NAVODAYA VIDYALAYA SAMITI
Pre Board-I Examination (2025-26)
MARKING SCHEME

Class :XII **Subject : MATHAMATICS** **Set – 1**

Time : 180 MIN **Maximum Marks**

Q. No.	**Section – A** Question	Marks
1.	C	1
2.	C	1
3.	B	1
4.	C	1
5.	B	1
6.	A	1
7.	C	1
8.	B	1
9.	B	1
10.	D	1
11.	A	1
12.	D	1
13.	D	1
14.	B	1
15.	**A**	1
16.	C	1
17.	B	1
18.	C	1
19 & 20		
19	A	1
20	A **Section - B**	1

21.

ANS: Given x = 10(t – sin t) ⇒ $\frac{dx}{dt}$ = 10(1 – cos t) y = 12(1 – cos t) ⇒ $\frac{dy}{dx}$ = 12 sin t

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{12\sin t}{10(1-\cos t)}$$

$$= \frac{12\times 2\sin\frac{t}{2}\cos\frac{t}{2}}{10\times 2\sin^2\frac{t}{2}} = \frac{6}{5}\cot\frac{t}{2}$$ 1

$$\left.\frac{dy}{dx}\right]_{t=\frac{2\pi}{3}} = \frac{6}{5}\cot\frac{\pi}{3} = \frac{6}{5}\times\frac{1}{\sqrt{3}} = \frac{6}{5\sqrt{3}}$$ 1

OR

$y = \log(x + \sqrt{a^2+x^2})$

$\frac{dy}{dx} = \frac{1}{(x+\sqrt{a^2+x^2})}\cdot\frac{d}{dx}(x + \sqrt{a^2+x^2})$ 1

$\frac{dy}{dx} = \frac{1}{(x+\sqrt{a^2+x^2})}\cdot\left(1 + \frac{2x}{2\sqrt{a^2+x^2}}\right) = \frac{1}{(x+\sqrt{a^2+x^2})}\cdot\left(\frac{(x+\sqrt{a^2+x^2})}{\sqrt{a^2+x^2}}\right) = \frac{1}{\sqrt{a^2+x^2}}$ 1

22.

$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right) = \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$ 1

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right) = \frac{\pi}{4}-x$ 1

23.

$$\frac{dy}{dx} = 2x(x-2)\frac{d}{dx}[x(x-2)]$$

$$\frac{dy}{dx} = 4x(x-2)(x-1)$$

$\frac{dy}{dx} = 0 \Rightarrow x = 0, 1, 2$ 1

Interval	sign of $\frac{dy}{dx} = 4x(x-2)(x-1)$ ...(i)	Nature of $y = f(x)$
$(-\infty, 0]$	Take $x = -1$ (say). Then from (i), $\frac{dy}{dx} = (-)(-)(-) = (-)$ (or $= 0$ at $x = 0$) i.e., ≤ 0	∴ $f(x)$ is decreasing ↓
$[0, 1]$	Take $x = \frac{1}{2}$ (say). Then from (i), $\frac{dy}{dx} = (+)(-)(-) = (+)$ (or $= 0$ at $x = 0, x = 1$) i.e., ≥ 0	∴ $f(x)$ is increasing ↑
$[1, 2]$	Take $x = 1.5$ (say). Then from (i), $\frac{dy}{dx} = (+)(-)(+)$	∴ $f(x)$ is decreasing ↓

1

24.

$\int\sqrt{1+\sin 2x}\,dx = \int\sqrt{(\sin x+\cos x)^2}\,dx$ 1

$\int(\sin x+\cos x)\,dx$ = -cos x + sin x +c 1

25.

$$3^2+4^2+5^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$9+16+25+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$50+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$ 1

