

Time Allowed: 3 Hours]

[Maximum Marks: 80

**General Instructions:****Read the following instructions very carefully and strictly follow them:**

- (i) This Question paper contains 38 questions. All questions are compulsory.
- (ii) This Question paper is divided into five Sections – A, B, C, D and E.
- (iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
- (iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
- (v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
- (vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
- (vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
- (ix) Use of calculators is **not** allowed.

**SECTION – A****(This section comprises of multiple choice questions (MCQs) of 1 mark each)****Select the correct option (Question 1 - Question 18):**

1. The minor of the element of second row and third column ( $a_{23}$ ) in the determinant  $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$  is  
[NCERT Part-I, Page 84]  
(a) -22                      (b) 13                      (c) -13                      (d) 22
2. The area of the triangle with vertices (-1, 2), (4, 0) and (3, 9) is  
[NCERT Part-I, Page 82]  
(a)  $\frac{43}{2}$  sq units              (b)  $-\frac{43}{4}$  sq units              (c) 21 sq units              (d) 42 sq units
3. If  $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$ , then the value of y is  
[NCERT Part-I, Page 41]  
(a) 11                      (b) 3                      (c) -3                      (d) 1
4. The principal value of  $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$  is  
[NCERT Part-I, Page 19]  
(a)  $\frac{\pi}{3}$                       (b)  $-\frac{\pi}{3}$                       (c)  $\frac{\pi}{2}$                       (d)  $-\frac{\pi}{2}$

5. Let  $A = \{1, 2, 3, \dots, n\}$  and  $B = \{a, b\}$ . Then the number of surjections from  $A$  into  $B$  is  
**[Conceptual Application]**  
 (a)  ${}^n P_2$  (b)  $2^n - 2$  (c)  $2^n - 1$  (d) None of these
6. The area bounded by the curve  $2x^2 + y^2 = 2$  is  
**[Conceptual Application]**  
 (a)  $\pi$  sq units (b)  $\sqrt{2}\pi$  sq units (c)  $\frac{\pi}{2}$  sq units (d)  $2\pi$  sq units
7. The area bounded by the lines  $|x| - 2 + |y| = 0$  is  
**[Conceptual Application]**  
 (a) 1 sq unit (b) 2 sq units (c) 4 sq units (d) 8 sq units
8. The value of  $\int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x} + \sqrt{21-x}} dx$  is  
**[NCERT Part-II, Page 274]**  
 (a)  $\frac{21}{2}$  (b) 0 (c)  $\frac{5}{2}$  (d) None of these
9. The value of  $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \tan^3 x} dx$  is  
**[NCERT Part-II, Page 274]**  
 (a) 0 (b) 1 (c)  $\frac{\pi}{4}$  (d)  $\frac{\pi}{2}$
10. The absolute maximum value of  $y = x^3 - 3x + 2$  in  $0 \leq x \leq 2$  is  
**[NCERT Part-I, Page 172]**  
 (a) 4 (b) 6 (c) 2 (d) 0
11. It is given that at  $x = 1$ , the function  $f(x) = x^4 - 62x^2 + ax + 9$  attains its maximum value, on the interval  $[0, 2]$ . The value of  $a$  is  
**[NCERT Part-I, Page 166]**  
 (a) 20 (b) -120 (c) 120 (d) 52
12. A function ' $f$ ' is said to be continuous at  $x = a$ , if  
**[NCERT Part-I, Page 105]**  
 (a)  $\lim_{x \rightarrow a} f(x)$  exists (b)  $\lim_{x \rightarrow a^-} f(x)$  does not exist  
 (c)  $f(a)$  exists (d) None of these
13. If  $y = a \sin^3 t$ ,  $x = a \cos^3 t$ , then  $\frac{dy}{dx}$  at  $t = \frac{3\pi}{4}$  is  
**[NCERT Part-I, Page 134-135]**  
 (a) -1 (b)  $-\frac{1}{\sqrt{3}}$  (c)  $-\sqrt{3}$  (d) 1
14. The general solution of the differential equation  $\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$  is  
**[NCERT Part-II, Page 322-323]**  
 (a)  $y = Ce^{-\frac{x^2}{2}}$  (b)  $y = Ce^{\frac{x^2}{2}}$  (c)  $y = (x + C)e^{\frac{x^2}{2}}$  (d)  $y = (C - x)e^{\frac{x^2}{2}}$
15. Degree of the differential equation  $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$  is  
**[NCERT Part-II, Page 302]**  
 (a) 2 (b) 1 (c) 0 (d) not defined
16. Integrating factor of the differential equation  $\frac{dy}{dx} = x + y$  is  
**[NCERT Part-II, Page 324]**  
 (a) -1 (b) 1 (c)  $e^{-x}$  (d)  $e^{-y}$
17. Given  $P(\bar{A}) = 0.4$ ,  $P(B) = 0.2$  and  $P(A/B) = 0.5$ , then  $P(A \cup \bar{B})$  is  
**[NCERT Part-II, Page 408]**  
 (a) 0.35 (b) 0.9 (c) 0.65 (d) 0.55
18. Mother, father and son line up at random for a family picture. If  $A$  is the event 'Son on one end' and  $B$  is the event 'Father in the middle', then  $P(A/B)$  is  
**[NCERT Part-II, Page 408]**  
 (a)  $\frac{1}{2}$  (b) 1 (c) 2 (d)  $\frac{1}{3}$

## ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

- (a) Both  $A$  and  $R$  are true and  $R$  is the correct explanation of  $A$ .
- (b) Both  $A$  and  $R$  are true but  $R$  is not the correct explanation of  $A$ .
- (c)  $A$  is true but  $R$  is false.
- (d)  $A$  is false but  $R$  is true.

