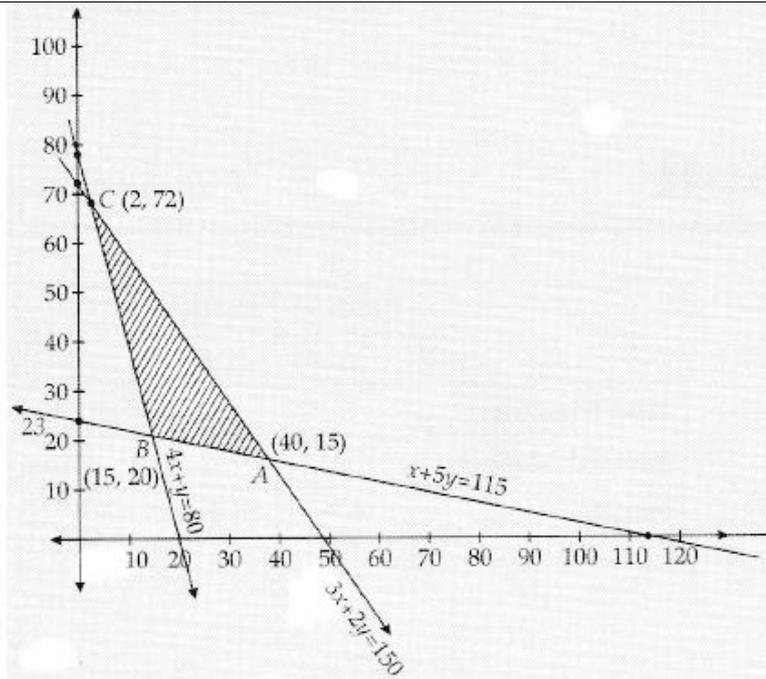




22	<p>Let <math>\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}</math> and <math>\vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}</math></p> <p>Vector <math>\vec{c}</math> perpendicular to both <math>\vec{a}</math> &amp; <math>\vec{b} = \vec{a} \times \vec{b}</math></p> $= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{bmatrix}$ $= \hat{i} + 10\hat{j} + 17\hat{k}$ <p>The unit vector <math>\hat{c}</math> in the direction of <math>\vec{c} = \frac{\vec{c}}{ \vec{c} }</math></p> $= \frac{\hat{i} + 10\hat{j} + 17\hat{k}}{\sqrt{390}}$ <p>Required vector = <math>5\hat{c} = 5\left(\frac{\hat{i} + 10\hat{j} + 17\hat{k}}{\sqrt{390}}\right)</math></p>	<p>1</p> <p>1/2</p> <p>1/2</p>
23	$\int e^x \left( \frac{\sqrt{1 + \sin 2x}}{1 + \cos 2x} \right) dx = \int e^x \left( \frac{\sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x}}{2 \cos^2 x} \right) dx.$ $= \int e^x \left( \frac{\sqrt{(\sin x + \cos x)^2}}{2 \cos^2 x} \right) dx.$ $= \int e^x \left( \frac{\sin x + \cos x}{2 \cos^2 x} \right) dx.$ $= \int e^x \left( \frac{1}{2} \tan x \sec x + \frac{1}{2} \sec x \right) dx.$ $= \frac{1}{2} \int e^x (\sec x + \sec x \tan x) dx$ $= \frac{1}{2} e^x (\sec x) + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
24	<p>Given that the function is continuous everywhere. For the function to be continuous at <math>x = -\frac{\pi}{2}</math>, the left hand limit and right and limit must be equal.</p> <p>The left hand limit is , LHL = <math>\lim_{x \rightarrow -\frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow -\frac{\pi}{2}^-} -2 \sin x</math></p> $= 2$ <p>The right hand limit is , RHL = <math>\lim_{x \rightarrow -\frac{\pi}{2}^+} f(x) = \lim_{x \rightarrow -\frac{\pi}{2}^+} A \sin x + B</math></p> $= -A + B$ <p><math>\therefore -A + B = 2</math> _____ (1)</p> <p>Continuity at <math>x = \frac{\pi}{2}</math></p> <p>LHL = <math>\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^-} A \sin x + B</math></p>	<p>1/2</p> <p>1/2</p> <p>1/2</p>

