Answer key

1) A) 9 Sec

2) c) $\log\left|1 - \cot\frac{x}{2}\right| + C$

3) $a)\ [2\pi\ , 3\pi\ ]$

4) c) 1

5) b)25

6) (c) 5

7) (d) ±8

8) (d) 0,1,-1.

9) (a) π

10) $d)\ \dfrac{-1}{2at^3}$

11) (c) 1/x

12) (c) 1/5

13) b) 112

14) b) 3,1

15) c)2^9A

16) c) 1

17) c) 36a

18) d) 4

19) (c) A is true but R is false.

20) (d) A is false but R is true.

SECTION-B

21)

$\sin^{-1}\dfrac{8}{17} + \sin^{-1}\dfrac{3}{5}$

$= \tan^{-1}\dfrac{8}{\sqrt{17^2 - 8^2}} + \tan^{-1}\dfrac{3}{\sqrt{5^2 - 3^2}}\quad \cdots\left[\sin^{-1}\dfrac{p}{h} = \tan^{-1}\dfrac{p}{\sqrt{h^2 - p^2}}\right]$

$= \tan^{-1}\dfrac{8}{\sqrt{289 - 64}} + \tan^{-1}\dfrac{3}{\sqrt{25 - 9}}$

$= \tan^{-1}\dfrac{8}{\sqrt{225}} + \tan^{-1}\dfrac{3}{\sqrt{16}}$

$= \tan^{-1}\dfrac{8}{15} + \tan^{-1}\dfrac{3}{4}$

$= \tan^{-1}\left(\dfrac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \times \frac{3}{4}}\right)\quad \cdots\left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1 - x \times y}\right)\right]$

$= \tan^{-1}\left[\dfrac{\frac{32+45}{60}}{1 - \frac{24}{60}}\right]$

$= \tan^{-1}\dfrac{77}{36}$

22)

Put x = sin θ, y = sin Φ

∴ θ = $\sin^{-1}$ x, Φ = $\sin^{-1}$ y

$\sqrt{1 - \sin^2\theta} + \sqrt{1 - \sin^2\phi} = a(\sin\theta - \sin\phi)$

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$2\cos\left(\dfrac{\theta+\phi}{2}\right)\cdot\cos\left(\dfrac{\theta-\phi}{2}\right) = 2a\cos\left(\dfrac{\theta+\phi}{2}\right)\cdot\sin\left(\dfrac{\theta-\phi}{2}\right)$

$\dfrac{\cos\left(\frac{\theta-\phi}{2}\right)}{\sin\left(\frac{\theta-\phi}{2}\right)} = a$

$\cot\left(\dfrac{\theta-\phi}{2}\right) = a$

$\dfrac{\theta-\phi}{2} = \cot^{-1}a$

$\theta - \phi = 2\cot^{-1}a$

$\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

Differentiating w.r.t. x, we get

$\dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{\sqrt{1-y^2}}\cdot\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

23)

On differentiating both sides w.r.t. x, we get

$$f'(x) = (x-1)^3 \cdot \frac{d}{dx}(x-2)^2$$

$$+(x-2)^2 \cdot \frac{d}{dx}(x-1)^3 \quad \left[\because \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \right]$$

$$f'(x) = (x-1)^3 \cdot 2(x-2)$$
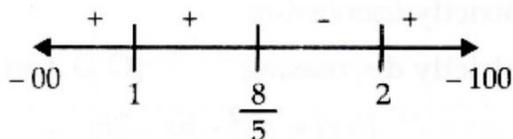$$+ (x-2)^2 \cdot 3(x-1)^2$$
$$= (x-1)^2 (x-2) [2(x-1) + 3(x-2)]$$
$$= (x-1)^2 (x-2) (2x-2+3x-6)$$

or $f'(x) = (x-1)^2 (x-2) (5x-8)$

Now, put $f'(x) = 0$

or $(x-1)^2 (x-2) (5x-8) = 0$

Either $(x-1)^2 = 0$ or $x-2 = 0$ or $5x-8 = 0$

$$
\begin{array}{ccccc}
+ & + & - & + \\
\end{array}
$$

$-00 \qquad 1 \qquad \dfrac{8}{5} \qquad 2 \qquad -100$

$\therefore \qquad x = 1, \dfrac{8}{5}, 2$

Interval	$f'(x) = (x-1)^2$ $(x-2)(5x-8)$	Sign of $f'(x)$
$x < 1$	$(+)(-)(-)$	+ve
$1 < x < \dfrac{8}{5}$	$(+)(-)(-)$	+ve
$\dfrac{8}{5} < x < 2$	$(+)(-)(+)$	-ve
$x > 2$	$(+)(+)(+)$	+ve

