STUDENT ID

No. of pages - 24

(M)

PRE-BOARD EXAMINATION (2025-26)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours
समय : 3 घंटे

Maximum Marks : 80
अधिकतम अंक - 80

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए।

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड-ख में प्रश्न संख्या 21 से 25 तक अति लघुउत्तरीय (VSA) प्रकार के दो-दो अंक के प्रश्न हैं।
5. खंड-ग में प्रश्न संख्या 26 से 31 तक लघुउत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच-पाँच अंकों के प्रश्न हैं।
7. खंड-ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के 3 प्रश्नों में, खण्ड-घ के 2 प्रश्नों में, खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section-A, question No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section-B, question No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section-C, question No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section-D, question No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section-E, question No. 36 to 38 are Case Study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section-E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises of Multiple Choice Questions (MCQs) of 1 mark each.

1. Let the Relation R on the set A = {1, 2, 3, 4, 5, 6} be defined as : 1

$R = \{(a, b) : a - b \text{ is even}\}$

which of the following statements is true about relation R?

(a) R is reflexive and symmetric but not transitive

(b) R is symmetric and transitive but not reflexive

(c) R is reflexive, symmetric and transitive

(d) R is neither reflexive nor transitive

2. If we compare the principal ranges of $y = \sin^{-1}x$ and $y = \tan^{-1}x$, then which of the following statements is correct : 1

(a) Both have same range

(b) Range of $\sin^{-1}x \subset$ range of $\tan^{-1}x$

(c) Range of $\tan^{-1}x \subset$ range of $\sin^{-1}x$

(d) Range of $\sin^{-1}x \cap$ range of $\tan^{-1}x = \{\} = \phi$

3. If $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$ is a skew-symmetric matrix, then find the value of

$$\frac{(aqz) + (b+p+2)(c+x+3)}{(r+y+1)} :$$ 1

(a) 0 (b) 2

(c) 3 (d) 6

4. If $A=\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$, then A.(adj A) =

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & -3 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$

5. If P, Q and PQ are matrices of order 3×2, mXn and 3×4 respectively they number of elements in matrix Q is : 1

(a) 4 (b) 6

(c) 8 (d) 12

6. If A is a square matrix of order 3 and $|A| = 5$, then value of $|\text{adj } A| + |2A|$ is : 1

(a) 65 (b) 40

(c) 45 (d) 25

7. If $A=\begin{bmatrix} \sin 2\alpha & -\cos 2\alpha \\ \cos 2\alpha & \sin 2\alpha \end{bmatrix}$ and $A + A^T = I$, then the value of $\sin(\alpha - 15°)$

(where $\alpha \neq 15°$ and $\alpha < 180°$) 1

(a) 0.5 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

8. If $y = 4\log\sqrt{\sin x}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is : 1

(a) 1 (b) 2

(c) 4 (d) 0

9. The value of K for which :

$$f(x)=\begin{cases}4+\cos x & , \quad x \geq 0 \\ x+10k & , \quad x<0\end{cases}$$

is continuous function, is: 1

(a) 0.25 (b) 0.4

(c) 0.5 (d) 1

10. Find the angle (θ), ($0<\theta<\pi$) for which the rate of increase of θ is twice the rate of decrease of $\sin\theta$. 1

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{2\pi}{6}$

11. $\int \cot^2 x\, dx$ equals : 1

(a) $-\cot x - x + c$ (b) $\cot x - x + c$

(c) $-\cot x + x + c$ (d) $\cot x + x + c$

12. If $\int e^x \left(\log x + \frac{1}{x^2}\right) dx = e^x f(x) + c$, then $f(2) + f\left(\frac{1}{2}\right) =$ 1

(a) $\frac{17}{4}$ (b) $\frac{-17}{4}$

(c) $\frac{5}{2}$ (d) $\frac{-5}{2}$

13. Which of the following differential equations have same/equal order and degree: 1

(a) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ (b) $\frac{dy}{dx} = \left(\frac{d^2y}{dx^2}\right)^3$