	$\text{RHL} = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} \cos x = 0$ $\therefore A + B = 0 \quad (2)$ <p>From (1) &amp; (2), <math>A = -1, B = 1</math></p>	$= A + B$ $\frac{1}{2}$
25	$y = (x + \sqrt{x^2 - 1})^2$ $\frac{dy}{dx} = 2(x + \sqrt{x^2 - 1}) \left(1 + \frac{1}{2\sqrt{x^2 - 1}} 2x\right)$ $= 2(x + \sqrt{x^2 - 1}) \left(\frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}\right)$ $= 2 \frac{(x + \sqrt{x^2 - 1})^2}{\sqrt{x^2 - 1}}$ $= \frac{2y}{\sqrt{x^2 - 1}}$ $(x^2 - 1) \left(\frac{dy}{dx}\right)^2 = 4y^2$ <p>OR</p> <p>Let <math>u = \cos^{-1}(2x^2 - 1)</math></p> <p>Then <math>u = \cos^{-1}(2\cos^2\theta - 1)</math></p> $= 2\theta = 2\cos^{-1}x$ $\therefore \frac{du}{dx} = \frac{-2}{\sqrt{1 - x^2}}$ <p>Let <math>v = \cos^{-1}x</math></p> $\therefore \frac{dv}{dx} = -\frac{1}{\sqrt{1 - x^2}}$ $\therefore \frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = 2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
<b>SECTION C</b>		
26	<p>Given that <math>\frac{dA}{dt} = 5\text{mm}^2/\text{s}</math> &amp; <math>r = 8\text{mm}</math></p> <p>ie, <math>\frac{d}{dt}(4\pi r^2) = 5 \Rightarrow 4\pi \times 2r \frac{dr}{dt} = 5</math></p> $\Rightarrow \frac{dr}{dt} = \frac{5}{8\pi r}$ $\therefore \frac{dr}{dt} \text{ (at } r = 8) = \frac{5}{64\pi} \text{ mm/s}$ <p>Therefore <math>\frac{dv}{dt} = \frac{d}{dt} \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi \times \frac{3r^2 dr}{dt}</math></p> $= 20\text{mm}^3/\text{s}.$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
27	$ x + 1  = \begin{cases} x + 1, & \text{if } x \geq -1 \\ -x - 1, & \text{if } x < -1 \end{cases}$ $\int_{-4}^2  x + 1  dx = \int_{-4}^{-1} -(x + 1) dx + \int_{-1}^2 (x + 1) dx$	1 $\frac{1}{2}$

	$= \left[-\frac{x^2}{2} - x\right]_{-4}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^2$ $= 9$ <p>The value of the integral is <b>9</b>. The integral represents the <b>total area</b> between the graph of <math>y =  x + 1 </math> and the x-axis on the interval <math>[-4, 2]</math>.</p> <p>.OR</p> <p>To find the points where the parabola <math>y = x^2</math> and the line <math>y = x</math> intersect, set the equations equal to each other and solve for <math>x</math>.</p> $x^2 = x \Rightarrow x(x - 1) = 0$ <p><math>\Rightarrow x = 0</math> or <math>x = 1</math></p> <p><math>\therefore</math> The required area between the curves <math>y = x^2</math> and <math>y = x</math> is</p> $A = \int_0^1 x - x^2 dx$ $= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$ $= \frac{1}{2} - \frac{1}{3}$ $= \frac{1}{6} \text{ sq. units}$	<p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p>
28	<p><math>E_1</math>: Female candidate  <math>E_2</math>: Male candidate  A: Candidate have distinction</p> <p><math>P(E_1) = \frac{2}{3}, P(E_2) = \frac{1}{3}</math></p> <p><math>P\left(\frac{A}{E_1}\right) = 0.35, P\left(\frac{A}{E_2}\right) = 0.40</math></p> <p><math>P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)</math></p> $= \frac{2}{3} \times 0.35 + \frac{1}{3} \times 0.40$ $= \frac{70 + 40}{300}$ $= \frac{11}{30}$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
29	<p><math>Z = 6x + 3y</math></p> <p><math>4x + y \geq 80</math></p> <p><math>x + 5y \geq 115</math></p> <p><math>3x + 2y \leq 150</math></p> <p><math>x, y \geq 0</math></p>	



Corner Points	coordinates	Z = 6x + 3y
A	(15, 20)	150 Minimum
B	(2, 72)	228
C	(40, 15)	285

The minimum value of Z is 150 at (15,20)

30

Given equation is  $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$

Let  $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$

Any point on the given line is of the form  $x = 3\lambda - 2, y = 2\lambda - 1, z = 2\lambda + 3$

Therefore the point Q on the line is  $Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$

According to the problem the distance between P and Q is 5 units.