19. **Assertion (A):** The area of parallelogram with diagonals  $\vec{a}$  and  $\vec{b}$  is  $\frac{1}{2}|\vec{a} \times \vec{b}|$  [Conceptual Application]

**Reason (R):** If  $\vec{a}$  and  $\vec{b}$  represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating  $|\vec{a} \times \vec{b}|$

20. **Assertion (A):**  $\sin^{-1}(0.76)$  is defined. [NCERT Part-I, Page 19]

**Reason (R):**  $\sin^{-1}(0.76)$  is defined because  $\sin^{-1}x$  is defined for all real numbers.

## SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the general solution of the differential equation  $\frac{dy}{dx} = \frac{x+1}{2-y}$ , ( $y \neq 2$ ). [NCERT Part-II, Page 306-307]

22.  $A$  and  $B$  throw a coin in turn till one of them gets a 'tail' and wins the game. If  $A$  starts the game find the probability of  $B$  winning the game. [Conceptual Application]

OR

There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. What is the probability that least one of the selected persons is non-violent. [Conceptual Application]

23. For any two vectors  $\vec{a}$  and  $\vec{b}$ , prove that  $(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$ . [NCERT Part-II, Page 356, 363]

24. Find the value of  $\tan^2(\sec^{-1}3) - \cot^2(\operatorname{cosec}^{-1}4)$ . [Conceptual Application]

25. For a square matrix  $A$ , show that  $A + A'$  is a symmetric matrix and  $A - A'$  is a skew symmetric matrix. [NCERT Part-I, Page 63]

OR

Solve the following matrix equation for  $x$ ,  $[x \ 1] \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$ . [NCERT Part-I, Page 94-95]

## SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If  $(\sin x)^y = x + y$ , find  $\frac{dy}{dx}$ . [NCERT Part-I, Page 122-123]

27. Find the intervals in which the function  $f$  given by  $f(x) = \sin^4 x + \cos^4 x$  in  $\left[0, \frac{\pi}{2}\right]$  is increasing or decreasing. [NCERT Part-I, Page 152-153]

28. Evaluate  $\int_0^a \frac{1}{x + \sqrt{a^2 - x^2}} dx$  [NCERT Part-II, Page 235-236, 274]

OR

Find the general solution of the differential equation,  $2x^2 \frac{dy}{dx} - 2xy + y^2 = 0$ .

[NCERT Part-II, Page 313-314]

29. Show that the relation  $R$  on the set  $A = \{x \in Z : 0 \leq x \leq 12\}$ , given by  $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$  is an equivalence relation. Find the equivalence classes related to 1 and 3 respectively.

[NCERT Part-I, Page 2, 4]

30. Find the area of the region in the first quadrant enclosed by the  $x$ -axis, the line  $x = \sqrt{3}y$  and the circle  $x^2 + y^2 = 4$ .

[Conceptual Application]

OR

Using the method of integration, find the area of the triangle  $ABC$ , coordinates of whose vertices are  $A(1, 2)$ ,  $B(2, 0)$  and  $C(4, 3)$ .

[Conceptual Application]

31. Differentiate  $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$  w.r.t.  $x$ .

[NCERT Part-I, Page 130]

OR

Show that the function  $f(x) = |x - 3|$ ,  $x \in R$  is continuous but not differentiable at  $x = 3$ .

[NCERT Part-I, Page 118-119]

## SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If  $\vec{\alpha} = 3\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$ , then express  $\vec{\beta}$  in the form  $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ , where  $\vec{\beta}_1$  is parallel to  $\vec{\alpha}$  and  $\vec{\beta}_2$  is perpendicular to  $\vec{\alpha}$ .

[Conceptual Application]

OR

Find the vector and Cartesian equations of the line passing through the point  $A(1, 2, -4)$  and

perpendicular to the lines  $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-1}{1} = \frac{y+2}{-3} = \frac{z-3}{5}$ . [Conceptual Application]

33. If  $A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , find  $\text{adj } A$  and verify that  $A(\text{adj } A) = (\text{adj } A)A = |A| I_3$ .

[NCERT Part-I, Page 88]

OR

Determine the product  $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$  and state how it can be used to solve the system

of equations  $x - y + z = 4$ ;  $x - 2y - 2z = 9$ ;  $2x + y + 3z = 1$ .

[NCERT Part-I, Page 94-95]

34. Find the general solution of the differential equation:  $(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$

[NCERT Part-II, Page 322-323]

35. Solve the following linear programming problem (LPP) graphically. [NCERT Part-II, Page 397-398]

Maximise:  $Z = 0.08x + 0.10y$

subject to the constraints,  $x \geq 2000$ ,  $y \geq 4000$ ,

$x + y \leq 12000$ .

## SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

### Case Study - 1

36. Based on a situation and complaint, higher authorities decided to talk to concerned persons as the situation relates to confidential matter of the organisation. [Conceptual Application]

From previous experience, organisation know, that person  $A$  speaks truth in 85% of the cases and person  $B$  tells a lie in 30% of the cases.

- (i) What is the probability of  $B$  speaking the truth?
- (ii) What is the probability of  $A$  speaking the truth?
- (iii) Organisation decided to have them face to face and ask questions from both, what is the probability that they contradict each other?

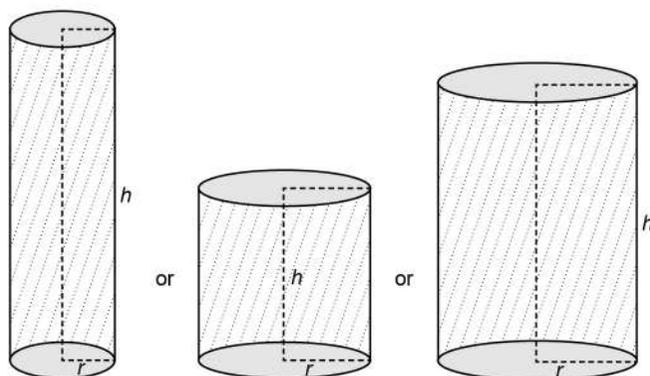
OR

- (iii) In what per cent of cases both are speaking truth?

