MARKING SCHEME
PRE-BOARD EXAMINATION (2025-26)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**
समय : 3 घंटे **अधिकतम अंक - 80**

GENERAL INSTRUCTIONS:

1. Evaluation is to be done as per instructions provided in the marking scheme. Marking scheme should be strictly adhered to and religiously followed. However, while evaluating, answer which are based on latest information or knowledge and/or are innovative they may be assessed for their correctness otherwise and marks to be awarded to them.

2. If a student has attempted an extra question, answer of the question deserving more marks should be retained and other answer scored out.

3. A full scale (0-80) has to be used. Please do not hesitate to award full marks if the answer deserve it.

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SECTION-A

1. (c) $(A+B)^{-1} = B^{-1}+A^{-1}$ 1

2. (c) It will have both maximum and minimum values as feasible region is bounded. 1

3. (a) one-one but not onto 1

4. (d) $(-\infty,-2)\cup(2,\infty)$ 1

5. (a) Skew-symmetric matrix 1

6. (c) order 3, degree 4 1

7. (a) (–2, 0, –5) 1

8. (d) x log x – x + c 1

9. (d) $\dfrac{y^2}{x(1-y\log x)}$ 1

10. (b) reflexive but not symmetric and transitive 1

11. (c) 4 1

12. (a) one-one and onto 1

13. (c) $\pi+e^{\frac{\pi}{2}}+e^{\frac{-\pi}{2}}$ 1

14. (b) an increasing function 1

15. (a) $\dfrac{1}{12}$ 1

16. (d) 13 units 1

17. (b) $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=1$ 1

18. (d) 7 is maximum at (2, 8) and min. at (7, 1). 1

19. (d) (A) is false, but (R) is true. 1

20. (a) Both (A) and (R) are true and (R) is the correct explanation of (A). 1

SECTION-B

21. (a) Yes, R is transitive relation. ½

Justification for transitive relation 1

Pre-images of 4 = {1, 2, 3, 4} ½

OR

(b) Range = R – {2} 1

Co-domain = R ½

Range $\neq$ co-domain $\Rightarrow$ f is not onto ½

22. (a) $\cot^{-1}\left[2\cos\left\{2\sin^{-1}\left(\frac{1}{2}\right)\right\}\right] = \cot^{-1}\left[2\cos\frac{\pi}{3}\right]$ 1

$$= \frac{\pi}{4}$$ 1

OR

(b) For domain, $-1 \leq 3x - 4 \leq 1$ ½

$\Rightarrow 1 \leq x \leq \frac{5}{3}$ or domain $= \left[1, \frac{5}{3}\right]$ 1

Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ½

23. $\int e^{3y} dy = \int e^{4x+5} dx$ 1

Gen. Solution : $\frac{e^{3y}}{3} = \frac{e^{4x+5}}{4} + c$ ½

Particular solution : $\frac{e^{3y}}{3} = \frac{e^{4x+5}}{4} + \frac{1}{3} - \frac{e^5}{4}$

Or $4e^{3y} = 3e^{4x+5} + 4 - 3e^5$ ½

24. $\frac{dS}{dt} = 5\,\text{min}^2/\text{s} \Rightarrow \frac{dr}{dt} = \frac{5}{8\pi r}$ 1