$$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = -50$$

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = \frac{-50}{2}$$ 1

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -25$$

Q. No.	Section - C	Marks
	OR $(a+\lambda b)\cdot c = 0$ $((2-\lambda)\mathbf{i}+(2+2\lambda)\mathbf{j}+(3+\lambda)\mathbf{k})\cdot(3\mathbf{i}+\mathbf{j}+0\mathbf{k})=0$ $(2-\lambda)(3)+(2+2\lambda)(1)+(3+\lambda)(0)=0$ $6-3\lambda+2+2\lambda=0$ $8-\lambda=0$ $\lambda=8$	
26.	$x\sqrt{1+y}+y\sqrt{1+x}=0$ $x\sqrt{1+y}=-y\sqrt{1+x}$ $x^2(1+y)=y^2(1+x)$ $x^2-y^2=y^2x-x^2y$ $(x+y)(x-y)=-xy(x-y)$ $x+y=-xy$; $\quad x=-y-xy$ $y(1+x)=-x$; $\quad y=-\frac{x}{1+x}$ $\quad$ [$since, x\neq y$] $\frac{dy}{dx}=-\left\{\frac{(1+x)\times 1-x(0+1)}{(1+x)^2}\right\}$ $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$ OR If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$ Solution: We have, $\sin y = x\sin(a+y)$ $x=\frac{\sin y}{\sin(a+y)}$ Differentiating both sides with respect to y, we get $\frac{dx}{dy}=\frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{\sin^2(a+y)}$ $\frac{dx}{dy}=\frac{\sin(a+y-y)}{\sin^2(a+y)}$ $\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}=\frac{\sin^2(a+y)}{\sin a}$	1 1 1
27.	$\int\frac{1-\sin 2x}{x+\cos^2 x}dx = \int\frac{1}{t}dt$ $=\log\lvert t\rvert+C=\log\lvert x+\cos^2 x\rvert+C$ Let $x+\cos^2 x = t$ $\Rightarrow (1-2\cos x\sin x)\,dx = dt$ $\Rightarrow (1-\sin 2x)\,dx = dt$	1 1 1
28.	The given differential equation $e^x\tan y\,dx + (1-e^x)\sec^2 y\,dy = 0$ $\Rightarrow e^x\tan y\,dx = (e^x-1)\sec^2 y\,dy$ Separating the variables $\frac{e^x}{(e^x-1)}dx=\frac{\sec^2 y\,dy}{\tan y}$ On integrating both sides we get $\int\frac{e^x}{(e^x-1)}dx=\int\frac{\sec^2 y\,dy}{\tan y}\Rightarrow \log\lvert e^x-1\rvert=\log\lvert\tan y\rvert+\log\lvert C\rvert$ OR $\frac{dy}{dx}+\sec^2 x\,.y=\tan x\,.\sec^2 x$ which is of the form $\frac{dy}{dx}+Py=Q$ Here P = $\sec^2 x$ Q= $\tan x\,.\sec^2 x$ Integrating factor (I.F) = $e^{\int P\,dx}=e^{\int\sec^2 x\,dx}=e^{\tan x}$ The solution of the linear differential equation is given by	1 1 1 1 1

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y (I.F) $=\int [Q\,.(I.F)]\,dx + C$

$\Rightarrow y\, e^{\tan x} = \int \tan x \,.\sec^2 x\, e^{\tan x}\, dx + C$ ----(1)

Put tanx =t $\sec^2 x$ dx =dt

$y\, e^{\tan x} = \int t\, e^t\, dt + C$ $\Rightarrow y\, e^{\tan x} = t\, e^t - e^t + C$

$\Rightarrow y\, e^{\tan x} = e^t(t-1) + C$ $\Rightarrow y\, e^{\tan x} = e^{\tan x}(\tan x - 1) + C$

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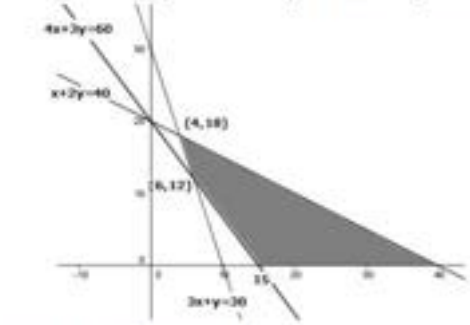
Subject to $x + 2y \le 40, 3x + y \ge 30, 4x + 3y \ge 60, x, y \ge 0$



<table>
<tr><th>Corner Pts</th><th>Z=20x+10y</th></tr>
<tr><td>(15,0)</td><td>300</td></tr>
<tr><td>(40,0)</td><td>800</td></tr>
<tr><td>(4,18)</td><td>260</td></tr>
<tr><td>(6,12)</td><td>240</td></tr>
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imum Value of Z is 240 at (6,12)

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SOL: Given $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

A Unit vector perpendicular to $\vec{a}$ and $\vec{b}$ $= \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$

$= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21) = 19\hat{j} - 19\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = 19\sqrt{2}$

A Unit vector perpendicular to $\vec{a}$ and $\vec{b}$ $= \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{19\hat{j} - 19\hat{k}}{19\sqrt{2}} = \frac{\hat{j} - \hat{k}}{\sqrt{2}}$

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<tr><td>31</td><td>

(i) P(A and B): $P(A \text{ and } B) = P(A) \cdot P(B) = 0.3 \cdot 0.6 = 0.18$

(ii) P(A and not B): $P(A \text{ and not } B) = P(A) \cdot P(\text{not } B) = 0.3 \cdot 0.4 = 0.12$

(iii) P(A or B):
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.3 + 0.6 - 0.18 = 0.9 - 0.18 = 0.72$

