10

SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. The order of matrix $\begin{bmatrix} 3 \\ x^2 \\ -5 \end{bmatrix}$ is **[NCERT Part-I, Page 36-37]**

(*a*) 2×3 (*b*) 1×3

(*c*) 5×3 (*d*) 3×1

2. Maximum value of $\begin{vmatrix} 1 & 1 & 1+\cos\theta \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1 \end{vmatrix}$ is **[Integrated Question]**

(*a*) $-\frac{1}{2}$ (*b*) $\frac{1}{2}$

(*c*) $\frac{3}{4}$ (*d*) $-\frac{3}{4}$

3. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then value of x is (are) **[NCERT Part-I, Page 77]**

(*a*) 2 (*b*) 4

(*c*) ±4 (*d*) ±2

4. The function f defined by $f(x) = \begin{cases} kx^2, & x \geq 1 \\ 4, & x < 1 \end{cases}$ is continuous at $x = 1$, then value of k is **[NCERT Part-I, Page 105]**

(*a*) 4
(*b*) –4
(*c*) 2
(*d*) $\frac{1}{4}$

5. Direction ratios of the line passing through points (3, 2, 5) and (1, 3, 9) such that line makes an acute angle with x-axis are : **[NCERT Part-II, Page 379-380]**

(*a*) 2, 1, –4
(*b*) 2, –1, –4
(*c*) –2, 1, 4
(*d*) –2, –1, 4

6. The product of the order and degree of differential equation **[NCERT Part-II, Page 301-302]**

$$8\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^3 - 2x = 0$$

(*a*) 3
(*b*) 2
(*c*) 6
(*d*) not defined

7. For a given LPP, the solution of a given constraint does not include point (1, 2). Which of the following constraint belongs to LPP? **[Conceptual Application]**

(*a*) $3x + y - 7 \leq 0$
(*b*) $3x \geq 5$
(*c*) $y - 1 \geq 0$
(*d*) $8x + 7y - 23 \leq 0$

8. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, then $|2\vec{c} + \vec{a} + \vec{b}|$ is equal to **[Conceptual Application]**

(*a*) $\sqrt{3}$
(*b*) $\sqrt{5}$
(*c*) 2
(*d*) $\sqrt{6}$

9. If $f'(x) = x + \frac{1}{x}$, then $f(x)$ is **[NCERT Part-II, Page 227]**

(*a*) $x + \log|x| + C$
(*b*) $x^2 + \log|x| + C$
(*c*) $\frac{x^2}{2} + \log|x| + C$
(*d*) $\frac{x}{2} - \log|x| + C$

10. The matrix $\begin{bmatrix} 0 & 2 & -4 \\ -2 & 3a & 7 \\ 4 & a & 0 \end{bmatrix}$ represent a skew-symmetric matrix for **[NCERT Part-I, Page 63]**

(*a*) $a = -7$
(*b*) $a = 0$
(*c*) $a = 2$
(*d*) no value of a

11. For a given LPP, the objective function is $Z = ax + by$, $a, b > 0$ and corner points of the feasible region are $A(2, 1)$, $B(3, 5)$ and $C(0, 7)$. If the value of objective function at B is 2 more than sum of its values at A and C, then the relation between a and b is **[Conceptual Application]**

(*a*) $a = 3b + 2$
(*b*) $a = 3b - 2$
(*c*) $a = 3b$
(*d*) $3a = b$

12. For the vector $\vec{r} = 2\hat{j} - 3\hat{k} - \hat{i}$, vector component of $\vec{r}$ along z-axis is **[Conceptual Application]**

(*a*) $2\hat{j}$
(*b*) –3
(*c*) $-3\hat{k}$
(*d*) –1

13. If A is a skew symmetric matrix of order 3 and $|A| = x$, then $(2024)^x$ is equal to **[Conceptual Application]**

(a) 2024 (b) $\frac{1}{2024}$

(c) $(2024)^2$ (d) 1

14. Events A and B are such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$, then $P(\overline{A} \cap B)$ is

[Conceptual Application]

(a) 0.15 (b) 0.5

(c) 0.1 (d) 0.9

15. Integrating factor for the differential equation, $x\frac{dy}{dx} + 2y = x^2$ is **[NCERT Part-II, Page 322-323]**

(a) $2 \log |x|$ (b) $2x$

(c) x^2 (d) $\frac{2}{x}$

16. $\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} \end{vmatrix}$ is equal to **[Conceptual Application]**

(a) $(\vec{a} \times \vec{b})^2$ (b) $(\vec{a} \cdot \vec{b})^2$

(c) $\vec{a} \cdot \vec{b}$ (d) $|\vec{a} \times \vec{b}|$

17. The side of an equilateral triangle is increasing at the rate of 2 cm/s. The rate at which area increases when side is 10 cm is **[NCERT Part-I, Page 147]**

(a) $10 \text{ cm}^2/\text{s}$ (b) $\sqrt{3} \text{ cm}^2/\text{s}$

(c) $10\sqrt{3} \text{ cm}^2/\text{s}$ (d) $\frac{10}{3} \text{ cm}^2/\text{s}$

18. A line makes angles α, β, γ with x, y and z-axis respectively, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to

[NCERT Part-II, Page 377-378]

(a) 1 (b) 0

(c) 2 (d) -1

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Let 'f' be a function defined by, $f(x) = |2x - 3|$, $x \in R$, then **[NCERT Part-I, Page 105, 118-119]**

Assertion (*A*): function 'f' is differentiable at $x = 0$.

Reason (*R*): An absolute function, $f(x) = |x - a|$ where $x \in R$, is continuous at $x = a$ but not differentiable at $x = a$.

20. **Assertion(*A*):** The relation $R = \{(1, 3)\}$ defined on set $A = \{1, 2, 3\}$ is a transitive relation.