$\frac{dV}{dt} = \frac{5r}{2}$ ½

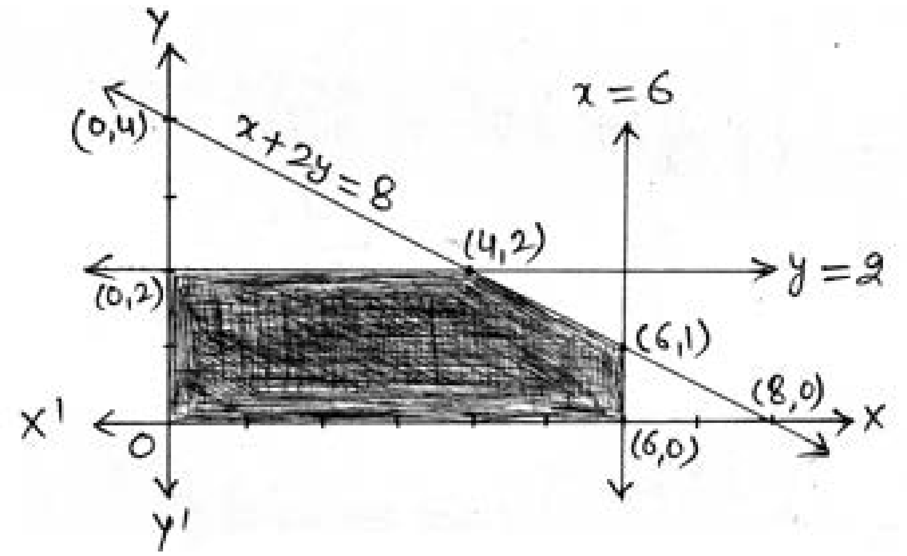
$\left(\frac{dV}{dt}\right)_{(r=8mm)} = 20mm^3/2$ ½

25. Symm. matrix $= \frac{1}{2}(A + A^T) = \begin{bmatrix} 9 & 4 \\ 4 & 5 \end{bmatrix}$ 1

Skew-symm. matrix $= \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ 1

SECTION-C

26.



1½

Corner Points	Max: Z = 7x + 8y
(0, 0)	0
(6, 0)	42
(6, 1)	50
(4, 2)	44
(0, 2)	16

→ Maximum (at (6, 1))

1

Hence, Z_{max} = 50 at point (6, 1) ½

27. (a) $\frac{3x-4}{(x^2-1)(x-2)} = \frac{7}{6(x+1)} + \frac{1}{2(x-1)} + \frac{2}{3(x-2)}$ 2

Hence, $\int \frac{3x-4}{(x^2-1)(x-2)} dx = \frac{7}{6}\log|x+1| + \frac{1}{2}\log|x-1| + \frac{2}{3}\log|x-2| + C$ 1

OR

(b) $\int_0^4 (|x-3| + |x-5|)\, dx$

$= -\int_0^3 (x-3)dx + \int_3^4 (x-3)dx - \int_0^4 (x-5)dx$ 1

$= \left[3x - \frac{x^2}{2}\right]_0^3 + \left[\frac{x^2}{2} - 3x\right]_3^4 + \left[5x - \frac{x^2}{2}\right]_0^4$ 1

$= \frac{9}{2} + \frac{1}{2} + 12 = 17$ 1

28. $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$ ½

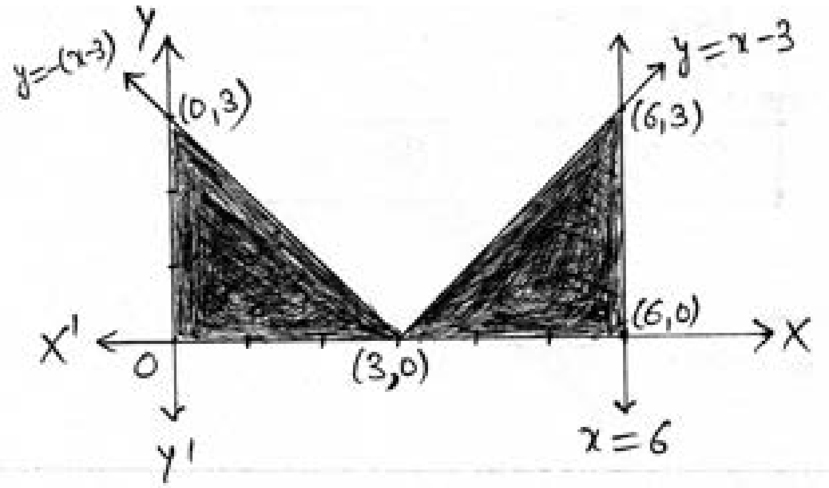
$P = \frac{2x}{1+x^2},\ Q = \frac{4x^2}{1+x^2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = e^{\log_{(1+x)^2}} = 1 + x^2$ 1

General solution : $y(1+x^2) = \int 4x^2 dx + c$ 1

$\Rightarrow y(1+x^2) = \frac{4x^3}{3} + c$ ½

29.