(iv) P(neither A nor B):
$P(\text{neither } A \text{ nor } B) = P(\text{not } A \text{ and not } B) = P(\text{not } A) \cdot P(\text{not } B) = 0.7 \cdot 0.4 = 0.28$

OR

$$P(\text{neither solves}) = P(A') \times P(B') = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$$

The probability that the problem is solved is $1 - P(\text{neither solves})$.

$$P(\text{problem is solved}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$P(\text{exactly one solves}) = P(A)P(B') + P(B)P(A') = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

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1

2

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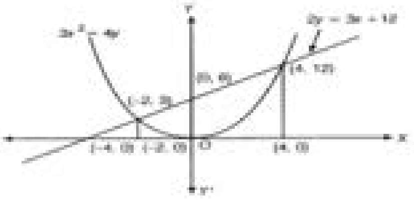
32.	Shortest Distance between the lines = $\frac{\|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})\|}{\|\vec{b_1}\times\vec{b_2}\|}$	1
	$\vec{a_1} = -\hat{i}-\hat{j}-\hat{k}$ $\quad \vec{a_2} = 3\hat{i}+5\hat{j}+7\hat{k}$ $\quad \vec{a_2}-\vec{a_1} = 4\hat{i}+6\hat{j}+8\hat{k}$ $\vec{b_1} = 7\hat{i}-6\hat{j}+\hat{k}$ $\quad \vec{b_2} = \hat{i}-2\hat{j}+\hat{k}$	
	$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \hat{i}(-6+2)-\hat{j}(7-1)+\hat{k}(-14+6) = -4\hat{i}-6\hat{j}-8\hat{k}$	2
	$\|\vec{b_1}\times\vec{b_2}\| = \sqrt{16+36+64} = \sqrt{116}$ $SD = \frac{\|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})\|}{\|\vec{b_1}\times\vec{b_2}\|} = \left\|\frac{(4\hat{i}+6\hat{j}+8\hat{k})\cdot(-4\hat{i}-6\hat{j}-8\hat{k})}{\sqrt{116}}\right\|$	
	$SD = \left\|\frac{(4)(-4)+(6)(-6)+(8)(-8)}{\sqrt{116}}\right\| = \left\|\frac{-16-36-64}{\sqrt{116}}\right\| = \frac{116}{\sqrt{116}} = \sqrt{116} = 2\sqrt{29}$	2
	OR	
	Sol: Let P(1,2,1) be the given point and L be the foot of the perpendicular from P to the given line AB (as shown in the figure above).	
	Let's put $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3} = \lambda$. Then, $x = \lambda+3, y = 2\lambda-1, z = 3\lambda+1$	1
	Let the coordinates of the point L be $(\lambda+3, 2\lambda-1, 3\lambda+1)$. So, direction ratios of PL are$(\lambda+3-1, 2\lambda-1-2, 3\lambda+1-1)$ *i.e.*, $(\lambda+2, 2\lambda-3, 3\lambda)$ Direction ratios of the given line are 1,2 and 3, which is perpendicular to PL. Therefore, we have,	2
	$(\lambda+2)\cdot 1+(2\lambda-3)\cdot 2+3\lambda\cdot 3 = 0 \Rightarrow 14\lambda = 4 \Rightarrow \lambda = 2/7$ Then, $\lambda+3 = \frac{2}{7}+3 = \frac{23}{7}$; $2\lambda-1 = 2\left(\frac{2}{7}\right)-1 = -\frac{3}{7}$; $3\lambda+1 = 3\left(\frac{2}{7}\right)+1 = \frac{13}{7}$	
	Therefore, coordinates of the point L are $\left(\frac{23}{7}, -\frac{3}{7}, \frac{13}{7}\right)$.	2
33.	$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ $= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$ $= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$	
	If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ then $A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$	2
	$\therefore X = A^{-1}B = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$	2
	$X = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$	1
	$\Rightarrow x = 3, y = -2, z = -1$	
34.		