(c) $\frac{d^2y}{dx^2} + y = 0$ (d) $\frac{dy}{dx} + y = 0$

14. Solution of the differential equation $dy - ydx = 0$ is : 1

(a) $y = c.e^x$ (b) $y = \log x + c$

(c) $y = e^x + c$ (d) $y = e^x + x + c$

15. The area (in square units) of the region bounded by the curve $y = |x-3|$, x-axis, $x = 3$ and $x = 7$ is : 1

(a) 16 (b) 2

(c) 4 (d) 8

16. The number of corner points of the feasible region determined by constraints $x \geq 0$, $y \geq 0$, $x + y \geq 219$ is : 1

(a) 0 (b) 1

(c) 2 (d) 3

17. The common region determined by all the constraints of a linear programming problem is called : 1

(a) an unbounded region (b) an optimal region

(c) a bounded region (d) a feasible region

18. The vector with terminal point A(5, 4, 3) and initial point B(3, 5, 4) is : 1

(a) $2\hat{i} + \hat{j} + \hat{k}$ (b) $-2\hat{i} + \hat{j} + \hat{k}$

(c) $2\hat{i} - \hat{j} - \hat{k}$ (d) $-2\hat{i} - \hat{j} + \hat{k}$

Questions number 19 and 20 are Assertion and Reason questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R), select the correct answer from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason(R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (A) is true.

19. Assertion (A): A line in space cannot be drawn perpendicular to my and z-ores simultaneously. 1

Reason (R): For any line making angles α, β, γ with the positive directions of x, y, and z ores respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

20. Assertion (A): If two event A and B are mutually exclusive, then $P\left(\frac{A}{B}\right) = 0, P(B) \neq 0$. 1

Reason (A): Mutually exclusive events cannot occur together i.e., $P(A \cap B) = 0$.

SECTION-B

This Section comprises of Very Short Answer type questions (VSA) of 2 marks each (Question number 21 to 25).

21. (a) Find the Principal value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right) + \cos^{-1}\left(\cos\frac{3\pi}{5}\right)$ 2

OR

(b) Find the domain and Principal Branch Range of $y = \sin^{-1}(3-4x)$

22. (a) If $\frac{1}{e^y(x+1)} = 1$, then prove that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ 2

OR

(b) The radius of a cylinder is increasing at the rate of 3 cm/sec and its height is decreasing at the rate of 5 cm/sec. Find the rate of change of its volume, when radius in 4 cm and height is 7 cm.

23. Find: $\int\frac{x^2+x+1}{x^2+x^2}\,dx$ 2

24. If $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=4,|\vec{b}|=3$ and $|\vec{c}|=2$, then find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ 2

25. Find the angle between the two lines whose equations are $6x = -y = -4z$ and $2x = 3y = -z$. 2

SECTION-C

This Section contains question numbers 26 to 31 of Short Answer (SA) type questions. Each question carries 3 marks each.

26. (a) Find the intervals in which the function $f(x)=3x^4-4x^3-12x^2+5$ is : 3

(i) Strictly increasing

(ii) Strictly decreasing

OR

(b) Find the Maximum Slope of the curve $y=-x^3+3x^2+9x-30$

27. (a) Evaluate : $\int_{-2}^{1}|x^3-x|\,dx$ 3

OR

(b) Find : $\int(\sin^{-1}x-\cos^{-1}x)dx$

28. (a) Solve the differential equation: $\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}$ 3

OR

(b) Find the general solution of the differential equation $\frac{dy}{dx}=\frac{2x^2+y}{x}$

29. If E and F are two independent events with P(E) = x, P(F) = 2x and P (Exactly one of E, F) = $\frac{5}{9}$, then find the value of x. 3

30. Consider the linear programming problem, where the objective function $z = x + 4y$ needs to be minimized subject to constraints 3

$$2x + y \geq 1000$$
$$x + 2y \geq 800$$
$$x, y \geq 0$$

Draw a neat graph of the feasible region and find the minimum value of z.