### Case Study - 2

37. A company is launching a new product and decided to pack the product in the form of a closed right circular cylinder of volume  $432\pi$  cubic units and having minimum surface area as shown. They tried different options and tried to get the solution by answering the questions given below:

[NCERT Part-I, Page 160]



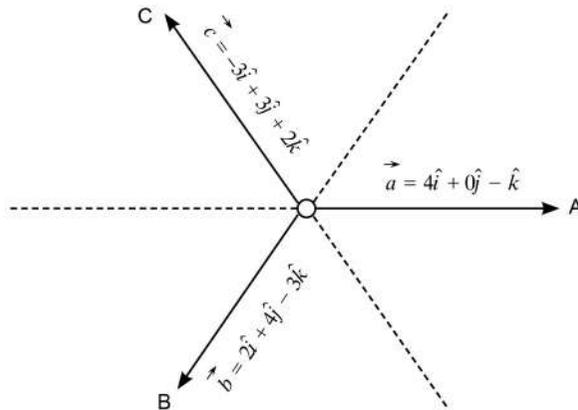
- (i) If  $r$  is radius of base of cylinder and  $h$  is the height of cylinder then establish the relation between  $r$  and  $h$ .
- (ii) Find total surface area in terms of  $r$  only.
- (iii) Find the radius  $r$  for minimum surface area.

OR

- (iii) Find the minimum surface area.

**Case Study - 3**

**38.** Teams  $A, B, C$  went for playing a tug of war game. Teams  $A, B, C$  have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below). [Conceptual Application]



Team  $A$  pulls along vector  $\vec{a} = 4\hat{i} - \hat{k}$

Team  $B$  pulls along vector  $\vec{b} = 2\hat{i} + 4\hat{j} - 3\hat{k}$

Team  $C$  pulls along vector  $\vec{c} = -3\hat{i} + 3\hat{j} + 2\hat{k}$

- (i) What are direction cosines of line along which team  $A$  pulls?
- (ii) What is the magnitude of the resultant force exerted by teams?

# SOLUTIONS

1. (b), as minor of  $a_{23} = M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13$

2. (a), as

$$\begin{aligned} \text{area of triangle} &= \frac{1}{2} \begin{vmatrix} -1 & 2 & 1 \\ 4 & 0 & 1 \\ 3 & 9 & 1 \end{vmatrix} \\ &= \frac{1}{2} [-1(0-9) - 2(4-3) + 1(36-0)] \\ &= \frac{1}{2} [9 - 2 + 36] = \frac{43}{2} \text{ sq units.} \end{aligned}$$

3. (b), as  $y + 2x = 7, -x = -2 \Rightarrow x = 2, y = 3$

4. (b), Let

$$\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \sin \theta = -\frac{\sqrt{3}}{2} = -\sin \frac{\pi}{3} = \sin\left(-\frac{\pi}{3}\right)$$

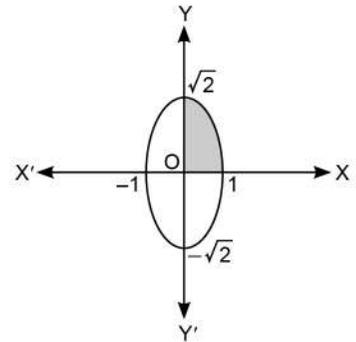
$$\Rightarrow \theta = -\frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

5. (b)  $2^n - {}^2C_1(2-1)^n + {}^2C_2(2-2)^n = 2^n - 2$

6. (b) The given equation is,

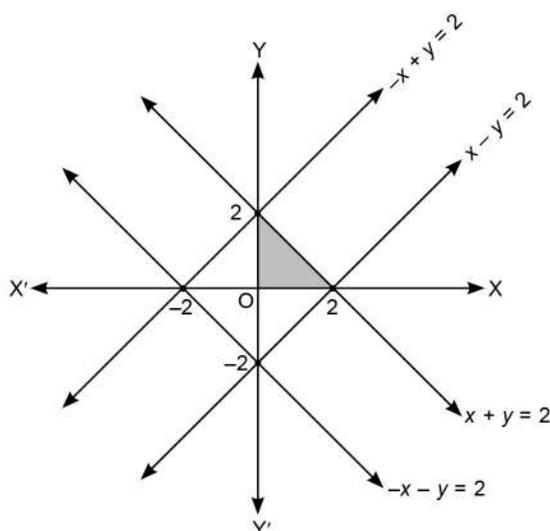
$$2x^2 + y^2 = 2 \Rightarrow \frac{x^2}{1^2} + \frac{y^2}{(\sqrt{2})^2} = 1$$

The above equations represents a vertical ellipse having major axis along y-axis and minor-axis along x-axis. Also, the length of semi-minor axis is 1 unit and length of semi-major axis is  $\sqrt{2}$  units.



$$\begin{aligned} \text{Required area} &= 4 \int_0^1 \sqrt{2-2x^2} dx \\ &= 4\sqrt{2} \int_0^1 \sqrt{1-x^2} dx \\ &= 4\sqrt{2} \left[ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_0^1 \\ &= 4\sqrt{2} \left[ 0 + \frac{1}{2} \times \frac{\pi}{2} - 0 \right] \\ &= \sqrt{2} \pi \text{ sq units} \end{aligned}$$

7. (d)



$$\begin{aligned} \text{Required area} &= 4 \int_0^2 (2-x) dx = 4 \left[ 2x - \frac{x^2}{2} \right]_0^2 \\ &= 4[4-2] = 8 \text{ sq units} \end{aligned}$$

8. (c) **Hint:** Use  $\int_a^b f(a+b-x) dx = \int_a^b f(x) dx$

$$\begin{aligned} 9. (c) \text{ Let } I &= \int_0^{\frac{\pi}{2}} \frac{1}{1+\tan^3 x} dx \\ \Rightarrow I &= \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx \quad \dots(i) \\ \Rightarrow I &= \int_0^{\frac{\pi}{2}} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\cos^3\left(\frac{\pi}{2}-x\right) + \sin^3\left(\frac{\pi}{2}-x\right)} dx \\ \Rightarrow I &= \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \quad \dots(ii) \end{aligned}$$

Adding (i) and (ii), we get

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} dx \\ \Rightarrow 2I &= \left[ x \right]_0^{\pi/2} \\ \Rightarrow I &= \frac{\pi}{4} \end{aligned}$$

