13 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(*iii*) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions,*** *carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$ and $x \in N$, then the value of x is **[NCERT Part-I, Page 77]**

(*a*) 1 (*b*) 5 (*c*) 2 (*d*) 4

2. If $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, then A^2 is **[NCERT Part-I, Page 50-51]**

(*a*) $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 4 & 0 \\ 4 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 4 \\ 0 & 4 \end{bmatrix}$ (*d*) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

3. If matrices A and B are inverse of each other then **[NCERT Part-I, Page 69]**

(*a*) $AB = BA$ (*b*) $AB = BA = I$ (*c*) $AB = BA = O$ (*d*) $AB = O, BA = I$

4. If $y = A \cos 2x + B \sin 2x$, then $\frac{d^2y}{dx^2}$ equals **[NCERT Part-I, Page 137]**

(*a*) y (*b*) $-y$ (*c*) $-4y$ (*d*) $4y$

5. A function $f(x) = \frac{x}{x-5}$, is not a continuous function for x equal to **[NCERT Part-I, Page 105]**

(*a*) 5 (*b*) –5 (*c*) $R - \{5\}$ (*d*) 0

6. If $y = 2^{\sqrt{x}}$, then $\frac{dy}{dx}$ is **[NCERT Part-I, Page 130]**

(*a*) $\frac{2^{\sqrt{x}}}{2\sqrt{x}}$ (*b*) $\sqrt{x}\ 2^{\sqrt{x}-1}$ (*c*) $\frac{2^{\sqrt{x}}}{2\sqrt{x}} \log_e 2$ (*d*) $\sqrt{x} \cdot 2^{\sqrt{x}-1} \log x^2$

7. If $f(x) = \frac{1}{4x^2 + 2x + 1}$, then its maximum value is **[Conceptual Application]**

(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$ (*c*) 1 (*d*) $\frac{5}{4}$

8. The area bounded by the line $y = 4x$, the y-axis and the line $y = 2$ is **[Conceptual Application]**

(*a*) 2 sq units (*b*) 4 sq units (*c*) $\frac{1}{4}$ sq units (*d*) $\frac{1}{2}$ sq units

9. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is **[Conceptual Application]**

(*a*) 4 sq units (*b*) 3 sq units (*c*) 2 sq units (*d*) 1 sq unit

10. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ equals to **[NCERT Part-II, Page 235-236]**

(*a*) $2e^{\sqrt{x}} + C$ (*b*) $\frac{1}{\sqrt{x}} + C$ (*c*) $2\sqrt{x} + C$ (*d*) $e^{\sqrt{x}} + C$

11. $\int \frac{\sin^6 x}{\cos^8 x} dx$ is equal to **[NCERT Part-II, Page 241]**

(*a*) $\frac{\sin^7 x}{\cos^9 x} + C$ (*b*) $\frac{1}{7} \tan^7 x + C$ (*c*) $\tan^6 x + C$ (*d*) $\sec^8 x + C$

12. A line makes equal angles with coordinate axes, direction cosines of line are **[NCERT Part-II, Page 377-378]**

(*a*) 1, 1, 1 (*b*) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

(*c*) $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ (*d*) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

13. If the direction cosines of a given line are $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ then, the value of k is **[NCERT Part-II, Page 377-378]**

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\pm\frac{1}{\sqrt{3}}$ (*c*) 1 (*d*) $\pm\sqrt{3}$

14. Given vector $\vec{a}$, then $-2\vec{a}$ is a vector whose **[NCERT Part-II, Page 346]**

(*a*) magnitude is twice that of $\vec{a}$ and direction is same as that of $\vec{a}$

(*b*) magnitude is twice that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(*c*) magnitude is same as that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(*d*) None of these

15. Position vectors of points A and B are $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$. Then $\overrightarrow{AB}$ equal to **[NCERT Part-II, Page 339]**

(*a*) $3\vec{a}$ (*b*) $-\vec{a} + 2\vec{b}$ (*c*) $\vec{a} - 2\vec{b}$ (*d*) None of these

16. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$ and $(\vec{a} \times \vec{b})$ is a unit vector. The angle between $\vec{a}$ and $\vec{b}$ is **[NCERT Part-II, Page 364]**

