BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION

PRE-BOARD EXAMINATION -1 (2025 − 2026)

GRADE XII

Date: 12.12.2025 **SET – 1** **Max Marks: 80**

Subject: Mathematics (041) **Time: 3 hours**

ANSWER KEY

SECTION : A

1) c) $3\pi/2$
2) d) 15
3) b) 1
4) d) 2^8
5) d) diag $(1\ \frac{1}{5}\ \frac{1}{4})$
6) d) 1
7) b) 1
8) a) $(1, -10)$
9) c) $y \geq \sqrt{2}$
10) b) 3
11) c) 0
12) d) $\frac{e^x}{(x-1)^2} + C$
13) a) $\sqrt{3}$
14) b) $3\hat{i} + \hat{j} + 6\hat{k}$
15) c) $3\theta = \pi$
16) b)
17) b) 1
18) c) $P(A \cap B) = \frac{1}{2} P(B)$
19) c)
20) b).

SECTION - B.

21) a) $\cot^2(\operatorname{cosec}^{-1} 3)$

$= \operatorname{cosec}^2(\operatorname{cosec}^{-1} 3) - 1$

$= 3^2 - 1$

$= 8$

; $\sin^2(\cos^{-1}\frac{1}{3})$

$= 1 - \cos^2(\cos^{-1}\frac{1}{3})$

$= 1 - (\frac{1}{3})^2$

$= \frac{8}{9}$

$\therefore \quad 8 + \frac{8}{9} = \frac{80}{9}$

21) b) $RHS = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ put $x = \tan^2 y$

$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2 y}{1+\tan^2 y}\right) = \frac{1}{2}\cos^{-1}(\cos 2y) = \frac{1}{2}\cdot 2y$

$= y$

$y = \tan^{-1}\sqrt{x} = LHS$ //

22) $x = a\sin^3\theta$; $y = b\cos^3\theta$

$\frac{dx}{d\theta} = a(3\sin^2\theta\cos\theta)$ $\quad \frac{dy}{d\theta} = b(-3\cos^2\theta\sin\theta)$

$\therefore \frac{dy}{dx} = -\frac{b}{a}\cot\theta$

$\frac{d^2y}{dx^2} = \frac{b}{a}\cdot\operatorname{cosec}^2\theta\left(\frac{d\theta}{dx}\right) = \frac{b}{a}(\operatorname{cosec}^2\theta)\left(\frac{1}{a\,3\sin^2\theta\cos\theta}\right).$

$\frac{d^2y}{dx^2} = \frac{b}{3a^2\sin^4\theta\cos\theta.}$ put $\theta = \pi/4$

$= \frac{b}{3a^2\left(\frac{1}{\sqrt{2}}\right)^4\left(\frac{1}{\sqrt{2}}\right)} = \frac{4\sqrt{2}\,b}{3a^2}$ //

23)

$\frac{2x+3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}.$

$2x+3 = Ax(x+3) + B(x+3) + C(x^2)$

Solve for coeff's, $A+C=0$, $3A+B=2$, $3B=3$

$B=1$, $A=\frac{1}{3}$, $C=-\frac{1}{3}.$

$\frac{2x+3}{x^2(x+3)} = \frac{1}{3}\cdot\frac{1}{x} + \frac{1}{x^2} - \frac{1}{3}\cdot\frac{1}{x+3}$

$\int \frac{2x+3}{x^2(x+3)}dx = \frac{1}{3}\log|x| - \frac{1}{x} - \frac{1}{3}\log|x+3| + c$

$= \frac{1}{3}\log\left|\frac{x}{x+3}\right| - \frac{1}{x} + C$

23) (OR)
(b)

$$\text{Area} = \int_{-2}^{-0.4} |5x+2|\,dx + \int_{-0.4}^{2} |5x+2|\,dx$$

$$= \int_{-2}^{-0.4} -(5x+2)\,dx + \int_{-0.4}^{2} (5x+2)\,dx$$

$$= \left(-\frac{5x^2}{2} - 2x\right)_{-2}^{-0.4} + \left(\frac{5x^2}{2} + 2x\right)_{-0.4}^{2}$$

$$= 6.4 + 14.4$$

$$= 20.8$$

24) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ (put $x=5$)

$f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$ $(f(x+y) = f(x) \times f(y))$

$= \lim_{h \to 0} \frac{f(5) \times f(h) - f(5)}{h} = \lim_{h \to 0} f(5) \left\{ \frac{f(h)-1}{h} \right\}$

