

KENDRIYA VIDYALAYA SANGATHAN , CHENNAI REGION

PRE BOARD-I EXAM 2024-25

Class: XII Session: 2024-25

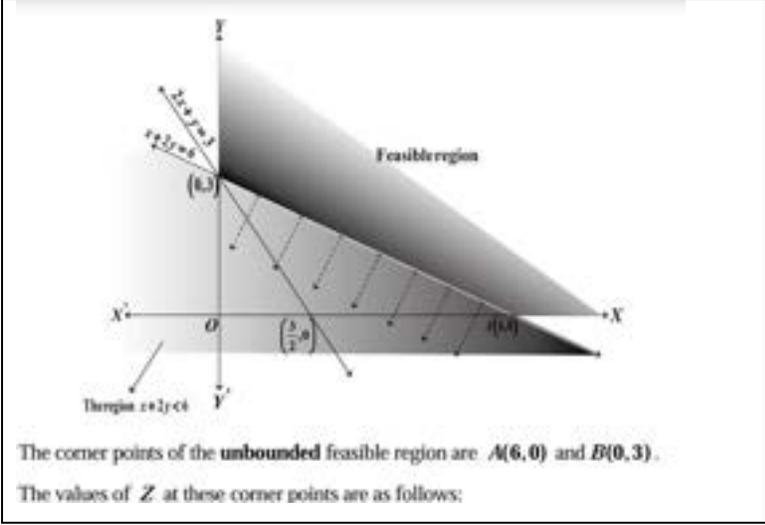
Subject: Mathematics (041)

Marking Scheme (Theory)

S. No.	Objective Type Question Sec A	Marks
1	(b) no such x and y possible	1
2	(a) $k = 3, p = n$	1
3	(d) (0, 2)	1
4	(b) B	1
5	(d) $y = -e^x + c$	1
6	(d) -25	1
7	(a) Skew symmetric matrix	1
8	(b) $P(A'B') = [1 - P(A)] [1 - P(B)]$	1
9	(d) $5\hat{i} - 10\hat{j} + 10\hat{k}$	1
10	(c) 5	1
11	(b) half plane that neither contains the origin nor the points on the line $2x + 3y = 6$	1
12	(b) $e^x \sec x + c$	1
13	(a) 0	1
14	(d) 0	1
15	(d) $\frac{1}{2}$	1
16	(c) z is maximum at (40,15), minimum at (15,20)	1
17	(b) $\frac{3}{4t}$	1
18	(d) $\frac{5}{2}$ sq units	1
19	(d) Assertion is incorrect, reason is correct.	1
20	(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.	1
SECTION B <u>VERY SHORT ANSWER TYPE QUESTIONS(VSA)</u> (Each question carries 2 marks)		
21	Required value = $\frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4}$ $= \frac{5\pi}{4}$	1 1

22	<p>As f is continuous at $x = 2 \Rightarrow \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x) = f(2)$</p> $\lim_{x \rightarrow 2^+} 3x = \lim_{x \rightarrow 2^-} (2x + 2) = k$ $\Rightarrow k = 6$ <p>OR</p> <p>Here, $\frac{ax}{a\theta} = a(1 - \cos \theta)$ and $\frac{dy}{d\theta} = a(-\sin \theta)$</p> $\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-a\sin \theta}{a(1 - \cos \theta)} = \frac{-\sin \theta}{(1 - \cos \theta)}$ $\therefore \left. \frac{dy}{dx} \right _{\theta = \frac{\pi}{3}} = \frac{-\sin \frac{\pi}{3}}{1 - \cos \frac{\pi}{3}} = \frac{-\sqrt{3}/2}{1 - (1/2)} = -\sqrt{3}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
23	$y^2 = 8x$ then $2y \frac{dy}{dt} = 8 \frac{dx}{dt}$ $\frac{dy}{dt} = \frac{dx}{dt}$ (given) Solving above $2y = 8$ $Y = 4$ Solving we get point (2,4)	<p>1</p> <p>1</p>
24	<p>The projection of vector \vec{a} on the vector \vec{b} is given by</p> $\frac{1}{ \vec{b} } (\vec{a} \cdot \vec{b}) = \frac{(2 \times 1 + 3 \times 2 + 2 \times 1)}{\sqrt{(1)^2 + (2)^2 + (1)^2}} = \frac{10}{\sqrt{6}} = \frac{5}{3} \sqrt{6}$ <p>OR $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \hat{i} - 2\hat{j} - 6\hat{k}$</p> <p>and $\vec{a} \times \vec{b} = \sqrt{1^2 + (-2)^2 + (-6)^2} = \sqrt{41}$</p> <p>Unit vector along $\vec{a} \times \vec{b} = \frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} } = \frac{\hat{i} - 2\hat{j} - 6\hat{k}}{\sqrt{41}} = \frac{1}{\sqrt{41}}\hat{i} - \frac{2}{\sqrt{41}}\hat{j} - \frac{6}{\sqrt{41}}\hat{k}$</p>	<p>1</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
25	$ \vec{a} + \vec{b} + \vec{c} ^2 = 0 \Rightarrow \vec{a} ^2 + \vec{b} ^2 + \vec{c} ^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -\frac{3}{2}$	<p>1</p> <p>1</p>
<p>SECTION C SHORT ANSWER TYPE QUESTIONS(SA) (Each question carries 3 marks)</p>		
26	$f'(x) = \frac{3}{2} \times 4x^3 - 12x^2 - 90x$ $\Rightarrow f'(x) = 6x^3 - 12x^2 - 90x = 6x(x^2 - 2x - 15) = 6x(x - 5)(x + 3)$	<p>1</p>

