MARKING SCHEME
PRE-BOARD EXAMINATION (2025-26)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**

समय : 3 घंटे **अधिकतम अंक - 80**

GENERAL INSTRUCTIONS:

1. Evaluation is to be done as per instructions provided in the marking scheme. Marking scheme should be strictly adhered to and religiously followed. However, while evaluating, answer which are based on latest information or knowledge and/or are innovative they may be assessed for their correctness otherwise and marks to be awarded to them.

2. If a student has attempted an extra question, answer of the question deserving more marks should be retained and other answer scored out.

3. A full scale (0-80) has to be used. Please do not hesitate to award full marks if the answer deserve it.

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SECTION-A

1. (c) R is reflexive, symmetric and transitive 1

2. (c) Range of $\tan^{-1} x \subset$ range of $\sin^{-1}x$, As $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)\subset\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ 1

3. (d) 6 1

As $b + p = 0, c + x = 0, r + y = 0, a = q = z = 0$

4. (d) $\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$ 1

As A.(adj A) = |A| I = 13 I

5. (c) 8 1

As $m = 2, n = 4$ so, order of matrix $Q = 2 \times 4$ 1

6. (a) 65 1

As $|\text{adj } A| = |A|^2 = 25$ and $|2A| = 8|A| = 40$ 1

7. (c) $\frac{\sqrt{3}}{2}$ 1

As $2\sin 2\alpha = 1 \Rightarrow \sin 2\alpha = \frac{1}{2} \Rightarrow 2\alpha = 150° \Rightarrow \alpha = 75°$

So, $\sin(\alpha - 15°) = \sin 60° = \frac{\sqrt{3}}{2}$

8. (b) 2 1

As $y = 2\log(\sin x) \Rightarrow \frac{dy}{dx} = 2\cot x \Rightarrow \left.\frac{dy}{dx}\right|_{x=\pi/4} = 2$

9. (c) 0.5 1

As LHL = RHL $\Rightarrow 5 = 10k \Rightarrow k = 0.5$

10. (d) $\frac{2\pi}{6}$ 1

As $\frac{d\theta}{dt} = -2\frac{d}{dt}(\sin\theta) \Rightarrow \cos\theta = \frac{-1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

11. (a) $-\cot x - x + c$ 1

As $\int \cot^2 x \, dx = \int (\operatorname{cosec}^2 x - 1)\, dx = -\cot x - x + c$

12. (d) $\frac{-5}{2}$ 1

As $\int e^x \left(\log x - \frac{1}{x} + \frac{1}{x} + \frac{1}{x^2} \right) dx = e^x \left(\log x - \frac{1}{x} \right) + c \Rightarrow f(x) = \log x - \frac{1}{x}$

$\therefore f(2) + f\left(\frac{1}{2}\right) = \log 2 - \frac{1}{2} + \log\frac{1}{2} - 2 = \frac{-5}{2}$

13. (d) $\frac{dy}{dx} = y$ 1

As (d) part has order = 1 = degree

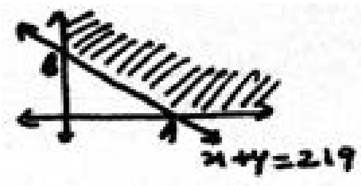
14. (a) $y = c.e^x$ 1

As $\int \frac{dy}{y} = \int dx \Rightarrow \log y = x + A \Rightarrow y = e^x.e^A \Rightarrow y = c.e^x$

15. (d) 8 1

As, Area $= \int_3^7 (x-3)dx = \frac{(7-3)^2}{2} = 8$

16. (c) 2 2



$\Rightarrow$ A and B are two corner points.