$f'(5) = 2 \lim_{h \to 0} \left\{ \frac{f(h)-1}{h} \right\}$ — ①

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$

$3 = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$

$3 = \lim_{h \to 0} \frac{f(h) - 1}{h}$ — ②

$\Big(\because f(x+y) = f(x) f(y)$
$x = 0,\ y = 5$
$\cancel{f(5)} = f(0)\, \cancel{f(5)}$
$f(0) = 1 \Big)$

from ① & ②

$f'(5) = 2 \times 3 = 6$

25) Given $\hat{a}\cdot\hat{b} = \hat{a}\cdot\hat{c} = 0$

Thus $\hat{a}$ is perpendicular to $\hat{b}$ & $\hat{c}$

A unit vector perpendicular to $\hat{b}$ & $\hat{c}$

$$= \pm \frac{(\hat{b}\times\hat{c})}{|\hat{b}\times\hat{c}|} = \pm \frac{(\hat{b}\times\hat{c})}{|\vec{b}||\hat{c}|\sin\frac{\pi}{6}} = \pm 2(\hat{b}\times\hat{c})$$

SECTION C

26) $x + 2y = 100$ —(i)

$2x - y = 0$ —(ii)

$2x + y = 200$ —(iii)

For eqn (i)

x	0	100
y	50	0

For eqn (ii)

x	0	50
y	0	100

For eqn (iii)