Then we have,  $(3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 5^2$

$$17\lambda^2 - 34\lambda = 0 \Rightarrow \lambda = 0, 2$$

Therefore the points are  $(-2, -1, 3)$  &  $(4, 3, 7)$

OR

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k}), \hat{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$b_1 = (2\hat{i} - 2\hat{j} + \hat{k}) \& b_2 = (\hat{i} + 2\hat{j} + 2\hat{k})$$

The vector perpendicular to the given two lines is  $b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$

$$= -6\hat{i} - 3\hat{j} + 6\hat{k}$$

	<p>Therefore vector equation of the required line is</p> $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$	1
31	<p>Consider the given function <math>\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)</math>  Let <math>x = \sin\theta</math> and <math>y = \sin\phi</math>  Then the given equation becomes, <math>\sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\phi} = a(\sin\theta - \sin\phi)</math>  <math>\Rightarrow \sqrt{\cos^2\theta} + \sqrt{\cos^2\phi} = a(\sin\theta - \sin\phi)</math>  <math>\therefore \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)</math>  <math>\Rightarrow 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = 2a\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}</math>  <math>\Rightarrow \frac{\tan(\theta-\phi)}{2} = \frac{1}{a}</math>  <math>\Rightarrow \frac{\theta-\phi}{2} = \tan^{-1}\left(\frac{1}{a}\right)</math>  <math>\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\tan^{-1}\left(\frac{1}{a}\right)</math>  Differentiate both sides w.r.to x  <math>\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} = 0</math>  <math>\Rightarrow \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}</math>  <b>OR</b>  <math>e^y(x+1) = 1</math>  Differentiate both sides w.r.t x  <math>e^y\left(\frac{dy}{dx}\right) \times (x+1) + e^y = 0</math>  Dividing both sides by <math>e^y</math>, we get <math>(x+1)\frac{dy}{dx} + 1 = 0</math> _____ (1)  Differentiating again w.r.t x, <math>\frac{dy}{dx} + (x+1)\frac{d^2y}{dx^2} = 0</math>  <math>\Rightarrow (x+1)\frac{d^2y}{dx^2} = -\frac{dy}{dx}</math>  <math>\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{x+1} \frac{dy}{dx}</math>  <math>\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2</math> (From(1))</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p>
	<b>SECTION D</b>	
32.	<p>The given differential equation is <math>(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}</math>  The given linear equation can be converted into the form  <math display="block">\frac{dy}{dx} + Py = Q</math>  Divide the equation by <math>1+x^2</math>, we get  <math>\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}</math>, where <math>P(x) = \frac{1}{1+x^2}</math>, <math>Q(x) = \frac{e^{\tan^{-1}x}}{1+x^2}</math>  Integrating factor, <math>IF = e^{\int P dx}</math>  <math>= e^{\int \frac{1}{1+x^2} dx}</math>  <math>= e^{\tan^{-1}x}</math></p>	<p>1/2</p> <p>1</p> <p>1</p>