1

Area of shaded region = $\int_0^6 |x-3|\,dx$ ½

$= -\int_0^3 (x-3)dx + \int_3^6 (x-3)dx$ ½

$= \left[3x - \frac{x^2}{2}\right]_0^3 + \left[\frac{x^2}{2} - 3x\right]_3^6$ ½

$= \frac{9}{2} + \frac{9}{2} = 9$ sq.units ½

For Visually Impaired Students Only :

$9x^2 + 16y^2 = 144 \Rightarrow y = \pm\frac{3}{4}\sqrt{16-x^2}$ 1

Required area = $4\int_0^4 y\ dx = 3\int_0^4 \sqrt{16-x^2}\ dx$ ½

$$=3\left[\frac{x}{2}\sqrt{16-x^2}+8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$ 1

$$=12\pi \text{ sq.units}$$ ½

30. (a) $\left.\begin{array}{l}\vec{a_1}=\hat{i}-2\hat{j}+3\hat{k};\ \ \vec{b_1}=-\hat{i}+\hat{j}-2\hat{k}\\ \vec{a_2}=\hat{i}-\hat{j}-\hat{k};\ \ \ \ \vec{b_2}=\hat{i}+2\hat{j}-2\hat{k}\end{array}\right\}$ ½

$$\vec{a_2}-\vec{a_1}=\hat{j}-4\hat{k}$$

$$\vec{b_1}\times\vec{b_2}=2\hat{i}-4\hat{j}-3\hat{k}$$ 1

$$\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right)=8$$ ½

$\therefore$ Given lines are not parallel and

$$\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right)\neq 0$$

$\therefore$ Given lines are skew lines ½

Shortest distance = $\frac{8}{\sqrt{29}}$ units or $\frac{8\sqrt{29}}{29}$ units ½

OR

(b) $\vec{a}+\vec{b}=-\vec{c}$ ½

Squaring both sides,

$|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=|\vec{c}|^2$ 1

$\Rightarrow \cos\theta=\frac{1}{2}$ 1

$\Rightarrow \theta=\frac{\pi}{3}$ ½

31. (a) (i) Probability that problem is not solved

$=P(\bar{A}).P(\bar{B})$

$=\frac{2}{3}\times\frac{3}{5}=\frac{2}{5}$ 1

Probability that problem is solved

$=\frac{1-2}{5}=\frac{3}{5}$ ½

(ii) Probability that exactly one of them solves the problem =

$P(A).P(\bar{B})+P(\bar{A}).P(B)$ ½

$=\frac{1}{3}\times\frac{3}{5}+\frac{2}{3}\times\frac{2}{5}$ ½

$=\frac{7}{15}$ ½

OR

(b) Let A : Prerna buys a colouring book.

B : Prerna buys a box of colours.

$$P(A) = 0.7,\ P(B) = 0.2,\ P\left(\frac{A}{B}\right) = 0.3$$ 1

(i) $$P(A \cap B) = P(B).P\left(\frac{A}{B}\right) = 0.06$$ 1

(ii) $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{3}{35}$$ 1

SECTION-D

32. Let I $I = \int_0^{\pi} \log \sin x \, dx = 2\int_0^{\pi/2} \log \sin x \, dx$ ....(1) ½

On applying property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, we get

$$I = 2\int_0^{\pi/2} \log \cos x \, dx \qquad ...(2)$$ ½

On adding equation (1) and (2), we get

$$\not{2} I = \not{2}\int_0^{\pi/2} \log\left(\frac{2\sin x \cos x}{2}\right) dx$$ 1

$$\Rightarrow I=\int_0^{\pi/2}\log\sin 2x\ dx-\log 2\int_0^{\pi/2}1\ dx$$ ½

$$\left\{\text{Put } 2x=t\Rightarrow dx=\frac{1}{2}dt\right\}$$ ½

$$\Rightarrow I=\frac{1}{2}\int_0^{\pi}\log\sin x.dt-\frac{\pi}{2}\log 2$$ ½

$$\Rightarrow I=\int_0^{\pi/2}\log\sin x.dx-\frac{\pi}{2}\log 2$$

$$\Rightarrow I=\frac{I}{2}-\frac{\pi}{2}\log 2 \quad \text{[By equation.(1)]}$$ 1

$$\Rightarrow I-\pi\log 2$$ ½

33. Let, number of students in sports, music and drama clubs are x, y and z respectively.

ATQ,

$$\left.\begin{array}{l} x=y+z \quad \Rightarrow x-y-z=0 \qquad ...(1) \\ y=\dfrac{x}{2}+20\Rightarrow -x+2y+0.z=40 \quad ...(2) \\ x+y+z=180 \qquad ...(3) \end{array}\right\}$$ 1½