x	100	0
y	0	200

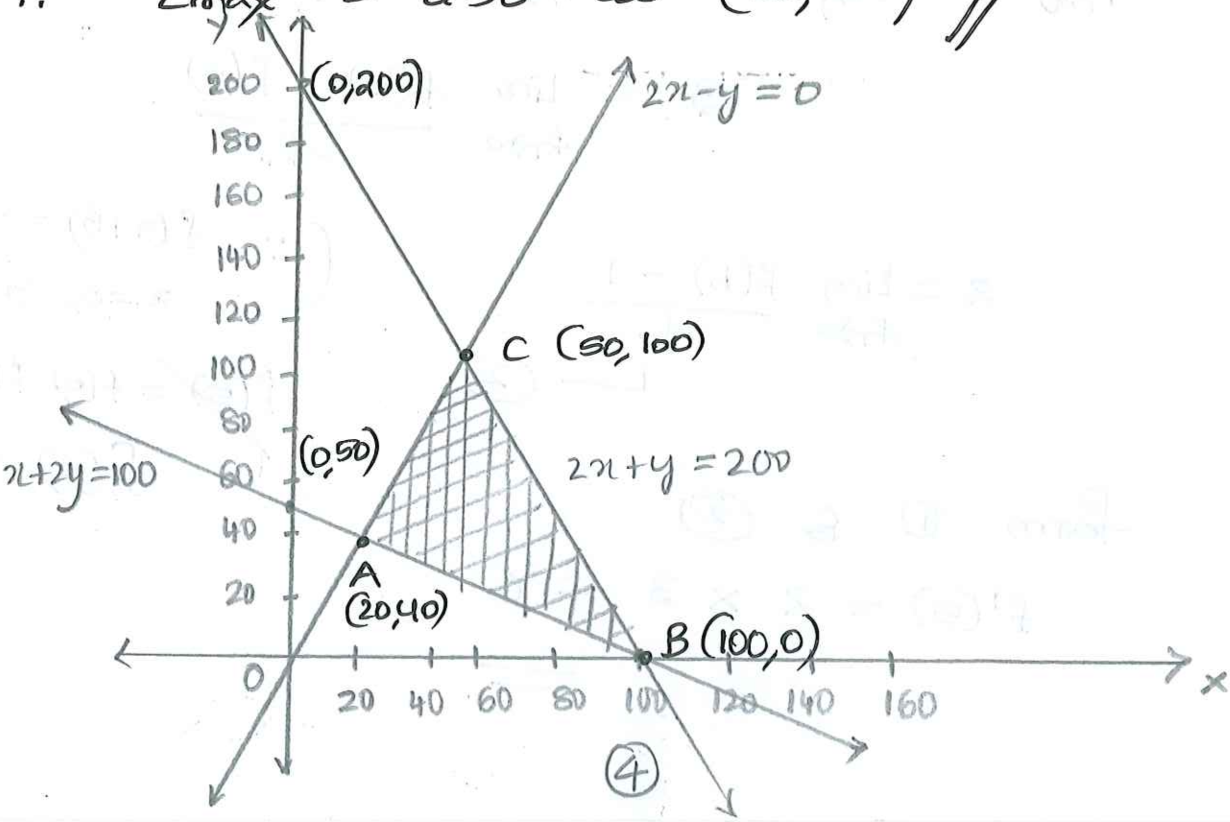
Maximum $Z = x + 2y$

For $A(20,40)$ $Z = 20 + 2(40) = 100$

$B(100,0)$ $Z = 100 + 0 = 100$

$C(50,100)$ $Z = 50 + 2(100) = 250$

$\therefore Z_{max} = 250$ at $(50,100)$ //



27) a) $A = \int_{-\sqrt{2}}^{\sqrt{3}} \sqrt{4-x^2}\, dx = \left\{ \frac{x}{2} \sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) \right\}_{-\sqrt{2}}^{\sqrt{3}}$

$= \left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right) - \left(\frac{-\sqrt{2}}{2}\sqrt{2} + 2\left[-\frac{\pi}{4}\right]\right)$

$= \left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right) - \left(-1 - \frac{\pi}{2}\right)$

OR

$= 1 + \frac{\sqrt{3}}{2} + \frac{7\pi}{6}$ Sq units //

b) Area $= \int_0^1 x^2\, dx + \int_1^3 x\, dx$

$= \left(x^3/3\right)_0^1 + \left(x^2/2\right)_1^3 = \frac{1}{3} + 4 = \frac{13}{3}$ sq units //

28) $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda$

$x = 3\lambda - 1, \quad y = 5\lambda - 3, \quad z = 7\lambda - 5.$

So, coordinates of a general point on this line are $(3\lambda-1, 5\lambda-3, 7\lambda-5)$

$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$

$x = \mu + 2, \quad y = 3\mu + 4, \quad z = 5\mu + 6$

Coordinates $(\mu+2, 3\mu+4, 5\mu+6)$.

If the lines intersect, then they have a common point.

$3\lambda - 1 = \mu + 2$ — ①, $\quad 5\lambda - 3 = 3\mu + 4$ — ②, $\quad 7\lambda - 5 = 5\mu + 6$ — ③

Solving ① & ② $\quad \lambda = 1/2, \quad \mu = -3/2$

Subst λ, μ values in eqn ③

$7(1/2) - 5 = 5(-3/2) + 6$

$-3/2 = -3/2$

∴ Given lines intersect each other.

P.o.i are $(1/2, -1/2, -3/2)$ //

28) b). $\vec{r_1} = \vec{a_1} + \lambda \vec{b_1}$ $\qquad \vec{a_1} = \hat{i} - \hat{j} + 0\hat{k}$

$\vec{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{r_2} = \vec{a_2} + \mu \vec{b_2}$ $\qquad \vec{a_2} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b_2} = 3\hat{i} + \hat{j} + 0\hat{k}$

$(\vec{a_2} - \vec{a_1}) = -2\hat{i} + 3\hat{j} + 2\hat{k}.$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= -\hat{i} + 3\hat{j} - 7\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-1)^2 + (3)^2 + (-7)^2}$$

$$= \sqrt{59}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -3.$$

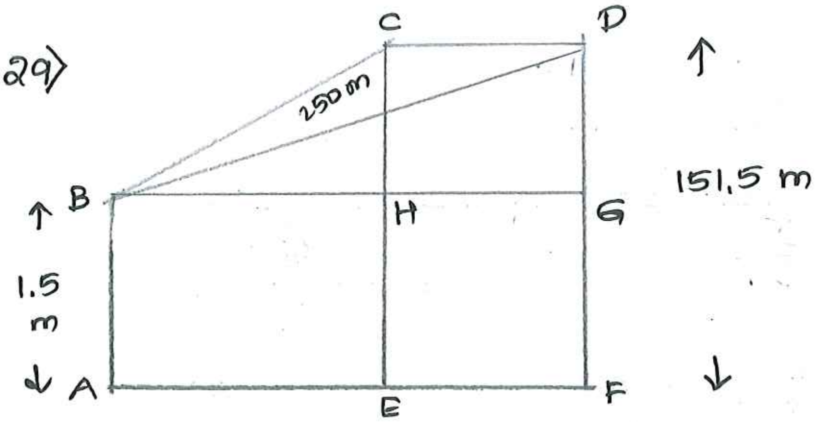
Shortest distance b/w the lines

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$d = \left| -\frac{3}{\sqrt{59}} \right|$$

$$d = \frac{3}{\sqrt{59}} \text{ units}$$

29)