Reason(*R*): A singleton relation in a given set is transitive. **[NCERT Part-I, Page 2]**

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)+2\tan^{-1}(1)$. **[Conceptual Application]**

22. Differentiate $\log_2\left(\sqrt{x-a}+\sqrt{x-b}\right)$ with respect to x. **[Conceptual Application]**

OR

If $y=\log(\log x^2)$, find y_2. **[NCERT Part-I, Page 137]**

23. Show that the function $f(x)=x^2$ is neither increasing nor deceasing on R. **[NCERT Part-I, Page 153]**

OR

Find the intervals in which the function f given by $f(x)=\sin 3x$, $0\le x\le\frac{\pi}{2}$, is increasing or decreasing.

[NCERT Part-I, Page 153]

24. Evaluate : $\int \sin x\cdot\sin 3x\,dx$ **[NCERT Part-II, Page 241]**

25. The volume of a sphere is increasing at the rate of 3 cm^3/s. Find the rate of increase of its surface area when its radius is 2 cm. **[NCERT Part-I, Page 147-148]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int \frac{1}{\cos(x+a)\sin(x+b)}\,dx$ **[NCERT Part-II, Page 241]**

27. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green is tossed once. Let A be the event "number obtained is even" and B be the event "number obtained is red". Are the events A and B independent?

[NCERT Part-II, Page 418]

28. Evaluate: $\int_0^{\pi}\frac{x\sin x}{1+\sin x}\,dx$ **[NCERT Part-II, Page 274, 241]**

OR

Evaluate: $\int_0^{\frac{\pi}{4}}\log(1+\tan x)\,dx$ **[NCERT Part-II, Page 274]**

29. Find the general solution of the differential equation: **[NCERT Part-II, Page 313-314]**

$$x\frac{dy}{dx}=y(\log y-\log x+1)$$

OR

If $y=\log\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$, find $\frac{d^2y}{dx^2}$. **[NCERT Part-II, Page 137]**

30. Solve the following LPP graphically: **[NCERT Part-II, Page 397-398]**

Maximize $Z=3x+y$

Subject to constraints

$x\ge 5, y\ge 1, x+y-8\le 0, x\ge 0, y\ge 0$

OR

Solve the following LPP graphically: **[NCERT Part-II, Page 397-398]**

Minimize $Z = 5x + 10y$

Subject to constraints

$x + 2y \leq 120, \quad x + y \geq 60 \quad x - 2y \geq 0, \quad x \geq 0, y \geq 0$

31. If $y = e^x (\sin x + \cos x)$, prove that **[NCERT Part-I, Page 137]**

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. The area of the region bounded by the line $y = mx$ ($m > 0$), the curve $x^2 + y^2 = 4$ and x-axis in first quadrant is $\frac{\pi}{2}$ unit^2. Using integration, find the value of m. **[Conceptual Application]**

33. Consider $f : R^+ \to [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$. Prove that it is bijective function. **[NCERT Part-I, Page 7]**

OR

Determine whether the relation R defined on set R of all real number as **[NCERT Part-I, Page 2]**

$R = \{(a, b) : a, b \in R$ and $a - b + \sqrt{5} \in S\}$ where 'S' is set of all irrational numbers is reflexive, symmetric, transitive relation.

34. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve system of equations:

$x + 2y - 3z = 6, 3x + 2y - 2z = 3, 2x - y + z = 2.$ **[NCERT Part-I, Page 94-95]**

35. Show that the lines $\frac{x+1}{3} = \frac{y+2}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-4}{5}$ intersect. Also find their point of intersection. **[Conceptual Application]**

OR

Find the length and the foot of the perpendicular drawn from the point (2, –1, 5) on the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. **[Conceptual Application]**

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Some friends are playing a game using cards numbered 1 to 8. Card are kept upside down so that numbers are not visible and also all cards look similar. The probability of a card numbered X is shown in the below. If $\Sigma P(X) = 1$ **[Conceptual Application]**

X	1	2	3	4	5	6	7	8
$P(X)$	p	$2p$	$2p$	p	$2p$	p^2	$2p^2$	$7p^2 + p$

Using above information, answer the following questions:

(*i*) Find the value of p.

(*ii*) Find the probability of choosing a card numbered more than 6.

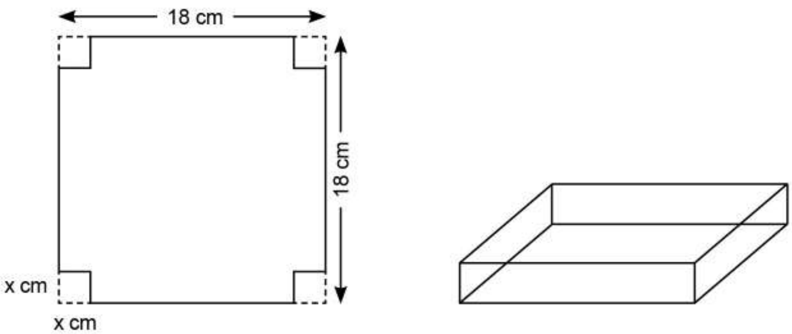
(*iii*) Find the probability of choosing a card bearing a number that is a multiple of 3.

OR

(*iii*) Find the probability of choosing an even numbered card.

Case Study - 2

37.



A company placed orders to make open cuboidal boxes from square sheets of cardboards of length 18 cm. Boxes are made by cutting the equal squares from four corners of cardboard and folding up the flaps. If a square of side x cm is cut from each of the 4 corners of cardboard, then **[Conceptual Application]**

(*i*) what is length, breadth and height of the box?

(*ii*) what is volume of the box in terms of x?

