23. $6y = x^3 + 2$

$\frac{dy}{dt} = \frac{3x^2}{6}\frac{dx}{dt} = 2\frac{dx}{dt}\left(\frac{1}{2}\right)$

$x^2 = 4$
$x = \pm 2$ $\left.\right\}\left(\frac{1}{2}\right)$

$x = 2,\ y = \frac{10}{6} = \frac{5}{3}$

$x = -2,\ y = \frac{-6}{6} = -1$

Pts $= (2, \frac{5}{3})$ & $(-2, -1)$ $\left.\right\}(1)$

(OR) $f(x) = x^3 - 12x^2 + kx - 8$

$f'(x) = 3x^2 - 24x + k = 0.$

$f'(2) = 12 - 48 + k = 0$

$k = 36$ (1)

$f(0) = -8$

$f(2) = 8 - 48 + 72 - 8 = 24$ $\left.\right\}(1)$

$f(3) = 27 - 108 + 108 - 8 = 19$

$\therefore f(2)$ is max.

(25) $f'(x) = e^x + (x-1)e^x$ (1)

$= e^x[x] > 0$ for all $x > 0$ $\left.\right\}(1)$

$\therefore$ inc for all $x > 0$

SECTION C

(26) $\int \frac{\sin x\, dx}{2\cos^2 x - 1} = \int \frac{dt}{2t^2 - 1}$ (1)

$-\frac{1}{2}\int \frac{dt}{t^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{-\sqrt{2}}{2\times 2}\log\left|\frac{t - \frac{1}{\sqrt{2}}}{t + \frac{1}{\sqrt{2}}}\right| + c$ (1)

$= \frac{-\sqrt{2}}{4}\log\left|\frac{\sqrt{2}t - 1}{\sqrt{2}t + 1}\right| + c = \frac{\sqrt{2}}{4}\log\left|\frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}\right| + c$ (1)

(OR) $\int \frac{A(-2x - 3) + B}{4 - 3x - 2x^2}\, dx$ $\left(\frac{1}{2}\right)$

$-2A = 2$ $\quad -3A + B = 1$ $\left.\right\}\left(\frac{1}{2}\right)$

$A = -1$ $\quad B = -2$

$I = -\int \frac{(-2x - 3)\, dx}{4 - 3x - x^2} + 2\int \frac{dx}{x^2 + 3x - 4}$ (1)

$= -\log|4 - 3x - x^2| + 2\int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{5}{2}\right)^2} = -\log|4 - 3x - x^2|$

$+ \frac{2}{5}\tan^{-1}\frac{2x + 3}{5} + c$ (1)

(28) $\int (\sin y + y\cos y)\,dy = \int (2x\log x + x)\,dx$ (½)

$-\cos y + y\sin y + \cos y = \frac{x^2}{2}\log x - \frac{x^2}{4} + \frac{x^2}{2} + c$ (1)

$\frac{\pi^2}{2} = \frac{\pi^2}{8}\log(1) + \frac{1}{4} + c$ (1)

$c = \frac{\pi^2}{2} - \frac{1}{4}$

PS $y\sin y = \frac{x^2}{2}\log x + \frac{x^2}{4} + \frac{\pi^2}{2} - \frac{1}{4}$ (½)

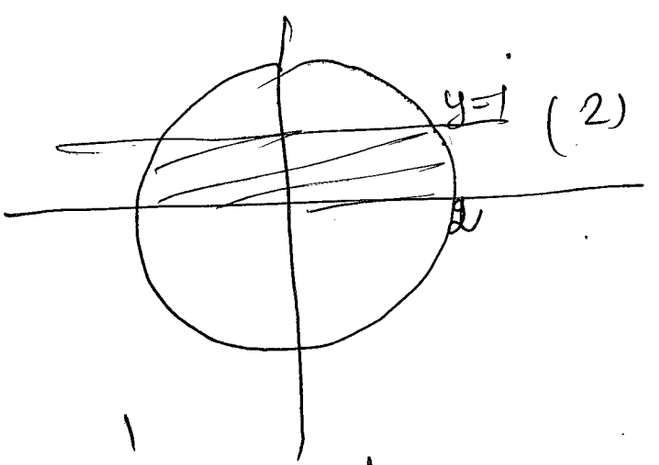
(35) $AB = 8I$ (½) $\Rightarrow A^{-1} = \frac{1}{8}B$ (1)

Matrix Eqn $AX = C$ (1)

$X = \frac{1}{8}BC = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ (1½)

$\therefore x = 3,\ y = -2,\ z = -1$ (1)

(32) $y = \sqrt{4 - x^2}$ $\quad x = \sqrt{4 - y^2}$



$A = 2\int_0^1 \sqrt{4 - y^2}\,dy$

$2\left[\frac{y}{2}\sqrt{4 - y^2} + 2\sin^{-1}\frac{y}{2}\right]_0^1$

$= 2\left[\frac{\sqrt{3}}{2} + \frac{2\pi}{6} - 0\right]$