$AF = x$ m

$BG = x$ m

$\frac{dx}{dt} = 10$ m/s

$BG^2 + GD^2 = BD^2$

$x^2 + (150)^2 = (250)^2$

$x^2 = 62500 - 22500$

$x^2 = 40000$

$BG^2 + GD^2 = BD^2$

$x^2 + (150)^2 = y^2$

$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$

$2 \times 200 \times 10 = 2 \times 250 \times \frac{dy}{dt}$

$dy/dt = 8$ m/s

30) a)

$x\cos(p+y) + \cos p.\sin(p+y) = 0$

$\div \cos(p+y)$

$x + \cos p.\tan(p+y) = 0$

$\tan(p+y) = -\frac{x}{\cos p}$

$\sec^2(p+y).\left(\frac{dy}{dx}\right) = -\frac{1}{\cos p}$

$\frac{dy}{dx} = -\frac{1}{\cos p} \times \cos^2(p+y)$

$\cos p.\frac{dy}{dx} = -\cos^2(p+y)$ //

(OR)

30) b) $y = x^{\sin x}.(\sin x)^x + a^x.e^x$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ — ①

$u = x^{\sin x} \cdot (\sin x)^{x}$

$\log u = \log\left(x^{\sin x} \cdot \sin x^{x}\right)$

$= \log\left(x^{\sin x}\right) + \log\left(\sin x\right)^{x}$

$\log u = \sin x . \log x + x . \log(\sin x)$

D w.r.t x.

$\frac{du}{dx} = u \left\{ \frac{\sin x}{x} + \cos x . \log x + x \cot x + \log(\sin x) \right\}$ —— ②

$v = a^{x} . e^{x}$

$\frac{dv}{dx} = a^{x} e^{x} (1 + \log a)$ —— ③

Substitute ②, ③ in ① //

31) Let A be event that a person uses public transport.
B be event that a person uses a bicycle.

$P(A) = 0.50$

$P(B) = 0.35$

$P(A \cap B) = 0.20$

i) $P(A \cap B') = P(A) - P(A \cap B) = 0.30$

ii) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.20}{0.50} = \frac{0.2}{0.5} = \frac{2}{5}$

$= 0.40$

iii) $P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - 0.65$

$= 0.35$ //

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \left(P(A \cup B) = 0.65\right)$.

SECTION - D

32)
$$BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I.$$

The given system of linear equations is :

$$\left.\begin{aligned} 2y + 2z &= 8 \\ x - y &= -1 \\ 2x + 3y + 4z &= 20 \end{aligned}\right\}$$ rearranged and written in the matrix form $$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 20 \end{bmatrix}$$

The matrix form now
$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 20 \\ 8 \end{bmatrix}$$

$$BX = D$$

$$X = B^{-1}.D$$

$$X = \frac{1}{6} A.D$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ 20 \\ 8 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 1$

$y = 2$

$z = 3$ //

33) a) $I = \int_0^{\pi/4} \frac{x}{1+\cos 2x + \sin 2x}\,dx$ —— ①

$\because \int_0^a f(x) = \int_0^a f(a-x)\,dx.$

$I = \int_0^{\pi/4} \frac{(\pi/4 - x)}{1+\cos 2\left(\frac{\pi}{4}-x\right) + \sin 2\left(\frac{\pi}{4}-x\right)}\,dx$

$I = \int_0^{\pi/4} \frac{\pi/4 - x}{1+\cos 2x + \sin 2x}\,dx$ —— ②

① + ②

$2I = \frac{\pi}{4}\int_0^{\pi/4} \frac{1}{1+\cos 2x + \sin 2x}\,dx = \cdot$

$I = \frac{\pi}{8}\int_0^{\pi/4} \frac{\sec^2 x}{2(1+\tan x)}\,dx$

Put $1+\tan x = t$

$\sec^2 x\,dx = dt$

$t = 01, \ t = 2$

$I = \frac{\pi}{8}\int_1^2 \frac{1}{2t}\,dt = \frac{\pi}{8}\times\frac{1}{2}\left\{\log|t|\right\}_1^2 = \frac{\pi}{16}\left\{\log 2 - \log 1\right\}$

$= \frac{\pi}{16}\log 2$ //

OR

(b)

