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No. of pages - 24

PRE-BOARD EXAMINATION (2025-26)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**

समय : 3 घंटे **अधिकतम अंक - 80**

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए।

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड-ख में प्रश्न संख्या 21 से 25 तक अति लघुउत्तरीय (VSA) प्रकार के दो-दो अंक के प्रश्न हैं।
5. खंड-ग में प्रश्न संख्या 26 से 31 तक लघुउत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच-पाँच अंकों के प्रश्न हैं।
7. खंड-ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के 3 प्रश्नों में, खण्ड-घ के 2 प्रश्नों में, खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section-A, question No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section-B, question No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section-C, question No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section-D, question No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section-E, question No. 36 to 38 are Case Study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section-E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

1. If A and B are invertible matrices, then which of the following is **NOT** correct: 1

(a) $(A.B)^{-1} = B^{-1}.A^{-1}$ (b) $\text{adj } A = |A|.A^{-1}$

(c) $(A+B)^{-1} = B^{-1}+A^{-1}$ (b) $|A^{-1}|=|A|^{-1}$

2. If the feasible region of a linear programming problem with objective function $Z = ax + by$ is bounded, then which of the following is correct: 1

(a) It will only have a maximum value.

(b) It will only have a minimum value.

(c) It will have both maximum and minimum values.

(d) It will have neither maximum nor minimum value.

3. If $f : N \to N$ defined by $f(x) = x^2 + 5$, then f is: 1

(a) one-one but not onto (b) onto but not one-one

(c) one-one and onto both (d) neither one-one nor onto

4. $f(x) = x^3 - 12x + 5$ is strictly increasing on: 1

(a) R (b) $(-2, 2)$

(c) $(-\infty,0)\cup(2,\infty)$ (d) $(-\infty,-2)\cup(2,\infty)$

5. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is **NOT** : 1

(a) Skew-symmetric matrix (b) Diagonal Matrix

(c) Scalar Matrix (d) Unit Matrix

6. The order and degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^5 = \sin x$ are: 1

(a) order 3, degree not defined
(b) order 4, degree 3
(c) order 3, degree 4
(d) order 5, degree 2

7. Line $\frac{x-1}{3} = \frac{y-7}{7} = \frac{z-5}{10}$ intersects XZ plane at point: 1

(a) $(-2, 0, -5)$
(b) $(3, 0, 10)$
(c) $(3, 7, 10)$
(d) $(1, 0, 5)$

8. $\int \log x \, dx$ is equal to: 1

(a) $\frac{1}{x} + c$
(b) $x. \log x + c$
(c) $x \log x + x + c$
(d) $x \log x - x + c$

9. If $y = x^y$, then $\frac{dy}{dx} =$ 1

(a) $\frac{y}{x(1-\log x)}$
(b) $\frac{y^2}{x(1-\log x)}$
(c) $\frac{y}{x(1+y\log x)}$
(d) $\frac{y^2}{x(1-y\log x)}$

10. Let A = {1, 2, 3}, then relation R on set A defined by R = {(1, 1), (1, 2), (3, 1), (2, 2), (1, 3), (3, 3)} is : 1

(a) reflexive and transitive but not symmetric
(b) reflexive but not symmetric and transitive
(c) reflexive and symmetric but not transitive
(d) transitive but not reflexive and symmetric

11. If $\begin{vmatrix} 14 & 16 \\ 18 & 20 \end{vmatrix} = k \begin{vmatrix} 7 & 8 \\ 9 & 10 \end{vmatrix}$, then the value of k is: 1

(a) 1
(b) 2
(c) 4
(d) 8

12. $f : R \rightarrow R$ defined by $f(x) = x^3$ is : 1

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

13. $\int_{-\pi/2}^{\pi/2} \left[\tan^3 x + 1 + e^x.(\sin x + \cos x\right] dx =$ 1

(a) $e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}}$ (b) $\pi + e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}}$

(c) $\pi + e^{\frac{\pi}{2}} + e^{\frac{-\pi}{2}}$ (d) $e^{\frac{\pi}{2}} + e^{\frac{-\pi}{2}}$