10. (a), as  $y' = 3x^2 - 3$ , for a point of absolute maximum or minimum,  $y' = 0 \Rightarrow x = \pm 1$ .  
Now,  $x = -1$  will be rejected as  $-1 \notin [0, 2]$

$$y|_{x=0} = 2, y|_{x=1} = 1 - 3 + 2 = 0, y|_{x=2} = 8 - 6 + 2 = 4$$

11. (c),  $f'(x) = 4x^3 - 124x + a$

As function attains maximum at  $x = 1 \in [0, 2]$

$$\therefore f'(1) = 0 \Rightarrow 4 - 124 + a = 0$$

$$\Rightarrow a = 120$$

12. (d), continuous if  $\lim_{x \rightarrow a} f(x) = f(a)$

13. (d), as 
$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{3a \sin^2 t \cos t}{-3a \cos^2 t \sin t}$$
$$= -\tan t$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{t = \frac{3\pi}{4}} = 1$$

14. (c) 
$$\frac{dy}{dx} - x \cdot y = e^{x^2/2}$$
  
So, 
$$P(x) = -x; Q(x) = e^{x^2/2}$$
$$\text{I.F.} = e^{\int P dx} = e^{\int -x dx}$$
$$= e^{-x^2/2}$$

Solution is given by,

$$y \times \text{I.F.} = \int (Q \times \text{I.F.}) dx$$
$$\Rightarrow ye^{-x^2/2} = \int e^{x^2/2} \cdot e^{-x^2/2} dx$$
$$\Rightarrow \frac{y}{e^{x^2/2}} = x + C$$
$$\Rightarrow y = (x + C) \cdot e^{x^2/2}$$

15. (d), as equation cannot be written as polynomial of differentials.

16. (c), as 
$$\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$$
  
I.F. =  $e^{\int (-1) dx} = e^{-x}$

17. (b), as 
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = 0.5 \times 0.2 = 0.10$$
$$P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A \cap \bar{B})$$
$$= P(A) + 1 - P(B) - P(A) + P(A \cap B)$$
$$= 1 - P(B) + P(A \cap B)$$
$$= 1 - 0.2 + 0.1 = 0.9$$

18. (b) Consider the events.  $A$  : son on one end  $\Rightarrow A = \{MFS, FMS, SFM, SMF\}$

$B$  : Father in the middle  $\Rightarrow B = \{MFS, SFM\}$

$A \cap B = \{MFS, SFM\}$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)} = \frac{2}{2} = 1$$

19. (c), Assertion is true but the Reason is false.

20. (c), Assertion is true but the Reason is false.

21. 
$$\frac{dy}{dx} = \frac{x+1}{2-y}, (y \neq 2)$$

$$\Rightarrow \int (2-y) dy = \int (x+1) dx$$

$$\Rightarrow 2y - \frac{y^2}{2} = \frac{x^2}{2} + x + C \text{ is the required solution.}$$

22. Success  $S$  : getting a tail

$$P(S) = \frac{1}{2}, P(\bar{S}) = \frac{1}{2}$$

$B$  can win in 2nd, 4th, 5th, ... turn

$$\begin{aligned} \therefore P(B \text{ wins}) &= P(\bar{S})P(S) + [P(\bar{S})]^3P(S) + \dots \\ &= \frac{1}{2} \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} + \dots \\ &= \frac{\frac{1}{4}}{1 - \left(\frac{1}{2}\right)^2} = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3} \quad \left[ \text{As, } a + ar + ar^2 + \dots = \frac{a}{1-r} \right] \end{aligned}$$

**OR**

Group of 50 people, 20 believe in non-violence

$$\begin{aligned} P(\text{at least one selected is non-violent}) &= P(1 \text{ selected as non-violent}) + P(\text{both selected are non-violent}) \\ &= P(1) + P(2) = \frac{{}^{30}C_1 \times {}^{20}C_1}{{}^{50}C_2} + \frac{{}^{30}C_0 \times {}^{20}C_2}{{}^{50}C_2} = \frac{30 \times 20 \times 2}{50 \times 49} + \frac{20 \times 19}{50 \times 49} = \frac{120}{245} + \frac{38}{245} = \frac{158}{245} \end{aligned}$$

**Alternate:**  $P(\text{at least one selected in non-violent}) = 1 - P(\text{none selected is non-violent})$

$$\begin{aligned} &= 1 - \frac{{}^{30}C_2 \times {}^{20}C_0}{{}^{50}C_2} \\ &= 1 - \frac{30 \times 29}{50 \times 49} \\ &= 1 - \frac{87}{245} = \frac{158}{245} \end{aligned}$$

$$\begin{aligned} 23. \quad (\vec{a} \times \vec{b})^2 &= |\vec{a} \times \vec{b}|^2 = (|\vec{a}| |\vec{b}| \sin \theta)^2 \\ &= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2 \theta) \\ &= |\vec{a}|^2 |\vec{b}|^2 - |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta \\ &= \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2 \end{aligned}$$

$$\begin{aligned} 24. \quad \tan^2(\sec^{-1}3) - \cot^2(\operatorname{cosec}^{-1}4) &= [\sec^2(\sec^{-1}3) - 1] - [\operatorname{cosec}^2(\operatorname{cosec}^{-1}4) - 1] \\ &= (3)^2 - 1 - (4)^2 + 1 = 9 - 16 = -7 \end{aligned}$$

$$\begin{aligned} 25. \quad \text{Consider} \quad (A + A')' &= A' + (A')' \\ &= A' + A = (A + A') \end{aligned}$$

As  $(A + A')' = A' + A$ , Hence,  $A + A'$  is a symmetric matrix.

$$\text{Consider} \quad (A - A')' = A' - (A')' = A' - A$$

$$\Rightarrow (A - A')' = -(A - A')$$

Hence,  $A - A'$  is a skew symmetric matrix.

