

Time Allowed: 3 Hours]

[Maximum Marks: 80

General Instructions:**Read the following instructions very carefully and strictly follow them:**

- (i) This Question paper contains 38 questions. All questions are compulsory.
- (ii) This Question paper is divided into five Sections – A, B, C, D and E.
- (iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
- (iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
- (v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
- (vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
- (vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
- (ix) Use of calculators is *not* allowed.

SECTION – A**(This section comprises of multiple choice questions (MCQs) of 1 mark each)****Select the correct option (Question 1 - Question 18):**

1. The domain of $\sin^{-1}\sqrt{x-1}$ is [NCERT Part-I, Page 19]
 (a) $[-1, 1]$ (b) $[0, 1]$ (c) $[-1, -2]$ (d) $[1, 2]$
2. Let $f: R \rightarrow R$ defined by $f(x) = 3x$. Choose the correct answer. [NCERT Part-I, Page 7]
 (a) f is one-one and onto (b) f is many-one and onto
 (c) f is one-one but not onto (d) f is neither one-one nor onto
3. $\int_0^1 \log(1+x)dx =$ [NCERT Part-II, Page 259-260, 268]
 (a) $\log 4$ (b) $\log\left(\frac{4}{e}\right)$ (c) $\log e^4$ (d) $\frac{1}{\log 4}$
4. $\int \frac{\cot x}{\sqrt[3]{\sin x}} dx =$ [NCERT Part-II, Page 235-236]
 (a) $\frac{-3}{\sqrt[3]{\sin x}} + C$ (b) $\frac{-2}{\sin^3 x} + C$ (c) $\frac{3}{\sin^{\frac{1}{3}} x} + C$ (d) $\frac{2}{\cos x} + C$
5. The area bounded by the curve $y = |x|$, $y = 0$ and $x = 3$ in first quadrant is [Conceptual Application]
 (a) $\frac{9}{2}$ sq units (b) 9 sq units (c) $\frac{25}{2}$ sq units (d) 25 sq units

6. The area bounded by $y^2 = x$ and $x = 3$ is given by [Conceptual Application]
 (a) $\int_0^3 (3 - y^2) dy$ (b) $\int_0^3 \sqrt{y} dy$ (c) $2 \int_0^{\sqrt{3}} (3 - y^2) dy$ (d) $2 \int_0^3 (y^2 + 3) dy$
7. If $y = \sin 5x$, then $\frac{d^2y}{dx^2}$ is [NCERT Part-I, Page 137]
 (a) $-\sin 5x$ (b) $-25 \sin 5x$ (c) $-\cos 5x$ (d) $-25 \cos 5x$
8. $\frac{d}{dx}(x^x)$ is equal to [NCERT Part-I, Page 130]
 (a) x^{x-1} (b) $x \log x$ (c) $x^x(1 + \log x)$ (d) xx^{x-1}
9. Diameter of a sphere is $\frac{3}{2}(2x + 5)$, the rate of change of its surface area with respect to x is [NCERT Part-I, Page 147]
 (a) $18\pi(2x + 5)$ (b) $\frac{3}{2}$ (c) $9\pi(2x + 5)$ (d) $\frac{3}{4}$
10. The edge of a cube is increasing at the rate of 0.3 cm/s, the rate of change of its surface area when edge is 3 cm is [NCERT Part-I, Page 147]
 (a) 10.8 cm (b) 10.8 cm^2 (c) $10.8 \text{ cm}^2/\text{s}$ (d) 10.8 cm/s
11. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is [NCERT Part-II, Page 302]
 (a) 1 (b) 2 (c) 3 (d) Not defined
12. The direction cosines of line of support of vector $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ are [NCERT Part-II, Page 349]
 (a) $\langle 1, -2, 2 \rangle$ (b) $\langle \frac{1}{3}, \frac{-2}{3}, \frac{2}{3} \rangle$ (c) $\langle \frac{1}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}} \rangle$ (d) $\langle -1, 2, 2 \rangle$
13. The position vector of a point through which the line $\vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$ passes is [NCERT Part-II, Page 382]
 (a) $\langle 2, -1, 4 \rangle$ (b) $\hat{i} - \hat{j} - \hat{k}$ (c) $2\hat{i} - \hat{j} + 4\hat{k}$ (d) $(3, -2, 3)$
14. If $\vec{a} \times \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $|\vec{a}| = 2, |\vec{b}| = 1$, then the angle between \vec{a} and \vec{b} is [NCERT Part-II, Page 363-364]
 (a) 30° (b) 60° (c) 90° (d) 120°
15. The line joining the points $(0, 5, 4)$ and $(1, 3, 6)$ has direction ratios proportional to [NCERT Part-II, Page 379-380]
 (a) $\langle -1, 2, -2 \rangle$ (b) $\langle 1, 2, 2 \rangle$ (c) $(1, -2, 0)$ (d) $(1, 3, 0)$
16. If \vec{a} and \vec{b} are non-zero vectors, such that $\vec{a} \cdot \vec{b} = 0$, then [NCERT Part-II, Page 356]
 (a) \vec{a} is parallel to \vec{b} (b) \vec{a} and \vec{b} are collinear
 (c) \vec{a} is perpendicular to \vec{b} (d) None of these
17. If A and B are events such that $P(A|B) = P(B|A)$, then [NCERT Part-II, Page 408]
 (a) $A \subset B$ but $A \neq B$ (b) $A = B$ (c) $A \cap B = \phi$ (d) $P(A) = P(B)$
18. If $P(A) = \frac{1}{2}, P(B) = 0$, then $P(A|B)$ is [NCERT Part-II, Page 408]
 (a) 0 (b) $\frac{1}{2}$ (c) not defined (d) 1

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

- (a) Both A and R are true and R is the correct explanation of A .
- (b) Both A and R are true but R is not the correct explanation of A .
- (c) A is true but R is false.
- (d) A is false but R is true.

