing. **[BOARD 2023]**
- Find the intervals in which the function $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is decreasing. **[COMPARTMENT 2023]**
- Find the intervals in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing. **[BOARD 2025]**
- Determine whether the function $f(x) = x^2 - 6x + 3$ is increasing or decreasing in $[4, 6]$. **[BOARD 2024]**
- If $f(x) = x + \frac{1}{x}$, $x \geq 1$ show that f is an increasing function. **[BOARD 2025]**
- Find the interval in which $f(x) = x + \frac{1}{x}$ is always increasing, $x \neq 0$. **[BOARD 2025]**
- Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$ is strictly decreasing in $(\frac{\pi}{2}, \pi)$. **[BOARD 2023]**
- Show that the function $f(x) = \frac{4 \sin x}{2 + \cos x} - x$ is an increasing in $[0, \frac{\pi}{2}]$. **[BOARD 2024]**

12. Show that the function $f(x) = \sin x + \cos x$ is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$. **[BOARD 2024]**
13. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain. **[BOARD 2024]**
14. If $f(x) = a(\tan x - \cot x)$, where $a > 0$, then find whether $f(x)$ is increasing or decreasing function in its domain. **[BOARD 2023]**
15. Find the sub-intervals in which $f(x) = \log(2+x) - \frac{x}{2+x}$, $x > -2$ is increasing or decreasing. **[BOARD 2023]**
16. Find the values of 'a' for which $f(x) = x^2 - 2ax + b$ is an increasing function for $x > 0$. **[BOARD 2025]**
17. Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on $[2, 4]$. **[BOARD 2025]**
18. Find the values of 'a' for which $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$ is decreasing on \mathbb{R} . **[BOARD 2025]**
19. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on \mathbb{R} . **[BOARD 2025]**
20. Determine the values of x for which $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function. **[BOARD 2025]**
21. Determine the values of x for which $f(x) = \frac{2}{x} - 5$, $x \neq 0$ is increasing or decreasing. **[BOARD 2025]**
22. Find the maximum and minimum values of the function given by $f(x) = 5 + \sin 2x$. **[BOARD 2023]**
23. Find local maximum value and local minimum value (whichever exists) for the function $f(x) = 4x^2 + \frac{1}{x}$, $x \neq 0$. **[BOARD 2024]**
24. Find the local maxima and local minima of the function $f(x) = \frac{8}{3}x^3 - 12x^2 + 18x + 5$. **[BOARD 2025]**
25. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$, $x \neq 0$ respectively, find the value of M-m. **[BOARD 2024]**
26. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima. **[BOARD 2024]**

27. If the product of two positive numbers is 9, find the numbers so that the sum of their squares is minimum. **[BOARD 2023]**
28. Find the absolute maximum and minimum values of the function
 $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}, x \in [0, 1]$. **[BOARD 2024]**
29. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius. **[BOARD 2023]**
30. The area of the circle is increasing at a uniform rate of $2 \text{ cm}^2/\text{sec}$. How fast is the circumference of the circle increasing when the radius $r = 5 \text{ cm}$?
[BOARD 2024]
31. If equal sides of an isosceles triangle with fixed base 10 cm are increasing at the rate of 4 cm/sec, how fast is the area of triangle increasing at an instant when all sides become equal? **[BOARD 2023]**
32. The volume of a cube is increasing at the rate of $6 \text{ cm}^3/\text{s}$. How fast is the surface area of cube increasing, when the length of an edge is 8 cm? **[BOARD 2024]**
33. The surface area of the cube increase at the rate of $72 \text{ cm}^2/\text{sec}$. Find the rate of change of its volume, when the edge of the cube measures 3 cm. **[BOARD 2024]**
34. Surface area of the balloon (spherical), when air is blown into it, increases at a rate of $5 \text{ mm}^2/\text{s}$. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing. **[BOARD 2025]**
35. Sand is pouring from a pipe at the rate of $15 \text{ cm}^3/\text{min}$. The falling sand forms a cone on the ground such that the height of the cone is always one-third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4 cm? **[BOARD 2024]**
36. A cylindrical water container has developed a leak at the bottom. The water is leaking at the rate of $5 \text{ cm}^3/\text{s}$ from the leak. If the radius of the container is 15 cm, find the rate at which the height of water is decreasing inside the container, when the height of water is 2 meter. **[BOARD 2025]**
37. The radius of a cylinder is decreasing at a rate of $2 \text{ cm}/\text{s}$ and altitude is increasing at the rate of $3 \text{ cm}/\text{s}$. Find the rate of change of volume of this cylinder when its radius is 4 cm and altitude is 6 cm. **[BOARD 2025]**
38. Let the volume of metallic hollow sphere be constant. If the inner radius increases at the rate of $2 \text{ cm}/\text{s}$, find the rate of increase of the outer radius when the radii are 2 cm and 4 cm respectively. **[BOARD 2025]**