Since, $f(x)$ is a polynomial function, so it is continuous at $x = 1, \dfrac{8}{5}, 2$. Hence, $f(x)$ is

(i) increasing on intervals $\left(-\infty, \dfrac{8}{5} \right]$ and $[2, \infty)$

0R

b)

Let x, y be the length and breadth of a rectangle and given perimeter is P ....(say)

ie. $2(x + y) = P$

$$y = \frac{P}{2} - x$$

Area of a rectangle 'A' = xy

$$A = x \left(\frac{P}{2} - x \right) = \frac{P}{2}x - x^2$$

$$\frac{dA}{dx} = \frac{P}{2} - 2x$$

For maximum or minimum,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \frac{P}{2} - 2x = 0$$

$$x = \frac{P}{4}$$

Now, $\dfrac{d^2A}{dx^2} = -2$

At $x = \frac{P}{4}$, $\dfrac{d^2A}{dx^2} < 0$

∴ Area of the rectangle is maximum when $x = \dfrac{P}{4}$

Now, $y = \dfrac{P}{2} - x = \dfrac{P}{2} - \dfrac{P}{4} = \dfrac{P}{4}$

∴ Length of a rectangle $= \dfrac{P}{4}$

Breadth of a rectangle $= \dfrac{P}{4}$

Since Length = Breadth, the rectangle is a square.

24)

$\overrightarrow{OA} = 4\hat{i} + 3\hat{k}$

$\overrightarrow{OB} = \hat{k}$

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$= \left(4\hat{i} + 3\hat{k}\right) - \hat{k}$

$= 4\hat{i} + 2\hat{k}$

$\left|\overrightarrow{BA}\right| = \sqrt{16 + 4}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$\widehat{BA} = \dfrac{4}{2\sqrt{5}}\hat{i} + \dfrac{2}{2\sqrt{5}}\hat{k}$

$= \dfrac{2}{\sqrt{5}}\hat{i} + \dfrac{1}{\sqrt{5}}\hat{k}$

So, the angles made by the vector $\overrightarrow{BA}$ with the x, y and the z axes are respectively $\cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right), \dfrac{\pi}{2}, \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$.

25)

Here, $\vec{a} = 2\lambda^2\hat{i} + 4\lambda\hat{j} + \hat{k}$

and $\vec{b} = 7\hat{i} - 2\hat{j} + \lambda\hat{k}$

If θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, then

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

for θ to be obtuse
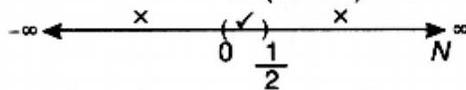
$$\cos\theta < 0 \text{ or } \vec{a}\cdot\vec{b} < 0$$

$$(2\lambda^2 \hat{i} + 4\lambda \hat{j} + \hat{k}) \cdot (7\hat{i} - 2\hat{j} + \lambda\hat{k}) < 0$$

$$14\lambda^2 - 8\lambda + \lambda < 0$$

$$14\lambda^2 - 7\lambda < 0$$

$$2\lambda^2 - \lambda < 0$$

$$\lambda(2\lambda - 1) < 0$$

$$\lambda \in \left(0, \frac{1}{2}\right)$$

SECTION C

26)

consider $\dfrac{x^2}{(x^2 + 2)(2x^2 + 1)}$

Put $x^2 = t$ (For finding partial fractions only)

$$\frac{t}{(t + 2)(2t + 1)} = \frac{A}{t + 2} + \frac{B}{2t + 1}$$

t=A(2t+1)+B(t+2)