19. **Assertion (A):** For a bounded feasible region maximum or minimum of objective function lies at the corner points of the feasible region. [NCERT Part-II, Page 398-399]

Reason (R): Maximum or minimum lies at the corner points of feasible region.

20. **Assertion (A):** The domain of the function $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is $[-1, 1]$ [NCERT Part-I, Page 19, 21, 24]

Reason (R): $\sin^{-1}x, \cos^{-1}x$ are defined for $|x| \leq 1$ and $\tan^{-1}x$ is defined for all x .

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. For the differential equation, find a particular solution satisfying the given condition
 $(1 + \sin^2 x) dy + (1 + y^2) \cos x dx = 0$, given that, when $x = \frac{\pi}{2}, y = 0$. [NCERT Part-II, Page 306-307]
22. A and B throw a pair of dice turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is $\frac{9}{17}$. [Conceptual Application]

OR

Assume that each born child is equally likely to be a boy or a girl. If a family has two children what is the conditional probability that both are girls, given that [NCERT Part-II, Page 408]

- (i) the youngest is a girl.
 - (ii) at least one is a girl.
23. Find the length and the foot of the perpendicular drawn from the point $(2, -1, 5)$ on the line
 $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. [Conceptual Application]
24. Find the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$. [NCERT Part-I, Page 27]
25. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$, hence find A^5 .

[NCERT Part-I, Page 47, 51]

OR

Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$. [NCERT Part-I, Page 51, 41]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $x = \frac{1}{y}$, then show that $\frac{dx}{dy} = -\sqrt{\frac{1+x^4}{1+y^4}}$. [NCERT Part-I, Page 122-123]
27. A point on the hypotenuse of a right-angled triangle is at distance a and b from the sides. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$ [NCERT Part-I, Page 166]
28. Evaluate $\int_0^1 |3x - 2| dx$. [Conceptual Application]

OR

Find the general solution of the differential equation, $x \cos y dy = (xe^x \log x + e^x) dx$.

[NCERT Part-II, Page 306-307]

29. Let S be the set of points in a plane and R be a relation in S defined as $R = \{(A, B) : d(A, B) < 2\}$, where $d(A, B)$ represents the distance between the points A and B . Is R an equivalence relation? [Conceptual Application]
30. Using integration find the area of the region: $\{(x, y) : 9x^2 + y^2 \leq 36 \text{ and } 3x + y \geq 6\}$. [Conceptual Application]

OR

Make a rough sketch of the region given below and find its area using integration

$\{(x, y) : 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$.

[Conceptual Application]

31. Prove that the function $y = \text{Sgn} [x - 1]$ is not differentiable at $x = 1$. [NCERT Part-I, Page 119]

OR

Find $\frac{dy}{dx}$, if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

[NCERT Part-I, Page 119]

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find the distance of the point $A(1, 8, 4)$ and its image in the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. [Conceptual Application]

OR

If the vertices A, B, C of a ΔABC are $(1, 2, 3), (-1, 0, 0)$ and $(0, 1, 2)$ respectively, what is the magnitude of $\angle ABC$? [NCERT Part-II, Page 356]

33. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$. [NCERT Part-I, Page 90]

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = O$. [Conceptual Application]

34. Show that the differential equation $2y e^{xy} dx + (y - 2x e^{xy}) dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 0$ when $y = 1$. [NCERT Part-II, Page 312-314]
35. Solve the following linear programming problem graphically. [NCERT Part-II, Page 397-398]
 Maximise $Z = x + y$
 subject to constraints $x \geq 3, y \geq 4$ and $6x + 9y \leq 72$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions: [Conceptual Application]

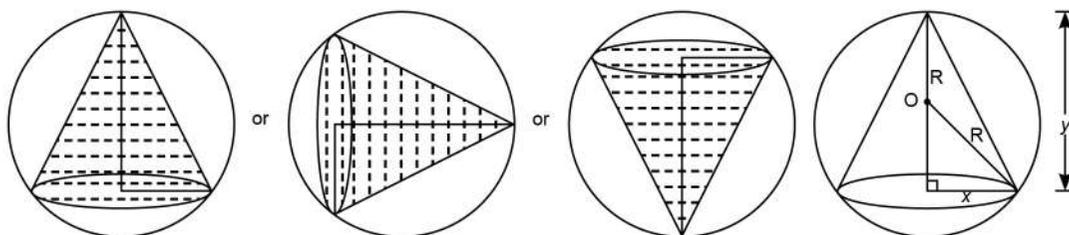
- (i) Find the probability that the age of the selected student is a composite number.
- (ii) Let X be the age of the selected student. What can be the value of X ?
- (iii) A student was selected and his age was found to be less than 16, what is the probability that his age is 13?

OR

- (iii) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number.

