First Pre Board (2025-26)

Class: 12

Subject: Mathematics

Marking Scheme(SET-A)

Section A

1. **(d)5**
2. **(c) 24** **[1]**
3. **(b)** $\left[\frac{-\pi}{2}, 0\right]$ **[1]**
4. **(a) 0** **[1]**
5. **(b) 2** **[1]**
6. **(c) 8** **[1]**
7. **(d)** $\frac{11}{4}$ **[1]**
8. **(d) 1.5** **[1]**
9. **(a)** $\frac{\cos a}{\cos^2(a+y)}$ **[1]**
10. **(d) -16x** **[1]**
11. **(d)** $[2, \infty)$ **[1]**
12. **(b)** $e^x.\sec x + c$ **[1]**
13. **(d) 0** **[1]**
14. **(d)** $\frac{4}{3}$ **[1]**
15. **(b) 1** **[1]**
16. **(a) 3** **[1]**
17. **(c)** $\left(\frac{1}{2}, \frac{1}{4}\right)$ **[1]**
18. **(c) a=3, b=5** **[1]**
19. **(a)** **[1]**
20. **(a)** **[1]**

Section B

21. **As f is continuous at $x = -1$**

$\therefore \lim_{x \to -1} f(x) = f(-1)$ $\left[\frac{1}{2}\right]$

$\Rightarrow \lim_{x \to -1} \frac{x^2-2x-3}{x+1} = f(-1)$

$\frac{1}{2}$

$\Rightarrow \lim_{x \to -1} \frac{(x+1)(x-3)}{(x+1)} = k$ **[1]**

$\Rightarrow -1 - 3 = k$

$\Rightarrow k = -4$ **[1]**

[OR]

$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ $\left[\frac{1}{2}\right]$

<u>LHL: -</u>

$\lim_{x \to o^-} \frac{f(x)-f(0)}{x-0} = \lim_{x \to o^-} \frac{(-x^2)-0}{x-0} = \lim_{x \to o^-} (-x) = 0$ **[1]**

<u>RHL: -</u>

$\lim_{x \to o^+} \frac{f(x)-f(0)}{x-0} = \lim_{x \to o^-} \frac{x^2-0^2}{x-0} = \lim_{x \to o^+} (x) = 0$ **[1]**

$\because$ **<u>LHL</u>=<u>RHL</u>=**$f'(0)$

$\therefore f$ is differentiable at $x = 0$ $\left[\frac{1}{2}\right]$

22. $\because A(adjA) = |A|I$ $\left[\frac{1}{2}\right]$

$|A| = 2(10 - 9) = 2 \times 1 = 2$ **[1]**

$\therefore A(adjA) = 2I$ or $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $\left[\frac{1}{2}\right]$

23. $let\ x = tan\theta$

$\Rightarrow \theta = \tan^{-1} x$ $\left[\frac{1}{2}\right]$

$\therefore \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \sin^{-1}\left(\frac{\tan\theta}{\sqrt{1+tan^2\theta}}\right) = \sin^{-1}\left(\frac{\tan\theta}{sec\theta}\right) = \sin^{-1}(sin\theta) = \theta$ **[1]**

$\theta = \tan^{-1} x$ $\left[\frac{1}{2}\right]$

[OR]

$\because -1 \le \sqrt{x-1} \le$ $\left[\frac{1}{2}\right]$

$\Rightarrow 0 \le x - 1 \le 1$ **[1]**

$1 \le x \le 2$

$\therefore$Domain is $x \in [1, 2]$ $\left[\frac{1}{2}\right]$

24. $f'(x) = 2x - 2a$ $\left[\frac{1}{2}\right]$

$\because f$ is an increasing function

$\therefore f'(x) \ge 0$ $\left[\frac{1}{2}\right]$

$\Rightarrow 2x - 2a \ge 0$ $\left[\frac{1}{2}\right]$

$\Rightarrow a \le x$ as $x > 0, a \le 0$ $\left[\frac{1}{2}\right]$

25. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\Rightarrow y = \pm\frac{2}{3}\sqrt{9 - x^2}$ $\left[\frac{1}{2}\right]$

$\therefore$Required area $= 2\int_0^3 \frac{2}{3}\sqrt{9 - x^2}$ $\left[\frac{1}{2}\right]$

$= \frac{4}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$ $\left[\frac{1}{2}\right]$

