

GENERAL INSTRUCTIONS:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed.

SECTION- A

1. The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is 1
(a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$
2. If A is a square matrix of order 2 such that $\det(A) = 4$, then $\det(4adjA)$ is equal to: 1
(a) 16 (b) 64 (c) 256 (d) 512
3. For any square matrix A of order 2, if $A \cdot (adj A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$ then the value of $|A|$ is 1
(a) 8 (b) 4 (c) 0 (d) 64
4. If I is a unit matrix, then 3I will be 1
(a) a null matrix (b) a unit matrix
(c) a triangular matrix (d) a scalar matrix
5. If A is a 3 x 3 matrix, whose elements are given by $a_{ij} = \frac{1}{3}|-3i + j|$, then write 1
the value of a_{23} .
(a) -1 (b) 1 (c) 2 (d) -2
6. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is a non-singular matrix and $a \in A$, then the set A is 1
(a) R (b) {0} (c) {4} (d) R - {4}
7. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is: 1
(a) 1 (b) -1 (c) $-2\sqrt{\pi}$ (d) $2\sqrt{\pi}$

- 8 The value of k if the function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $x = 0$ 1
- (a) 1 (b) 2 (c) 3 (d) 4

- 9 $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ 1
- (a) $-\cot(xe^x) + C$ (b) $\tan(xe^x) + C$ (c) $\tan(e^x) + C$ (d) $\cot(e^x) + C$

- 10 The function $f(x) = x^2 - 4x + 6$ is increasing in the interval 1
- (a) $(0,2)$ (b) $(-\infty, 2]$ (c) $[1,2]$ (d) $(2, \infty)$

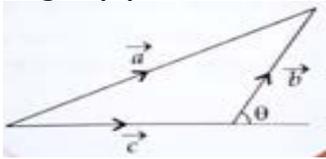
- 11 $\int_{-1}^1 \frac{|x|}{x} dx, x \neq 0$ is equal to 1
- (a) -1 (b) 0 (c) 1 (d) 2

- 12 The order and degree of the differential equation $(1 + 3\frac{dy}{dx})^{2/3} = 4\frac{d^3y}{d^3x}$ are 1
- (a) $(1, 2/3)$ (b) $(3, 1)$ (c) $(3, 3)$ (d) $(1, 2)$

- 13 The corner points of the feasible region determined by the system of linear constraints are $(0, 10), (5, 5), (15, 15), (0, 20)$. Let $z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of z occurs at both the points $(15, 15)$ and $(0, 20)$ is 1
- (a) $p = q$ (b) $p = 2q$ (c) $q = 2p$ (d) $q = 3p$

- 14 If the projection of $\vec{a} = \alpha\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 6 units, then α is 1
- (a) -12 (b) -5 (c) 12 (d) 5

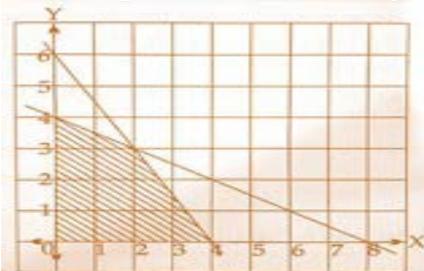
- 15 If $\vec{a}, \vec{b}, \vec{c}$ are side vector of triangle such that $|\vec{a}| = \sqrt{37}, |\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle (θ) between \vec{b} and \vec{c} is 1



- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

- 16 If a vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct 1
- (a) $\vec{a} \parallel \vec{b}$ (b) $\vec{a} \perp \vec{b}$ (c) $|\vec{a}| < |\vec{b}|$ (d) $|\vec{a}| = |\vec{b}|$

- 17 Shown below is the feasible region of a maximisation problem whose objective function is given by $z = 5x + 3y$. 1



- (a) $(0,0)$ (b) $(0,4)$ (c) $(2,3)$ (d) $(4,0)$

- 18 If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P\left(\frac{E}{F'}\right)$ is equal to : 1
- (a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{7}{9}$