In matrix form,

$$\underset{A}{\begin{bmatrix}1 & -1 & -1\\ -1 & 2 & 0\\ 1 & 1 & 1\end{bmatrix}}\underset{X}{\begin{bmatrix}x\\ y\\ z\end{bmatrix}}=\underset{B}{\begin{bmatrix}0\\ 40\\ 180\end{bmatrix}}$$ ½

$\Rightarrow A.X = B \Rightarrow X = A^{-1}.B \quad ...(4)$ ½

$|A| = 4 \Rightarrow A^{-1}$ exists as $|A| \neq 0$

$$\text{adj.}A = \begin{bmatrix} 2 & 0 & 2 \\ 1 & 2 & 1 \\ -3 & -2 & 1 \end{bmatrix}$$ 1

$$A^{-4} = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & 2 & 1 \\ -3 & -2 & 1 \end{bmatrix}$$ ½

By equation (4),

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & 2 & 1 \\ -3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$$ ½

On comparing,

$x = 90, y = 65, z = 25$

Hence, number of students in sports, music and drama clubs are 90, 65 and 25 respectively. ½

34. (a) Put $x = \sin A$ and $y = \sin B$ 1

$\therefore \cos A + \cos B = a(\sin A - \sin B)$ ½

$$\Rightarrow \cancel{2}\cancel{\cos\left(\frac{A+B}{2}\right)}.\cos\left(\frac{A-B}{2}\right) = \cancel{2}a\cancel{\cos\left(\frac{A+B}{2}\right)}.\sin\left(\frac{A-B}{2}\right)$$ 1

$\Rightarrow A - B = 2\cot^{-1} a$ 1½

On difference w.r.t. x –

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}.\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$ 1

OR

(b) $\frac{dx}{d\theta} = \frac{a\cos^2\theta}{\sin\theta}$ 2

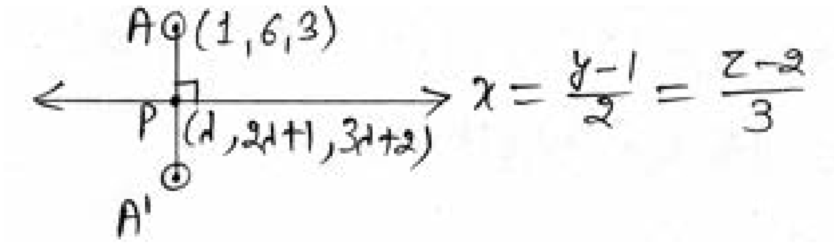
$\frac{dy}{d\theta} = \cos\theta$ ½

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{1}{a}\tan\theta$ ½

$\frac{d^2y}{dx^2} = \frac{1}{a}\sec^2\theta.\frac{d\theta}{dx} = \frac{\sin\theta}{a^2\cos^4\theta}$ 1½

$\left(\frac{d^2y}{dx^2}\right)_{\left(\theta=\frac{\pi}{4}\right)} = \frac{2\sqrt{2}}{a^2}$ ½

35. (a)



Let $x = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

$\Rightarrow x = \lambda,\ y = 2\lambda + 1,\ z = 3\lambda + 2$

Let $P \equiv (\lambda,\ 2\lambda + 1,\ 3\lambda + 2)$ ½

$\overrightarrow{AP} = (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}$ ½

$\therefore \overrightarrow{AP} \perp \text{line} \Rightarrow= (\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow \lambda = 1$ 1½