For Visually Impaired students only :

The corner points of the feasible region determined by some system of linear in equations are (0, 0), (5, 0), (3, 4) and (0, 5). Let $z = ax + by$, where a, b > 0. Find the condition on a and b so that the maximum of z occurs at both points (3, 4) and (0, 5).

31. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$ 3

SECTION-D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. (a) Use integration to find the Area of the region enclosed by curve $y = -x^2$ and the straight lines x = - 3, x = 2 and y = 0. Sketch a rough figure to illustrate the bounded region. 5

OR

(b) Using integration, find the Area enclosed by the curve $25x^2 + 16y^2 = 400$.

For Visually Impaired students only :

(a) Evaluate the integral, $\int_1^2 (x^3 + 2)dx$ and explain what physical quantity represents by I on the graph. 5

OR

(b) Evaluate: $I = \int_0^{\pi} |\sin x| \, dx$ and explain what does it represents on the graph.

33. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations

$3x + 4y + 7z = 14,\ 2x - y + 3z = 4,\ x + 2y - 3z = 0.$ 5

34. (a) If $y^x + x^y + x^x = a^b$, then find $\frac{dy}{dx}$ 5

OR

(b) Find the value of $\int \frac{x+2}{2x^2 + 6x + 5} dx$

35. Find the shortest distance between the lines ℓ_1, and ℓ_2 given by: 5

$\ell_1 : \vec{r} = (\hat{i} + 2\hat{j} - +4\hat{k}) + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$

$\ell_2 : \vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(10\hat{i} + 15\hat{j} + 30\hat{k})$

SECTION-E

This section comprises 3 case-study based questions of 4 marks each.

36. A mathematics teacher gave the following four functions to her students to analyse their nature and type:

- $f_1 : \mathbb{R} \to \mathbb{R}, f_1(x) = x^2$
- $f_2 : \mathbb{R} \to \mathbb{R}, f_2(x) = 2x + 3$
- $f_3 : \mathbb{R} \to [-1, 1], f_3(x) = x^3$
- $f_4 : \mathbb{R} \to \mathbb{R}, f_4(x) = 2025$

Students were asked to study these functions and answer the following questions :

(a) Which of the above functions is/are not one-one? 1

(b) Which of the above functions is/are onto? 1

(c) (i) From the given functions, which is one-one and onto both. Give reasons in support of your answer. 2

OR

(ii) From the given functions which is/are many-one and into. Give reason in support of your answer.

37. A steel wire of total length 28 metres is to be cut into two parts. One part is used to form a square and the other part is used to form a circle.

We want to find how the wire should be cut so that the combined area of both figures is minimum.

Let the length used for making the circle be x meters and the length used for the square will be (28-x) metres.

On the basis of above information, answer the following questions:

(i) Form an expression for the total Area A(x) and express in terms of x. 1

(ii) Find the valve of $\frac{d}{dx}(A(x))$ 1

(iii) (a) Using the first derivative test, find the value of x for which the total area is minimum.

Hence, find the corresponding lengths of two pieces. 2

OR

(b) Using the second derivative test. Find the value of x for which the total area is minimum.

Hence, find the corresponding lengths of two pieces. 2

38. In a community of 1000 people, individuals are classified into three categories based on their health conditions :

- 600 people have good health
- 300 people have average health
- 100 people have poor health

The probabilities of contracting a certain infections disease for each health group are :

A_1 : Good health : 25%

A_2 : Average health : 35%

A_3 : Poor health : 50%

Based upon the above information, the following questions:

(i) What is the probability that a randomly selected person from the community has contracted the disease? 2

(ii) Given that the person has not contracted the disease. What is the probability that the person is from category average health group? 2