$= \frac{4}{3}\left[\left(0 + \frac{9}{2}\sin^{-1} 1 - 0\right)\right]$

$= 3\pi\ sq.units$ $\left[\frac{1}{2}\right]$

Section C

26. Show that R is reflexive $\left[\frac{1}{2}\right]$

Show that R is symmetric **[1]**

Show that R is transitive **[1]**

$\because R$ is reflexive, symmetric as well as transitive

$\therefore R$ is an equivalence relation $\left[\frac{1}{2}\right]$

[OR]

<u>for one-one: -</u>

Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-1}{x_1-2} = \frac{x_2-1}{x_2-2} \Rightarrow x_1 = x_2$ **[1]**

$\therefore f$ is one-one $\left[\frac{1}{2}\right]$

<u>For onto: -</u>

$f(x) = y \Rightarrow \frac{x-1}{x-2} = y$

$\therefore x = \frac{2y-1}{y-1}$ **[1]**

$\therefore$ Range of $f = R - \{1\} =$ codomain B

$\therefore f$ is onto $\left[\frac{1}{2}\right]$

27. $\log(\cos x)^y = \log(\cos y)^x$

$\Rightarrow y \log\cos x = x \log\cos y$ **[1]**

Differentiate both sides w.r.t x

$\Rightarrow y\frac{1}{\cos x}(-\sin x) + \log\cos x\frac{dy}{dx} = x\frac{1}{\cos x}(-\sin y)\frac{dy}{dx}$ **[1]**

$\Rightarrow (\log\cos x + x\tan y)\frac{dy}{dx} = \log\cos y + y\tan x$

$\Rightarrow \frac{dy}{dx} = \frac{\log\cos y + y\tan x}{\log\cos x + x\tan y}$ **[1]**

[OR]

$\frac{dy}{d\theta} = a\sin\theta, \frac{dx}{d\theta} = a(1-\cos\theta)$ $\left[\frac{1}{2}\right]$

$\frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$ **[1]**

$\frac{d^2y}{dx^2} = -\frac{cosec^2\frac{\theta}{2}}{2a(1-\cos\theta)}$ **[1]**

$\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{3}} = -\frac{4}{a}$ $\left[\frac{1}{2}\right]$

28. $I = \int\frac{x}{1+\cos x}dx + \int\frac{\sin x}{1+\cos x}dx$

$I = \int\frac{x}{2cos^2\frac{x}{2}}dx + \int\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos\frac{x}{2}\cos\frac{x}{2}}dx$ **[1]**

$I = \frac{1}{2}\int xsec^2\frac{x}{2}dx + \int\tan\frac{x}{2}dx$

$I = \left[\left(x\int sec^2\frac{x}{2}dx - \int\frac{d(x)}{dx}\int sec^2\frac{x}{2}dx\right)dx\right] + \int\tan\frac{x}{2}dx$

$I = \frac{1}{2}x.2\tan\frac{x}{2} - \frac{1}{2}\int 1\left(2\tan\frac{x}{2}\right)dx + \int\tan\frac{x}{2}$ **[1]**

$I = x\tan\frac{x}{2} - \frac{1}{2}\int\tan\frac{x}{2}dx + \frac{1}{2}\int\tan\frac{x}{2}dx$ **[1]**

$I = x\tan\frac{x}{2} + c$

$\therefore \int_0^{\frac{\pi}{4}}\frac{x+\sin x}{1+\cos x}dx = \left[x\tan\frac{x}{2}\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{4}\tan\frac{\pi}{8} - 0 = \frac{\pi}{4}\tan\frac{\pi}{8}$ **[1]**

29. $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 4 & 2\\ 3 & -2 & 7\end{vmatrix} = 32\hat{\imath} - \hat{\jmath} - 14\hat{k}$ **[1]**

$\because \vec{d}$is perpendicular to both $\vec{a}$ and $\vec{b}$

$\therefore \vec{d} = \lambda(\vec{a}\times\vec{b}) = \lambda(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$ $\left[\frac{1}{2}\right]$

$\because \vec{c}.\vec{d} = 18$ (given)

$\therefore (2\hat{\imath} - \hat{\jmath} + 4\hat{k})(32\lambda\hat{\imath} - \lambda\hat{\jmath} - 14\lambda\hat{k}) = 18$ $\left[\frac{1}{2}\right]$