ASSERTION-REASON BASED QUESTIONS

In each of the following questions (Q.NO. 19 and Q.NO.20), a statement of Assertion(A) is given followed by a corresponding statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both assertion(A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
 (b) Both assertion (A) and reason(R) are true but reason(R) is not the correct explanation of assertion(A).
 (c) Assertion (A) is true but reason(R) is false.
 (d) Assertion(A) is false but reason(R) is true

- 19 **Assertion (A):** If two lines in space are parallel, then the shortest distance between them is zero. 1

Reason(R): The shortest distance between two skew lines is given by

$$\left| \frac{(a_2 - a_1) \cdot (b_1 \times b_2)}{|b_1 \times b_2|} \right|$$

Where a_1, a_2 are positions vectors of points on the lines and \vec{b}_1, \vec{b}_2 are their direction vectors.

- 20 **Assertion (A) :** The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$ 1

Reason(R) : Domain of $\cot^{-1}x$ is $R - \{-1,1\}$.

SECTION B (VSA)

- 21 Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. 2

OR

Find the domain of $f(x) = \sin^{-1}(-x^2)$.

- 22 If $x = e^{\frac{y}{x}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$. 2

- 23 Differentiate $2^{\cos^2 x}$ w.r.t. $\cos^2 x$ 2

- 24 Evaluate $\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ 2

OR

Evaluate $\int x\sqrt{1+2x} dx$

- 25 A vector \vec{a} makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find vector \vec{a} . 2

SECTION - C (SA)

- 26 The volume of a spherical balloon is increasing at the rate of 25 cm³/s. Find the rate of change of its surface area at the instant when radius is 5 cm. 3

- 27 Solve the following linear programming problem graphically: 3

Maximise $Z = x + 2y$

$$x - y \geq 0,$$

Subject to the constraints: $x - 2y \geq -2$

$$x \geq 0, y \geq 0$$

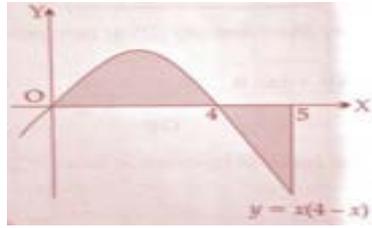
- 28 If $x = a \left(\cos\theta + \log \tan \frac{\theta}{2} \right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$. 3

OR

If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.

- 29 Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$. 3

OR



Find the area bounded by the curve, $y = x(4 - x)$ and the x -axis from $x = 0$ to $x = 5$ as shown in the figure given above.

- 30 Verify that lines given by $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. 3

OR

During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$, and $\vec{F} = 12\hat{i} + 18\hat{j}$, respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

- 31 Two dice are thrown simultaneously. Given that the sum of the numbers on the dice is 9, find the probability that the first die showed an even number. 3

SECTION -D (LA)

- 32 Solve the differential equation $(\tan^{-1}y - x) dy = (1 + y^2) dx$. 5

- 33 If A is a 3×3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ 5

- 34 Find the integral: $I = \int \frac{\cos x}{(4 + \sin^2 x)(5 - 4\cos^2 x)} dx$ 5

OR

Evaluate the integral: $I = \int_0^\pi \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

- 35 Find the image A' of the point $A(1,6,3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A' . 5

OR

Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that distance from point $Q(2,4,-1)$ is 7 units. Also, Find the equation of line joining P and Q .

SECTION-E (CASE BASED)

- 36 A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$. 1+1+2



Based on the above, answer the following :

- (i) How many relations can be there from S to J ?
- (ii) How many one-one functions can be there from set S to set J ?
- (iii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective.

OR

Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S . Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

- 37 A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ $4000 (\text{depth})^2$. Suppose the side of the square plot is x metres and depth is h metres. 1+1+2



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On the basis of the above information, answer the following questions:

- (i) Write cost $C(h)$ as a function in terms of h .
- (ii) Find critical point.
- (iii) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool?

OR

Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost.

38 A bank offers loans to its customers on different types of interest rates namely, fixed rate, floating rate, and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate, or variable rate with probabilities 10%, 20%, and 70% respectively. A customer after availing a loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate, and variable rate is 5%, 3%, and 1% respectively.

2+2



Based on the above information, answer the following:

(i) What is the probability that a customer after availing the loan will default on the loan repayment?

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?