**OR**

$$\text{Consider} \quad [x \ 1] \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$$

$$\Rightarrow [x-2 \ 0] = [0 \ 0]$$

$$\Rightarrow x-2 = 0 \Rightarrow x = 2$$

26. Given  $(\sin x)^y = x + y$

On taking log on both sides, we get

$$y \cdot \log(\sin x) = \log(x + y)$$

Differentiating with respect to  $x$ , we get

$$\Rightarrow y \cdot \frac{1}{\sin x} \cdot \cos x + \log(\sin x) \frac{dy}{dx} = \frac{1}{x+y} \cdot \left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \log(\sin x) \frac{dy}{dx} - \frac{1}{x+y} \frac{dy}{dx} = \frac{1}{x+y} - y \cot x$$

$$\Rightarrow \left[ \log(\sin x) - \frac{1}{x+y} \right] \frac{dy}{dx} = \frac{1}{x+y} - y \cot x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y \cot x}{\log(\sin x) - \frac{1}{x+y}} = \frac{1 - (x+y)y \cot x}{(x+y)\log(\sin x) - 1}$$

27.  $f'(x) = 4 \sin^3 x \cos x - 4 \cos^3 x \sin x = -\sin 4x$

...(i)

$$f'(x) = 0 \Rightarrow \sin 4x = 0 \Rightarrow 4x = 0, \pi, 2\pi$$

$$\Rightarrow x = 0, \frac{\pi}{4}, \frac{\pi}{2}$$

$$(i) 0 < x < \frac{\pi}{4} \Rightarrow 0 < 4x < \pi$$

$\Rightarrow 4x \in \text{I, IInd quadrant}$

From (i),  $f'(x) < 0 \Rightarrow f$  is decreasing

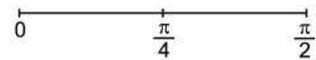
$$(ii) \frac{\pi}{4} < x < \frac{\pi}{2} \Rightarrow \pi < 4x < 2\pi$$

$\Rightarrow 4x \in \text{III, IVth quadrant.}$

From (i),  $f'(x) > 0 \Rightarrow f$  is increasing.

$\therefore f$  is decreasing for  $\left(0, \frac{\pi}{4}\right)$ ,

increasing for  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$



28. Consider  $\int_0^a \frac{1}{x + \sqrt{a^2 - x^2}} dx$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{a \sin \theta + \sqrt{a^2 - a^2 \sin^2 \theta}} \cdot a \cos \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{a \cos \theta}{a (\sin \theta + \cos \theta)} d\theta$$

Let  $x = a \sin \theta$

$\Rightarrow dx = a \cos \theta d\theta$

When  $x = 0$ ,  $\sin \theta = 0 \Rightarrow \theta = 0$

When  $x = a$ ,  $\sin \theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos \theta}{\sin \theta + \cos \theta} d\theta \quad \dots(i)$$

Using property  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ , we get

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin \theta}{\cos \theta + \sin \theta} d\theta \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \frac{\sin \theta + \cos \theta}{\sin \theta + \cos \theta} d\theta = \int_0^{\frac{\pi}{2}} 1 \cdot d\theta \\ &= [\theta]_0^{\frac{\pi}{2}} = \frac{\pi}{2} \end{aligned}$$

$$\Rightarrow I = \frac{\pi}{4}$$

OR

Consider equation  $2x^2 \frac{dy}{dx} - 2xy + y^2 = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy - y^2}{2x^2} \quad (\text{homogeneous as combined degree of each term is same}) \quad \dots(i)$$

Let  $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

From (i), we get

$$v + x \frac{dv}{dx} = \frac{2x^2v - x^2v^2}{2x^2} = \frac{2v - v^2}{2}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{2v - v^2}{2} - v = \frac{2v - v^2 - 2v}{2} = -\frac{v^2}{2}$$

$$\Rightarrow -\int v^{-2} dv = \int \frac{1}{2x} dx$$

$$\Rightarrow \frac{1}{v} = \frac{1}{2} \log |x| + C$$

$$\Rightarrow \frac{x}{y} = \frac{1}{2} \log |x| + C \text{ is the required solution.}$$

29. Given set  $A = \{x \in Z : 0 \leq x \leq 12\}$

$R = \{(a, b) : a, b \in A \text{ and } |a - b| \text{ is divisible by } 4\}$

**For reflexive:** For  $a \in A$ ,  $(a, a) \in R$

$\Rightarrow |a - a|$  is divisible by 4  $\Rightarrow 0$  is divisible by 4, true.

Hence, reflexive.

**For symmetric:** For  $a, b \in A$ ,  $(a, b) \in R$

$\Rightarrow |a - b|$  is divisible by 4

$\Rightarrow |-(b - a)|$  is divisible by 4

$\Rightarrow |b - a|$  is divisible by 4  $\Rightarrow (b, a) \in R$

As  $(a, b) \in R \Rightarrow (b, a) \in R$ , for  $a, b \in A$

Hence, symmetric.

**For transitive:** For  $a, b, c \in A$

Let  $(a, b) \in R$  and  $(b, c) \in R$

$\Rightarrow |a - b|$  is divisible by 4 and  $|b - c|$  is divisible by 4

$\Rightarrow (a - b)$  is divisible by 4 and  $(b - c)$  is divisible by 4

$\Rightarrow (a - b) + (b - c) = a - c$  is divisible by 4

$\Rightarrow |a - c|$  is divisible by 4.

$\Rightarrow (a, c) \in R$ . Hence, transitive.

...(i)

[using (i)]

As the relation  $R$  is reflexive, symmetric and transitive, hence, relation  $R$  is an equivalence relation.

Equivalence class related to 1, is  $[1]$ .

If  $a$  belongs to  $[1]$ , then

$$|a - 1| \text{ is a multiple of } 4 \Rightarrow a - 1 = 4\lambda \Rightarrow a = 1 + 4\lambda$$

$$\therefore [1] = \{1, 5, 9\}$$

Equivalence class related to 3 is  $[3]$ .

If  $a$  belongs to  $[3]$ , then

$$|a - 3| \text{ is a multiple of } 4 \Rightarrow a - 3 = 4\lambda \Rightarrow a = 3 + 4\lambda$$

$$\therefore [3] = \{3, 7, 11\}$$

30. Given circle is  $x^2 + y^2 = 4$  and line  $x = \sqrt{3}y$

Eliminating  $y$  from both equations, we get

$$x^2 + \frac{x^2}{3} = 4 \Rightarrow \frac{4x^2}{3} = 4$$

$$\Rightarrow x = \sqrt{3} \text{ (in 1st quadrant)}$$

On plotting the circle and line, we have to find the shaded area.