14. If $f : R \rightarrow R$ is defined as $f(x) = 2x - \sin x$, then f is : 1

(a) a decreasing function (b) an increasing function

(c) maximum at $x = \frac{\pi}{2}$ (d) maximum at $x = 0$

15. A pair of dice is thrown two times. The probability of obtaining the sum greater than 10 is : 1

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{3}{14}$

16. The distance of the point with position vector $5\hat{i} + 4\hat{j} + 12\hat{k}$ from y-axis is : 1

(a) 4 units (b) 5 units

(c) 12 units (d) 13 units

17. $y = e^{\log x}$ is a solution of differential equation: 1

(a) $\frac{dy}{dx} = e^x$ (b) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 1$

(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} = \frac{dy}{dx}$

18. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 8), (5, 3) and (7, 1). If $Z = 13x + 15y$ be objective function, then: 1

(a) Z is maximum at (2, 8) and min. at (5, 3).

(b) Z is maximum at (7, 1) and min at (2, 8).

(c) 7 is maximum at (5, 3) and min at (7, 1).

(d) 7 is maximum at (2, 8) and min. at (7, 1).

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. **ASSERTION (A) :** The principal value of $\cos^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$ is $\frac{2\pi}{5}$. 1

REASON (R): The range of principal value branch of the function $f(x) = \cos^{-1} x$ is $[0, \pi]$.

20. **ASSERTION (A):** $f(x) = \begin{cases} 3x - 8 & , \quad x \le 5 \\ 2kx & , \quad x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{7}{10}$. 1

REASON (R): for a function f to be continuous at $x = a$, $\lim_{x \to a^-}(f(x)) = \lim_{x \to a^+}(f(x)) = f(a)$

SECTION-B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Check whether relation R on the set of natural numbers N defined by $R = \{(a,b) : a \leq b \text{ where } a, b \in N\}$ is transitive or not. Justify your answer. Write all pre-images of 4 also. 1½+½

OR

(b) Check whether the function $f : R - \{4\} \rightarrow R$ defined by $f(x) = \frac{2x-3}{x-4}$ is onto or not. Justify your answer. 2

22. (a) Evaluate: $\cot^{-1}\left[2\cos\left\{2\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$ 2

OR

(b) Find the domain and range of $f(x) = \sin^{-1}(3x-4)$

23. Find the particular solution of differential equation $\frac{dy}{dx} = e^{4x-3y+5}$; given that y = o at x = 0. 2

24. Surface area of a spherical balloon, when air is blown into it, increases at a rate of $5mm^2/s$. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing. 2

25. Express the matrix $A = \begin{bmatrix} 9 & 6 \\ 2 & 5 \end{bmatrix}$ as the sum of a Symmetric matrix and skew-symmetric matrix. 2

SECTION-C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. Solve the following linear programming problem graphically :

Maximise : $Z = 7x + 8y$

Subject to the constraints :

$x + 2y \leq 8;\ x \leq 6;\ y \leq 2;\ x, y \geq 0$ 3

For Visually Impaired Students Only:

The objective function Z = 5x + 2y of a lineal programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are A (700, 0), B(1300, 0), C(900, 400) and D(500, 200). Find the point at which Z is maximum and the point at which Z is minimum. Also, find the corresponding maximum and minimum values of Z. 3

27. (a) Find : $\int \frac{3x-4}{(x^2-1)(x-2)} dx$ 3

OR

(b) Evaluate : $\int_0^4 (|x-3| + |x-5|)\, dx$

28. Find the general solution of differential equation $(1+x^2).\frac{dy}{dx} + 2xy = 4x^2$ 3

29. Sketch the graph of y = |x − 3| and find the area of the region enclosed by the given curve, x-axis, between x = 6 and x = 0 , using integration. 3

For Visually Impaired Students Only:

Find the area enclosed within the curve $9x^2 + 16y^2 = 144$ using integration.