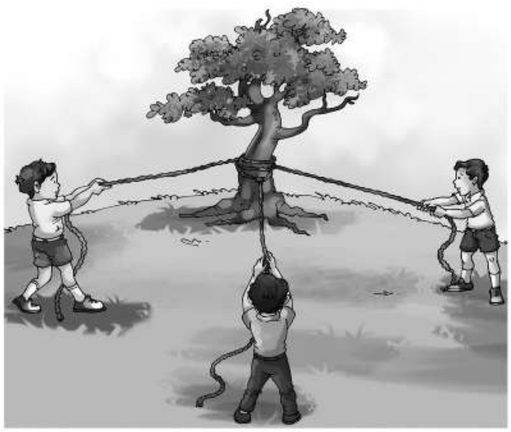
(*iii*) for what value of x, volume of box is maximum?

OR

(*iii*) find the maximum volume of the box.

Case Study - 3

38.



Three friends Andy, Ben and Chris were playing in the park after school. They tied three ropes from a point marked $P(-2, 3, 7)$ and other end of three ropes were held from the end by Andy, Ben and Chris who were standing at the positions $A(0, 5, 4)$, $B(-3, 6, 8)$ and $C(1, 5, -2)$ respectively. **[NCERT Part-II, Page 347, 356]**

Based on the above information, answer the following questions:

(*i*) Find unit vector along $\overrightarrow{BC}$.

(*ii*) Find the angle 'θ' between the vectors $\overrightarrow{BP}$ and $\overrightarrow{CP}$.

SOLUTIONS

1. (*d*) 3×1

2. (*b*) $\begin{vmatrix} 1 & 1 & 1+\cos\theta \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1(\sin\theta) - 1(0) + (1+\cos\theta)(-\sin\theta)$

$$= \sin\theta - \sin\theta - \sin\theta\cos\theta = -\sin\theta\cos\theta$$

$$= -\frac{1}{2}\sin 2\theta$$

Now, $-1 \le \sin 2\theta \le 1$

$\Rightarrow$ $\frac{-1}{2} \le \frac{-1}{2}\sin 2\theta \le \frac{1}{2}$

So, maximum value of given determinant is $\frac{1}{2}$.

3. (*a*) As, $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$

$\Rightarrow$ $2x^2 + 6x - 6x = 8$

$\Rightarrow$ $x^2 = 4 \Rightarrow x = \pm 2$

As, $x \in N$, so we take $x = 2$.

4. (*a*) Since f is continuous at $x = 1$,

$\therefore$ $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow$ $\lim_{x \to 1^-} (4) = \lim_{x \to 1^+} (kx^2) = k(1)^2$

$\Rightarrow$ $4 = k = k \Rightarrow k = 4$

5. (*b*) Let $\langle l, m, n \rangle$ be the dc's and $\langle a, b, c \rangle$ be the dr's of the line passing through points (3, 2, 5) and (1, 3, 9)

As, line makes an acute angle with x-axis, so $l > 0$.

Now, dr's of a line are always proportional to dc's.

Now, $l > 0 \Rightarrow a > 0$

$\therefore$ dr's of the given line are $\langle 3-1, 2-3, 5-9 \rangle$ i.e. $\langle 2, -1, -4 \rangle$.

6. (*b*) order = 2, degree = 1

Product = $2 \times 1 = 2$

7. (*b*) As (1, 2) does not satisfy inequation $3x \ge 5$.

8. (*d*) As $|2\vec{c} + \vec{a} + \vec{b}|^2 = (2\vec{c} + \vec{a} + \vec{b}) \cdot (2\vec{c} + \vec{a} + \vec{b})$

$$= 4|\vec{c}|^2 + |\vec{a}|^2 + |\vec{b}|^2 + 4\vec{c}\cdot\vec{a} + 2\vec{a}\cdot\vec{b} + 4\vec{b}\cdot\vec{c}$$

$$= 4 + 1 + 1 + 0 + 0 + 0$$

$$= 6$$

$\Rightarrow$ $|2\vec{c} + \vec{a} + \vec{b}| = \sqrt{6}$

9. (*c*) $$f(x) = \int\left(x+\frac{1}{x}\right)dx = \frac{x^2}{2} + \log|x| + C$$

10. (*d*) As for skew-symmetric matrix, $a_{ij} = -a_{ji}\ \forall\ i, j$. So, $a = 0$ and $a = -7$

$\therefore$ No value of 'a'.

11. (*a*)
$$Z_B = Z_A + Z_C + 2$$
$$\Rightarrow \quad 3a + 5b = 2a + b + 7b + 2$$
$$\Rightarrow \quad a = 3b + 2$$

12. (*c*) Vector component of $\vec{r}$ along z-axis $= -3\vec{k}$

13. (*d*) If A is a skew symmetric matrix of odd order, then $|A| = 0$.

So, $|A| = x = 0$. Then $(2024)^x = (2024)^0 = 1$

14. (*c*)
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.4 + 0.3 - 0.5 = 0.2$$
$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$
$$= 0.3 - 0.2 = 0.1$$

15. (*c*) The differential equation is $\frac{dy}{dx} + \frac{2}{x}y = x$

Comparing with $\frac{dy}{dx} + Py = Q$,

we get $P = \frac{2}{x}$ and $Q = x$

$$\text{Integrating factor} = e^{\int P dx} = e^{\int \frac{2}{x}dx} = e^{2\log|x|}$$
$$= e^{\log|x|^2} = x^2$$

16. (*a*) On evaluating the determinant,
$$\Delta = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$$
$$= |\vec{a}|^2|\vec{b}|^2 - \{|\vec{a}||\vec{b}|\cos\theta\}^2$$
$$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$$
$$= |\vec{a}|^2\,|\vec{b}|^2\sin^2\theta$$
$$= \{|\vec{a}|^2\,|\vec{b}|^2\sin\theta\}^2$$
$$= |\vec{a}\times\vec{b}|^2$$
$$= (\vec{a}\times\vec{b})^2$$