	<p>For strictly increasing, $f'(x) > 0$</p> $\Rightarrow 6x(x - 5)(x + 3) > 0$ $\Rightarrow x \in (-3, 0) \cup (5, \infty)$ <p>For strictly decreasing, $f'(x) < 0$</p> $\Rightarrow 6x(x - 5)(x + 3) < 0$ $\Rightarrow x \in (-\infty, -3) \cup (0, 5)$	1 1
27	<p>Let V and S be the volume and surface area of a cube of side x cm respectively.</p> <p>Given $\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$</p> <p>We require $\left. \frac{dS}{dt} \right _{x=10 \text{ cm}}$</p> <p>Now $V = x^3$</p> $\Rightarrow \frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} \Rightarrow 9 = 3x^2 \cdot \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{9}{3x^2} = \frac{3}{x^2}$ <p>Again, $\because S = 6x^2$</p> <p>[By formula for surface area of a cube]</p> $\Rightarrow \frac{dS}{dt} = 12 \cdot x \cdot \frac{dx}{dt} = 12x \cdot \frac{3}{x^2} = \frac{36}{x}$ $\Rightarrow \left. \frac{dS}{dt} \right _{x=10 \text{ cm}} = \frac{36}{10} = 3.6 \text{ cm}^2/\text{sec}.$	1 1 1
28	<p>We $x^3 - x \geq 0$ on $[-1, 0]$ and $x^3 - x \leq 0$ on $[0, 1]$ and that $x^3 - x \geq 0$ on $[1, 2]$.</p> $\int_{-1}^2 x^3 - x dx = \int_{-1}^0 (x^3 - x) dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) dx$ $= \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_1^2$ $= -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + 2 - \frac{1}{4} + \frac{1}{2} = \frac{3}{2} - \frac{3}{4} + 2 = \frac{11}{4}$ <p>OR</p> <p>Let $x + 3 = A \frac{d}{dx} (5 - 4x - 2x^2) + B = A(-4 - 4x) + B$</p> <p>On comparing the coefficients of like term,</p> <p>We have $A = -\frac{1}{4}$ and $B = 2$</p> $\Rightarrow I = -\frac{1}{2} \sqrt{5 - 4x - 2x^2} + \sqrt{2} \sin^{-1} \left[\sqrt{\frac{2}{7}} (x + 1) \right] + C$	1 1 1 1 1

<p>29</p>	<p>Let $\vec{d} = x\hat{i} + y\hat{j} + z$ \vec{d} is perpendicular to $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$ $\therefore \vec{d} \cdot \vec{b} = 0$ and $\vec{d} \cdot \vec{c} = 0$ hence $x - 4y + 5z = 0$ (1) and $3x + y - z = 0$(2) Also, $\vec{d} \cdot \vec{a} = 21$, where $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$ $\Rightarrow 4x + 5y - z = 21$(3)</p> <p>Eliminating z from (i) and (ii), we get $16x + y = 0$ Eliminating z from (ii) and (iii), we get $x + 4y = 21$ Solving we get $x = \frac{-1}{3}, y = \frac{16}{3}$ and $z = \frac{13}{3}$ $\therefore \vec{d} = \frac{-1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ is the required vector.</p> <p>OR</p> <p>Any point on the line (1) is $(3r - 1, 5r - 3, 7r - 5)$ Any point on the line(2) is $(k + 2, 3k + 4, 5k + 6)$ Let these points be common point of intersection So $3r - 1 = k + 2, 5r - 3 = 3k + 4, 7r - 5 = 5k + 6$ On solving these, we get $r = \frac{1}{2}, k = -\frac{3}{2}$ \therefore Lines (i) and (ii) intersect and their point of intersection is $(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2})$</p>	<p>$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$</p> <p>$\frac{1}{2}$ $\frac{1}{2}$</p> <p>1 1</p>
<p>30</p>	 <p>The corner points of the unbounded feasible region are $A(6, 0)$ and $B(0, 3)$. The values of Z at these corner points are as follows:</p>	<p>1</p>