Case Study - 2

37. A company dealing in gift items decided to make three-dimensional gift items and a core team was given the responsibility for the same. After discussions core team suggested to insert a right circular cone in a sphere touching the surface of a sphere and at the same time gets illuminated when taking rounds. They presented the different dimensions also keeping the cost factor into account. The gift item looks like as shown [Conceptual Application]



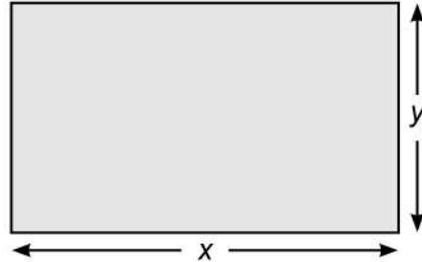
- (i) If a cone of base radius x and height y is inscribed in sphere of radius R then establish relation between x, y and R .
- (ii) What is the volume of a cone in terms of y alone?
- (iii) What should be the length of altitude of a cone so that the volume of inscribed cone is maximum?

OR

- (iii) Find the maximum volume of a cone inscribed in a sphere.

Case Study - 3

38. Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2 . [NCERT Part-I, Page 34-35, 94]



- (i) Express the given situation in the form of linear equation in 2 variables.
(ii) Express above system of equations in matrix form and hence find dimensions of plot.

SOLUTIONS

1. (d), Let $y = \sin^{-1} \sqrt{x-1}$

$$\begin{aligned} \therefore & -1 \leq \sqrt{x-1} \leq 1 \\ \Rightarrow & 0 \leq \sqrt{x-1} \leq 1 \\ \Rightarrow & 0 \leq x-1 \leq 1 \\ \Rightarrow & 1 \leq x \leq 2 \\ & x \in [1, 2] \end{aligned}$$

2. (a)

3. (b) Let $I = \int_0^1 \log(1+x) dx = \int_0^1 \log(1+x) \cdot \underset{\ominus}{1} dx = [x \log(1+x)]_0^1 - \int_0^1 \frac{x}{x+1} dx$
 $= \log 2 - \int_0^1 \left(1 - \frac{1}{x+1}\right) dx = \log 2 - [x - \log|x+1|]_0^1 = \log 2 - 1 + \log 2$
 $= \log 4 - \log e = \log\left(\frac{4}{e}\right).$

4. (a) Let $I = \int \frac{\cot x}{\sqrt[3]{\sin x}} dx \Rightarrow I = \int \frac{\cos x}{(\sin x)^{4/3}} dx$

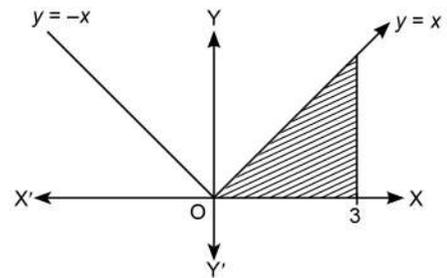
put $\sin x = t \Rightarrow \cos x dx = dt$

$\therefore I = \int t^{-4/3} dt \Rightarrow I = \frac{-3}{t^{1/3}} + C$

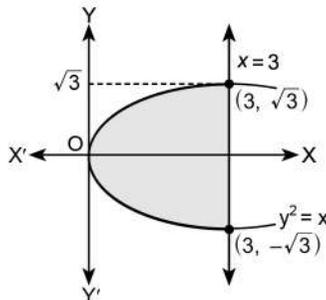
$\Rightarrow I = \frac{-3}{\sqrt[3]{\sin x}} + C$

5. (a), as we have to find shaded area

$$\begin{aligned} \text{Area} &= \int_0^3 |x| dx = \int_0^3 x dx (\because |x| = x, x \geq 0) \\ &= \left[\frac{x^2}{2} \right]_0^3 = \frac{9}{2} \text{ sq units} \end{aligned}$$



6. (c) Eliminating 'x' from the given equation $y^2 = x$ and $x = 3$, we get $y = \pm \sqrt{3}$.



So, points of intersection are: $(3, \sqrt{3})$ and $(3, -\sqrt{3})$.

Required area = $2 \int_0^{\sqrt{3}} (3 - y^2) dy$

7. (b), as $y = \sin 5x$

$$\begin{aligned} \frac{dy}{dx} &= 5 \cos 5x \\ \frac{d^2y}{dx^2} &= 5 \cdot \frac{d}{dx} (\cos 5x) \\ &= 5 \cdot (-5 \sin 5x) = -25 \sin 5x \end{aligned}$$

8. (c), as
$$\frac{d}{dx}(x^x) = \frac{d}{dx}(e^{x \log x})$$

$$= e^{x \log x} \left(x \cdot \frac{1}{x} + \log x \right) = x^x(1 + \log x)$$

9. (c), as $r = \frac{3}{4}(2x + 5)$ and $S = 4\pi r^2$,
then
$$S = 4 \cdot \pi \left[\frac{3}{4}(2x + 5) \right]^2 = \frac{9}{4} \pi (2x + 5)^2$$

$\therefore \frac{dS}{dx} = \frac{9}{4} \pi \cdot 2(2x + 5) \cdot 2 = 9\pi(2x + 5)$

10. (c), as $\frac{dx}{dt} = 0.3$ cm/s, x is edge of a cube.

then
$$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \cdot \frac{dx}{dt} = 12x \times 0.3$$

$\Rightarrow \frac{dS}{dt} = 3.6x$

and
$$\left. \frac{dS}{dt} \right|_{x=3} = 3.6 \times 3 = 10.8 \text{ cm}^2/\text{s}$$

11. (d), as equation cannot be represented as a polynomial of derivatives.