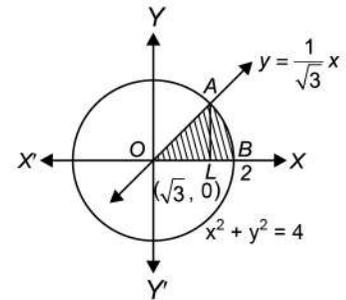
$$\therefore \text{Area} = \text{ar}(OAL) + \text{ar}(LAB)$$

$$= \int_0^{\sqrt{3}} \frac{1}{\sqrt{3}}x \, dx + \int_{\sqrt{3}}^2 \sqrt{4-x^2} \, dx$$

$$= \left[ \frac{1}{\sqrt{3}} \cdot \frac{x^2}{2} \right]_0^{\sqrt{3}} + \left[ \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^2$$

$$= \frac{1}{2\sqrt{3}} \times 3 - 0 + (0 + 2 \sin^{-1} 1) - \left( \frac{\sqrt{3}}{2} \sqrt{1} + 2 \sin^{-1} \frac{\sqrt{3}}{2} \right)$$

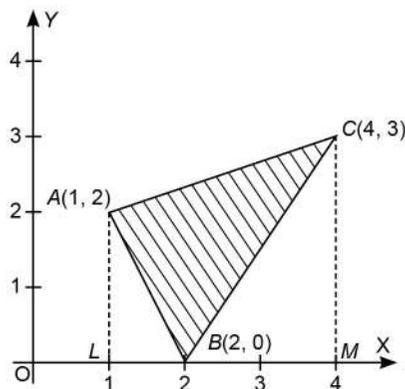
$$= \frac{\sqrt{3}}{2} + 2 \cdot \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \cdot \frac{\pi}{3} = \pi - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq units}$$



OR

The vertices of a triangle are  $A(1, 2)$ ,  $B(2, 0)$  and  $C(4, 3)$ . Plotting graph, we notice we have to find area of shaded region.

$$\text{ar}(ABC) = \text{ar}(LACM) - \text{ar}(\Delta LAB) - \text{ar}(\Delta BCM)$$



Equation of  $AB$ :  $A(1, 2)$ ,  $B(2, 0)$

$$y - 0 = \frac{2-0}{1-2}(x-2) \Rightarrow y = -2x + 4$$

Equation of BC: B(2, 0), C(4, 3)

$$y - 0 = \frac{3-0}{4-2}(x-2) \Rightarrow y = \frac{3}{2}x - 3$$

Equation of AC: A(1, 2), C(4, 3)

$$y - 3 = \frac{2-3}{1-4}(x-4)$$

$$\Rightarrow y - 3 = \frac{1}{3}x - \frac{4}{3} \Rightarrow y = \frac{1}{3}x + \frac{5}{3}$$

$$\begin{aligned} \therefore \text{ar}(ABC) &= \int_1^4 y_{AC} dx - \int_1^2 y_{AB} dx - \int_2^4 y_{BC} dx \\ &= \int_1^4 \left(\frac{1}{3}x + \frac{5}{3}\right) dx - \int_1^2 (-2x + 4) dx - \int_2^4 \left(\frac{3}{2}x - 3\right) dx \\ &= \left[\frac{x^2}{6} + \frac{5x}{3}\right]_1^4 - \left[-x^2 + 4x\right]_1^2 - \left[\frac{3x^2}{4} - 3x\right]_2^4 \\ &= \left(\frac{16}{6} + \frac{20}{3}\right) - \left(\frac{1}{6} + \frac{5}{3}\right) - (-4 + 8) + (-1 + 4) - (12 - 12) + (3 - 6) \\ &= \frac{15}{6} + \frac{15}{3} - 4 + 3 - 0 - 3 = \frac{45}{6} - 4 \\ &= \frac{45 - 24}{6} = \frac{21}{6} = \frac{7}{2} \text{ sq units} \end{aligned}$$

31. Let  $y = x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$

On differentiating both sides w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \frac{d}{dx} (x^{x \cos x}) + \frac{d}{dx} \left(\frac{x^2 + 1}{x^2 - 1}\right) \quad \dots(i)$$

Consider  $u = x^{x \cos x} \Rightarrow \log u = x \cos x \cdot \log x$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \cos x \cdot \frac{1}{x} + x \log x (-\sin x) + \log x \cdot 1 \cdot \cos x$$

$$\Rightarrow \frac{d}{dx} (x^{x \cos x}) = x^{x \cos x} [\cos x - x \log x \cdot \sin x + \log x \cos x] \quad \dots(ii)$$

$$\begin{aligned} \text{Consider, } \frac{d}{dx} \left(\frac{x^2 + 1}{x^2 - 1}\right) &= \frac{(x^2 - 1) \cdot \frac{d}{dx} (x^2 + 1) - (x^2 + 1) \cdot \frac{d}{dx} (x^2 - 1)}{(x^2 - 1)^2} \\ &= \frac{(x^2 - 1) 2x - (x^2 + 1) 2x}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2} \quad \dots(iii) \end{aligned}$$

Substituting from (ii) and (iii) in (i), we get

$$\frac{dy}{dx} = x^{x \cos x} [\cos x - x \log x \cdot \sin x + \log x \cdot \cos x] - \frac{4x}{(x^2 - 1)^2}$$

OR

$$\text{Given function } f(x) = |x - 3| = \begin{cases} x - 3, & x \geq 3 \\ -x + 3, & x < 3 \end{cases}$$

For continuity at  $x = 3$ ,

$$\text{LHL} = \lim_{x \rightarrow 3} f(3 - h) = \lim_{h \rightarrow 0} \{-(3 - h) + 3\} = \lim_{h \rightarrow 0} h = 0$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 3} f(3 + h) = \lim_{h \rightarrow 0} \{(3 + h) - 3\} \\ &= \lim_{h \rightarrow 0} h = 0 \end{aligned}$$

$$f(3) = 3 - 3 = 0$$

$$\text{As } \lim_{x \rightarrow 3} \text{LHL} = \lim_{x \rightarrow 3} \text{RHL} = f(3),$$

hence, function is continuous at  $x = 3$ .