Corner point	Value of the objective function $Z = x + 2y$
A(6,0)	6
B(0,3)	6

We observe the region $x + 2y < 6$ have no points in common with the unbounded feasible region. Hence the minimum value of $z = 6$.

It can be seen that the value of Z at points **A** and **B** is same. If we take any other point on the line $x + 2y = 6$ such as (2,2) on line $x + 2y = 6$, then $Z = 6$.

Thus, the minimum value of Z occurs for more than 2 points, and is equal to 6 .

31

Let $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$

Required probability = $1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$$

$$= 1 - [1 - P(A)][1 - P(B)][1 - P(C)]$$

$$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{5}{6} = 1 - \frac{5}{12} = \frac{7}{12}$$

OR

Solution Let E be the event that the man reports that six occurs in the throwing of the die and let S_1 be the event that six occurs and S_2 be the event that six does not occur.

Then $P(S_1) = \frac{1}{6}$, $P(S_2) = \frac{5}{6}$

$P(E|S_1)$ = Probability that the man speaks the truth = $\frac{3}{4}$

$P(E|S_2)$ = Probability that the man does not speak the truth = $1 - \frac{3}{4} = \frac{1}{4}$

Now $P(S_1 | E) = \frac{P(S_1)P(E|S_1)}{P(S_1)P(E|S_1) + P(S_2)P(E|S_2)}$

1

1

1

$$\frac{1}{2} + \frac{1}{2}$$

1

1

1

1

$$= \frac{\frac{1 \times 3}{6 \times 4}}{\frac{1 \times 3}{6 \times 4} + \frac{5 \times 1}{6 \times 4}} = \frac{1}{8} \times \frac{24}{8} = \frac{3}{8}$$

SECTION D
LONG ANSWER TYPE QUESTIONS(LA)
(Each question carries 5 marks)

32 $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Gives identity matrix of order 3.

Hence $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Now the system of equation can be written in the form $AX = B$ gives $x = 0, y = 5$ and $z = 3$

33 $A = \left| \int_{-4}^{-1} (x + 1) dx \right| + \int_{-1}^2 (x + 1) dx = 9 \text{ sq.un}$

34 $\frac{dy}{dt} = \frac{(\sqrt{\cos 2t}) \cdot \frac{d}{dt}(\cos^3 t) - (\cos^3 t) \cdot \frac{d}{dt}(\sqrt{\cos 2t})}{(\sqrt{\cos 2t})^2}$

Simplifying:

$1\frac{1}{2}$

$$\frac{dy}{dt} = \frac{-3 \cos^2 t \sin t \sqrt{\cos 2t} + \cos^3 t \cdot \frac{\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

$$\text{And } \frac{dx}{dt} = \frac{(\sqrt{\cos 2t}) \cdot \frac{d}{dt}(\sin^3 t) - (\sin^3 t) \cdot \frac{d}{dt}(\sqrt{\cos 2t})}{(\sqrt{\cos 2t})^2}$$

Simplifying:

$$\frac{dx}{dt} = \frac{3 \sin^2 t \cos t \sqrt{\cos 2t} + \sin^3 t \cdot \frac{\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

Now substituting $\frac{dy}{dt}$ and $\frac{dx}{dt}$ into the formula for $\frac{dy}{dx}$:

This simplifies to:

$$\frac{dy}{dx} = \frac{-3 \cos^2 t \sin t \sqrt{\cos 2t} + \cos^3 t \cdot \frac{\sin 2t}{\sqrt{\cos 2t}}}{3 \sin^2 t \cos t \sqrt{\cos 2t} + \sin^3 t \cdot \frac{\sin 2t}{\sqrt{\cos 2t}}}$$

$$\frac{dy}{dx} = \frac{-(4 \cos^3 x - 3 \cos x)}{3 \sin x - 4 \sin^3 x} = -\cot x$$