17. (*c*) Let 'x' be the side and 'A' be the area of the equilateral Δ at any time 't'.
$$\frac{dx}{dt} = 2 \text{ cm/s}$$
Now, $$A = \frac{\sqrt{3}}{4}x^2 \Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}x\cdot\frac{dx}{dt}$$
$$\left.\frac{dA}{dt}\right]_{x=10} = \frac{\sqrt{3}}{2}\times 10\times 2 = 10\sqrt{3}\ \text{cm}^2/\text{s}$$

18. (*d*) As, $$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\Rightarrow \quad \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$$
$$\Rightarrow \quad \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

19. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

20. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

21.
$$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 2\tan^{-1}(1) = -\frac{\pi}{4} + 2\cdot\frac{\pi}{4} = \frac{\pi}{4}$$

22.
$$y = \log_2(\sqrt{x-a} + \sqrt{x-b}) = \frac{\log(\sqrt{x-a} + \sqrt{x-b})}{\log 2}$$
$$\frac{dy}{dx} = \frac{1}{\log 2}\cdot\frac{1}{(\sqrt{x-a} + \sqrt{x-b})}\cdot\left\{\frac{1}{2\sqrt{x-a}}\cdot 1 + \frac{1}{2\sqrt{x-b}}\cdot 1\right\}$$
$$= \frac{1}{\log 2}\cdot\frac{1}{(\sqrt{x-a} + \sqrt{x-b})}\cdot\frac{1}{2}\left\{\frac{\sqrt{x-b} + \sqrt{x-a}}{\sqrt{x-a}\,\sqrt{x-b}}\right\}$$
$$= \frac{1}{2\log 2\cdot\left(\sqrt{x-a}\cdot\sqrt{x-b}\right)}$$

OR

$$y = \log(\log x^2)$$

Differentiating w.r.t. x both sides,
$$y_1 = \frac{1}{\log x^2}\cdot\frac{1}{x^2}\cdot 2x = \frac{2}{x\log x^2}$$
$$\Rightarrow \quad y_1 = \frac{2}{x\cdot 2\log x} = \frac{1}{x\log x}$$

Differentiating again w.r.t. x both sides,
$$y_2 = \frac{x\log x\times 0 - 1\left\{x\times\frac{1}{x} + \log x\right\}}{(x\log x)^2} = \frac{-(1+\log x)}{(x\log x)^2}$$

23. Consider function $f(x) = x^2$

$\Rightarrow \quad f'(x) = 2x$

For increasing function, $\quad f'(x) \geq 0$

$\Rightarrow \quad 2x \geq 0 \Rightarrow x \geq 0$

For decreasing function, $\quad f'(x) \leq 0$

$\Rightarrow \quad 2x \leq 0 \Rightarrow x \leq 0$

So, f decreases on $(-\infty, 0]$ and increases for $[0, \infty)$. Hence function is neither increasing nor decreasing on set of real numbers.

OR

We have, $\quad f(x) = \sin 3x$

$\Rightarrow \quad f'(x) = 3\cos 3x \quad$ ...(*i*)

For critical points, $\quad f'(x) = 0$

$\Rightarrow \quad \cos 3x = 0$

$\Rightarrow \quad 3x = \frac{\pi}{2}, \frac{3\pi}{2}$

So, $\quad x = \frac{\pi}{6}$ and $\frac{\pi}{2}$

Case I: When $0 < x < \frac{\pi}{6}$ i.e. $0 < 3x < \frac{\pi}{2}$

In this case, we have

$\cos 3x > 0$

$\Rightarrow \quad 3\cos 3x > 0$

$\Rightarrow \quad f'(x) > 0$

$\therefore f(x)$ is increasing on $\left(0, \frac{\pi}{6}\right)$.

Case II: When $\frac{\pi}{6} < x < \frac{\pi}{2}$ i.e. $\frac{\pi}{2} < 3x < \frac{3\pi}{2}$

In this case, we have

$$\cos 3x < 0$$

$$\Rightarrow \quad 3\cos 3x < 0$$

$$\Rightarrow \quad f'(x) < 0$$

$\therefore f(x)$ is decreasing on $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$.

24.
$$\int \sin x \cdot \sin 3x\, dx = \frac{1}{2}\int (2\sin 3x \sin x)\, dx$$
$$= \frac{1}{2}\int (\cos 2x - \cos 4x)\, dx$$
$$= \frac{1}{2}\left[\frac{\sin 2x}{2} - \frac{\sin 4x}{4}\right] + C$$
$$= \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$$

25. Let r be the radius, V be the volume and S be the surface area of the sphere at any instant t.

$$\frac{dV}{dt} = 3 \text{ cm}^3/\text{s}$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 3 \Rightarrow 4\pi r^2 \frac{dr}{dt} = 3$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{3}{4\pi r^2} \quad \text{...}(i)$$

Now, $S = 4\pi r^2$

$$\Rightarrow \quad \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} = 8\pi r \cdot \frac{3}{4\pi r^2} = \frac{6}{r} \quad \text{[using } (i)\text{]}$$

$$\left.\frac{dS}{dt}\right]_{r=2} = \frac{6}{2} = 3 \text{ cm}^2/\text{s}$$

26. $\int \frac{1}{\cos(x+a)\sin(x+b)}dx$

$$= \frac{1}{\cos(a-b)}\int \frac{\cos\{(x+a)-(x+b)\}}{\cos(x+a)\sin(x+b)}dx$$

$$= \frac{1}{\cos(a-b)}\int \frac{\cos(x+a)\cos(x+b) + \sin(x+a)\sin(x+b)}{\cos(x+a)\sin(x+b)}dx$$

$$= \frac{1}{\cos(a-b)}\int \{\cot(x+b) + \tan(x+a)\}dx$$

$$= \frac{1}{\cos(a-b)}[\log|\sin(x+b)| + \log|\sec(x+a)|] + C$$

27. Let S be the sample space.

$\therefore \quad S = \{1, 2, 3, 4, 5, 6\}$

So, $\quad n(S) = 6$

A : number obtained is even.

$$A = \{2, 4, 6\}$$

So, $n(A) = 3$

So, $P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B : number obtained is red.