12. (b), as
$$\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\Rightarrow \hat{a} = \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k},$$

therefore, DC's: $\left\langle \frac{1}{3}, \frac{-2}{3}, \frac{2}{3} \right\rangle$

13. (c), as equation of line is $\vec{r} = \vec{a} + \lambda\vec{b}$, \vec{a} represents position vector of a point through which line passes.

Hence, the position vector of the point = $2\hat{i} - \hat{j} + 4\hat{k}$.

14. (b), as
$$|\vec{a} \times \vec{b}| = |\hat{i} + \hat{j} + \hat{k}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|}$$

$$= \frac{\sqrt{3}}{2 \times 1} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$\Rightarrow \theta = 60^\circ$

15. (a), as dr's of line joining points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$
or $\langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

16. (c)

17. (d) as, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

18. (c) as, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

19. (b), Both A and R are correct but R is not the correct explanation of A .

20. (a), Both A and R are true and R is the correct explanation of A .

21. As, $(1 + \sin^2 x)dy + (1 + y^2) \cos x dx = 0$

$$\Rightarrow \frac{1}{1 + y^2} dy = -\left(\frac{\cos x}{1 + \sin^2 x}\right) dx$$

$$\Rightarrow \int \frac{dy}{1 + y^2} = \int \frac{-\cos x}{1 + \sin^2 x} dx$$

$$\Rightarrow \tan^{-1} y = -\tan^{-1}(\sin x) + C$$

...(i)

When $x = \frac{\pi}{2}, y = 0$ then from (i), $0 = -\frac{\pi}{4} + C \Rightarrow C = \frac{\pi}{4}$

Substituting $C = \frac{\pi}{4}$ in (i), we get

$$\tan^{-1} y + \tan^{-1}(\sin x) = \frac{\pi}{4} \text{ as the required solution.}$$

22. S : getting a total of 9 = $\{(3, 6), (4, 5), (5, 4), (6, 3)\}$

$$P(S) = \frac{4}{36} = \frac{1}{9}. P(\bar{S}) = \frac{8}{9}$$

A can win in 1st, 3rd, 5th, 7th, throws

$$P(A \text{ wins}) = P(S) + [P(\bar{S})]^2 P(S) + [P(\bar{S})]^4 P(S) + \dots$$

$$= \frac{1}{9} + \left(\frac{8}{9}\right)^2 \cdot \frac{1}{9} + \left(\frac{8}{9}\right)^4 \cdot \frac{1}{9} + \dots$$

$$= \frac{\frac{1}{9}}{1 - \frac{64}{81}} = \frac{9}{17}$$

$$\left[\begin{array}{l} \text{sum of infinite GP} \\ a + ar + ar^2 + \dots = \frac{a}{1-r} \end{array} \right]$$

OR

Sample space for two children is $S = \{BB, BG, GB, GG\}$

(i) E : the youngest is a girl = $\{BG, GG\}$

F : both are girls = $\{GG\}$

$E \cap F = \{GG\}$

Required probability,

$$P(F/E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

(ii) H : at least one is a girl = $\{BG, GB, GG\}$

F : both are girls = $\{GG\}$

$F \cap H = \{GG\}$

Required probability,

$$P(F/H) = \frac{P(F \cap H)}{P(H)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

23. General point on the line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda \text{ (say) is}$$

$$Q(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$$

...(i)

Direction ratios of PQ are

$$10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5$$

$$\text{i.e. } 10\lambda + 9, -4\lambda - 1, -11\lambda - 13$$

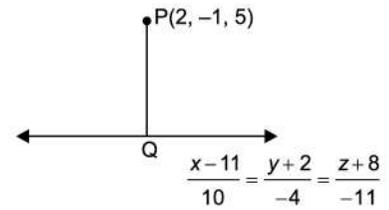
If PQ is perpendicular to the given line, then

$$10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$$

$$\Rightarrow 237\lambda = -237 \Rightarrow \lambda = -1$$

Substituting in (i), we get the foot of perpendicular as $Q(1, 2, 3)$.

$$\text{Length of perpendicular, } PQ = \sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2} = \sqrt{1+9+4} = \sqrt{14}.$$



24. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) + \cos^{-1}\left(-\cos\frac{\pi}{3}\right) + \tan^{-1}\left(\tan\frac{\pi}{6}\right)$$

$$= \frac{\pi}{3} + \cos^{-1}\left\{\cos\left(\pi - \frac{\pi}{3}\right)\right\} + \frac{\pi}{6}$$

$$= \frac{\pi}{2} + \frac{2\pi}{3} = \frac{7\pi}{6}$$

25. Given

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}; f(x) = x^2 - 4x + 7$$

$$f(A) = A^2 - 4A + 7I$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

\Rightarrow

$$A^2 = 4A - 7I$$

...(i)

Consider

$$A^5 = A \cdot A^2 \cdot A^2 = A(4A - 7I)(4A - 7I)$$

$$= A(16A^2 - 28AI - 28IA + 49I^2)$$

$$A^5 = A(16A^2 - 56A + 49I)$$

$$= A(64A - 112I - 56A + 49I)$$

[using (i)]

$$= A(8A - 63I) = 8A^2 - 63AI$$

$$= 8(4A - 7I) - 63A = 32A - 56I - 63A$$

$$= -31A - 56I = -31 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -62 & -93 \\ 31 & -62 \end{bmatrix} - \begin{bmatrix} 56 & 0 \\ 0 & 56 \end{bmatrix}$$

$$= \begin{bmatrix} -62-56 & -93-0 \\ 31-0 & -62-56 \end{bmatrix} = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$$

OR

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

For multiplication to define, X should be of order 2×2 .