For differentiability at  $x = 3$ ,

$$\begin{aligned} \text{LHD} &= \lim_{x \rightarrow 3} \frac{f(3 - h) - f(3)}{-h} = \lim_{h \rightarrow 0} \frac{(-3 + h + 3) - (0)}{-h} \\ &= \lim_{h \rightarrow 0} \frac{h}{-h} = \lim_{h \rightarrow 0} (-1) = -1 \end{aligned}$$

$$\begin{aligned} \text{RHD} &= \lim_{x \rightarrow 3} \frac{f(3 + h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{(3 + h - 3) - (0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h}{h} = \lim_{h \rightarrow 0} (1) = 1 \end{aligned}$$

As  $\lim_{x \rightarrow 3} \text{LHD} \neq \lim_{x \rightarrow 3} \text{RHD}$ . Hence, function is not derivable (differentiable) at  $x = 3$ .

$$32. \vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2, \text{ where } \vec{\beta}_1 \parallel \vec{\alpha} \text{ and } \vec{\beta}_2 \perp \vec{\alpha} \quad \dots (i)$$

$$\text{Let } \vec{\beta}_1 = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}) \quad \dots (ii)$$

$$\therefore 2\hat{i} + \hat{j} - 4\hat{k} = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{\beta}_2 \quad [\text{from } (i)]$$

$$\Rightarrow \vec{\beta}_2 = (2 - 3\lambda)\hat{i} + (1 - 4\lambda)\hat{j} + (-4 - 5\lambda)\hat{k} \quad \dots (iii)$$

$$\text{Also } \vec{\beta}_2 \perp \vec{\alpha} \Rightarrow \vec{\beta}_2 \cdot \vec{\alpha} = 0$$

$$\Rightarrow 3(2 - 3\lambda) + 4(1 - 4\lambda) + 5(-4 - 5\lambda) = 0$$

$$\Rightarrow 50\lambda = -10 \Rightarrow \lambda = -\frac{1}{5}$$

Substituting for  $\lambda$  in (ii) and (iii), we get

$$\vec{\beta}_1 = -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) = -\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j} - \hat{k}$$

$$\text{and } \vec{\beta}_2 = \left(2 + \frac{3}{5}\right)\hat{i} + \left(1 + \frac{4}{5}\right)\hat{j} + (-4 + 1)\hat{k}$$

$$\Rightarrow \vec{\beta}_2 = \frac{13}{5}\hat{i} + \frac{9}{5}\hat{j} - 3\hat{k}$$

$$\therefore 2\hat{i} + \hat{j} - 4\hat{k} = \left(-\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j} - \hat{k}\right) + \left(\frac{13}{5}\hat{i} + \frac{9}{5}\hat{j} - 3\hat{k}\right)$$

OR

Let line through the point  $A(1, 2, -4)$  be  $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$  ...*(i)*

If line *(i)* is perpendicular to the lines

$$\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x-1}{1} = \frac{y+2}{-3} = \frac{z-3}{5}$$

then

$$2a + 3b + 4c = 0 \quad \dots\text{(ii)}$$

and

$$a - 3b + 5c = 0 \quad \dots\text{(iii)}$$

Solving *(ii)* and *(iii)*, we get

$$\frac{a}{15+12} = \frac{-b}{10-4} = \frac{c}{-6-3} \Rightarrow \frac{a}{27} = \frac{b}{-6} = \frac{c}{-9}$$

DR's are 9, -2, -3

From *(i)*, line in Cartesian form is  $\frac{x-1}{9} = \frac{y-2}{-2} = \frac{z+4}{-3}$

Let  $\frac{x-1}{9} = \frac{y-2}{-2} = \frac{z+4}{-3} = \lambda$  (say)

General point on the line is  $(9\lambda + 1, -2\lambda + 2, -3\lambda - 4)$

Position vector of point on the line is

$$\vec{r} = (9\lambda + 1)\hat{i} + (-2\lambda + 2)\hat{j} + (-3\lambda - 4)\hat{k}$$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(9\hat{i} - 2\hat{j} - 3\hat{k})$  is equation of line in vector form.

33.  $|A| = \begin{vmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2 \alpha + \sin^2 \alpha = 1$  (expanding along  $C_3$ )

Let  $A_{ij}$  be the cofactors of elements in  $|A|$ . Then,

$$\begin{aligned} A_{11} &= \cos \alpha, & A_{12} &= -\sin \alpha, & A_{13} &= 0 \\ A_{21} &= \sin \alpha, & A_{22} &= \cos \alpha, & A_{23} &= 0 \\ A_{31} &= 0, & A_{32} &= 0, & A_{33} &= 1 \end{aligned}$$

$$\therefore \text{adj } A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}' = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Consider } A(\text{adj } A) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha + 0 & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha + 0 & 0 + 0 + 0 \\ \cos \alpha \sin \alpha - \sin \alpha \cos \alpha + 0 & \sin^2 \alpha + \cos^2 \alpha + 0 & 0 + 0 + 0 \\ 0 - 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 = |A| I_3$$

$$\begin{aligned}
(\text{adj } A)A &= \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha + 0 & -\sin \alpha \cos \alpha + \sin \alpha \cos \alpha + 0 & 0 + 0 + 0 \\ -\sin \alpha \cos \alpha + \sin \alpha \cos \alpha + 0 & \sin^2 \alpha + \cos^2 \alpha + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 = |A| I_3
\end{aligned}$$

$$\therefore A(\text{adj } A) = (\text{adj } A)A = |A| I_3.$$

**OR**

Consider  $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

and  $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I \quad \dots(i)$$

Consider equations

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

Corresponding matrix equation is

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\Rightarrow BX = C \text{ is matrix equation.}$$

Its solution is  $X = B^{-1}C$

... (ii)