On Solving we get A=2/3, B=-1/3

$$\frac{t}{(t + 2)(2t + 1)} = \frac{\frac{2}{3}}{t + 2} + \frac{-\frac{1}{3}}{2t + 1}$$

$$\frac{x^2}{(x^2 + 2)(2x^2 + 1)} = \frac{\frac{2}{3}}{t + 2} + \frac{-\frac{1}{3}}{2t + 1}$$

$$I = \int \left[\frac{\frac{2}{3}}{t + 2} + \frac{-\frac{1}{3}}{2t + 1} \right] dx$$

$$= \frac{2}{3} \int \frac{1}{x^2 + 2} dx - \frac{1}{3} \int \frac{1}{2x^2 + 1} dx$$

$$= \frac{2}{3} \int \frac{1}{x^2 + \left(\sqrt{2}\right)^2} dx - \frac{1}{6} \int \frac{1}{x^2 + \left(\frac{1}{\sqrt{2}}\right)^2} dx$$

$$= \frac{\sqrt{2}}{3} \tan^{-1} \left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3\sqrt{2}} \tan^{-1} \left(\sqrt{2}x\right) + c$$

27)

$$= \int \sqrt{\frac{\sin(x-a)}{\sin(x+a)} \times \frac{\sin(x-a)}{\sin(x-a)}} \, dx$$

$$= \int \frac{\sin(x-a)}{\sqrt{\sin^2 x - \sin^2 a}} \, dx$$

$$= \int \frac{(\sin x \cos a - \cos x \sin a)}{\sqrt{\sin^2 x - \sin^2 a}} \, dx$$

$$= \cos a \int \frac{\sin x}{\sqrt{\sin^2 x - \sin^2 a}} \, dx$$

$$- \sin a \int \frac{\cos x}{\sqrt{\sin^2 x - \sin^2 a}} \, dx$$

$$= \cos a \int \frac{\sin x}{\sqrt{\cos^2 a - \cos^2 x}} \, dx$$

$$- \sin a \int \frac{\cos x}{\sqrt{\sin^2 x - \sin^2 a}} \, dx$$

$$= -\cos a \sin^{-1}\left(\frac{\cos x}{\cos a}\right)$$

$$- \sin a \log\left|\sin x + \sqrt{\sin^2 x - \sin 2a}\right| + c$$

28)

Let $I = \int_0^\pi \frac{x}{1 + \sin x} \, dx$ ...(i)

On using property

$$\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$$

$$\therefore I = \int_0^\pi \frac{\pi - x}{1 + \sin(\pi - x)} \, dx$$

$$I = \int_0^\pi \frac{\pi - x}{1 + \sin x} \, dx \quad \text{...(ii)}$$

Adding equations (i) and (ii), we get

$$2I = \int_0^\pi \frac{\pi}{1 + \sin x} dx$$

$$= \pi \int_0^\pi \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx \quad ...[\text{Multiplying and dividing by } (1 - \sin x)]$$

$$= \pi \int_0^\pi \frac{1 - \sin x}{1 - \sin^2 x} dx = \pi \int_0^\pi \frac{1 - \sin x}{\cos^2 x} dx$$

$$= \pi \left[\int_0^\pi \frac{1}{\cos^2 x} dx - \int_0^\pi \frac{\sin x}{\cos^2 x} dx \right]$$

$$= \pi \left[\int_0^\pi \sec^2 x \, dx - \int_0^\pi \sec x \tan x \, dx \right]$$

$$= \pi \left[[\tan x]_0^\pi - [\sec x]_0^\pi \right]$$

$$= \pi[0 - (-1 - 1)]$$

$$= 2\pi$$

$$\therefore I = \frac{2\pi}{2} = \pi.$$

OR

b)

$$\int_{-a}^a \sqrt{\frac{a - x}{a + x}} dx$$

Let $I = \int_{-a}^a \sqrt{\frac{a - x}{a + x}} dx$

$$= \int_{-a}^a \sqrt{\frac{(a - x)(a - x)}{(a + x)(a - x)}} dx$$

$$= \int_{-a}^a \frac{a - x}{\sqrt{a^2 - x^2}} dx$$

$$= \int_{-a}^a \frac{a}{\sqrt{a^2 - x^2}} dx - \int_{-a}^a \frac{x}{\sqrt{a^2 - x^2}} dx$$

[but $\dfrac{a}{\sqrt{a^2 - x^2}}$ is an is an even function and $\dfrac{x}{\sqrt{a^2 - x^2}}$ is an odd function]

$$= 2a. \left[\sin^{-1} \left(\frac{x}{a} \right) \right]_0^a$$

$$= 2a. \left[\sin^{-1} 1 - \sin^{-1} 0 \right]$$

$$= 2a \left[\frac{\pi}{2} - 0 \right]$$

$$\int_{-a}^a \sqrt{\frac{a - x}{a + x}} . dx = \pi a$$

29)

a)

This is a linear differential equation in y.