Let
$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Now,
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix}$$

$$\therefore \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Using equality of matrices, we get

$$a + 4b = -7, 2a + 5b = -8, 3a + 6b = -9$$

Solving first two equations we get $a = 1, b = -2$. This also satisfies the third equation.

Also $c + 4d = 2, 2c + 5d = 4$ and $3c + 6d = 6$

Solving first two equations, we get $c = 2, d = 0$ which satisfies third equation also.

Hence $a = 1, b = -2, c = 2, d = 0$ and
$$X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

26.

$$\begin{aligned} x = \frac{1}{y} \Rightarrow \frac{dx}{dy} &= -\frac{1}{y^2} = -\frac{1}{y^2} \sqrt{\frac{1+y^4}{1+y^4}} \\ &= -\sqrt{\frac{1}{y^4} \cdot \frac{(1+y^4)}{(1+y^4)}} = -\sqrt{\frac{\frac{1}{y^4} + 1}{1+y^4}} \\ &= -\sqrt{\frac{x^4 + 1}{1+y^4}} = -\sqrt{\frac{1+x^4}{1+y^4}} \end{aligned}$$

27. Let $\triangle AOB$ is the right angled \triangle in which $\angle AOB = 90^\circ$ and $\angle OAB = \theta$. Let P be the point on hypotenuse AB . Draw $PL \perp OA$ and $PM \perp OB$. Let $PL = a$ and $PM = b$. Now, $\angle BPM = \angle OAB = \theta$ (corresponding \angle s).

Now, Hypotenuse (h) = $AB = AP + PB$... (i)

Now
$$\frac{AP}{a} = \operatorname{cosec} \theta \Rightarrow AP = a \operatorname{cosec} \theta$$

and
$$\frac{BP}{b} = \sec \theta \Rightarrow BP = b \sec \theta$$

From (i),
$$h = a \operatorname{cosec} \theta + b \sec \theta$$
 ... (ii)

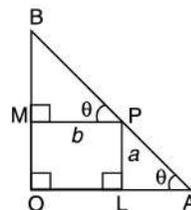
$$\frac{dh}{d\theta} = a(-\operatorname{cosec} \theta \cot \theta) + b(\sec \theta \tan \theta)$$

For minimum h ,
$$\frac{dh}{d\theta} = 0 \Rightarrow a \operatorname{cosec} \theta \cot \theta = b \sec \theta \tan \theta$$

$$\Rightarrow \frac{a}{b} = \tan^3 \theta \Rightarrow \tan \theta = \left(\frac{a}{b}\right)^{1/3}$$

$$\frac{d^2h}{d\theta^2} = -a[\operatorname{cosec} \theta(-\operatorname{cosec}^2 \theta) + \cot \theta(-\operatorname{cosec} \theta \cot \theta)] +$$

$$b[\sec \theta \sec^2 \theta + \tan \theta \sec \theta \tan \theta]$$



Clearly, $\frac{d^2h}{d\theta^2} > 0$ for $\tan \theta = \left(\frac{a}{b}\right)^{\frac{1}{3}}$

\therefore h is minimum for $\tan \theta = \left(\frac{a}{b}\right)^{\frac{1}{3}}$

$$\Rightarrow \sin \theta = \frac{a^{1/3}}{\sqrt{a^{2/3} + b^{2/3}}}, \cos \theta = \frac{b^{1/3}}{\sqrt{a^{2/3} + b^{2/3}}}$$

Putting in (ii), we get $h = a \cdot \frac{\sqrt{a^{2/3} + b^{2/3}}}{a^{1/3}} + b \cdot \frac{\sqrt{a^{2/3} + b^{2/3}}}{b^{1/3}} = (a^{2/3} + b^{2/3})^{3/2}$

28. Consider $\int_0^1 |3x - 2| dx = -\int_0^{2/3} (3x - 2) dx + \int_{2/3}^1 (3x - 2) dx$

$$= \left[-\frac{3x^2}{2} + 2x \right]_0^{2/3} + \left[\frac{3x^2}{2} - 2x \right]_{2/3}^1$$

$$= \left(-\frac{3}{2} \times \frac{4}{9} + \frac{4}{3} \right) - (0) + \left(\frac{3}{2} - 2 \right) - \left(\frac{3}{2} \times \frac{4}{9} - \frac{4}{3} \right) = \frac{2}{3} - \frac{1}{2} + \frac{2}{3} - \frac{4}{9} + \frac{4}{3} = \frac{5}{6}$$

OR

Consider equation,

$$x \cos y dy = (xe^x \log x + e^x) dx$$

$$\Rightarrow x \cos y dy = e^x (x \log x + 1) dx$$

$$\Rightarrow \cos y dy = e^x \left(\frac{x \log x + 1}{x} \right) dx$$

Integrating both sides, we get

$$\int \cos y dy = \int e^x \left(\log x + \frac{1}{x} \right) dx \quad [\text{using } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C]$$

$\Rightarrow \sin y = e^x \cdot \log x + C$, is the required solution where C is constant of integration.