$\Rightarrow (2)\times(32) + (1)\times(\lambda) + (4)\times(-14\lambda) = 18$

$\Rightarrow \lambda(64 + 1 - 56) = 18$

$\Rightarrow \lambda = \frac{18}{9}$

$\Rightarrow \lambda = 2$ $\left[\frac{1}{2}\right]$

$\therefore \vec{d} = 2(32\hat{\imath} - \hat{\jmath} - 14\hat{k}) = 64\hat{\imath} - 2\hat{\jmath} - 28\hat{k}$ $\left[\frac{1}{2}\right]$

30. Correct graph with shaded region $\left[1\frac{1}{2}\right]$

Corner Points	Value of Z=3x+9y
(0,20)	180⟶ maximum
(0,10)	90
(5,5)	60

(15,15)	180⟶ maximum

∴Z maximum=180 at infinitely many points lying on the line joining points (0,20) & (15,15). $\left[\frac{1}{2}\right]$

31. P(A)= $\frac{2}{7}$ and P(B)= $\frac{4}{7}$

∴P(A')= $1-\frac{2}{7}=\frac{5}{7}$ and P(B')= $1-\frac{4}{7}=\frac{3}{7}$ [1]

∴P (one of them coming to school in time)=P(A). P(B') +P(A'). P(B) [1]

$=\frac{2}{3}\times\frac{3}{7}+\frac{5}{7}\times\frac{4}{7}=\frac{6}{49}+\frac{20}{49}=\frac{26}{49}$ [1]

[OR]

Let A and B be two events such that

A=Selection of committee having exactly 2 boys

B=Selection of committee having at least one girl

To find- P(A/B)

∵P(B)= $\frac{4_{c_1}\times 7_{c_3}+4_{c_2}\times 7_{c_2}+4_{c_3}\times 7_{c_1}+4_{c_4}}{11_{c_4}}$

⇒P(B)= $\frac{140+126+28+1}{330}=\frac{295}{330}=\frac{59}{55}$ [1]

⇒P(A∩B)= $\frac{4_{c_2}\times 7_{c_2}}{11_{c_4}}=\frac{126}{330}=\frac{21}{55}$ [1]

∵P(A/B)= $\frac{P(A\cap B)}{P(B)}=\frac{\frac{21}{55}}{\frac{59}{56}}=\frac{21}{55}\times\frac{66}{59}=\frac{126}{295}$ [1]

Section D

32. $|A|=1\neq 0\Rightarrow |A|^{-1}$ exists [1]

(adj$A$)= $\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}^T=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$ [1]

$A^{-1}=\frac{adjA}{|A|}=\frac{1}{1}\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$ [1]

In the given equations

$CX=B$ where $C=\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}=A^T, B=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

$\Rightarrow X=C^{-1}B=\left(A^T\right)^{-1}B=\left[A^{-1}\right]^T B$ [1]

$\Rightarrow X=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}=\begin{bmatrix}-30+16+14\\ -20+8+7\\ -40+16+21\end{bmatrix}$

$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ -5\\ 3\end{bmatrix}$

$\therefore x=0, y=-5, c=-3$ [1]

33. Let $5x+3=A.d\left(x^2+4x+10\right)+B$

$A=\frac{5}{2}$ and $B=-7$

Now,

$$I=\int\frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}dx=\frac{5}{2}\int\frac{(2x+4)}{\sqrt{x^2+4x+10}}dx-7\int\frac{dx}{\sqrt{x^2+4x+10}}$$

$\therefore I=\frac{5}{2}I_1-7I_2$ [1]

$I_1=\int\frac{(2x+4)}{\sqrt{x^2+4x+10}}dx=\int\frac{dt}{\sqrt{t}}$ [where $t=x^2+4x+10$

$I_1=2\sqrt{t}+c_1$ [1]

$I_1 = 2\sqrt{x^2+4x+10} + c_1$

$I_2 = \int \frac{dx}{\sqrt{x^2+4x+10}} = \int \frac{dx}{\sqrt{(x+2)^2+(\sqrt{6})^2}} = \log(x+2) + \sqrt{x^2+4x+10} + c_2$ **[1]**

$\therefore I = \frac{5}{2} \times 2\sqrt{x^2+4x+10} - 7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + \left(\frac{5}{2}c_1 - 7c_2\right)$

$\Rightarrow I = 5\sqrt{x^2+4x+10} - 7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + C$

[OR]