Integrating factor:
IF = e^(∫ sec²(x) dx) = e^(tan x).

Multiply the equation by the integrating factor:
d/dx [y · e^(tan x)] = tan(x) sec²(x) e^(tan x).

Integrate:
y · e^(tan x) = (tan x - 1) e^(tan x) + C.

Thus,
y = tan x - 1 + C e^(-tan x).

Apply initial condition y(0) = 0:
0 = 0 - 1 + C ⟹ C = 1.

Particular Solution:
y(x) = tan x − 1 + e^(−tan x)

OR

b)

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} \qquad ...(1)$$

Let F (x,y) = $\dfrac{y + \sqrt{x^2 + y^2}}{x}$

∴ F(λx, λy) = $\dfrac{\lambda x + \sqrt{(\lambda x)^2 + (\lambda y)^2}}{\lambda x}$

$$= \frac{y + \sqrt{x^2 + y^2}}{x} = \lambda^0 . F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

y = vx

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of v and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + (vx)^2}}{x}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$$

$$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$
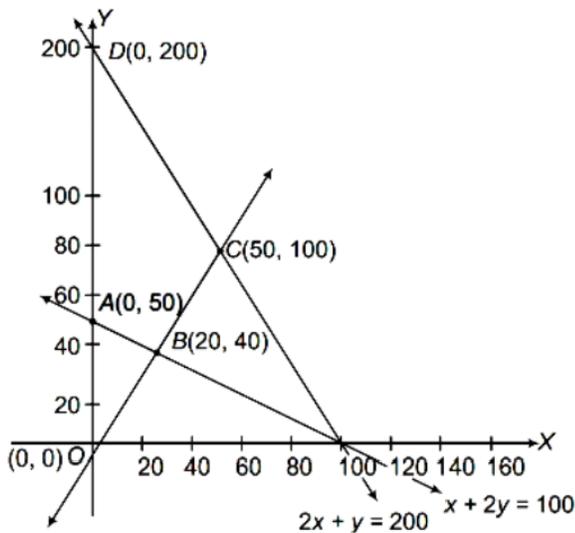
Integrating both sides, we get:

$$\log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log C$$

$$\Rightarrow \log\left|\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right| = \log|Cx|$$

$$\Rightarrow \log\left|\frac{y + \sqrt{x^2 + y^2}}{x}\right| = \log|Cx|$$

$$\Rightarrow y + \sqrt{x^2 + y^2} = Cx^2$$

30)

The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200). The values of Z at these points are as follows:

Corner point	Z = x + 2y
A(0, 50)	100 → Minimum
B(20, 40)	100 → Minimum
C(50, 100)	250
D(0, 200)	400 → Maximum

The maximum value of Z is 400 at D(0, 200) and the minimum value of Z is 100 at all the points on the line segment joining A(0, 50) and B(20, 40).

31)

Group-I		Group-II		Group-III	
Girls	Boys	Girls	Boys	Girls	Boys
3	1	2	2	1	3

Let G_1, G_2, G_3 denote events for selecting a girl and B_1, B_2, B_3 denote events for selecting a boy from 1st, 2nd and 3rd groups respectively.

Then $P(G_1) = \dfrac{3}{4}$, $P(G_2) = \dfrac{2}{4}$, $P(G_3) = \dfrac{1}{4}$

$P(B_1) = \dfrac{1}{4}$, $P(B_2) = \dfrac{2}{4}$, $P(B_3) = \dfrac{3}{4}$

Where $G_1, G_2, G_3, B_1, B_2,$ and B_3 are mutually exclusive events.