Now, $B = \{1, 2, 3\}$

So, $n(B) = 3$

$\therefore$ $P(B) = \frac{3}{6} = \frac{1}{2}$

Now, $A \cap B = \{2\} \Rightarrow n(A \cap B) = 1$

$\therefore$ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$

As, $P(A \cap B) \neq P(A) \cdot P(B)$

So, events A and B are not independent.

28. $$I = \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx \qquad ...(i)$$

Using property $\int_0^a f(x)dx = \int_0^a f(a - x)\, dx$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin (\pi - x)}{1 + \sin (\pi - x)}\, dx$$

$$= \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1 + \sin x} dx \qquad ...(ii)$$

$\therefore$ $$2I = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x}\, dx \qquad \text{[On adding } (i) \text{ and } (ii)]$$

$$= \pi \int_0^{\pi} \frac{\sin x\,(1 - \sin x)}{(1 + \sin x)\,(1 - \sin x)} dx$$

$$= \pi \int_0^{\pi} (\sec x \tan x - \tan^2 x)\, dx$$

$$= \pi \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1)\, dx$$

$$= \pi \Big[\sec x - \tan x + x\Big]_0^{\pi}$$

$$= \pi[(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0)]$$

$$2I = \pi(-2 + \pi)$$

$\Rightarrow$ $$I = \frac{\pi}{2}\,(\pi - 2) = \pi\left(\frac{\pi}{2} - 1\right)$$

OR

Let $$I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx \qquad ...(i)$$

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right) dx\right] \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right]$$

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right] dx$$

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1 + \tan x}\right) dx \qquad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{4}} \left[\log(1+\tan x) + \log\left(\frac{2}{1+\tan x}\right)\right] dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{4}} [\log(1+\tan x) + \log 2 - \log(1+\tan x)] dx$$

$$\Rightarrow \quad 2I = \log 2 \int_0^{\frac{\pi}{4}} dx$$

$$\Rightarrow \quad 2I = \log 2 \times \Big[x\Big]_0^{\frac{\pi}{4}}$$

$$\Rightarrow \quad 2I = \frac{\pi}{4} \log 2$$

$$\Rightarrow \quad I = \frac{\pi}{8} \log 2$$

29. We have, $\quad \dfrac{dy}{dx} = \dfrac{y}{x}\left(\log\dfrac{y}{x} + 1\right) \quad$ ...(i)

Let $\quad y = vx \;\Rightarrow\; \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore$ From (i), we get

$$v + x\frac{dv}{dx} = v(\log v + 1) = v\log v + v$$

$$\Rightarrow \quad x\frac{dv}{dx} = v\log v$$

$$\Rightarrow \quad \frac{dv}{v\log v} = \frac{dx}{x}$$

On integrating both sides,

$$\int \frac{1}{v\log v} dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log|\log v| = \log|x| + \log|C| \Rightarrow \log|\log v| = \log|Cx| \Rightarrow \log v = xC$$

$\Rightarrow \quad \log\left(\dfrac{y}{x}\right) = xC$ is the required solution.

OR

$$y = \log\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$$

$$\Rightarrow \quad y = \log\sqrt{\frac{2\sin^2 x}{2\cos^2 x}}$$

$$\Rightarrow \quad y = \log(\tan x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sec^2 x}{\tan x} = 2\text{cosec } 2x$$

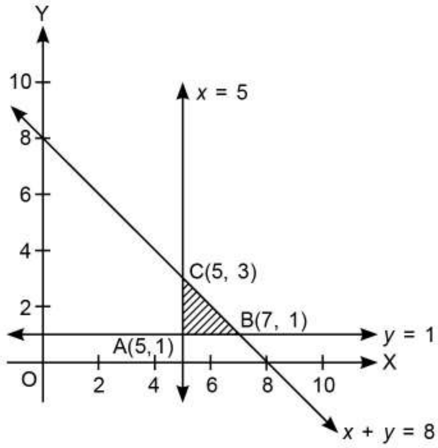
$$\Rightarrow \quad \frac{d^2y}{dx^2} = -4\cot 2x \cdot \text{cosec } 2x$$

30. To maximize, $Z = 3x + y$

subject to constraints,

$$x \geq 5, y \geq 1, x + y - 8 \leq 0, x \geq 0, y \geq 0$$

On plotting the inequations on graph, we notice shaded portion is feasible solution.



Possible points for maximum Z are $A(5, 1)$, $B(7, 1)$, $C(5, 3)$

Corner Points	$Z = 3x + y$	Values	
$A(5, 1)$	15 + 1	16	
$B(7, 1)$	21 + 1	22	← Maximum
$C(5, 3)$	15 + 3	18	

$\therefore$ Z is maximum at $B(7, 1)$ i.e. $x = 7, y = 1$.

Maximum value of $Z = 22$.