39. Consider the statement "There exists at least one value of $b \in \mathbb{R}$ for which $f(x) = \frac{b}{x}, b \neq 0$ is strictly increasing in $\mathbb{R} - \{0\}$."

State True or False. Justify.

[BOARD 2023]

40. Find the points on the curve $6y = x^3 + 2$ at which ordinate is changing 8 times as fast as abscissa.

[BOARD 2023]

41. Find points on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

[BOARD 2023]

42. For the curve $y = 5x - 2x^3$ if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?

[BOARD 2025]

3 marks:

1. Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

[BOARD 2024]

2. Find the intervals in which the function $f(x) = \sin 3x - \cos 3x, 0 < x < \frac{\pi}{2}$ is strictly increasing.

[BOARD 2025]

3. Find the value of 'a' for which $f(x) = \sqrt{3} \sin x - \cos x - 2ax + 6$ is decreasing in \mathbb{R} .

[BOARD 2025]

4. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left[0, \frac{\pi}{4}\right]$.

[BOARD 2025]

5. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval $[1, 2]$.

[BOARD 2024]

6. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.

[BOARD 2025]

7. Find dimensions of a rectangle of perimeter 12 cm which will generate maximum volume when swept along a circular rotation keeping the shorter side fixed as the axis.

[BOARD 2025]

8. Show that of all the rectangles with a fixed perimeter, the square has the greatest area.

[BOARD 2025]

9. The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm?

[BOARD 2025]

10. The area of an expanding rectangle is increasing at the rate of $48 \text{ cm}^2/\text{s}$. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth is 4.5 cm?

[BOARD 2025]

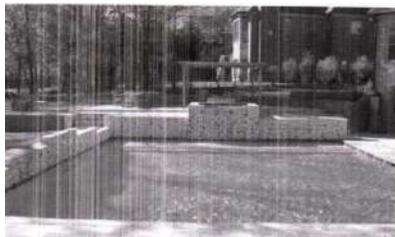
11. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius. **[BOARD 2025]**

5 marks:

1. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing. **[BOARD 2023]**
2. Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$. **[BOARD 2025]**
3. Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers. **[BOARD 2023]**
4. It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function $f(x)$ attains local maximum or local minimum values. **[BOARD 2024]**
5. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight. **[BOARD 2025]**
 - (i) Find the rate of growth of the plant with respect to sunlight. **2**
 - (ii) In how many days will the plant attain its maximum height? What is the maximum height? **3**
6. The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum. **[BOARD 2024]**

Case Based Questions:

1. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic meters. Cost of land is < 500 per square meter. The cost of digging increases with the depth and cost for the whole pool is < 4000 (depth)². **[COMPARTMENT 2023]**



Suppose the side of the square plot is x meters and depth is h meters.

On the basis of above information, answer the following questions:

- (i) Write cost $C(h)$ as a function in terms of h . 1
- (ii) Find critical point. 1
- (iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool? 2

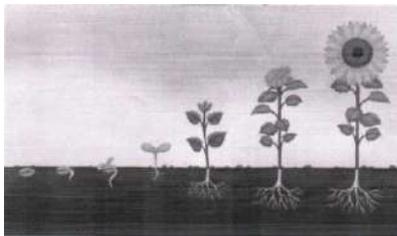
OR

- (b) Use first derivative test to find the depth of the pool so that cost of constructing is minimum. Also, find a relation between x and h for minimum cost. 2
2. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observes that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function **[COMPARTMENT 2023]**

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, 0 \leq x \leq 10$$

Where x is the number of days the plant is exposed to sunlight.