29. Relation $R = \{(A, B) : d(A, B) < 2\}$

For reflexive: For every $A \in S$, $d(A, A) = 0 < 2$

Hence $(A, A) \in R$, $\forall A \in S$. Hence, reflexive.

For symmetric: Let $(A, B) \in R \Rightarrow d(A, B) < 2$

$\Rightarrow d(B, A) < 2 \Rightarrow (B, A) \in R$, for all $A, B \in S$

Hence, symmetric.

For transitive: Let $(A, B) \in R$ and $(B, C) \in R$

Let $d(A, B) = 1.5$ and $d(B, C) = 1.7$ and A, B, C are collinear.

Then $d(A, C) = 1.5 + 1.7 = 3.2 > 2$

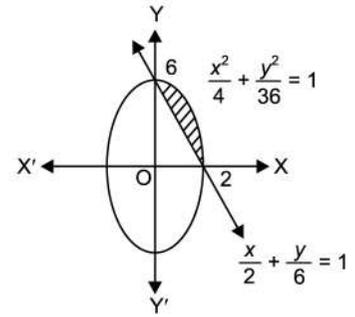
Hence $(A, B) \in R$, $(B, C) \in R \not\Rightarrow (A, C) \in R$ for all $A, B, C \in S$.

Hence, not transitive.

Hence, relation R is not an equivalence relation.

30. Consider corresponding equations

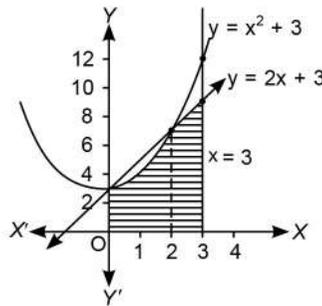
$$\begin{aligned} & 9x^2 + y^2 = 36 \\ \text{and} & 3x + y = 6 \\ \Rightarrow & \frac{x^2}{4} + \frac{y^2}{36} = 1 \\ \text{and} & \frac{x}{2} + \frac{y}{6} = 1 \end{aligned}$$



We have to find shaded area.

$$\begin{aligned} \text{Area} &= \int_0^2 \{ \sqrt{36 - 9x^2} - (6 - 3x) \} dx = 3 \int_0^2 (\sqrt{4 - x^2} - 2 + x) dx \\ &= 3 \left[\frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} - 2x + \frac{x^2}{2} \right]_0^2 \\ &= 3 \left[(0 + 2 \sin^{-1} 1 - 4 + 2) - 0 \right] = 3 \left[2 \cdot \frac{\pi}{2} - 2 \right] = 3(\pi - 2) \text{ sq units} \end{aligned}$$

OR



$$\text{Region} = \{(x, y) : 0 \leq y \leq x^2 + 3, 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$$

On plotting the inequations we have to find the area of the shaded portion.

Eliminating y from corresponding equations, we get

$$\begin{aligned} & x^2 + 3 = 2x + 3 \\ \Rightarrow & x = 0, 2 \\ \therefore & \text{area} = \int_0^2 (x^2 + 3) dx + \int_2^3 (2x + 3) dx \\ &= \left[\frac{x^3}{3} + 3x \right]_0^2 + [x^2 + 3x]_2^3 = \left(\frac{8}{3} + 6 \right) - (0) + (9 + 9) - (4 + 6) \\ &= \left(\frac{8}{3} + 6 + 18 - 10 \right) \text{ sq units} = \frac{50}{3} \text{ sq units} \end{aligned}$$

31. Given $y = \text{Sgn}(x - 1) = \begin{cases} 1, & x > 1 \\ 0, & x = 1 \\ -1, & x < 1 \end{cases}$

$$\text{LHD} = \lim_{x \rightarrow 1} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{-1 - 0}{-h} = \lim_{h \rightarrow 0} \frac{1}{h} \rightarrow \text{does not exist.}$$

As LHD at $x = 1$ does not exist, hence not differentiable.

OR

Consider $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Differentiating w.r.t. x , we get

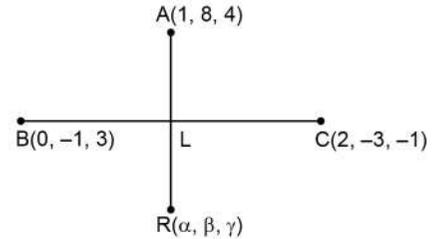
$$a(2x) + 2h \left\{ x \frac{dy}{dx} + y \cdot 1 \right\} + b(2y \frac{dy}{dx}) + 2g(1) + 2f \frac{dy}{dx} + 0 = 0$$

$$\Rightarrow 2ax + 2hx \frac{dy}{dx} + 2hy + 2by \frac{dy}{dx} + 2g + 2f \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(2hx + 2by + 2f) = -(2ax + 2hy + 2g)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(ax + hy + g)}{hx + by + f}$$

32. Let L be the foot of the perpendicular from the point $A(1, 8, 4)$ on the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$ DR's of BC : $2 - 0, -3 + 1, -1 - 3$
i.e. $2, -2, -4$ or $1, -1, -2$



$$\therefore \text{Equation of } BC: \frac{x-0}{1} = \frac{y+1}{-1} = \frac{z-3}{-2} = \lambda \text{ (say)}$$

Suppose the coordinates of any general point on BC is $(\lambda, -\lambda - 1, -2\lambda + 3)$.

