CSSC MATHEMATICS SET 1 MARKING SCHEME

SECTION-A

1 (D) 2. B 3. (D) 4. (B) (5) B (6) B (7) C

(8) B (9) D (10) B (11) (D) (12) B (13) C (14) C

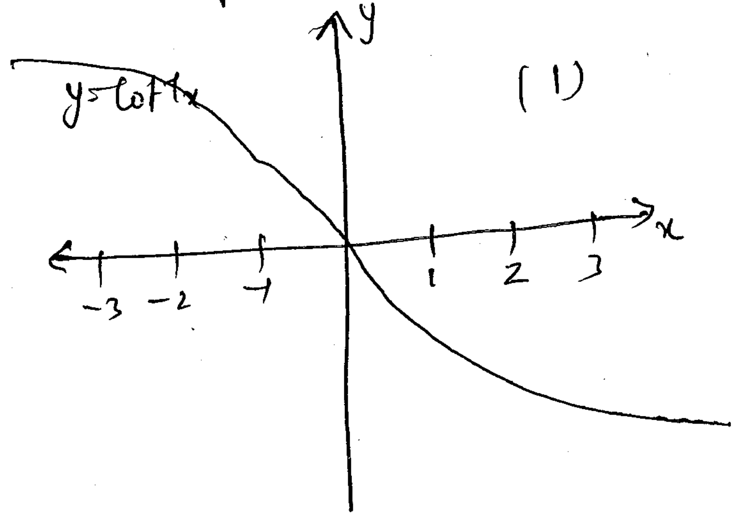
(15) C (16) C (17) C (18) D (19) A (20) D

SECTION-B

(21) $-\pi/6 + \pi/3 + \cancel{\pi} - \pi/4$ (1) OR (21) Principal value $= (0, \pi)$ (1)

$= -\pi/12$ (1)

(1)



(22) $f(x) = (x-2)\sqrt{x-1}$ in $[1, 9]$

$f'(x) = \sqrt{x-1} + \dfrac{x-2}{2\sqrt{x-1}}$ (½)

$= \dfrac{2(x-1) + x - 2}{2\sqrt{x-1}} = 0 \Rightarrow x = 4/3$

Critical points $= 1, 4/3, 9$ (½)

ABSOLUTE MAX $= 14\sqrt{2}$ when $x = 9$

Absolute min $= \dfrac{-2}{3\sqrt{3}}$ when $x = 4/3$ (1)

(23) $V = \frac{1}{3}\pi r^2 \cancel{x} + \frac{2}{3}\pi r^3$ (½)

$= \frac{4}{3}\pi r^3$ $(\because h = 3r,\ x = 2r)$

$= 4/3\, \pi \left(\frac{h^3}{27}\right)$ (1)

$\left(\dfrac{dV}{dh}\right)_{h=9} = 12\pi\, cm^2$ (½)

$\therefore$ Vol is inc. at $12\pi\, cm^2$ with respect to total ht.

(23) OR $V = x^3$, $\dfrac{dx}{dt} = 10$ cm/sec

$\dfrac{dV}{dt} = 3x^2 \dfrac{dx}{dt}$ (1) $\left(\dfrac{dV}{dt}\right)_{x=5} = 750\ cm^3/sec$ (1)

(27) $\int_0^{\pi/2} (2\log \cos x - \log \sin 2x)\, dx$

$= \int_0^{\pi/2} \log\left(\frac{\cos^{\cancel{2}} x}{2 \sin x \cancel{\cos x}}\right) dx = \int_0^{\pi/2} \log\left(\frac{\cot x}{2}\right) dx$ (1)

$\int_0^{\pi/2} \log(\cot x)\, dx - \int_0^{\pi/2} \log 2\, dx = \int_0^{\pi/2} \log(\tan x)\, dx - \int_0^{\pi/2} \log 2\, dx$ (1)

$= 0 - \frac{\pi}{2}\log 2 = -\frac{\pi}{2}\log 2$ (1)

(28) $(x^2 - y^2)\, dx + 2xy\, dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