$\therefore P \equiv (1,3,5)$ ½

P is mid-point of A and $A' \Rightarrow A' \equiv (1,0,7)$ 1

Equation of line joining A and A' is given by

$\frac{x-1}{0} = \frac{y-6}{6} = \frac{z-3}{-4}$ or $x = 1,\ 2y + 3z = 21$ 1

OR

(b) Arbitrary point on line $x+5=\frac{y+3}{4}=\frac{6-z}{9}$ is $P(\lambda-5,\ 4\lambda-3,\ 6-9\lambda)$ 1

ATQ, $PQ=7 \Rightarrow (\lambda-7)^2+(4\lambda-7)^2+(7-9\lambda)^2=7^2$

$\Rightarrow \lambda=1$ 2

Hence, $P \equiv (-4,1,-3)$ 1

Equation of line joining points P and Q is :

$\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$ 1

SECTION-E

36. (I) V = (36 – 2x). (36 – 2x) . x or $V = 4x\,(18-x)^2$ 1

(II) $\frac{dV}{dx}=12(6-x)(18-x)$ 1

(III) (a) Put $\frac{dV}{dx}=0 \Rightarrow x=6$ as $\neq 18$ ½

$\left(\frac{dV}{dx}\right)_{(x=6^-)}=+ve$ and $\left(\frac{dV}{dx}\right)_{(x=6^+)}=-ve$ ½

Hence, V is maximum at x = 6 ½

V_{max} = 3456 cubic cm ½

OR

(b) Put $\frac{dV}{dx} = 0 \Rightarrow x = 6$ as $x \neq 18$ ½

$\frac{d^2V}{dx^2} = 24(x - 12)$ ½

$\left(\frac{d^2V}{dx^2}\right)_{(x=6)} = -\text{ve Value} \Rightarrow V$ is maximum at $x = 6$ ½

$V_{max} = 3456$ cubic cm ½

37. (I) Required position vector of mid-point = $\frac{\vec{a} + \vec{b}}{2}$ 1

(II) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\vec{a} - 2\vec{b}$ ½

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 5\vec{a} - 3\vec{b}$ ½

(III) (a) $\vec{a}.\vec{b} = 1 \Rightarrow 1 \times 2 \times \cos\theta = 1 \Rightarrow \theta = \frac{\pi}{3}$ 1

$|\vec{a} \times \vec{b}| = 1 \times 2 \times \sin\frac{\pi}{3} = \sqrt{3}$ 1

OR

(b) $\vec{a}+\vec{b}=2\hat{i}+3\hat{k}$ and $\vec{a}-\vec{b}=2\hat{i}-2\hat{j}+5\hat{k}$ ½

$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=6\hat{i}-4\hat{j}-4\hat{k}$ 1

$\therefore \hat{n}=\dfrac{3\hat{i}-2\hat{j}-2\hat{k}}{\sqrt{17}}$ ½

38. Let A : The person has contracted the disease.

ATQ,

$$P(A_1)=\frac{7}{10};\ P(A_2)=\frac{1}{5};\ P(A_3)=\frac{1}{10}$$

$$P\left(\frac{A}{A_1}\right)=\frac{1}{4};\ P\left(\frac{A}{A_2}\right)=\frac{7}{20};\ P\left(\frac{A}{A_3}\right)=\frac{1}{2}$$

(I) $P(A)=\dfrac{7}{10}\times\dfrac{1}{4}+\dfrac{1}{5}\times\dfrac{7}{20}+\dfrac{1}{10}\times\dfrac{1}{2}=\dfrac{59}{200}$ 2

(II) Using Baye's Theorem,

$$P\left(\frac{A_2}{\bar{A}}\right)=\frac{P(A_2).P\left(\frac{\bar{A}}{A_2}\right)}{P(A_1).P\left(\frac{\bar{A}}{A_1}\right)+P(A_2).P\left(\frac{\bar{A}}{A_2}\right)+P(A_3).P\left(\frac{\bar{A}}{A_3}\right)}$$ ½

$$=\frac{\frac{1}{5}\times\frac{13}{20}}{\left(\frac{7}{10}\times\frac{3}{4}+\frac{1}{5}\times\frac{13}{20}+\frac{1}{10}\times\frac{1}{2}\right)}$$ 1

$$=\frac{26}{141}$$ ½