On the basis of above information, answer the following questions:

- (i) What are the critical points of the function $f(x)$? 2
- (ii) Using second derivative test, find minimum value of the function. 2
3. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank. A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2 \text{ cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45° . **[BOARD 2023]**



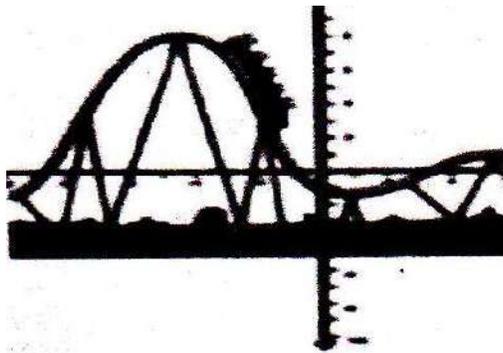
On the basis of above information, answer the following questions:

- (i) Find the volume of water in the tank in terms of its radius r . 1
- (ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. 1
- (iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. 2

OR

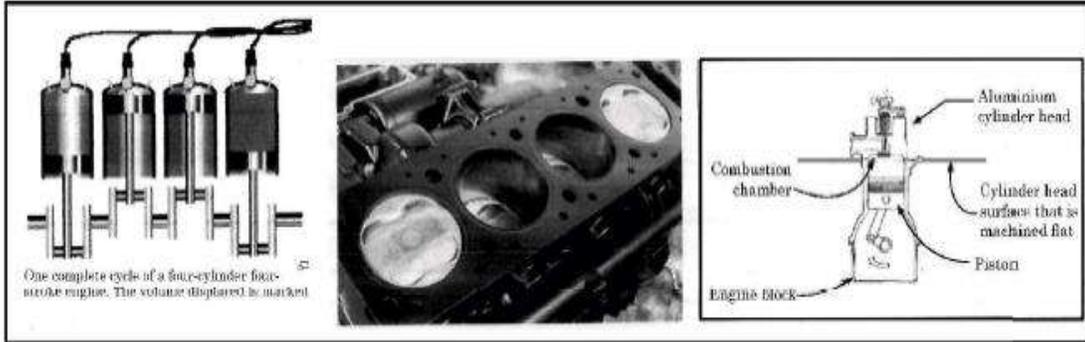
- (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm. 2

4. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following: **[BOARD 2023]**



On the basis of above information, answer the following questions:

- (i) Find the value of 'a'. 2
- (ii) Find $f''(x)$ at $x = 1$. 2
5. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore. The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm². **[BOARD 2023]**



On the basis of above information, answer the following questions:

- (i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r . **1**
- (ii) Find $\frac{dV}{dr}$. **1**
- (iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

- (b) For maximum volume, $h > r$. State true or false and justify. **2**

6. The use electric vehicles will curb air pollution in the long run.



The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

Where t represents the time and $t = 1, 2, 3 \dots$ corresponds to year 2001, 2002, 2003, ... respectively.

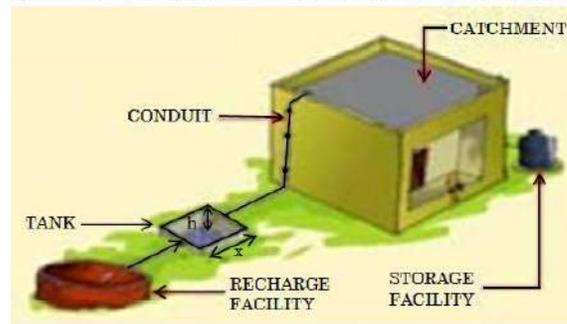
[BOARD 2023]

On the basis of above information, answer the following questions:

- (i) Can the above function be used to estimate number of vehicles in the year 2000? Justify. **2**
- (ii) Prove that the function $V(t)$ is an increasing function. **2**

7. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3 . The cost of land is Rs 5000 per square meter and cost of digging increases with depth and for the whole tank, it is Rs $40,000 h^2$, where h is the depth of the tank in meters. x is the side of the square base of the tank in meters. **[BOARD 2023]**