	<p>Therefore solution is, <math>y(IF) = \int Q(x) \cdot (IF) dx + C</math></p> $\therefore y \cdot e^{\tan^{-1} x} = \int \frac{e^{\tan^{-1} x}}{1+x^2} \cdot e^{\tan^{-1} x} dx$ $= \int \frac{e^{2 \tan^{-1} x}}{1+x^2} dx$ <p>Let <math>\tan^{-1} x = t</math>, then <math>\frac{1}{1+x^2} = dt</math></p> $\therefore \int \frac{e^{2 \tan^{-1} x}}{1+x^2} dx = \int e^{2t} dt = \frac{e^{2t}}{2} + c$ $\therefore y \cdot e^{\tan^{-1} x} = \frac{1}{2} [e^{2 \tan^{-1} x}] + c$ <p>OR</p> <p>The given differential equation is <math>2xydy = (x^2 + y^2)dx</math>._____ (1)</p> <p>To solve this homogeneous equation, rewrite the equation into the form <math>\frac{dy}{dx} = f\left(\frac{y}{x}\right)</math></p> <p>Divide both sides of eq(1) by <math>dx</math> &amp; <math>2xy</math>, we get</p> $\frac{dy}{dx} = \frac{(x^2+y^2)}{2xy} \Rightarrow \frac{dy}{dx} = \frac{1+\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)} \text{_____ (2)}$ <p>Let <math>y = vx</math> and differentiate both sides w. r. to <math>x</math></p> $\frac{dy}{dx} = v + x \frac{dv}{dx} \text{_____ (3)}$ <p>Equate (2) &amp; (3)</p> $v + x \frac{dv}{dx} = \frac{1+v^2}{2v} \Rightarrow x \frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1-v^2}{2v}$ $\frac{2v}{v^2-1} dv = -\frac{dx}{x}$ <p>Integrate both sides, we get</p> $\log(v^2-1) + \log x = \log C$ $\Rightarrow (v^2-1)x = C$ $\Rightarrow y^2 - x^2 = cx$	<p>1</p> <p>1½</p> <p>1</p> <p>1</p> <p>1 ½</p> <p>1 ½</p>
33	<p>Let <math>I = \int_0^\pi \frac{4x \sin x}{1+\cos^2 x} dx</math> _____ (1)</p> <p>Using the property that <math>\int_0^a f(x) dx = \int_0^a f(a-x) dx</math></p> <p>Therefore <math>I = \int_0^\pi \frac{4(\pi-x) \sin(\pi-x)}{1+\cos^2(\pi-x)} dx</math></p> $I = \int_0^\pi \frac{4(\pi-x) \sin x}{1+\cos^2 x} dx \text{_____ (2)}$ $= \int_0^\pi \frac{4\pi \sin x}{1+\cos^2 x} dx - \int_0^\pi \frac{4x \sin x}{1+\cos^2 x} dx$ $= \int_0^\pi \frac{4\pi \sin x}{1+\cos^2 x} dx - I$ $2I = \int_0^\pi \frac{4\pi \sin x}{1+\cos^2 x} dx$ $I = \frac{1}{2} \times 4\pi \int_0^\pi \frac{\sin x}{1+\cos^2 x} dx$	<p>½</p> <p>2</p> <p>1</p> <p>1½</p>

	<p>Let <math>\cos x = t, \sin x dx = -dt</math>  If <math>x = 0, t = 1</math> and if <math>x = \pi, t = -1</math>  So the integral becomes, <math>2\pi \int_1^{-1} \frac{-dt}{1+t^2} = 2\pi \int_{-1}^1 \frac{dt}{1+t^2}</math>  <math>= 2\pi(\tan^{-1} t)_{-1}^1</math>  <math>= 2\pi \times \frac{\pi}{2} = \pi^2.</math></p> <p>OR</p> <p>Let <math>I = \int \frac{1}{x[(\log x)^2 - 3\log x - 4]} dx</math>  Let <math>\log x = t</math>, then <math>\frac{1}{x} dx = dt</math>  Then <math>I = \int \frac{dt}{t^2 - 3t - 4}</math>  <math>= \int \frac{1}{(t - 4)(t + 1)} dt</math>  Apply integration by partial fraction  <math>\frac{1}{(t - 4)(t + 1)} = \frac{A}{t - 4} + \frac{B}{t + 1}</math>  To solve for A and B we get, <math>A = \frac{1}{5}</math> &amp; <math>B = -\frac{1}{5}</math>  <math>\therefore I = \int \frac{\frac{1}{5}}{t - 4} dt + \int \frac{-\frac{1}{5}}{t + 1} dt</math>  <math>= \frac{1}{5} \log(t - 4) - \frac{1}{5} \log(t + 1) + C</math>  <math>= \frac{1}{5} [\log(\log x - 4) - \log(\log x + 1)] + C</math>  <math>= \frac{1}{5} \left[ \log \left  \frac{\log x - 4}{\log x + 1} \right  \right] + C</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p>
34	<p>Given</p> <p><math>L_1</math>: The line passing through <math>(2, -1, 1)</math> and parallel to <math>\frac{x}{1} = \frac{y}{1} = \frac{z}{3}</math></p> <p><math>L_2: \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}</math></p> <p>The standard form of given equations of lines are</p> <p><math>L_1: \vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + 3\hat{k})</math></p> <p><math>L_2: \vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(2\hat{j} - \hat{k})</math></p>	<p>1</p>

$$a_1 = (2\hat{i} - \hat{j} + \hat{k}), \quad b_1 = (\hat{i} + \hat{j} + 3\hat{k})$$

$$a_2 = (\hat{i} + \hat{j} - 2\hat{k}), \quad b_2 = (2\hat{j} - \hat{k})$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 0 & 2 & -1 \end{vmatrix}$$