Let the coordinates of point L be $L(\lambda, -\lambda - 1, -2\lambda + 3)$... (i)

Direction ratios of AL are $\lambda - 1, -\lambda - 1 - 8, -2\lambda + 3 - 4$, i.e. $\lambda - 1, -\lambda - 9, -2\lambda - 1$

If AL is perpendicular to BC , then

$$1(\lambda - 1) - 1(-\lambda - 9) - 2(-2\lambda - 1) = 0$$

$$\Rightarrow \lambda - 1 + \lambda + 9 + 4\lambda + 2 = 0 \Rightarrow 6\lambda = -10 \Rightarrow \lambda = -\frac{5}{3}$$

Substituting in (i), we get

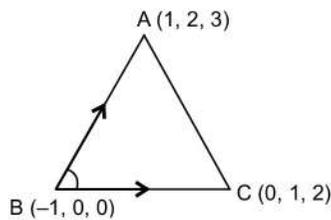
$$\text{Foot of the perpendicular as } L\left(-\frac{5}{3}, \frac{5}{3} - 1, \frac{10}{3} + 3\right), \text{ i.e. } L\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right).$$

Let $R(\alpha, \beta, \gamma)$ be image of $A(1, 8, 4)$ in the line BC , then

$$AR = 2AL = 2\sqrt{\left(-\frac{5}{3} - 1\right)^2 + \left(\frac{2}{3} - 8\right)^2 + \left(\frac{19}{3} - 4\right)^2} = 2\sqrt{\frac{64}{9} + \frac{484}{9} + \frac{49}{9}} = \frac{2}{3}\sqrt{597} \text{ units}$$

OR

$$\cos B = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|}$$



$$\vec{BA} = 2\hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{BC} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\cos B = \frac{(2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{17}\sqrt{6}} = \frac{2 + 2 + 6}{\sqrt{102}} = \frac{10}{\sqrt{102}}$$

$$\Rightarrow B = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

$$\therefore \angle ABC \text{ is } \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

33. Given

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Also,

$$(AB)^{-1} = B^{-1}A^{-1}$$

...(i)

A^{-1} is given, so we find B^{-1} .

$$\begin{aligned} |B| &= \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} \\ &= 1(3) - 2(-1) - 2(2) \\ &= 3 + 2 - 4 = 1 \neq 0. \end{aligned}$$

Matrix formed by cofactors of each element in $|B|$. B_{ij} is cofactor of element b_{ij} in $|B|$

$$B_{11} = +(3-0) = 3; \quad B_{12} = -(-1-0) = 1;$$

$$B_{13} = +(2-0) = 2$$

$$B_{21} = -(2-4) = 2; \quad B_{22} = +(1-0) = 1;$$

$$B_{23} = -(-2-0) = 2$$

$$B_{31} = +(0+6) = 6; \quad B_{32} = -(0-2) = 2;$$

$$B_{33} = +(3+2) = 5$$

$$\begin{aligned} \therefore \text{adj } B &= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \\ \Rightarrow B^{-1} &= \frac{\text{adj } B}{|B|} = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \end{aligned}$$

\therefore From (i)

$$\begin{aligned} (AB)^{-1} &= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \end{aligned}$$

OR

Given $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= 1(1-4) - 2(2-4) + 2(4-2) = -3 + 4 + 4 = 5 \neq 0.$$

Let A_{ij} be the cofactors of each element in $|A|$.

$$\begin{aligned}
A_{11} &= +(1-4) = -3; & A_{12} &= -(2-4) = 2; \\
A_{13} &= +(4-2) = 2 \\
A_{21} &= -(2-4) = 2; & A_{22} &= +(1-4) = -3; \\
A_{23} &= -(2-4) = 2 \\
A_{31} &= +(4-2) = 2; & A_{32} &= -(2-4) = 2; \\
A_{33} &= +(1-4) = -3
\end{aligned}$$

$$\therefore \text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \quad \dots(i)$$

Consider $A^2 - 4A - 5I = O$

Multiplying both sides by A^{-1} ,

$$A^{-1}(AA) - 4A^{-1}A - 5A^{-1}I = A^{-1}O$$

$$\Rightarrow (A^{-1}A)A - 4I - 5A^{-1} = O \Rightarrow IA - 4I - 5A^{-1} = O$$

$$\Rightarrow IA - 4I = 5A^{-1} \Rightarrow A - 4I = 5A^{-1}$$

$$\begin{aligned}
\text{LHS} &= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1-4 & 2-0 & 2-0 \\ 2-0 & 1-4 & 2-0 \\ 2-0 & 2-0 & 1-4 \end{bmatrix} \\
&= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = 5 \times \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = 5A^{-1} = \text{RHS}
\end{aligned}$$

Hence $A^2 - 4A - 5I = O$

34. Consider equation $2y e^{xy} dx + (y - 2x e^{xy}) dy = 0$

$$\Rightarrow \frac{dx}{dy} = \frac{2x e^{xy} - y}{2y e^{xy}} \quad \dots(i)$$

$$\text{Let } F(x, y) = \frac{2x e^{xy} - y}{2y e^{xy}}$$

$$F(\lambda x, \lambda y) = \frac{2\lambda x \cdot e^{\lambda x \lambda y} - \lambda y}{2\lambda y e^{\lambda x \lambda y}} = \frac{2x \cdot e^{\frac{x}{y}} - y}{2y e^{\frac{x}{y}}} = F(x, y)$$