Let E be the event that 1 girl and 2 boys are selected

$\therefore E = (G_1 \cap B_2 \cap B_3) \cup (B_1 \cap G_2 \cap B_3) \cup (B_1 \cap B_2 \cap G_3)$

$\therefore P(E) = P(G_1 \cap B_2 \cap B_3) + P(B_1 \cap G_2 \cap B_3) + P(B_1 \cap B_2 \cap G_3)$

$= P(G_1) \cdot P(B_2) \cdot P(B_3) + P(B_1) \cdot P(G_2) \cdot P(B_3) + P(B_1) \cdot P(B_2) \cdot P(G_3)$

$= \left(\dfrac{3}{4} \times \dfrac{2}{4} \times \dfrac{3}{4} \right) + \left(\dfrac{1}{4} \times \dfrac{2}{4} \times \dfrac{3}{4} \right) + \left(\dfrac{1}{4} \times \dfrac{2}{4} \times \dfrac{1}{4} \right)$

$= \dfrac{18 + 6 + 2}{64}$

$= \dfrac{26}{64}$

$= \dfrac{13}{32}$

SECTION D

32)

A = R – {3}, B = R – {1}

And,

f : A → B be defined by f (x) = x – 2/ x – 3 ∀ x ∈ A

Hence, f (x) = (x – 3 + 1)/ (x – 3) = **1** + 1/ (x – 3)

Let f(x₁) = f (x₂)

$$1 + \frac{1}{x_1 - 3} = 1 + \frac{1}{x_2 - 3}$$

$$\frac{1}{x_1 - 3} = \frac{1}{x_2 - 3}$$

$$x_1 = x_2$$

So, f (x) is an injective function.

Now let y = (x – 2)/ (x -3)

x – 2 = xy – 3y

x(1 – y) = 2 – 3y

x = (3y – 2)/ (y – 1)

y ∈ R – {1} = B

Thus, f (x) is onto or subjective.

Therefore, f(x) is a bijective function.

$$\frac{a - 2}{a - 3} = \frac{7}{6}$$

$$6a - 12 = 7a - 21$$

$$a = 9$$

33)

LHS= $A^3 - 6A^2 + 7A + 2I$

$$A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

=0

=RHS

$-2A^{-1} = A^2 - 6A + 7I$

$$-2A^{-1} = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 12 \\ 0 & 12 & 6 \\ 12 & 0 & 18 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$-2A^{-1} = \begin{pmatrix} 6 & 0 & -4 \\ 2 & -1 & -1 \\ -4 & 0 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{-1}{2} \begin{pmatrix} 6 & 0 & -4 \\ 2 & -1 & -1 \\ -4 & 0 & 2 \end{pmatrix}$$

OR

b)

Given system of equation is:

3x + 2y – 2z = 3

x + 2y + 3z = 6

And 2x – y + z = 2

or AX = B

i.e. $\begin{bmatrix} 3 & 2 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$

$\therefore X = A^{-1}B$

$$\therefore \text{adj } A = \begin{bmatrix} 5 & 5 & -5 \\ 0 & 7 & 7 \\ 10 & -11 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

$$|A| = 3(5) + 2(5) + (-2)(-5) = 35$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{35}\begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

Now $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$
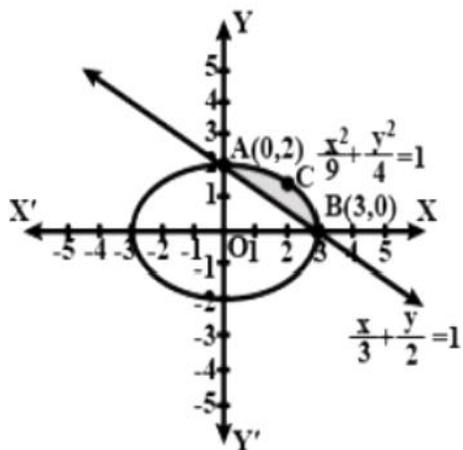
$$= \frac{1}{35}\begin{bmatrix} 15 + 20 \\ 15 + 42 - 22 \\ -5 + 42 + 8 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y = 1$ and $z = 1$

34)