17. (d) a feasible region 1

18. (c) $2\hat{i} - \hat{j} - \hat{k}$ 1

As $\overrightarrow{BA} = \text{P.V of } A - \text{P.V of } B = (5-3)\hat{i} + (4-5)\hat{j} + (3-4)\hat{k} = 2\hat{i} - \hat{j} - \hat{k}$

19. (a) Both Assertion (A) and Reason(R) are true and Reason (R) is the correct explanation of the Assertion (A). 1

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SECTION-B

21. (a) $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)+\cos^{-1}\left(\cos\frac{3\pi}{5}\right)$

$=\sin^{-1}\left(\sin\left(\pi-\frac{2\pi}{5}\right)\right)+\cos^{-1}\left(\cos\frac{3\pi}{5}\right)$ ½

$=\frac{2\pi}{5}+\frac{3\pi}{5}$ 1

$=\pi$ ½

OR

(b) As, $-1\le 3-4x\le 1\Rightarrow -4\le -4x\le -2\Rightarrow \frac{1}{2}\le x\le 1$

Domain of given function = $\left[\frac{1}{2},1\right]$ 1

Range of given function = $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ 1

22. (a) $e^{y}=\frac{1}{x+1}\Rightarrow e^{y}\frac{dy}{dx}=\frac{-1}{(x+1)^{2}}\Rightarrow \frac{dy}{dx}=\frac{-1}{x+1}$ 1

Now, $\frac{d^{2}y}{dx^{2}}=\frac{1}{(x+1)^{2}}=\left(\frac{-1}{x+1}\right)^{2}=\left(\frac{dy}{dx}\right)^{2}$ 1

(b) Given, $\frac{dr}{dt} = 3cm/sec, \frac{db}{dt} = -5cm/sec,\ r = 4, h = 7$ ½

$$V = \pi r^2 h \Rightarrow \frac{dV}{dt} = \pi\left[r^2 \frac{db}{dt} + 2r\frac{dr}{dt}h\right]$$ 1

$$\frac{dV}{dt} = 88\pi cm^3/sec$$ ½

23. $I = \int \frac{x^2 + x + 1}{x^2(x+1)} dx = \int \frac{1}{x+1} dx + \int \frac{1}{x^2} dx$ 1

$$I = \log|x+1| - \frac{1}{x} + c$$ 1

24. $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c})$ ½

$$0 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b} + \vec{c}.\vec{a})$$ 1

$$\therefore \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-29}{2}$$ ½

25. $\ell_1 : 6x = -y = -4z \Rightarrow \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ and $\ell_2 : 2x = 3y = -z \Rightarrow \frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ ½+½

Since $a_1a_2 + b_1b_2 + c_1c_2 = 2\times3 - 12\times2 + 3\times6 = 0$

So, ℓ_1 & $\ell_2 \Rightarrow$ Angle between the lines is 90° 1

SECTION-C

26. (a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x(x^2 - x - 2) = 12x(x+1)(x-2) = 0$ 1

Critical points are x = 0, x = 2, x = -1

(i) For strictly increasing function : $f'(x) > 0 \Rightarrow x \in (-1, 0) \cup (2, \infty)$ 1

(i) For strictly decreasing function : $f'(x) < 0 \Rightarrow x \in (-\infty, -1) \cup (0, 2)$ 1

OR

(b) Slope $= \frac{dy}{dx} = -3x^2 + 6x + 0 = m(x)(\text{let})$ 1

$m(x) = -3(x^2 - 2x - 3)$

$\frac{dm(x)}{dx} = -3(2x - 2) = 0 \Rightarrow x = 1$ 1

Since, $\frac{d^2m(x)}{dx^2} = -6 < 0$, slope is maximum when x = 1

Maximum slope = m(1) = 12 1

27. (a) $I=\int_{-2}^{1}(x-x^3)dx+\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}(x-x^3)dx$ 1

$=\left(\frac{x^2}{2}-\frac{x^4}{4}\right)_{-2}^{-1}+\left(\frac{x^4}{4}-\frac{x^2}{2}\right)_{-1}^{0}+\left(\frac{x^2}{2}-\frac{x^4}{4}\right)_{0}^{1}$ 1

$=\frac{1}{4}+2+\frac{1}{4}+\frac{1}{4}=\frac{11}{4}$ 1

OR

(b) $I=\int(\sin^{-1}x-\cos^{-1}x)dx=2\int\sin^{-1}x\,dx-\frac{\pi}{2}\int 1\,dx$ 1