$v + x\frac{dv}{dx} = \frac{\cancel{x^2}(v^2-1)}{2v\cancel{x^2}}$

$x\frac{dv}{dx} = \frac{v^2-1}{2v} - \frac{v}{1}$

$\frac{v^2-1-2v^2}{2v}$

$\frac{2v\, dv}{v^2+1} = \frac{-dx}{x}$

$\log\left|\frac{y^2}{x^2}+1\right| = -\log x + \log c$

$\boxed{\frac{y^2+x^2}{x} = C}$

(28) OR $(1+x^2)\, dy + 2xy\, dx = \cot x\, dx$

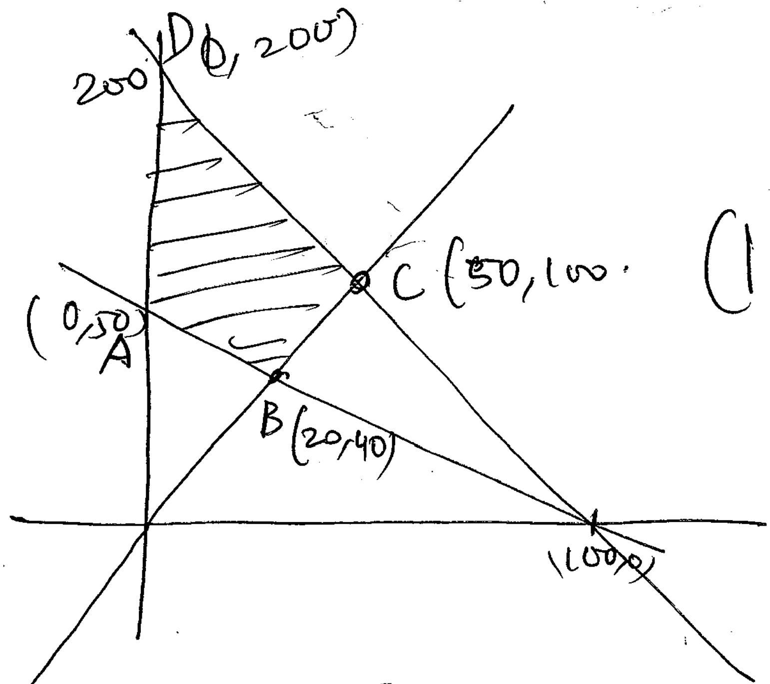
$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$ (1)

$IF = e^{\log(1+x^2)} = 1+x^2$ (1)

GS: $y(1+x^2) = \int \cot x\, dx + c$

$y(1+x^2) = \log \sin x + c$ (1)

(29)



(1)

CORNER POINTS

A	0	50	$Z = 100$
B	20	40	$Z = 100$
C	50	100	$Z = 250$
D	0	200	$Z = 400$

(1)

MAX $= 400$ when $x = 0, y = 200$. (1)

MIN $= 100$ at all points on the line segment AB

(OR).

(0, 0)	$Z = 0$
(52, 0)	$Z = 156$
44, 16	$Z = 196$ (Max)
0, 38	$Z = 152$

(2)

Max $= 196$. (1)

(30) $P(W) = P(WW) + P(BW)$ (1)

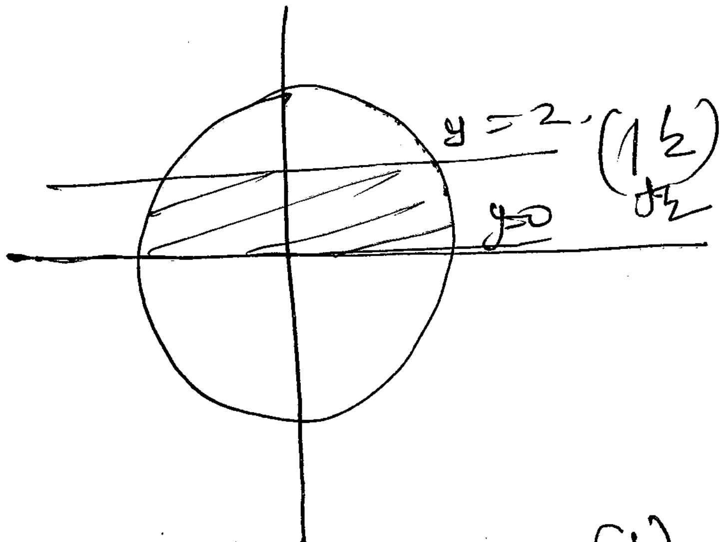
$= \frac{5}{9} \times \frac{6}{14} + \frac{4}{9} \times \frac{7}{14} = \frac{29}{63}$ (2)

(31) $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right] \quad 0 < x < \frac{\pi}{2}$.

$= \cot^{-1}\left[\frac{\cos\frac{x}{2} + \sin\frac{x}{2} + \cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2} - \cos\frac{x}{2} + \sin\frac{x}{2}}\right]$ (1) $= \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2}$ (1)

$\therefore \frac{dy}{dx} = \frac{1}{2}$ (1)

(32) $y^2 = 9 - x^2$, $y = 0$, $y = 2$



$A = 2\int_0^2 \sqrt{9 - y^2}\, dy$ (1)

$= \cancel{2}\left[\frac{y}{\cancel{2}}\sqrt{9-y^2} + \frac{9}{\cancel{2}}\sin^{-1}\frac{y}{3}\right]_0^2$ (1)

$= \cancel{2}\left[\frac{2\sqrt{5}}{\cancel{2}} + 9\sin^{-1}\frac{2}{3}\right]$. (1)

(33) $a - a = 0$ divisible by n.

$\therefore (a,a) \in R$ $\therefore$ reflexive (1)

Let $(a,b) \in R$ (1)

$\Rightarrow a - b = kn$

$\therefore b - a = -kn \Rightarrow b - a$ is div. by n.