OR

$$\text{Left-Hand Limit} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\sin(a+1)x + \sin x}{x}$$

Using the limit properties, we can rewrite this as:

$$\lim_{x \rightarrow 0^-} \left(\frac{\sin(a+1)x}{(a+1)x} \times (a+1) + \frac{\sin x}{x} \right)$$

$$\lim_{x \rightarrow 0^-} f(x) = (a+1) + 1 = a+2$$

$$\text{Right-Hand Limit } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}}$$

To simplify this, we can factor out $x^{1/2}$:

$$= \lim_{x \rightarrow 0^+} \frac{x^{1/2}((1+bx)^{1/2} - 1)}{bx^{1/2}}$$

$$= \lim_{x \rightarrow 0^+} \frac{(1+bx)^{1/2} - 1}{(1+bx) - 1}$$

$$= \frac{1}{2} (1)^{\frac{1}{2} - 1} = \frac{1}{2}$$

And $f(0) = c$

$$\text{now } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$1\frac{1}{2}$

2

2

2

1

	$\Rightarrow a + 2 = \frac{1}{2} = c$ <p>hence $a = -\frac{3}{2}, c = \frac{1}{2}$ and b is any non zero real no.</p> \Rightarrow	
35	$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$ $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$ $\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 2\hat{j} - \hat{k}$ $ \vec{b}_1 \times \vec{b}_2 = \sqrt{6}$ $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 1$ <p>S.D. = $d = \left \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{ \vec{b}_1 \times \vec{b}_2 } \right$</p> <p>Shortest distance $d = \frac{1}{\sqrt{6}}$</p> <p>The lines do not intersect</p> <p>OR</p> <p>Eq. of line $\vec{r} = \vec{a} + \lambda\vec{b}$</p> <p>Line passes through (1,2,-4) and let (a,b,c) be the D, Ratio of line then Eq of line is</p> $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$ <p>Line is perpendicular to the lines</p> $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ <p>$\vec{a} \times \vec{b}$ is perpendicular to \vec{a} and \vec{b} both</p> $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ $= 24\hat{i} + 36\hat{j} + 72\hat{k}$ <p>Hence D' Ratio of line is 24,36,72</p> <p>Eq. of line $\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72}$</p> $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(24\hat{i} + 36\hat{j} + 72\hat{k})$ $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$	<p>1</p> <p>1</p> <p>1</p> <p>2</p> <p>2</p> <p>2</p> <p>1</p>

SECTION E (3 case study questions carries 4 marks each)		
36	<p>Ans a) R is reflexive and transitive but not symmetric. b) No. Because $n(B)$ is greater than $n(A)$ c) R is neither reflexive nor symmetric nor transitive</p> <p>OR No. of relations = 2^{12}</p>	2 1 1 1
37	<p>1) Area, $A(x) = (10 + x)\sqrt{100 - x^2}$ 2) $A'(x) = \frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}}$ 3) Put $A'(x) = 0$ we get $x = 5$</p> $A''(x) = \frac{\sqrt{100 - x^2}(-4x - 10) - (-2x^2 - 10x + 100) \frac{(-2x)}{2\sqrt{100 - x^2}}}{100 - x^2}$ $= \frac{2x^3 - 300x - 1000}{(100 - x^2)^{\frac{3}{2}}} \text{ (on simplification)}$ $A''(5) = \frac{2(5)^3 - 300(5) - 1000}{(100 - (5)^2)^{\frac{3}{2}}} = \frac{-2250}{75\sqrt{75}} = \frac{-30}{\sqrt{75}} < 0$ <p>thus, area of trapezium is maximum at $x = 5$</p> <p>OR</p> <p>Maximum area of trapezium</p> $A(5) = (5 + 10)\sqrt{100 - (5)^2} = 15\sqrt{75} = 75\sqrt{3} \text{ cm}^2$	1 1 2 2
38	<p>ANS a) $K=0.14$</p> <p>b) A: even face is turn up B: face show 2 or 4 $S = \{1, 2, 3, 4, 5, 6\}$, $P(A) = P(2) + P(4) + P(6) = 0.24 + 0.18 + 0.14 = 0.56$ $B = \{2, 4\}$ $B \cap A = \{2, 4\}$ $P(B \cap A) = P(2) + P(4) = 0.24 + 0.18 = 0.42$ $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.42}{0.56} = \frac{3}{4} = 0.75$</p>	2 2