	Required are $= \int_{-2}^{4}\left(\frac{3x+12}{2} - \frac{3x^2}{4}\right)dx$ $=\left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4} = \frac{3\times4^2}{4} + 6\times4 - \frac{4^3}{4} - \left[\frac{3\times4}{4} - 12 + \frac{8}{4}\right]$ $=12 + 24 - 16 - 3 + 12 - 2 = 27$ sq units  	2 3
35.	$\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$ Set $x = 1$: $1^2 + 1 = B(1+3) \Longrightarrow 2 = 4B \Longrightarrow B = \frac{1}{2}$ Set $x = -3$: $(-3)^2 + 1 = C(-3-1)^2 \Longrightarrow 10 = 16C \Longrightarrow C = \frac{10}{16} = \frac{5}{8}$ Set $x = 0$: $0^2 + 1 = A(-1)(3) + B(3) + C(-1)^2 \Longrightarrow 1 = -3A + 3B + C$ Substitute the values of B and C: $1 = -3A + 3\left(\frac{1}{2}\right) + \frac{5}{8} \Longrightarrow 1 = -3A + \frac{12}{8} + \frac{5}{8} \Longrightarrow 1 = -3A + \frac{17}{8}$ $3A = \frac{17}{8} - 1 \Longrightarrow 3A = \frac{9}{8} \Longrightarrow A = \frac{3}{8}$ $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx = \int \left(\frac{3/8}{x-1} + \frac{1/2}{(x-1)^2} + \frac{5/8}{x+3}\right)dx$ We integrate each term separately: $\int \frac{3/8}{x-1}\,dx = \frac{3}{8}\ln\lvert x-1\rvert$ $\int \frac{1/2}{(x-1)^2}\,dx = \frac{1}{2}\int (x-1)^{-2}\,dx = \frac{1}{2}\,\frac{(x-1)^{-1}}{-1} = -\frac{1}{2(x-1)}$ $\int \frac{5/8}{x+3}\,dx = \frac{5}{8}\ln\lvert x+3\rvert$ Combining these results and adding the constant of integration, C_{int}: $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx = \frac{3}{8}\ln\lvert x-1\rvert - \frac{1}{2(x-1)} + \frac{5}{8}\ln\lvert x+3\rvert + C_{int}$ OR $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ Divide the numerator and denominator by $\cos^2 x$: $\int_0^{\frac{\pi}{2}} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$ Let $u = \tan x$, so $du = \sec^2 x\,dx$. The limits of integration change from $x = 0$ to $u = \tan(0) = 0$ and from $x = \frac{\pi}{2}$ to $u = \tan(\frac{\pi}{2}) = \infty$. $\int_0^{\infty} \frac{du}{a^2 + b^2u^2} = \frac{1}{b^2}\int_0^{\infty} \frac{du}{(a/b)^2 + u^2}$ Using the standard integral formula $\int \frac{dx}{c^2 + x^2} = \frac{1}{c}\arctan(\frac{x}{c})$, we get: $\frac{1}{b^2}\left[\frac{b}{a}\arctan\left(\frac{bu}{a}\right)\right]_0^{\infty} = \frac{1}{ab}(\arctan(\infty) - \arctan(0)) = \frac{1}{ab}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{2ab}$	1 2 2 1 2 2

36	(i) Given that $\frac{dv}{dt} = 100$ and $v = \frac{1}{3}\pi r^2 h$ or $v = \frac{2}{9}\pi r^3$ So, $\frac{dv}{dt} = 100 = \frac{2}{3}\pi r^2 \frac{dr}{dt}$ implies $\frac{dr}{dt} = \frac{150}{\pi r^2}$ Now area of the base is $A = \pi r^2$ and $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ Therefore, rate of change in area of base at radius = 5, is $\frac{dA}{dt} = 60\ cm^2$ /minute (ii) Given that $\frac{dh}{dt} = \frac{4}{11}$ and $h = 7$ Then $v = \frac{1}{3}\pi r^2 h = \frac{3\pi h^3}{4}$ and hence, $\frac{dv}{dt} = \frac{9}{4}\pi h^2 . \frac{dh}{dt}$ $\frac{dv}{dt} = 2 \times 7 \times 9 = 126\ cc/min$	2 2
37.	(i)Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R \Rightarrow$ R is a symmetric relation. (ii) Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2$ and Let $(l_2, l_3) \in R \Rightarrow l_2 \parallel l_3$ Since $l_1 \parallel l_2$ *and* $l_2 \parallel l_3, l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R$ Hence R is a transitive relation. (iii)(a) The set is $\{l : l \text{ is a line of type } y = 3x + c, c \in R\}$ (b) Let $(l_1, l_2) \in S \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in S$ $\Rightarrow$ S is symmetric. Let $(l_1, l_2) \in S \Rightarrow l_1 \perp l_2$ and Let $(l_2, l_3) \in S \Rightarrow l_2 \perp l_3$ $l_1 \perp l_2$ *and* $l_2 \perp l_3 \Rightarrow l_1 \parallel l_3$ $\Rightarrow (l_1, l_3)$ *is not an element of S.* S is not a transitive relation.	1 1 2
38.	$(a) A: Cab B: Metro C: Bike D: other$ $E: Late arrival$ $P(E) = P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C) + P(D).P(E/D)$ $= \frac{3}{10}.\frac{1}{4} + \frac{1}{5}.\frac{1}{3} + \frac{1}{10}.\frac{1}{12} + \frac{2}{5}.\frac{1}{10} = \frac{114}{600}$ $(b) P(B/E) = \frac{P(B).P(E/B)}{P(E)} = \frac{1/15}{114/600} = 40/114 = 20/57$	2 2