$5x-3 = A(4-4x) + B.$

Solve for A and B,

$A = -5/4$

$B = 2.$

$I = \int \frac{5x-3}{\sqrt{1+4x-2x^2}}\,dx = \int \frac{A(4-4x)+B}{\sqrt{1+4x-2x^2}}\,dx$

$I = \int \frac{-\frac{5}{4}(4-4x)}{\sqrt{1+4x-2x^2}}\,dx + \int \frac{2}{\sqrt{1+4x-2x^2}}\,dx$

$= -\frac{5}{4}\,2\sqrt{1+4x-2x^2} + 2\int \frac{1}{\sqrt{3-2(x-1)^2}}\,dx$

$\because \int \frac{1}{\sqrt{a^2-u^2}}\,dx$

$= -\frac{5}{2}\sqrt{1+4x-2x^2} + 2\sin^{-1}\left(\frac{\sqrt{2}(x-1)}{\sqrt{3}}\right) + c$

34) a) $\frac{dy}{dx} = \cos x - 2y$

$\frac{dy}{dx} + 2y = \cos x \quad ; \quad \frac{dy}{dx} + Py = Q \quad ; \quad P(x) = 2 \, , \; Q(x) = \cos x$

$$IF = e^{\int p\,dx} = e^{\int 2\,dx} = e^{2x}.$$

$$y\,IF \Rightarrow y\{e^{2x}\} = \int Q\,IF\,dx \Rightarrow \int e^{2x} . \cos x\,dx$$

$$y\,e^{2x} = \frac{e^{2x}}{5}(2\cos x + \sin x) + C$$

OR

$$y = \frac{1}{5}(2\cos x + \sin x) + C.e^{-2x}.$$

34) b)

$$\{\sqrt{(x+y)} + \sqrt{(x-y)}\}dx + \{\sqrt{x-y} - \sqrt{x+y}\}dy = 0$$

Let $\sqrt{x+y} = u$, $\sqrt{x-y} = v$

$$x = \frac{u^2+v^2}{2} \quad ; \quad y = \frac{u^2-v^2}{2}.$$

$$dx = u\,du + v\,dv \; ; \; dy = u\,du - v\,dv.$$

$$\therefore (u+v)(u\,du + v\,dv) + (v-u)(u\,du - v\,dv) = 0$$

$$2uv\,du + 2uv\,dv = 0$$

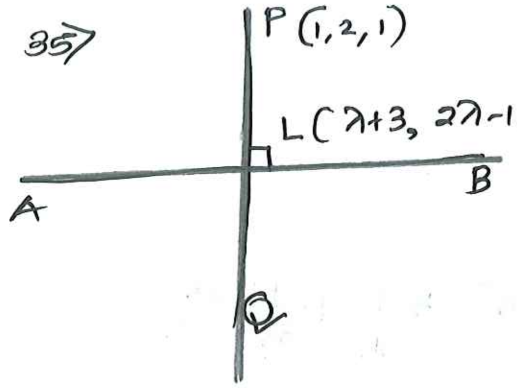
$$du + dv = 0$$

$$\int du + \int dv = \int 0$$

$$u + v = c$$

$$\sqrt{x+y} + \sqrt{x-y} = c$$

35)



Let P(1, 2, 1) be the given point.
Let L be the foot of $\perp^r$.

$$\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3} = \lambda.$$

$$x = \lambda+3, \quad y = 2\lambda-1, \quad z = 3\lambda+1$$

L be $(\lambda+3, 2\lambda-1, 3\lambda+1)$.

So, Direction ratios of PL are $(\lambda+2, 2\lambda-3, 3\lambda)$.

Direction ratios of given line are (1, 2, 3), which is $\perp^r$ to PL.

$$(\lambda+2)(1) + (2\lambda-3)(2) + (3\lambda)(3) = 0$$

$$\Rightarrow 14\lambda = 4 \Rightarrow \boxed{\lambda = 2/7}$$

$\therefore$ Coordinates of the point L are $\left(\frac{23}{7}, -\frac{3}{7}, \frac{13}{7}\right)$.