$\therefore (b,a) \in R$ $\therefore$ symmetric

Let $(a,b) \in R$, Let $(b,c) \in R$

(2)

$\Rightarrow a - b = k_1 n$

$b - c = k_2 n$

$a - c = n(k_1 + k_2)$ $\therefore$ divisible by n.

$\therefore (a,c) \in R$.

$\therefore$ Equivalence relation (1)

(32)

(OR) $f(x) = \frac{x-2}{x-3}$

1-1 $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$ (2)

$\cancel{x_1x_2} - 2x_2 - 3x_1 + \cancel{6} = \cancel{x_1x_2} - 2x_1 - 3x_2 + \cancel{6}$

$\Rightarrow x_1 = x_2$ ∴ one-to-one

$y = \frac{x-2}{x-3}$ (2½)

$xy - 3y = x - 2$

$x(y-1) = 3y - 2$

$x = \frac{3y-2}{y-1}$ defined for all $y \in$ co-domain ∴ onto

∴ One-to-one & onto (1/2)

(33) L_1 $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$, L_2: $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$

$\vec{a_1} = -\hat{i} + \hat{j} + 9\hat{k}$ $\vec{b_1} = 2\hat{i} + \hat{j} - 3\hat{k}$ (1½)

$\vec{a_2} = 3\hat{i} - 15\hat{j} + 9\hat{k}$ $\vec{b_2} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$SD = \frac{|(\vec{a_2} - \vec{a_1}) \cdot \vec{b_1} \times \vec{b_2}|}{|\vec{b_1} \times \vec{b_2}|}$ (½)

$\vec{a_2} - \vec{a_1} = 4\hat{i} - 16\hat{j}$ (½)

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = -16\hat{i} - 16\hat{j} - 16\hat{k}$ (1)

$|\vec{b_1} \times \vec{b_2}| = 16\sqrt{3}$ (½)

∴ $SD = \frac{192}{16\sqrt{3}} = 4\sqrt{3}$ (1)

(34) OR. $\vec{r} = \vec{a} + \lambda(\vec{b_1} \times \vec{b_2})$ (1)

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ (1)

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(24\hat{i} + 36\hat{j} + 72\hat{k})$ (2)

CE $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ (1)

(8)

(38). (i) $S = 2\pi r^2 + \frac{6000}{r}$ (1)

(ii) $S'(r) = 4\pi r - \frac{6000}{r^2} = 0$

$r = \left(\frac{1500}{\pi}\right)^{1/3}$ $\frac{1}{2} + \frac{1}{2}$

$S''(r) > 0$.

(iii) $h = 2\sqrt[3]{\frac{1500}{\pi}}$ (2)

OR Min $S = \frac{2\pi r^3 + 6000}{r} = 1153.84\ cm^2$ (1)

Min Cost $= \frac{1153.84}{100} = ₹11.538$ (1)

(35) $BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$ (1)

$B^{-1} = A/6$ $(\frac{1}{2})$

Eqn Matrix $BX = C$
$X = B^{-1}C$ (1)

$= \frac{1}{6}AC = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ (1)

$(\frac{1}{2})$

$\therefore x = 2,\ y = -1,\ z = 4$ (1)

SECTION-E

(36) (i) P(Grade A in all subjects) $= 0.2 \times 0.3 \times 0.5 = 0.03$ $(\frac{1}{2})$ $(\frac{1}{2})$

(ii) P(Grade A in no subjects) $= 0.8 \times 0.7 \times 0.5 = 0.280$ $(\frac{1}{2})$ $(\frac{1}{2})$

(iii) P(Grade A in 2 subjects) $=$
$0.2 \times 0.3 \times 0.5 + 0.3 \times 0.5 \times 0.8 + 0.2 \times 0.5 \times 0.7 = 0.22$ $(1\frac{1}{2})$ $(\frac{1}{2})$

OR P(Grade A in atleast 1 subject)

$1 - (ii) = 0.72$ (1)
(1)

(37) (i) Resultant displacement vector $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 2\hat{i} - \hat{j} + 5\hat{k}$. (1)

(ii) $\overrightarrow{AB} = 2\hat{i} - \hat{j} + 5\hat{k}$ $\quad \overrightarrow{AC} = -2\hat{i} + \hat{j} + 3\hat{k}$

$A = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 5 \\ -2 & 1 & 3 \end{matrix} \right\| = \frac{1}{2}\sqrt{(-9)^2 + (-16)^2 + 16}$

$= \frac{1}{2}\sqrt{81 + 256 + 16}$

$= \frac{1}{2}\sqrt{353}$

$\frac{-9\hat{i} - 16\hat{j} + 4\hat{k}}{\sqrt{353}}$ (2)

(3) $\hat{AC} = \frac{-2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{4+1+9}} = \frac{-2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ (2) (OR)