$I = \int_0^{\pi} \frac{x}{a^2cos^2x + b^2sin^2x} dx$ **(i)** **[1]**

$I = \int_0^{\pi} \frac{\pi - x}{a^2cos^2(\pi-x) + b^2sin^2(\pi-x)} dx$

$I = \int_0^{\pi} \frac{\pi - x}{a^2cos^2x + b^2sin^2x} dx$ **(ii)** **[1]**

Adding (i) and (ii)

$$I + I = \int_0^{\pi} \frac{\pi - x + x}{a^2cos^2x + b^2sin^2x} dx$$

$$2I = \int_0^{\pi} \frac{\pi}{a^2cos^2x + b^2sin^2x} dx$$

$I = \frac{\pi}{2} \times 2\int_0^{\frac{\pi}{2}} \frac{1}{a^2cos^2x + b^2sin^2x} dx$ $\left[\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx\right]$ **[1]**

$I = \pi\int_0^{\frac{\pi}{2}} \frac{sec^2x}{a^2 + b^2tan^2x} dx$ **(dividing numerator and denominator by cos^2x)**

$I = \frac{\pi}{b}\int \frac{dt}{a^2+(t)^2}$ $\begin{bmatrix} where\ t = b\tan x \\ when\ x = 0, t = 0 \\ when\ x = \frac{\pi}{2}, t = \infty \end{bmatrix}$ **[1]**

$$I = \frac{\pi}{b}\left[\frac{1}{a}\tan^{-1}\frac{t}{a}\right]_0^{\infty}$$

$$I = \frac{\pi}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right]_0^{\infty}$$

$$I = \frac{\pi}{ab}\left[\frac{\pi}{2} - 0\right]$$

$I = \frac{\pi^2}{ab}$ **[1]**

34. $x\frac{dy}{dx} + xcos^2\left(\frac{y}{x}\right) = y$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - cos^2\left(\frac{y}{x}\right)$ **(i)** **[1]**

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ **(ii)** $\left(\frac{1}{2}\right)$

$\therefore v - cos^2\left(\frac{y}{x}\right) = v + x\frac{dv}{dx}$

$\Rightarrow \sec^2 v\, dv = -\frac{dx}{x}$ **[1]**

$\Rightarrow \int \sec^2 v\, dv = -\int \frac{dx}{x}$ $\left(\frac{1}{2}\right)$

$\Rightarrow \tan v = -\log|x| + c$

$\Rightarrow \tan\frac{y}{x} = -\log|x| + c$ **[1]**

Putting $x = 1 \& y\frac{\pi}{4}$ **we get** $c = 1$

$\therefore$ **Particular solution is** $\tan\frac{y}{x} = -\log|x| + 1$ **[1]**

[OR]

$\Rightarrow (x^2+1)\dfrac{dy}{dx}+2xy=\sqrt{x^2+4}$

$\Rightarrow \frac{dy}{dx}+\frac{2x}{x^2+1}=\frac{\sqrt{x^2+4}}{x^2+1}$ **[1]**

$\Rightarrow P=\dfrac{2x}{x^2+1}\;\&\;Q=\dfrac{\sqrt{x^2+4}}{x^2+1}$

$\therefore I.F=e^{\int Pdx}=e^{\int\frac{2x}{x^2+1}dx}=e^{\log|x^2-1|}=x^2-1$ **[1]**

$\therefore$ **Solution of the differential equation is**

$y(I.F)=\int Q(I.F)\,dx$ $\left(\frac{1}{2}\right)$

$\Rightarrow y(x^2+1)=\int\frac{(\sqrt{x^2+4})(x^2-1)}{(x^2-1)}dx$

$\Rightarrow y(x^2+1)=\int\sqrt{x^2+4}\,dx$

$\Rightarrow y(x^2+1)=\int\sqrt{x^2+2^2}\,dx$ $\left(\frac{1}{2}\right)$

$\Rightarrow y(x^2+1)=\dfrac{x}{2}\sqrt{x^2+2^2}+\dfrac{4}{2}\log\left|x+\sqrt{x^2+2^2}\right|+c$

$\Rightarrow y(x^2+1)=\dfrac{x}{2}\sqrt{x^2+2^2}+2\log\left|x+\sqrt{x^2+2^2}\right|+c$

35. For the given lines

$\overrightarrow{a_1}=-\hat{\imath}+\hat{\jmath}+9\hat{k},\ \overrightarrow{b_1}=2\hat{\imath}+\hat{\jmath}-3\hat{k}$