OR

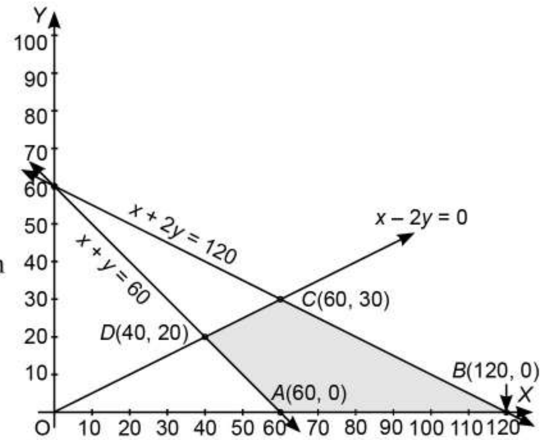
To minimise

$$Z = 5x + 10y$$

subject to the constraints

$$x \geq 0, y \geq 0, x - 2y \geq 0, x + y \geq 60, x + 2y \leq 120$$

Plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for minimum Z are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$



Corner Points	$Z = 5x + 10y$	Values	
$A(60, 0)$	300 + 0	300	← Minimum
$B(120, 0)$	600 + 0	600	
$C(60, 30)$	300 + 300	600	
$D(40, 20)$	200 + 200	400	

$\therefore$ Z is minimum at $A(60, 0)$. Hence, for $x = 60$ and $y = 0$, Z is minimum.

Minimum value of $Z = 300$.

31. We have, $y = e^x(\sin x + \cos x)$ ...(*i*)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^x (\cos x - \sin x) + e^x (\sin x + \cos x)$$

$$\Rightarrow \quad \frac{dy}{dx} = e^x (\cos x - \sin x) + y \quad ...(ii)$$

Again differentiating w.r.t. x, we get

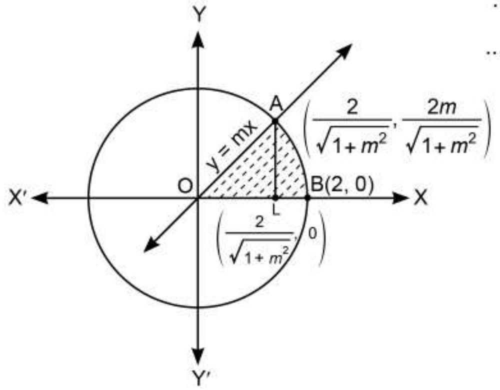
$$\frac{d^2y}{dx^2} = e^x (\cos x - \sin x) + e^x (-\sin x - \cos x) + \frac{dy}{dx}$$

$\Rightarrow$ $$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx} - y\right) - y + \frac{dy}{dx}$$ [from (*i*) and (*ii*)]

$\Rightarrow$ $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

32. Given curves are : $y = mx (m > 0)$ ...(*i*)

and $x^2 + y^2 = 4$ ...(*ii*)



Put $y = mx$ from (*i*) in (*ii*), we get

$$x^2 + m^2x^2 = 4$$

$\Rightarrow$ $$x^2(1 + m^2) = 4$$

$\Rightarrow$ $$x = \frac{2}{\sqrt{1+m^2}}$$

From (*i*), $$y = \frac{2m}{\sqrt{1+m^2}}$$

So, point of intersection of (*i*) and (*ii*) in 1st quadrant is $A\left(\frac{2}{\sqrt{1+m^2}}, \frac{2m}{\sqrt{1+m^2}}\right)$.

On plotting the given curves on graph, we notice that area of shaded region is to be found.

$\therefore$ $$\text{Area}(OAB) = \text{ar}(OAL) + \text{ar}(LAB)$$

$\Rightarrow$ $$\int_0^{\frac{2}{\sqrt{1+m^2}}} mx\, dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} \sqrt{4 - x^2}\, dx = \frac{\pi}{2}$$

$$\Rightarrow \left[\frac{mx^2}{2}\right]_0^{\frac{2}{\sqrt{1+m^2}}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{2}{\sqrt{1+m^2}}}^{2}$$

$$\Rightarrow \frac{m}{2} \cdot \frac{4}{1+m^2} + \{0 + 2\sin^{-1}(1)\} - \left\{\frac{1}{\sqrt{1+m^2}}\sqrt{4 - \frac{4}{1+m^2}} + 2 \cdot \sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right)\right\} = \frac{\pi}{2}$$

$\Rightarrow$ $$\frac{2m}{1+m^2} + 2 \times \frac{\pi}{2} - \frac{2m}{1+m^2} - 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{2}$$

$\Rightarrow$ $$2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{4}$$

$\Rightarrow$ $$\frac{1}{\sqrt{1+m^2}} = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\sqrt{1+m^2} = \sqrt{2}$$

$\Rightarrow$ $$1 + m^2 = 2 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$$

As, $m > 0$, so we take $m = 1$.

33. $$f(x) = 5x^2 + 6x - 9$$

For one-one: Let $x_1, x_2 \in R^+$.

Then, $$f(x_1) = f(x_2)$$

$$\Rightarrow \quad 5x_1^2 + 6x_1 - 9 = 5x_2^2 + 6x_2 - 9$$

$$\Rightarrow \quad 5(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$$

$$\Rightarrow \quad (x_1 - x_2)(5x_1 + 5x_2 + 6) = 0$$

Now, $x_1, x_2 \in R^+ \quad \therefore \quad 5x_1 + 5x_2 + 6 \neq 0$

Hence, $$x_1 - x_2 = 0$$

$$\Rightarrow \quad x_1 = x_2$$

$\therefore$ f is one-one function.