$$= -7\hat{i} + \hat{j} + 2\hat{k}$$

$$|b_1 \times b_2| = |-7\hat{i} + \hat{j} + 2\hat{k}|$$

$$= \sqrt{(-7)^2 + 1^2 + 2^2}$$

$$= \sqrt{54}$$

$$a_2 - a_1 = (\hat{i} + \hat{j} - 2\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$= -\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\text{Shortest distance} = \left| \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|} \right|$$

$$= \left| \frac{(-7\hat{i} + \hat{j} + 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{54}} \right|$$

$$= \frac{3}{\sqrt{54}} \text{ units}$$

OR

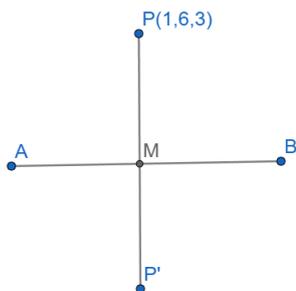
Let  $P(1,6,3)$  be the given point.

Let  $M$  be the foot of the perpendicular from 'P' to the given line AB.

$$\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) \text{-----(1)}$$

$$\Rightarrow \frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

Then the coordinates of  $M$  be  $(\lambda, 2\lambda + 1, 3\lambda + 2)$ ,  $\lambda$  is a scalar.



The direction ratios of PM are :

1

1/2

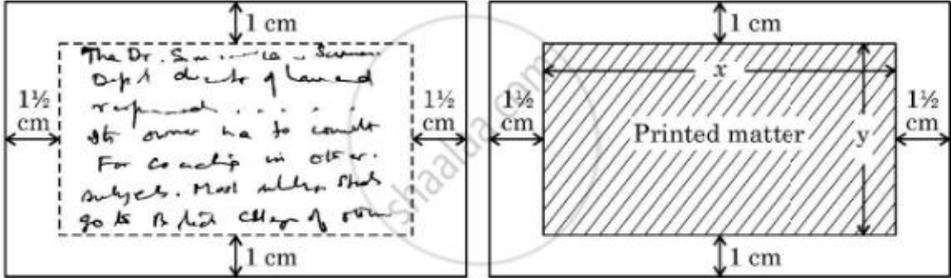
1/2

1

1

1 1/2

	<p> <math>\langle \lambda - 1, 2\lambda + 1 - 6, 3\lambda + 2 - 3 \rangle</math>            Ie, <math>\langle \lambda - 1, 2\lambda - 5, 3\lambda - 1 \rangle</math>            And direction ratios of line (1) are <math>\langle 1, 2, 3 \rangle</math>            Now, <math>PM \perp AB \Rightarrow (\lambda - 1) \cdot 1 + (2\lambda - 5) \cdot 2 + (3\lambda - 1) \cdot 3 = 0</math>  <math>\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0</math>  <math>\Rightarrow 14\lambda - 14 = 0</math>  <math>\Rightarrow \lambda = 1.</math>  <math>\therefore M</math> is <math>(1, 3, 5)</math>            Let <math>P'(\alpha, \beta, \gamma)</math> be the image of <math>P(1, 6, 3)</math>, then <math>M</math> is the midpoint of <math>PP'</math>.  <math>\therefore \frac{\alpha + 1}{2} = 1, \frac{\beta + 6}{2} = 3, \frac{\gamma + 3}{2} = 5 \Rightarrow \alpha = 1, \beta = 0, \gamma = 7.</math>            Hence the <math>(1, 0, 7)</math> is the image of <math>P(1, 6, 3)</math> in given line.            Equation of line joining <math>P</math> &amp; <math>P'</math> is, <math>\frac{x-1}{0} = \frac{y-6}{-6} = \frac{z-3}{4}</math> </p>	<p>1</p> <p>1 ½</p>
35	<p> <math>A = \begin{bmatrix} 1 &amp; 2 &amp; -3 \\ 2 &amp; 0 &amp; -3 \\ 1 &amp; 2 &amp; 0 \end{bmatrix}</math>  <math>\therefore  A  = -12 \neq 0</math>            Therefore <math>A</math> is invertible.  <math>Adj A = \begin{bmatrix} 6 &amp; -3 &amp; 4 \\ -6 &amp; 3 &amp; 0 \\ -6 &amp; -3 &amp; -4 \end{bmatrix}^T</math>  <math display="block">= \begin{bmatrix} 6 &amp; -6 &amp; 6 \\ -3 &amp; 3 &amp; -3 \\ 4 &amp; 0 &amp; -4 \end{bmatrix}</math>  <math>\therefore A^{-1} = -\frac{1}{12} \begin{bmatrix} 6 &amp; -6 &amp; 6 \\ -3 &amp; 3 &amp; -3 \\ 4 &amp; 0 &amp; -4 \end{bmatrix}</math>            The given equations are  <math>x + 2y - 3z = 1</math>  <math>2x - 3z = 2</math>  <math>x + 2y = 3</math>            Which can be written in matrix form as  <math>\begin{bmatrix} 1 &amp; 2 &amp; -3 \\ 2 &amp; 0 &amp; -3 \\ 1 &amp; 2 &amp; 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}</math>  <math>\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 &amp; 2 &amp; -3 \\ 2 &amp; 0 &amp; -3 \\ 1 &amp; 2 &amp; 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}</math>  <math>\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12} \begin{bmatrix} 6 &amp; -6 &amp; 6 \\ -3 &amp; 3 &amp; -3 \\ 4 &amp; 0 &amp; -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}</math> </p>	<p>1</p> <p>1</p> <p>½</p> <p>½</p> <p>1</p>