Let $Q(x_1, y_1, z_1)$ be the image of P(1, 2, 1) w.r.t to the given line. Then L is the midpoint of PQ.

$$\frac{1+x_1}{2} = \frac{23}{7}, \quad \frac{2+y_1}{2} = -\frac{3}{7}, \quad \frac{1+z_1}{2} = \frac{13}{7}$$

Hence image of the point P(1, 2, 1) to the given line Q

$$Q\left(\frac{39}{7}, -\frac{20}{7}, \frac{19}{7}\right).$$

The equation of the line joining P(1, 2, 1) and $Q\left(\frac{39}{7}, -\frac{20}{7}, \frac{19}{7}\right)$ is given by

$$\frac{x-1}{\frac{39}{7}-1} = \frac{y-2}{\frac{-20}{7}-2} = \frac{z-1}{\frac{19}{7}-1}$$

$$\frac{x-1}{\frac{32}{7}} = \frac{y-2}{-\frac{34}{7}} = \frac{z-1}{\frac{12}{7}}$$

$$\frac{x-1}{16} = \frac{y-2}{-17} = \frac{z-1}{6} \quad //$$

37) i) Given that $x = 3y$

volume of the brick $x.y.h = 1$

$3y^2h = 1$

ii) $S = 2(xy + yh + hx) = 2\left(3k^2 + k.\frac{1}{3k^2} + \frac{1}{3k^2}.3k\right)$

$= 2\left(3y^2 + \frac{4}{3k}\right).$

iii) $S = 2\left(3y^2 + \frac{4}{3k}\right)$

$\frac{ds}{dy} = 2\left(6y - \frac{4}{3y^2}\right)$; $\frac{d^2s}{dy^2} = 2\left(6 + \frac{8}{3k^3}\right)$

for $\frac{ds}{dy} = 0$, $4\left(3y - \frac{2}{3y^2}\right) = 0 \Rightarrow 9y^3 = 2$

$\therefore y = \left(\frac{2}{9}\right)^{1/3}$

Note that $\left(\frac{d^2s}{dk^2}\right) = 2\left\{3 \times 2 + \frac{8}{3 \times \frac{2}{9}}\right\} = 36 > 0$

S is minimum at $y = (2/9)^{1/3}$.

(OR)

iii) $S = 2\left(3y^2 + \frac{4}{3k}\right)$

$\frac{ds}{dk} = 2\left(6y - \frac{4}{3y^2}\right)$; $\frac{d^2s}{dy^2} = 2\left(6 + \frac{8}{3y^3}\right)$

for $\frac{ds}{dy} = 0 \Rightarrow y = (2/9)^{1/3}$.

Note $\frac{d^2s}{dy^2} = 36 > 0$

Surface area $= S = 2\left\{3y^2 + \frac{4}{3k}\right\}$ $\left(y = \left(\frac{2}{9}\right)^{1/3}\right)$

$= 4\left\{\frac{9}{2}\right\}^{1/3}$ square feet //

36) i) A relation is reflexive if every element is related to itself. But from the data, we see no department sends messages to itself.
So, the communication relation (X) is not reflexive.

ii) A relation is symmetric if whenever (A, B) is in the relation then (B, A) is also in the relation.

So, the communication relation (X) is not symmetric.

iii) From the data, the communication relation (X) as ordered pairs is given by

$X = \{(P, Q), (P, R), (P, S), (Q, R), (Q, T), (R, T), (S, T), (S, Q)\}$

Now Domain $= \{P, Q, R, S\}$

Range $= \{Q, R, S, T\}$.

Note that, $(P, R) \in X$, $(R, T) \in X$ but $(P, T) \notin X$

So it is not transitive.

(OR)

iii) A relation is a function if each input maps to exactly one output. Note that the communication relation (X) has (P, Q), (P, R) and (P, S) in it.

That is element P is mapped to more than one element.

Hence, the communication relation (X) is not a function//

38) Let E_1, E_2, E_3 respectively be the three screen time groups of students - Group A, B, C.
Also let S be the event that a student has poor sleep time.

$\therefore P(E_1) = 45\%$, $P(E_2) = 35\%$, $P(E_3) = 20\%$

$P(S|E_1) = 75\%$, $P(S|E_2) = 60\%$, $P(S|E_3) = 20\%$

i) By using total probability theorem,

$$P(S) = P(S|E_1) \times P(E_1) + P(S|E_2) \times P(E_2) + P(S|E_3) \times P(E_3)$$

$$= 0.75 \times 0.45 + 0.60 \times 0.35 + 0.20 \times 0.20$$

$$= 0.5875.$$

Hence 58.75 % of students suffer from poor sleep quality.

ii) Using Bayes Theorem, $P(E_1|S) = \dfrac{P(S|E_1) \times P(E_1)}{P(S)}$

$$= \frac{0.75 \times 0.45}{0.5875}$$

$$= \frac{0.3375}{0.5875}$$

$$= \frac{27}{47} \;//$$