



CLASS – XII

MAX. MARKS – 80

SUB. – MATHEMATICS (Code – 041)

TIME – 03 HOURS

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
3. **Section A** has **18 MCQ's and 02 Assertion-Reason** based questions of 1 mark each.
4. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 marks each.
5. **Section C** has **6 Short Answer (SA)-type** questions of 3 marks each.
6. **Section D** has **4 Long Answer (LA)-type** questions of 5 marks each.
7. **Section E** has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Let Z denote the set of integers, then the function $f: Z \rightarrow Z$ defined as $f(x) = x^3 - 1$ is (A) both on-one and onto (B) one-one but not onto (C) onto but not one-one (D) neither one-one nor onto
2. If $A = [a_{ij}]$ is a diagonal matrix, then which of the following is true?
(A) $a_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall i, j$
(C) $a_{ij} = 0$ if $i \neq j$ & $a_{ij} \neq 0$ if $i = j$ (D) $a_{ij} = 1, \forall i, j$
3. Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ be a square matrix such that $adj A = A$. Then $(p + q + r + s)$ is equal to (A) $2p$ (B) $2q$ (C) $2r$ (D) 0
4. If A and B are symmetric matrix of the same order, then $(AB' - BA')$ is a (A) Skew Symmetric matrix (B) Null matrix (C) Symmetric matrix (D) None of these

5. If $x, y \in R$, then the determinant $\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval

- (A) $[-\sqrt{2}, \sqrt{2}]$ (B) $[-1, 1]$ (C) $[-\sqrt{2}, 1]$ (D) $[-1, \sqrt{2}]$

6. The area of the triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 square units. The value of k will be

- (A) 9 (B) 3 (C) -9 (D) 6

7. The number of points at which the function $f(x) = \frac{1}{x - [x]}$ is not continuous is

- (A) 1 (B) 2 (C) 3 (D) None of these

8. Differential coefficient of $\sec(\tan^{-1}x)$ with respect to x is

- (A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\frac{x}{1+x^2}$ (C) $x\sqrt{1+x^2}$ (D) $\frac{1}{\sqrt{1+x^2}}$

9. The function $f(x) = \tan x - x$

- (A) Always increases (B) Always decreases
(C) Never increases (D) Sometimes increases and sometime decreases

10. $\int_{a+c}^{b+c} f(x) dx$ is equal to

- (A) $\int_a^b f(x-c) dx$ (B) $\int_a^b f(x+c) dx$ (C) $\int_a^b f(x) dx$ (D) $\int_{a-c}^{b-c} f(x) dx$

11. The solution of the differential equation $2x \frac{dy}{dx} - y = 3$ represents a family of

- (A) Straight lines (B) Circles (C) Parabolas (D) Ellipses

12. The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$ is

- (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{-\pi}{3}$ (D) $\frac{5\pi}{6}$

13. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$ then $\vec{a} \cdot \vec{b}$ is

- (A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) None of these

14. The equation of x-axis in space are

- (A) $x = 0, y = 0$ (B) $x = 0, z = 0$ (C) $x = 0$ (D) $y = 0, z = 0$

15. If a line makes equal acute angles with coordinate axes, then direction cosines of the line is

- (A) 1,1,1 (B) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (C) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ (D) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

16. The corner point of the feasible region determined by the system of linear constraints are (0,10), (5,5), (15,15) and (0,20) . let $Z = px + qy$, where $p, q > 0$.

Condition on p and q so that the maximum of z occurs at both the points (15,15) and (0,20) is

- (A) $p = q$ (B) $p = 2q$ (C) $q = 2p$ (D) $q = 3p$

17. The linear function which is to be optimized in the Linear Programming Problem is known as

- (A) constraints (B) optimal solution (C) objective function (D) decision variables

18. Let A and B be two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P\left(\frac{A}{B}\right) = 0.5$ then $P\left(\frac{A'}{B'}\right)$ equals

- (A) 1/10 (B) 3/10 (C) 3/8 (D) 6/7

In Questions number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is not the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true

19. Assertion (A) : Every scalar matrix is a diagonal matrix

Reason(R) : In a diagonal matrix , all the diagonal elements are zero.

20. Assertion (A) : Projection of \vec{a} on \vec{b} is same as projection of \vec{b} on \vec{a} .

Reason(R) : Angle between \vec{a} and \vec{b} is same as angle between \vec{b} and \vec{a}

SECTION B

This section comprises very short answer(VSA) type questions of 2 marks each.

21. Find the value of $\tan^{-1}(-1) + \sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.

22. (a) for what value of μ is the function defined by

$$f(x) = \begin{cases} \mu(x^2 - 2x) & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$$

Continuous at $x=0$?

OR (b) Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$

23. (a) Evaluate: $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

OR (b) Evaluate: $\int_0^4 |x - 1| dx$

24. Find the area of the parallelogram whose diagonals are $4\hat{i} - \hat{j} - 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$.

25. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) If $f(x) = \begin{cases} 3ax + b & \text{if } x < 1 \\ 11 & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$, find a and b

OR (b) If $y = x^{\sin x} + (\sin x)^{\cos x}$ then find $\frac{dy}{dx}$

27. (a) It is given that $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function $f(x)$ attains local maximum values.

OR

(b) A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

28. Find : $\int \frac{x^3}{x^4+3x^2+2} dx$

29. (a) Find: $\int e^{\tan^{-1}x} \left(\frac{1+x+x^2}{1+x^2} \right) dx$

OR

(b) Evaluate : $\int_0^{\pi/4} \log(1 + \tan x) dx$

30. Solve the linear programming problem graphically

Maximize $Z = 510x + 675y$

subject to the constraints:

$x + y \leq 300;$ $2x + 3y \leq 720;$ $x \geq 0, y \geq 0$

31. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive.

SECTION D

This section comprises long answer(LA) type questions of 5 marks each.

32. Show that the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also find the equivalence class [3]
33. (a) Find the critical points and hence find absolute maximum and minimum values of a function f given by $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [-1, 1]$.
- OR** (b) A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?
34. Using integration Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.
35. (a) Find the distance of a point $(2, 4, -1)$ from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

OR

- (b) Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study -1

1. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and unskilled themselves.



A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below:

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

Where x denotes the number of hours.

Based on the above information, answer the following questions:

- (i) Express the probability distribution given above in the form of a probability distribution table. 1
- (ii) Find the value of k . 1
- (iii) (a) Find the mean number of hours spent by the student. 2

OR

(b) Find $P(1 < X < 6)$.

2. It is known that, if the interest is compounded annually, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be $r\%$ per annum.



Based on the above information answer the following questions.

- iii) At what interest rate will Rs. 100 double itself in 10 years. 2
- iv) (a) How much will Rs. 1000 be worth at 5% interest after 10 years? 2

OR

(b) If the interest is compounded continuously at 5% per annum, in how many years will Rs. 100 double itself? 2

[Use $\ln 2 = 0.6931$; $e^{0.5} = 1.648$]

Case Study -3

3. The monthly income of two sisters Ojaswini and Tejaswini are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. Each sister saves ₹ 15,000 per month.



- a) Write the information in the matrix equation. 1
- b) Is the system of equation consistent? 1
- c) Find the monthly income of both sisters by matrix method. 2

OR

Find the monthly expenditure of both sisters by matrix method.