$I=2\left[x\sin^{-1}x-\int\frac{x}{\sqrt{1-x^2}}dx\right]-\frac{\pi}{2}x+c$ 1½

$I=2\sin^{-1}x+2\sqrt{1-x^2}-\frac{\pi}{2}x+c$ ½

28. (a) $\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}=\frac{y/x}{1+\left(\frac{y}{x}\right)^3}=f\left(\frac{y}{x}\right)$, (homogenous differential equation) ½

Let $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ ½

So, $v+x\frac{dv}{dx}=\frac{v}{1+v^3}\Rightarrow x\frac{dv}{dx}=\frac{v-v-v^4}{1+v^3}=\frac{-v^4}{1+v^3}$ 1

$\Rightarrow\int\left(\frac{1}{v^4}+\frac{1}{v}\right)dv=-\log|x|+c$

$\Rightarrow \dfrac{-1}{3v^3} + \log|v| = -\log|x| + c$

$\Rightarrow \boxed{\dfrac{-x^3}{3y^3} + \log y = c}$

OR

(b) $\dfrac{dy}{dx} - \dfrac{1}{x}.y = 2x$ (linear differential equation) ½

I.F $= \dfrac{1}{x}$ (integrating factor) 1

So, solution of given different equation $y.\left(\dfrac{1}{x}\right) = \int 2\,dx$ 1

$\therefore \boxed{y = 2x^2 + cx}$ ½

29. Given $P(E \cap F) = P(E) \times P(F)$ ½

Now, P (exactly one of E, F = P (only E) + P (only F) = P(E) + P(F) -2 $P(E \cap F)$ 1

$\Rightarrow \dfrac{5}{9} = x + 2x - 2x(2x)$

$\Rightarrow 36x^2 - 27x + 5 = 0$ ½

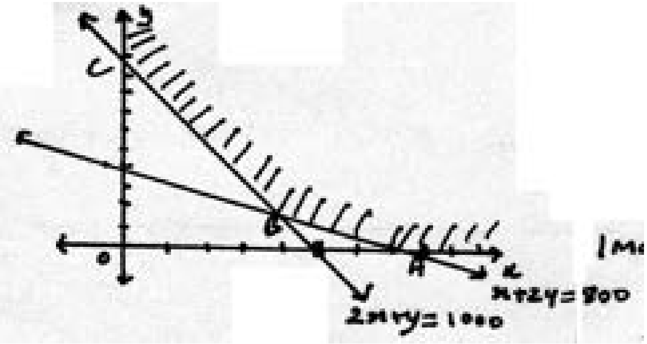
$\Rightarrow 36x^2 - 15x - 12x + 5 = 0$

$\Rightarrow 3x(12x - 5) - 1(12x - 5) = 0$

$\Rightarrow x = \dfrac{1}{3}$ or $\dfrac{5}{12}$ 1

30. Corner points of the festive region are A(800,0) B(400, 200) and C(0, 1000)

(1 mark for corner points)



(1 mark for graph)

$Z_A = 800 + 0 = 800$

$Z_B = 400 + 800 = 1200$

$Z_C = 0 + 4000 = 4000$

$\therefore$ Minimum value of Z = 800 when x = 800, y = 0 1

(Here, x + 4y < 800 has no common point with feasible region)

For visually impaired students

As, per condition given Z at (3, 4) = Z at (0, 5) 1

3a + 4b = 0a + 5b

$\boxed{3a = b}$ 2

31. As, $\hat{a}.\hat{b} = 0 \Rightarrow \hat{a} \perp \hat{b}$ and $\hat{a}.\hat{c} = 0 \Rightarrow \hat{a} \perp \hat{c}$

Thus, $\hat{a}$ is $\perp$ to the plane containing $\hat{b}$ and $\hat{c}$

$\hat{a} = \lambda(\hat{b} \times \hat{c})$ 1

$|\hat{a}| = |\lambda||\hat{b}||\hat{c}|\sin\frac{\pi}{6}$ 1

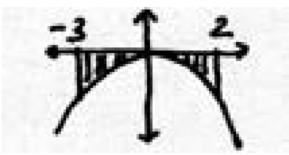
$\Rightarrow 1 = |\lambda| \times 1 \times 1 \times \frac{1}{2} \Rightarrow |\lambda| = 2 \Rightarrow \lambda = \pm 2$

