

Section C

26. Show that the relation R in the set $A = \{x \in \mathbb{Z}: 0 \leq x \leq 12\}$ given by $R = \{(a, b): a, b \in \mathbb{Z}, |a - b| \text{ is divisible by } 3\}$ is an equivalence relation.

[OR]

Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f: A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is a bijective function.

27. Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$

[OR]

If $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$, $a > 0$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

28. Find $\int_0^{\frac{\pi}{4}} \frac{x + \sin x}{1 + \cos x} dx$.

29. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} and $\vec{c} \cdot \vec{d} = 18$.

30. Solve the following linear programming problem graphically:

Maximise $Z = 3x + 9y$

Subject to constraints:

$$x + y \geq 10, x + 3y \leq 60, x \leq y, x \geq 0, y \geq 0.$$

31. The probability of two students A and B coming to school in time are $\frac{2}{7}$ and $\frac{4}{7}$ respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent. Find the probability of only one of them coming to school on time

[OR]

A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

Section D

32. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ then find A^{-1} . Hence solve the system of linear equations:

$$x - 2y = 10; 2x - y - z = 8 \text{ and } -2y + z = 7$$

33. Evaluate: $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$

[OR]

Evaluate: $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

34. Find the particular solution of the differential equation $x \frac{dy}{dx} + x \cos^2 \frac{y}{x} = y$; given that when $x = 1, y = \frac{\pi}{4}$.

[OR]

Find the general solution of the differential equation:

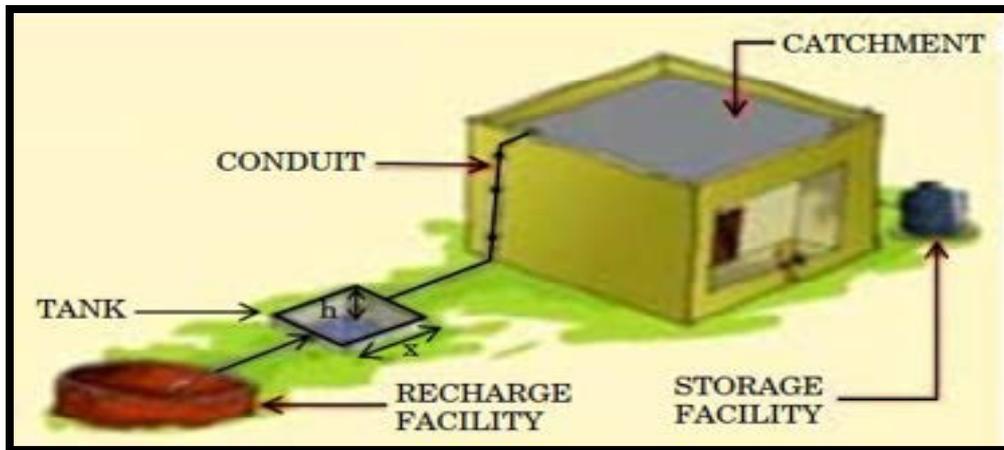
$$(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

35. Find the shortest distance between the lines: $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ and $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$

Section E (CASE STUDY)

36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3 . The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ $40,000 h^2$, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions:

(i) Find the total cost C of digging the tank in terms of x.

(ii) Find $\frac{dc}{dx}$.

(iii) Find the value of x for which cost C is minimum.

[OR]

Check whether the cost function C(x) expressed in terms of x is increasing or not, Where $x > 0$.

37. The equation of motion of a missile are $x = 3t$, $y = -4t$, $z = t$, where the time 't' is given in seconds, and the distance is measured in kilometres.



- (i) Write the path of the missile.
- (ii) Find the distance of the missile from the starting point $(0, 0, 0)$ in 5 seconds.
- (iii) If the position of the missile at a certain instant of the time is $(5, -8, 10)$. Find the height of the missile from the ground? (Ground considered as xy – plane)

[OR]

Find the value of k for which the lines $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{k}$ and $\frac{x-2}{-2} = \frac{y-3}{-1} = \frac{z-5}{7}$ are perpendicular?

38. A bank offers loans to its customers on different types of interest namely, fixed rate, floating rate, and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate, or variable rate with probabilities 10%, 20%, and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate, and variable rate is 5%, 3%, and 1% respectively.



Based on the above information, answer the following:

- (i) What is the probability that a customer after availing the loan will default on the loan repayment?
- (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

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