	$= \begin{bmatrix} 2 \\ \frac{1}{2} \\ \frac{2}{3} \end{bmatrix}$ $\therefore x = 2, y = \frac{1}{2}, z = \frac{2}{3}$	1
36	<p><math>E_1</math> : Event that a person has TB</p> <p><math>E_2</math>: Event that a person does not have TB</p> <p>A: The person is diagnosed to have TB</p> $P(E_1) = 0.001 = \frac{1}{1000}$ $P(E_2) = 1 - 0.001 = 0.999 = \frac{999}{1000}$ $P\left(\frac{A}{E_1}\right) = 0.99 = \frac{99}{100}, P\left(\frac{A}{E_2}\right) = 0.001 = \frac{1}{1000}$ <p>a) Number of persons out of 200,000 suffering from TB = <math>\frac{1}{1000} \times 200,000</math> = 200</p> <p>b) Required probability = <math>P\left(\frac{A}{E_1}\right) = \frac{99}{100}</math></p> <p>c) Required probability = <math>P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}</math></p> $= \frac{\frac{1}{1000} \times \frac{99}{100}}{\frac{1}{1000} \times \frac{99}{100} + \frac{999}{1000} \times \frac{1}{1000}}$ $= \frac{110}{221}$ <p>d) Required probability = <math>P\left(\frac{A}{E_2}\right) = 0.001</math></p>	<p>1</p> <p>1</p> <p>1 ½</p> <p>½</p>
37	 <p>a) Let <math>A(x)</math> be the area of the visiting card then, As <math>xy = 24</math> <math>A(x) = (x + 3)(y + 2)</math> <math>= 2x + 3y + xy + 6</math></p>	2

	$= 2x + 3\left(\frac{24}{x}\right) + x\left(\frac{24}{x}\right) + 6$ $= 2x + \frac{72}{x} + 30$ <p>b) <math>A'(x) = 2 - \frac{72}{x^2}</math> and <math>A''(x) = \frac{144}{x^3}</math></p> <p>Solving <math>A'(x) = 0 \Rightarrow 2 - \frac{72}{x^2} = 0</math></p> $\Rightarrow x = 6$ $A''(6) = \frac{144}{6^3} > 0$ <p><math>\therefore</math> Area of the card is minimum at <math>x = 6</math> and <math>y = 4</math></p> <p>The dimension of the card with minimum area is length = 6 cm and breadth = 4 cm</p>	2
38	<p>Here <math>A = \{Nitya, Rohit\}, B = \{1,2,3,4,5,6\}</math></p> <p>a) Number of relations from A to B = <math>2^{2 \times 6} = 2^{12}</math></p> <p>b) Number of reflexive relations on B = <math>2^{6^2 - 6} = 2^{30}</math></p> <p>c) I) <math>R =</math>  <math>\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (4,4), (5,5), (6,6)\}</math></p> <p>R is not an equivalence relation as <math>(1,2) \in R</math>, but <math>(2,1) \notin R</math></p> <p>OR</p> <p>II) <math>P = \{\}</math>, P is a null relation. Since null relation is not reflexive, p is not an equivalence relation.</p>	1 1 2 2