Hence, function is homogeneous, so corresponding differential equation is homogeneous.

$$\text{Let } \frac{x}{y} = v \Rightarrow x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}$$

From (i), we have

$$v + y \frac{dv}{dy} = \frac{2vy \cdot e^v - y}{2ye^v} = \frac{2ve^v - 1}{2e^v}$$

$$\Rightarrow y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v} = \frac{-1}{2e^v}$$

$$\Rightarrow e^y dy = -\frac{1}{2} \cdot \frac{dy}{y} \Rightarrow \int e^y dy = -\frac{1}{2} \int \frac{dy}{y}$$

$$\Rightarrow e^y = -\frac{1}{2} \log |y| + C$$

$$\Rightarrow \frac{x}{e^y} = -\frac{1}{2} \log |y| + C$$

... (ii)

Given $x = 0$, when $y = 1$, then from (ii), we get

$$e^0 = -\frac{1}{2} \log |1| + C$$

$$\Rightarrow 1 = 0 + C \Rightarrow C = 1$$

Substituting $C = 1$ in (ii), we get

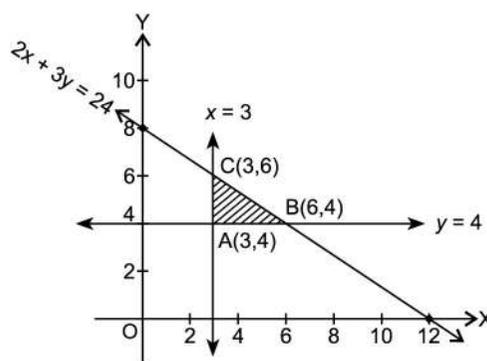
$$e^y = -\frac{1}{2} \log |y| + 1 \text{ is the particular solution.}$$

35. Plotting the graph of inequations, we get shaded portion as feasible solution.

Possible points for maximum Z are $A(3, 4)$, $B(6, 4)$ and $C(3, 6)$.

Points	$Z = x + y$	Values
$A(3, 4)$	$3 + 4$	7
$B(6, 4)$	$6 + 4$	10
$C(3, 6)$	$3 + 6$	9

← Maximum



$\therefore Z$ is maximum for $B(6, 4)$, i.e. $x = 6, y = 4$.

36. (i) Total students = 10

Composite numbers 16, 15, 14, 16, 16, 15

$$\text{Probability} = \frac{6}{10} = \frac{3}{5}$$

(ii) X can be 14, 15, 16, 17, 19

(iii) A : Age less than 16 : {14, 15, 15}

B : Age 13 : ϕ

$$A \cap B = \phi$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)} = \frac{0}{3} = 0.$$

OR

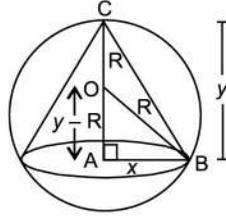
(iii) A : Age greater than 15 : 16, 16, 16, 17, 17, 19, 19

B : Age prime number : 17, 17, 19, 19

$A \cap B$: 17, 17, 19, 19

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{4}{10} = \frac{4}{10}$$

37. Let a cone of base radius x and height y be inscribed in a sphere of radius R .



$$(i) \quad R^2 = (y - R)^2 + x^2$$

$$x^2 + y^2 = 2Ry$$

$$(ii) \quad \text{Volume of the cone, } V = \frac{1}{3}\pi x^2 y$$

$$= \frac{1}{3}\pi y(2Ry - y^2) = \frac{1}{3}\pi(2Ry^2 - y^3)$$

$$(iii) \quad \frac{dV}{dy} = \frac{1}{3}\pi(4Ry - 3y^2)$$

$$\text{For maximum volume, } \frac{dV}{dy} = 0 \Rightarrow 4Ry = 3y^2$$

$$\Rightarrow y = \frac{4R}{3}$$

$$\frac{d^2V}{dy^2} = \frac{\pi}{3}(4R - 6y)$$

$$\left[\frac{d^2V}{dy^2} \right]_{y=\frac{4R}{3}} = \frac{\pi}{3} \left[4R - 6 \times \frac{4R}{3} \right] = \frac{-4}{3}\pi R < 0$$

So, Volume of cone is maximum when altitude of the cone is $\frac{4R}{3}$.

OR

$$(iii) \quad V_{\max} = \frac{1}{3}\pi \left[2R \left(\frac{4R}{3} \right)^2 - \left(\frac{4R}{3} \right)^3 \right]$$

$$= \frac{1}{3}\pi \left[\frac{32R^3}{9} - \frac{64R^3}{27} \right]$$

$$= \frac{32\pi R^3}{81} \text{ cubic units.}$$

38. (i) $x - y = 50, 2x + y = 550$

$$(ii) \quad \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$\text{Where } A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}; \quad X = \begin{bmatrix} x \\ y \end{bmatrix}; \quad B = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3 \neq 0$$

So, A^{-1} exists,

Let A_{ij} be cofactors of a_{ij} in $|A|$

$$\text{Now, } A_{11} = 1; \quad A_{12} = -2; \quad A_{21} = 1; \quad A_{22} = 1$$

So,
$$\text{adj}(A) = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

\therefore
$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

Now,
$$X = A^{-1}B$$

\Rightarrow
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

\Rightarrow
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 600 \\ 450 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

\therefore
$$x = 200; y = 150$$

So, dimensions of plot are: 200 m \times 150 m.