30. (a) Verify that lines

$\vec{r} = (1-\lambda)\hat{i} - (2-\lambda)\hat{j} - (2\lambda-3)\hat{k}$ and

$\vec{r} = (\mu+1)\hat{i} - (1-2\mu)\hat{j} - (2\mu+1)\hat{k}$

are skew-lines. Hence, find shortest distance between the lines. 3

OR

(b) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ such that $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|=7$, then find the angle between तो $\vec{a}$ and और $\vec{b}$.

31. (a) Probability of solving specific problem independently by A and B are $\frac{1}{3}$ and $\frac{2}{5}$ respectively. Find the probability that (i) Problem is solved (ii) exactly one of them solves the problem.

OR

(b) The probability thet Prerna buys a colouring book is 0.7 and that she buys a box of colours is 0.2. The probability that she buys a colouring book, given that she buys a box of colours, is 0.3. Find the probability that :

(i) she buys the colouring book and the box of colours.

(ii) she buys a box of colours given that she buys the colouring book. 3

SECTION-D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. Evaluate : $\int_0^{\pi} \log \sin x \cdot dx$ 5

33. A school wants to allocate students into three clubs: sports, Music and Drama; under following conditions:

- The number of students in sports club should be equal to the sum of the number of students in Music and Drama Club.
- The number of students in Music Club should be 20 more than half the number of students in Sports Club.
- The total number of students to be allocated in all three clubs are 180.

Find number of students allocated to different clubs, using matrix method. 5

34. (a) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$ 5

OR

(b) If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=\sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.

OR

35. (a) Find the image A' of the point A (1, 6, 3) in the line $x=\frac{y-1}{2}=\frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

OR

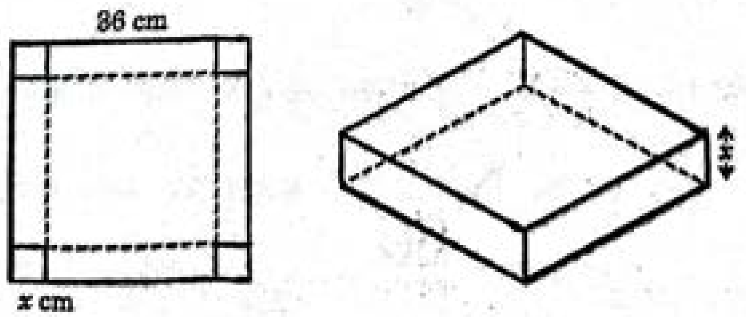
(b) Find a point P on the line $x+5=\frac{y+3}{4}=\frac{6-z}{9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining points P and Q.

SECTION-E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub-parts (I) and (II) of marks 2 each.

CASE-STUDY-1

36. Shalu has an expensive square-shaped piece of golden board of side 36 cm. She wants to turn it into a box without top by cutting a square from each corner and folding the flaps. Let x cm be the side of square, which is cut from each corner.



Based on the above information, answer the following questions :

(I) Find the expression for the volume (V) of open box in terms of x. 1

(II) Find $\frac{dV}{dx}$ 1

(III) (a) Find the maximum volume of the open box by first derivative test. 2

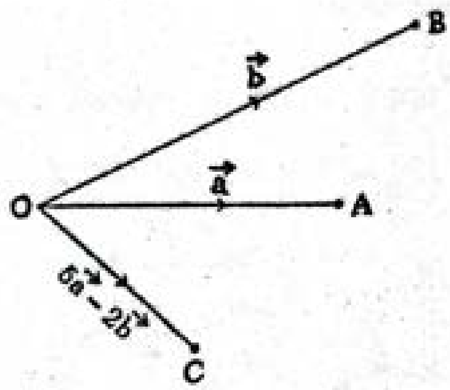
OR

(b) Find the maximum volume of the open box by second derivative test.

CASE STUDY-2

37. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations.

They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, fallowing straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based on the above information, answer the following questions :

(I) Find the position vector of mid-point of the vector joining point A to point B. 1

(II) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. 1

(III) (a) If $\vec{a}.\vec{b}=1$, distance of O to A is 1 km. and that from O to B is 2 km., then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also find $|\vec{a}\times\vec{b}|$.

OR

(b) If $\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, then find a unit vector perpendicular to $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$. 2

CASE STUDY-3

38. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health and

A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50% respectively.