For onto: Let $5x^2 + 6x - 9 = y$

$$\Rightarrow \quad 5x^2 + 6x - (9 + y) = 0$$

$$\Rightarrow \quad x = \frac{-6 \pm \sqrt{36 + 20(9 + y)}}{10} \qquad \text{(Using quadratic formula)}$$

$$\Rightarrow \quad x = \frac{-6 \pm \sqrt{36 + 180 + 20y}}{10}$$

$$= \frac{-6 \pm 2\sqrt{54 + 5y}}{10}$$

Here $$\frac{\sqrt{54 + 5y} - 3}{5} \in R^+ \text{ but } \frac{-\sqrt{54 + 5y} - 3}{5} \notin R^+ \qquad \text{(Rejected)}$$

$$\therefore \quad f(x) = 5\left(\frac{\sqrt{54 + 5y} - 3}{5}\right)^2 + 6\left(\frac{\sqrt{54 + 5y} - 3}{5}\right) - 9$$

$$= \left(\frac{54 + 5y + 9 - 6\sqrt{54 + 5y}}{5}\right) + \frac{6(\sqrt{54 + 5y} - 3)}{5} - 9$$

$$= \frac{63 + 5y - 6\sqrt{54 + 5y} + 6\sqrt{54 + 5y} - 18 - 45}{5}$$

$$= y$$

$\therefore$ f is onto

Hence f is a bijective function.

OR

$R = \{(a, b) : a - b + \sqrt{5} \in \text{S and } a, b \in \text{R}\}$

For reflexive: Let $a \in R$

Now, $$(a, a) \in R$$

$$\Rightarrow \quad a - a + \sqrt{5} = \sqrt{5} \in S$$

$\therefore$ R is reflexive relation.

For symmetric: Let $a, b \in R$

Take $a = \sqrt{5}$, $b = 1$.

Now, $$a - b + \sqrt{5} = \sqrt{5} - 1 + \sqrt{5}$$

$$= 2\sqrt{5} - 1 \in S$$

But $$b - a + \sqrt{5} = 1 - \sqrt{5} + \sqrt{5} = 1 \notin S$$

$\therefore$ $(b, a) \notin R$

$\therefore$ R is not symmetric relation.

For transitive:

Take $a = 1, b = \sqrt{2}, c = \sqrt{5}$

Now, $(a, b) \in R \Rightarrow 1 - \sqrt{2} + \sqrt{5} \in S$

And $(b, c) \in R \Rightarrow \sqrt{2} - \sqrt{5} + \sqrt{5} = \sqrt{2} \in S$

But $(a, c) \notin R$, as $1 - \sqrt{5} + \sqrt{5} = 1 \notin S$

$\therefore$ Given relation is reflexive but neither symmetric nor transitive.

34. Consider

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 1(0) - 2(7) - 3(-7)$$

$$= -14 + 21 = 7 \neq 0$$

So, A^{-1} exists.

Let A_{ij} be the cofactors of a_{ij} in $|A|$. Then,

$A_{11} = (-1)^2 (0) = 0, A_{12} = (-1)^3 (7) = -7, A_{13} = (-1)^4 (-7) = -7$

$A_{21} = (-1)^3 (-1) = 1, A_{22} = (-1)^4 (7) = 7, A_{23} = (-1)^5 (-5) = 5$

$A_{31} = (-1)^4 (2) = 2, A_{32} = (-1)^5 (7) = -7, A_{33} = (-1)^6 (-4) = -4$

Now,

$$\text{Adj } A = \begin{bmatrix} 0 & -7 & -7 \\ 1 & 7 & 5 \\ 2 & -7 & -4 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$\therefore$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \quad ...(i)$$

Consider equations

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

Corresponding matrix equation is,

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$\Rightarrow$ $$AX = B$$

Its solution is $X = A^{-1}B$.

$\Rightarrow$ $$X = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} \quad \text{[from } (i)\text{]}$$

$\Rightarrow$ $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 + 3 + 4 \\ -42 + 21 - 14 \\ -42 + 15 - 8 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow x = 1, y = -5, z = -5$ is solution of the given system of equations.

35. The given equations of lines are :

$$\frac{x+1}{3} = \frac{y+2}{5} = \frac{z+5}{7} = \lambda \text{ (say)} \qquad ...(i)$$

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-4}{5} = \mu \text{ (say)} \qquad ...(ii)$$

The coordinates of any general point on line (*i*) are given by $(3\lambda - 1, 5\lambda - 2, 7\lambda - 5)$.

Also, the coordinates of any general point on line (*ii*) are $(\mu + 2, 3\mu + 4, 5\mu + 4)$.

If lines intersect, then for some values of λ, μ they represent the same point.

i.e.,

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda = \mu + 3 \qquad ...(iii)$$

$$5\lambda - 2 = 3\mu + 4 \Rightarrow 5\lambda = 3\mu + 6 \qquad ...(iv)$$

$$7\lambda - 5 = 5\mu + 4 \Rightarrow 7\lambda = 5\mu + 9 \qquad ...(v)$$

Solving (*iii*) and (*iv*) we get

$$\lambda = \frac{3}{4} \text{ and } \mu = -\frac{3}{4}$$

Substituting the values of λ and μ in (*v*), we get

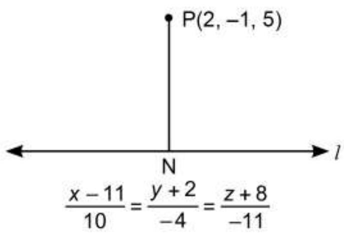
$$\frac{21}{4} = -\frac{15}{4} + 9 \Rightarrow \frac{21}{4} = \frac{21}{4}, \text{ true.}$$

Hence, for $\lambda = \frac{3}{4}$, $\mu = \frac{-3}{4}$ lines intersect.

Substituting $\lambda = \frac{3}{4}$ in (*i*) or $\mu = \frac{-3}{4}$ in (*ii*),

Coordinates of point of intersection are $\left(\frac{9}{4} - 1, \frac{15}{4} - 2, \frac{21}{4} - 5\right)$ *i.e.,* $\left(\frac{5}{4}, \frac{7}{4}, \frac{1}{4}\right)$.