$\Rightarrow \hat{a} = \pm 2(\hat{b} \times \hat{c})$ 1

SECTION-D

32. (a) Area $= \left|\int_{-3}^{2} y\, dx\right| = \left|\int_{-3}^{2} -x^2\, dx\right|$ 2

Area $= \left|\left[\frac{-x^3}{3}\right]_{-3}^{2}\right| = \frac{35}{3}$ sq.units 2



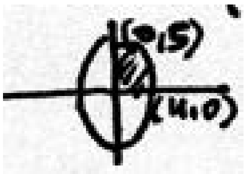
(1 mark for correct figure)

OR

(b) Area $= 4\times\int_0^4 \frac{5}{4}\sqrt{4^2-x^2}\,dx$ 2

$$=5\left[\frac{x}{2}\sqrt{4^2-x^2}+8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$=5\left[8\times\frac{\pi}{2}\right]=20\pi \text{ sq.units}$$ 2



(1 mark for figure)

For visually impaired students

(a) $I=\int_{-1}^{2}(x^3+2)dx=\left(\frac{x^4}{4}+2x\right)_{-1}^{2}$ 2

$$I=\left(\frac{16-1}{4}\right)+(4+2)=\frac{39}{4}$$ 2

This integral represents the area bounded by $y = x^3 + 2$ with x-axis

between $x = -1$ to $x = 2$ 1

OR

(b) $I = \int_0^{\pi} |\sin x)dx = \int_0^{\pi} \sin x \; dx = (-\cos x)_0^{\pi}$ 2

$I = -\cos\pi + \cos 0 = 2$ 2

This integral represents the area bounded by y = |sin x) with x-axis

between x 0 and x = π 1

33. |A| = 3 (-3) -2 (-26) +1 (19) = 62 $\neq 0$ (A^{-1} exists) ½

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{62}\begin{pmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{pmatrix}$$ 2

Now, given equation can be written as $\begin{pmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix}$

$$A^{T} \times = B$$

$\Rightarrow X = (A^{T})^{-1} B = (A^{-1}) T_B$ 1

$$\Rightarrow X = \frac{1}{62}\begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}\begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore x = 1, \; y = 1, \; z = 1$ ½+½+½

34. (a) $y^x + x^y + x^x = a^b \Rightarrow e^{x\log y} + e^{y\log x} + e^{x\log x} = a^b$ ½

On differentiating w.r.t. x, we get

$$\Rightarrow e^{x\log y}\left(\frac{x}{y}.\frac{dy}{dx} + \log y\right) + e^{y\log x}\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + e^{x\log x}(1+\log x) = 0$$ 1½+1½+1

$$\Rightarrow y^{x-1}.x\frac{dy}{dx} + y^x.\log y + x^{y-1}.y + x^y.\log x\frac{dy}{dx} + x^x(1+\log x) = 0$$

$$\therefore \frac{dy}{dx} = -\frac{\left[x^x(1+\log x) + y^x\log y + x^{y-1}.y\right]}{x.y^{x-1} + x^y\log x}$$ ½

OR

(b) $$I = \frac{1}{4}\int\frac{4x+8}{2x^2+6x+5}dx = \frac{1}{4}\int\frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int\frac{1}{2x^2+6x+5}dx$$ 1

$$I = \frac{1}{4}\log|2x^2+6x+5| + \frac{1}{4}\int\frac{dx}{x^2+3x+5_2}$$ 2

$$I = \frac{1}{4}\log|2x^2+6x+5| + \frac{1}{4}\int\frac{dx}{\left(x+\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$I = \frac{1}{4}\log|2x^2+6x+5| + \frac{1}{2}\tan^{-1}(2x+3) + c$$ 2

35. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + 2\lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 5\mu(2\hat{i} + 3\hat{j} + 6\hat{k}) = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu'(2\hat{i} + 3\hat{j} + 6\hat{k})$

Thus, we observe these line are parallel as d.r. are proportional.