Now we can use result (i) as

$$AB = 8I \Rightarrow \left(\frac{1}{8}A\right)B = I$$

$$\Rightarrow B^{-1} = \frac{1}{8}A$$

Now we can substitute  $B^{-1}$  in (ii) and proceed further by substituting for  $A$  and finding  $X$  and then  $x, y, z$ .

$$X = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow x = 3; y = -2; z = -1$$

34.  $(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$

$$\Rightarrow (x - e^{\tan^{-1}y}) \frac{dy}{dx} = -(1 + y^2)$$

$$\Rightarrow \frac{-(x - e^{\tan^{-1}y})}{1 + y^2} = \frac{dx}{dy} \Rightarrow \frac{dx}{dy} = -\frac{x}{1 + y^2} + \frac{e^{\tan^{-1}y}}{1 + y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{1}{1 + y^2} \cdot x = \frac{e^{\tan^{-1}y}}{1 + y^2}$$

Here,  $P(y) = \frac{1}{1 + y^2}, Q(y) = \frac{e^{\tan^{-1}y}}{1 + y^2}$

$$\text{I.F.} = e^{\int \frac{1}{1 + y^2} dy} = e^{\tan^{-1}y}$$

Solution is (I.F.) $x = \int$ (I.F.) $\cdot Q(y)dy$

$$e^{\tan^{-1}y}x = \int e^{\tan^{-1}y} \cdot \frac{e^{\tan^{-1}y}}{1 + y^2} dy = \int \frac{e^{2\tan^{-1}y}}{1 + y^2} dy$$

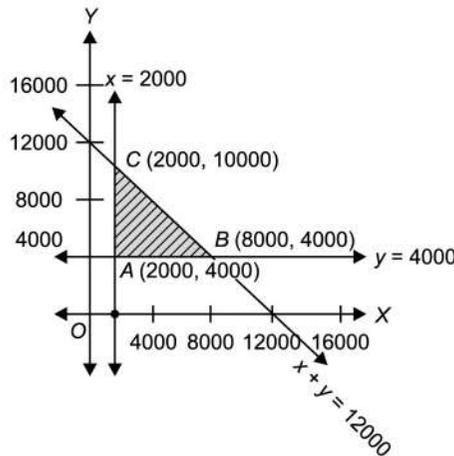
$$e^{\tan^{-1}y}x = \int e^{2t} dt = \frac{e^{2t}}{2} + C$$

$$e^{\tan^{-1}y}x = \frac{1}{2}e^{2\tan^{-1}y} + C$$

$$x = \frac{1}{2}e^{\tan^{-1}y} + C e^{-\tan^{-1}y}$$

$$\left| \begin{array}{l} \text{Let } \tan^{-1}y = t \\ \Rightarrow \frac{1}{1 + y^2} dy = dt \end{array} \right.$$

35. On plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for maximum  $Z$  are  $A(2000, 4000), B(8000, 4000)$  and  $C(2000, 10000)$ .



Points	$Z = 0.08x + 0.10y$	Values
$A(2000, 4000)$	$160 + 400$	560
$B(8000, 4000)$	$640 + 400$	1040
$C(2000, 10000)$	$160 + 1000$	1160 ← Maximum

$Z$  is maximum for  $C(2000, 10000)$ , i.e.  $x = 2000, y = 10000$ .

36. (i)  $P(B) = 1 - \frac{30}{100} = \frac{70}{100} = 0.7$

(ii)  $P(A) = 85\% = 0.85$

(iii)  $P(\text{contradiction}) = P(\overline{AB} \text{ or } \overline{A\overline{B}})$   
 $= \frac{85}{100} \times \frac{30}{100} + \frac{15}{100} \times \frac{70}{100}$   
 $= \frac{2550 + 1050}{10000} = \frac{3600}{10000} = \frac{9}{25}$

**OR**

(iii)  $P(AB) = \frac{85}{100} \times \frac{70}{100} = \frac{5950}{10000} = 59.5\%$

37. (i)  $\pi r^2 h = 432\pi$

$\Rightarrow r^2 h = 432$

(ii)  $S = 2\pi r h + 2\pi r^2$   
 $= 2\pi r \frac{432}{r^2} + 2\pi r^2 = \left( \frac{864\pi}{r} + 2\pi r^2 \right)$  sq units

(iii) For minimum surface area  $\frac{dS}{dr} = 0$  and  $\frac{d^2S}{dr^2} > 0$

$$\frac{dS}{dr} = \frac{-864\pi}{r^2} + 4\pi r$$

Now,  $\frac{dS}{dr} = 0 \Rightarrow r^3 = 216 \Rightarrow r = 6$  units

Now,  $\frac{d^2S}{dr^2} = \frac{1728\pi}{r^3} + 4\pi$

So,  $\frac{d^2S}{dr^2} > 0$  for  $r = 6$  units

OR

$$(iii) \quad S_{\min} = \frac{864\pi}{6} + 2\pi(6)^2 = 144\pi + 72\pi = 216\pi \text{ sq units}$$

38. (i) Direction ratios of line along which team A pulls 4, 0, -1

Direction cosines of the line along which team A pulls are

$$\frac{4}{\sqrt{4^2+0^2+(-1)^2}}, \frac{0}{\sqrt{4^2+0^2+(-1)^2}}, \frac{-1}{\sqrt{4^2+0^2+(-1)^2}} \text{ i.e. } \frac{4}{\sqrt{17}}, 0, \frac{-1}{\sqrt{17}}$$

$$(ii) \text{ Let } \vec{F} \text{ be the combined force, } \begin{aligned} \vec{F} &= \vec{F}_A + \vec{F}_B + \vec{F}_C \\ &= 4\hat{i} + 0\hat{j} - \hat{k} + 2\hat{i} + 4\hat{j} - 3\hat{k} - 3\hat{i} + 3\hat{j} + 2\hat{k} \\ &= 3\hat{i} + 7\hat{j} - 2\hat{k} \\ |\vec{F}| &= \sqrt{3^2 + 7^2 + (-2)^2} \\ &= \sqrt{9 + 49 + 4} = \sqrt{62} \text{ units} \end{aligned}$$