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



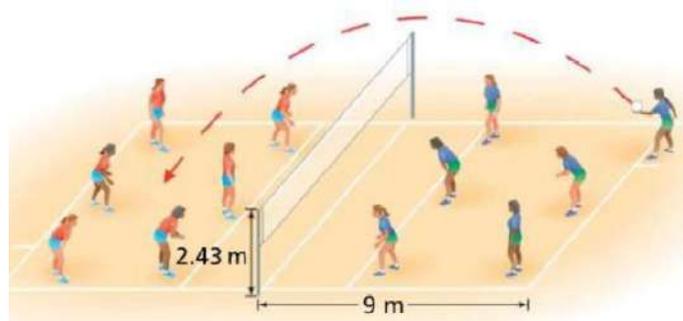
On the basis of above information, answer the following questions:

- (i) Find the total cost C of digging the tank in terms of x . **1**
- (ii) Find $\frac{dC}{dx}$. **1**
- (iii)(a) Find the value of x for which cost C is minimum. **2**

OR

- (b) Check whether the cost function $C(x)$ expressed in terms of x is increasing or not, where $x > 0$. **2**

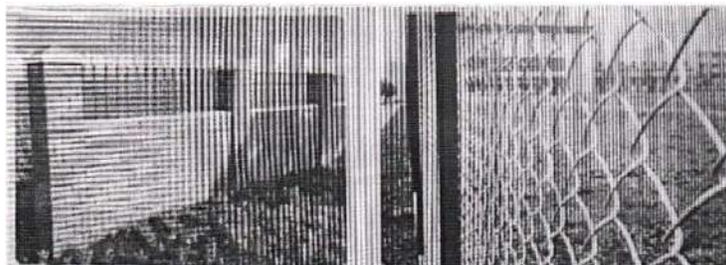
8. A volley ball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where $h(t)$ is the height of ball at any time t (in seconds), ($t \geq 0$). **[BOARD 2023]**



On the basis of above information, answer the following questions:

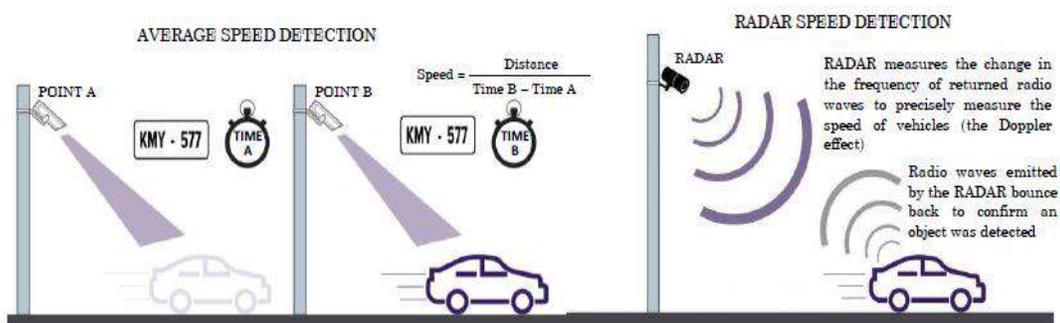
- (i) Is $h(t)$ a continuous function? Justify. **2**
- (ii) Find the time at which the height of the ball is maximum. **2**

9. Sooraj's father wants to construct a rectangular garden using brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 meters of fencing wire. **[BOARD 2023]**



On the basis of above information, answer the following questions:

- (i) Let 'x' meters denote the length of the side of the garden perpendicular to the brick wall and 'y' meters denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write $A(x)$, the area of the garden. **2**
- (ii) Determine the maximum value of $A(x)$. **2**
10. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark. **[BOARD 2024]**



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ .

On the basis of above information, answer the following questions:

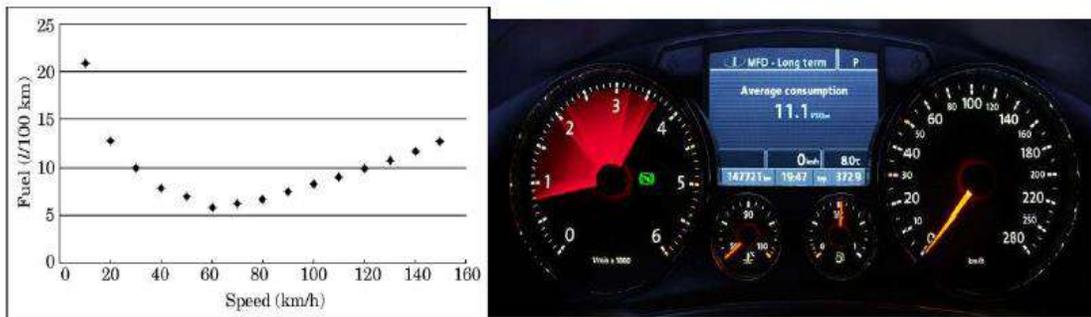
- (i) Express θ in terms of height of the camera installed on the pole and x. **1**
- (ii) Find $\frac{d\theta}{dx}$. **1**

- (iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. 2

OR

- (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. 2

11. Over speeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h. **[BOARD 2024]**



The relation between fuel consumption F ($\frac{l}{100}$ km) and speed V (km/hr) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of above information, answer the following questions:

- (i) Find F , when $V = 40$ km/hr. 1
- (ii) Find $\frac{dF}{dV}$. 1
- (iii) (a) Find the speed V for which fuel consumption F is minimum. 2

OR

- (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$. 2

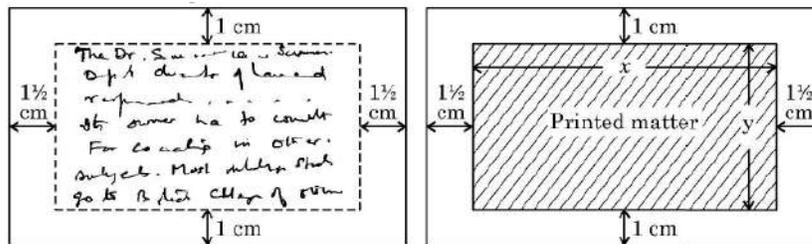
12. A store has been selling calculators at Rs 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$. **[BOARD 2024]**



On the basis of above information, answer the following questions:

- (i) Determine the number of units (x) that should be sold to maximize the revenue $R(x) = xp(x)$. Also, verify the result. **2**
- (ii) What rebate in price of calculator should the store give to maximize the revenue? **2**

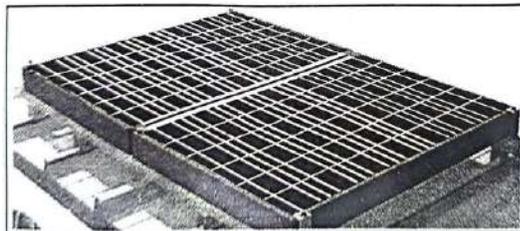
13. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm as shown below: **[BOARD 2024]**



On the basis of above information, answer the following questions:

- (i) Write the expression for the area of the visiting card in terms of x . **2**
- (ii) Obtain the dimensions of the card of minimum area. **2**

14. A technical company is designing a rectangular solar panel installation on a roof using 300 m of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.



Let the length of the side perpendicular to the partition be x m and with parallel to the partition be y m. **[BOARD 2025]**

On the basis of above information, answer the following questions:

- (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y . **1**
- (ii) Write the area of the solar panel as a function of x . **1**
- (iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. **2**
- (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 m of boundary material, considering the parallel partition. **2**

15. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x m from the start of the street can modelled by $f(x) = e^x \sin x$, where x is in meter.

[BOARD 2025]



On the basis of above information, answer the following questions:

- (i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$ **2**
- (ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflection. **2**

16. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

[BOARD 2025]

On the basis of above information, answer the following questions:

- (i) Taking length= breadth= x m and height= y m, express the surface area(S) of the box in terms of x and its volume (V), which is constant. **1**
- (ii) Find $\frac{dS}{dx}$. **1**

- (iii) (a) Find a relation between x and y such that the surface area (S) is minimum. **2**

OR

- (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$. **2**

17. A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.



On the basis of above information, answer the following questions:

- (i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. **1**
- (ii) Find the derivative of the area (A) with respect to the height on the wall (x) and find its critical point. **1**
- (iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. **2**
- (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s , then at what rate is the height on the wall (x) is increasing, when the foot of the ladder is 3 m away from the wall? **2**