So, $\vec{r} = \vec{a}_1 + \lambda'\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu' b$

$(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k} \Rightarrow |\vec{b}| = 7$ 1½

Now, $\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix} = -9\hat{i} + 14\hat{j} - 4\hat{k}$

$|\vec{b} \times (\vec{a}_2 - \vec{a}_1)| = \sqrt{293}$ 2

Thus, S.D. between the lines = $\left| \dfrac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right| = \dfrac{\sqrt{293}}{7}$ units ½

SECTION-E

36. (a) $f_1(x) = x^2$ and $f_4(x) = 2025$ are not one-one. 1

(b) $f_2(x) = 2x+3$ (on $\mathbb{R}$ to $\mathbb{R}$) is onto. 1

(c) (i) As, $f_2(x)$ and $f_3(x)$ are one-one function but since the range of $f_3(x) = x^3$ is $\mathbb{R} \neq$ co-domain $f_3(x)$.

Thus, $f_2(x) = 2x+3$ is one-one function and onto both.

[$f_2(a) = f_2(b) \Rightarrow a = b$ and range of f_2 = co-domain of f_2] 2

OR

(ii) Since $f_1(x)$ and $f_4(x)$ many-one from part (a).

Now, check for their range.

Range of $f_1(x) = [0,\infty) \neq$ codomain of $f_1(x)$

Range of $f_4(x) = \{2025\} \neq$ codomain of $f_4(x)$.

So, $f_1(x) = x^2$ (on $\mathbb{R}$ to $\mathbb{R}$) and $f_4(x) = 2025$ (on $\mathbb{R}$ to $\mathbb{R}$)

are many-one and into functions. 2

37. (i) Here $2\pi r = x \Rightarrow r = \dfrac{x}{2\pi}$ and $28 - x = 4a \Rightarrow a = \dfrac{28-x}{4}$

$A(x) = \pi\left(\dfrac{x^2}{4\pi^2}\right) + \left(\dfrac{28-x}{4}\right)^2 = \dfrac{x^2}{4\pi} + \dfrac{(28-x)^2}{16}$ 1

(ii) $\dfrac{d}{dx}\big(A(x)\big) = \dfrac{x}{2\pi} - \dfrac{(28-x)}{8}$ 1

(iii) Put $\dfrac{dA}{dx} = 0 \Rightarrow x = \dfrac{28\pi}{4+\pi}$m (for both the parts (a) and (b))

(a) $f'\left(\dfrac{28\pi}{4+\pi} - h\right) = -\text{ve}$ and $f'\left(\dfrac{28\pi}{4+\pi} + h\right) = +\text{ve}$

Case of minima $\Rightarrow A(x)$ is minimum at $x = \dfrac{28\pi}{4+\pi}$m

Thus, length of other piece $= \dfrac{112}{4+\pi}$m 2

(ii) $\frac{dA}{dx} = 0 \Rightarrow x = \frac{28\pi}{4+\pi} m$

$\frac{d^2A}{dx^2} = \frac{1}{2\pi} + \frac{1}{8} > 0$, so case of minima

A(x) is minimum at $x = \frac{28\pi}{\pi+4} m$

Length of other piece = $28 - x = \frac{112}{\pi+4} m$ 2

38. (i) P (contracted the disease) = $\frac{6}{10} \times \frac{25}{100} + \frac{3}{10} \times \frac{35}{100} + \frac{1}{10} \times \frac{50}{100}$

$P(c) = \frac{305}{1000} = 0.305$ ½

(ii) $P(\overline{c}) = 1 - P(c) = \frac{695}{1000}$

People from A_2 = 300, chance of not contracting = $\frac{65}{100}\left(1 - \frac{35}{100} = \frac{65}{100}\right)$

$P(A_2 \cap \overline{c}) = \frac{300}{1000} \times \frac{65}{100} = \frac{195}{1000}$ 1

$P\left(\frac{A_2}{\overline{c}}\right) = \frac{\frac{195}{1000}}{\frac{695}{1000}} = \frac{195}{695} = \frac{39}{139}$ ½