$\therefore$ Area BCAB = Area (OBCAO) − Area (OBAO)

$$= \int_0^3 2\sqrt{1 - \frac{x^2}{9}}\,dx - \int_0^3 2\left(1 - \frac{x}{3}\right)dx$$

$$= \frac{2}{3}\left[\int_0^3 \sqrt{9 - x^2}\,dx\right] - \frac{2}{3}\int_0^3 (3 - x)dx$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - \frac{2}{3}\left[3x - \frac{x^2}{2}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right] - \frac{2}{3}\left[9 - \frac{9}{2}\right]$$

$$= \frac{2}{3}\left[\frac{9\pi}{4} - \frac{9}{2}\right]$$

$$= \frac{2}{3} \times \frac{9}{4}(\pi - 2)$$

$$= \frac{3}{2}(\pi - 2) \text{ sq. units}$$

35)

We have:

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \ (say)$$

x=3λ−1, y=5λ−3 and z=7λ−5

So, the coordinates of a general point on this line are (3λ−1, 5λ−3, 7λ−5).

The equation of the second line is given below:

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \ (say)$$

x=μ+2, y=3μ+4 and z=5μ+6

So, the coordinates of a general point on this line are (μ+2, 3μ+4, 5μ+6).

If the lines intersect, then they have a common point.

So, for some values of λ and μ, we must have:

3λ−1=μ+2, 5λ−3=3μ+4 and 7λ−5=5μ+6

⇒ 3λ−μ=3, 5λ−3μ=7 and 7λ−5μ=11

Solving the first two equations, 3λ−μ=3 and 5λ−3μ=7, we get:

λ=1/2 and μ=−3/2

Since λ=1/2 and μ=−3/2 satisfy the third equation, 7λ−5μ=11, the given lines intersect each other.

When λ=1/2 in (3λ−1, 5λ−3, 7λ−5), the coordinates of the required point of intersection are (1/2, −1/2, −3/2)

case- study/passage based questions

36)

Let $P(E1)$ be the probability that many workers were not present for the job.

Let $P(E2)$ be the probability that all workers were present for the job.

Let $P(E)$ be the probability that the construction work is completed on time.

    i)   What is the probability that all the workers are present for the job?

          We know $P(E1)=0.65$ and $P(E2)=1-P(E1)=1-0.65=0.35$.

    ii)   What is the probability that construction will be completed on time?

          We are given $P(E|E1)=0.35$ and $P(E|E2)=0.80$.

          Using the law of total probability,

          $P(E)=P(E|E1)P(E1)+P(E|E2)P(E2)$

               $=0.35 \cdot 0.65 + 0.80 \cdot 0.35$

               $=0.2275+0.28=0.5075$.

For (iii) (a), we use Bayes' theorem:

$P(E1|E)=P(E)P(E|E1)/P(E1)$

$=\dfrac{0.35 \times 0.65}{0.5075}=\dfrac{\mathbf{0.2275}}{0.5075} \approx \mathbf{0.448}$.

OR

For (iii) (b), we use Bayes' theorem: $P(E2|E)=P(E)P(E|E2)/P(E2)$

$=\dfrac{\mathbf{0.80 \times 0.35}}{0.5075} = \dfrac{\mathbf{0.28}}{0.5075} \approx \mathbf{0.552}$.

37)

i) The length and width of the base of the box will be (18−2x) cm each, since x cm is cut from both sides of the length and width.

The volume V of the box can be expressed as:

$V = \text{length} \times \text{width} \times \text{height} = (18 - 2x)(18 - 2x)x = x(18 - 2x)^2$

$\dfrac{dV}{dx} = (18 - 2x)(18 - 6x)$

$(18-2x)(18-6x)=0$

This gives us $x=3$ cm or $x=9$ cm.

ii) $V''(x) = 24x - 144$

$V''(3) = -72 < 0$

Therefore, x=3 is point of Maxima

V(x) is maximum at x=3

38)

i)

$\overrightarrow{OA} = -4\,\hat{\imath}$

$\overrightarrow{AB} = \dfrac{5\hat{\imath}}{2} + \dfrac{5\sqrt{3}\,\hat{\jmath}}{2}$

ii)

$\left|\overrightarrow{AB}\right| = 5$

Unit vector of AB$= \dfrac{\hat{\imath}}{2} + \dfrac{\sqrt{3}\,\hat{\jmath}}{2}$

iii) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \dfrac{-3\hat{\imath}}{2} + \dfrac{5\sqrt{3}\,\hat{\jmath}}{2}$

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