$\overrightarrow{a_2}=3\hat{\imath}-15\hat{\jmath}+9\hat{k},\ \overrightarrow{b_2}=2\hat{\imath}-7\hat{\jmath}+5\hat{k}$

$\therefore \overrightarrow{a_1}-\overrightarrow{a_2}=4\hat{\imath}-16\hat{\jmath}$ **[1]**

$\overrightarrow{b_1}\times\overrightarrow{b_2}=-16\hat{\imath}-16\hat{\jmath}-16\hat{k}$ **[2]**

$\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=16\sqrt{3}$ **[1]**

$\therefore S.D=\left|\frac{(\overrightarrow{a_1}-\overrightarrow{a_2}).(\overrightarrow{b_1}\times\overrightarrow{b_2})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$

$\Rightarrow S.D=\left|\frac{(4\hat{\imath}-16\hat{\jmath}).(-16\hat{\imath}-16\hat{\jmath}-16\hat{k})}{16\sqrt{3}}\right|=\frac{192}{16\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{12\sqrt{3}}{3}=4\sqrt{3}$ **[1]**

36. (i) **Volume of tank** $=250m^3$

$\Rightarrow x^2\times h=250m^3$

$\Rightarrow h=\frac{250}{x^2}$ $\left[\frac{1}{2}\right]$

$\therefore$ **total cost of digging tank** $=40,000h^2+5,000x^2$

$\Rightarrow C(x)=40,000\left(\frac{250}{x^2}\right)^2+5,000x^2$

$\Rightarrow C(x)=40,000\times\frac{62,500}{x^4}+5,000x^2$ $\left[\frac{1}{2}\right]$

$(ii)\frac{dC(x)}{dx}=\ 40,000\times\frac{62,500}{x^4}+5,000x^2$

$\Rightarrow\frac{dC(x)}{dx}=\frac{-4\times40,000\times62,500}{x^5}+10,000x$ **[1]**

(iii) **For maximum/minimum cost** $\frac{dC(x)}{dx}=0$

$\Rightarrow x=10m$ **[1]**

$\Rightarrow \frac{d^2C(x)}{dx^2}_{\ at\ x=10}>0$

$\therefore C(x)$ **is minimum at** $x=10$ **[1]**

[OR]

$C(x)=5,000x^2+\dfrac{2,50,00,00,000}{x^4}$

$C'^{(x)}=10,000-\frac{(10)^{10}}{x^5}$ **[1]**

For increasing $C'x>0$

$\Rightarrow x>10$

So, for $C'(x)$ **to be increasing** $x>10$ **and** $C'(x)<0,\forall x\in(0,10)$.

$\therefore C(x)$ **is not increasing for** $x>0$. **[1]**

37. $(i)\frac{x}{3}=\frac{y}{-4}=\frac{z}{1}$ **path is straight line** **[1]**

(ii) **After** $5sec$ **position of the missile be**

$$x=3t=3\times 5=15$$
$$y=-4t=-4\times 5=-20$$
$$z=t=5$$

$\therefore$ **Point is** $(15,-20,5)$ **[1]**

Its distance from origin is $\sqrt{650}km.$ **[1]**

(iii) **Height of the missile from the ground**= $\sqrt{(5-5)^2(-8+8)^2(10-0)^2}=10km$ **[1]**

[OR]

Given lines are perpendicular if $2\times(-2)+3(-1)+7\times k=0$

$\Rightarrow k=1$ **[1]**

38. Let E_1, E_2, E_3 **be the events that a customer avails loan on fixed rate, floating rate, variable rate respectively. Also let E be the events that the person defaults on the loan.**

Now, $P(E_1)=10\%=\frac{1}{10}, P(E_2)=20\%=\frac{2}{10}, P(E_3)=70\%=\frac{7}{10}$

Also, $P(E|E_1)=5\%, P(E|E_2)=3\%, P(E|E_3)=1\%.$

$(i)P(E_1)=P(E_1)P(E|E_1)+P(E_2)P(E|E_2)+P(E_3)P(E|E_3)$ **[2]**

$\Rightarrow P(E)=0.018$ **[2]**

(ii) **By using Baye's theorem,** $(E_3|E)$=$\frac{(E_3|E)}{P(E_1)P(E|E_1)+P(E_2)P(E|E_2)+P(E_3)P(E|E_3)}$ **[2]**

$\Rightarrow (E_3|E)=\frac{7}{18}$ **[2]**