OR



The given equation of line '*l*' is,

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}.$$

Let $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$

$\Rightarrow \quad x = 10\lambda + 11, y = -4\lambda - 2, z = -11\lambda - 8$

Suppose the coordinates of any general point on the line '*l*' is $(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$.

Let *N* be the foot of the perpendicular drawn from the point *P*(2, –1, 5) on the given line '*l*'.

Suppose the coordinates of point *N* be $N(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$.

Now, dr's of line '*l*' are <10, – 4, – 11>

Also, dr's of $PN = \langle 10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5 \rangle$

$= \langle 10\lambda + 9, -4\lambda - 1, -11\lambda - 13 \rangle$

As, *PN* is perpendicular to line '*l*', then

$$10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$$

$$\Rightarrow \quad 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$$

$$\Rightarrow \quad 237\lambda = -237 \Rightarrow \lambda = -1$$

Now, we will put $\lambda = -1$ in the coordinates of *N*. So, coordinates of the foot of the perpendicular drawn from the point *P* on the given line '*l*' is

$N(-10 + 11, 4 - 2, 11 - 8)$ *i.e.* $N(1, 2, 3)$.

Length of perpendicular, $PN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$$= \sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

36. (*i*) As, $\Sigma P(X) = 1$

$\Rightarrow \quad p + 2p + 2p + p + 2p + p^2 + 2p^2 + 7p^2 + p = 1$

$\Rightarrow \quad 10p^2 + 9p - 1 = 0 \quad \Rightarrow \quad (10p - 1)(p + 1) = 0$

$\Rightarrow \quad 10p - 1 = 0 \text{ or } p + 1 = 0$

$\Rightarrow \quad p = \frac{1}{10} \text{ or } p = -1 \text{ (rejected)}$

(*ii*) $P(X > 6) = P(7) + P(8) = 2p^2 + 7p^2 + p$

$$= 9p^2 + p = \frac{9}{100} + \frac{1}{10} = \frac{19}{100} = 0.19$$

(*iii*) Required probability $= P(3) + P(6)$

$$= 2p + p^2 = \frac{2}{10} + \frac{1}{100}$$

$$= \frac{21}{100} = 0.21$$

OR

(*iii*) Required probability $= P(2) + P(4) + P(6) + P(8)$

$$= 2p + p + p^2 + 7p^2 + p = 8p^2 + 4p$$

$$= \frac{8}{100} + \frac{4}{10} = \frac{48}{100} = 0.48$$

37. (*i*) Length = $(18 - 2x)$ cm, breadth = $(18 - 2x)$ cm, height = x cm

(*ii*) Volume V = length × breadth × height = $x(18 - 2x)^2$ cm^3

(*iii*) As, $V = x(18 - 2x)^2$

$\Rightarrow \quad \frac{dV}{dx} = x \cdot 2(18 - 2x)(-2) + (18 - 2x)^2$

$$= (18 - 2x)(18 - 6x)$$

For maximum or minimum volume, $\frac{dV}{dx} = 0$

$\Rightarrow \quad (18 - 6x)(18 - 2x) = 0$

$\Rightarrow \quad x = 3 \text{ or } x = 9$

Now, $x = 9$ is rejected as length and breadth becomes 0 for $x = 9$.

Now, $\frac{d^2V}{dx^2} = (18 - 2x)(-6) + (18 - 6x)(-2)$

$$\left.\frac{d^2V}{dx^2}\right]_{x=3} = (18 - 2 \times 3)(-6) + (-2)(18 - 6 \times 3) = -72 < 0$$

$\therefore$ Volume is maximum at $x = 3$.

OR

(*iii*) Maximum Volume $= 3(18 - 6)^2 = 432$ cm^3

38. (*i*)

$$\overrightarrow{BC} = \text{Position vector of } C - \text{Position vector of } B$$

$$\Rightarrow \quad \overrightarrow{BC} = \left(\hat{i} + 5\hat{j} - 2\hat{k}\right) - \left(-3\hat{i} + 6\hat{j} + 8\hat{k}\right)$$

$$= 4\hat{i} - \hat{j} - 10\hat{k}$$

$$\text{unit vector along } \overrightarrow{BC} = \frac{\overrightarrow{BC}}{|\overrightarrow{BC}|}$$

$$= \frac{4\hat{i} - \hat{j} - 10\hat{k}}{\sqrt{16+1+100}} = \frac{4}{\sqrt{117}}\hat{i} - \frac{1}{\sqrt{117}}\hat{j} - \frac{10}{\sqrt{117}}\hat{k}$$

(*ii*)

$$\overrightarrow{BP} = \text{p.v. of } P - \text{p.v. of } B = \left(-2\hat{i} + 3\hat{j} + 7\hat{k}\right) - \left(-3\hat{i} + 6\hat{j} + 8\hat{k}\right)$$

$$= \hat{i} - 3\hat{j} - \hat{k}$$

$$\overrightarrow{CP} = \text{p.v. of } P - \text{p.v. of } C = \left(-2\hat{i} + 3\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 5\hat{j} - 2\hat{k}\right)$$

$$= -3\hat{i} - 2\hat{j} + 9\hat{k}$$

$$\cos\theta = \frac{\overrightarrow{BP}\cdot\overrightarrow{CP}}{|\overrightarrow{BP}||\overrightarrow{CP}|} = \frac{-3+6-9}{\sqrt{1+9+1}\,\sqrt{9+4+81}}$$

$$\Rightarrow \quad \cos\theta = \frac{-6}{\sqrt{11}\,\sqrt{94}} = \frac{-6}{\sqrt{1034}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{-6}{\sqrt{1034}}\right)$$