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{4}$ (*c*) $\frac{\pi}{6}$ (*d*) $\frac{\pi}{2}$

17. A pair of dice is thrown and it is known that the second die always exhibits an odd number. Then the probability that the sum obtained on two dice is 7, is **[NCERT Part-II, Page 406-408]**

(*a*) $\frac{1}{6}$ (*b*) $\frac{5}{6}$ (*c*) $\frac{1}{2}$ (*d*) none of these

18. A man is known to speak truth in 3 out of 4 times. He throws a dice and reports that it is a six. Then the probability that it is actually a six is **[NCERT Part-II, Page 406-408]**

(*a*) $\frac{4}{9}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{3}{8}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. **Assertion (*A*):** The smaller area enclosed by the ellipse $9x^2 + y^2 = 36$ and the line $3x + y = 6$ in first quadrant is $(3\pi - 6)$ square units. **[Conceptual Application]**

Reason (*R*): Area bounded by the curves $y = f(x), y = g(x)$ between $x = a$ and $x = b$ is $\int_a^b \{f(x) - g(x)\}dx$ $f(x) \geq g(x)$ for $x \in [a, b]$

20. **Assertion (*A*):** Both $\sin x$ and $\cos x$ are decreasing function in $\left(\frac{\pi}{2}, \pi\right)$. **[Conceptual Application]**

Reason (*R*): If a differentiable function decreases in an interval (a, b), then its derivative also decreases in (a, b).

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$. **[NCERT Part-II, Page 306-307]**

22. Let E and F be the events with $P(E) = \frac{3}{5}, P(F) = \frac{3}{10}$ and $P(E \cup F) = \frac{7}{10}$. Are E and F independent events? **[NCERT Part-II, Page 418]**

OR

Three balls are drawn one by one with replacement from a bag containing 5 white and 4 red balls. Find the probability of drawing 2 red and 1 white balls.

23. If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

[Conceptual Application]

24. Find the principal value of $\sec^{-1}(-2)$. **[NCERT Part-I, Page 22-23]**

25. Find A and B, if $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $2A + 3B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. **[Conceptual Application]**

OR

If $f(x) = x^2 - 4x + 1$, find $f(A)$, when $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $e^y = y^x$, show that $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$. **[NCERT Part-I, Page 130]**

27. Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is

(*i*) strictly increasing (*ii*) strictly decreasing **[NCERT Part-I, Page 152-153]**

28. Find $\int \frac{e^x}{(e^x - 1)^2 (e^x + 2)} dx$. **[Integrated Question]**

OR

Solve the differential equation, $\frac{dy}{dx} + 2y = xe^{4x}$ **[NCERT Part-II, Page 322-323]**

29. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}$, for all $x \in R$ is not one one. **[NCERT Part-I, Page 7]**

30. Using integration, find the area of the region bounded by the triangle whose vertices are (–1, 2), (1, 5) and (3, 4). **[Conceptual Application]**

OR

Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration. **[Conceptual Application]**

31. Discuss the differentiability of the function $f(x) = x|x|$ at $x = 0$. **[NCERT Part-I, Page 118-119]**

OR

If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$. **[NCERT Part-I, Page 134-135]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Show that the line through the points (1, – 1, 2) and (3, 4, – 2) is perpendicular to the line through the points(0, 3, 2) and (3, 5, 6). **[Conceptual Application]**

OR

Find the shortest distance between the lines **[NCERT Part-II, Page 387-388]**

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

33. Express the following matrix as the sum of a symmetric and a skew symmetric matrix and verify your result: **[NCERT Part-I, Page 64]**

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = O$. **[Conceptual Application]**

34. Find the general solution of the differential equation $\frac{dy}{dx} - y = \sin x$. **[NCERT Part-II, Page 322-323]**

35. Solve the following *LPP* graphically **[NCERT Part-II, Page 397-398]**

Minimise $Z = 5x + 10y$

subject to constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$ and $x, y \geq 0$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. During the interview a candidate is asked to pick three cards out of 8 black and 2 white cards which are kept upside down. Maximum marks are allotted to candidate who picks both the white cards and least marks are allotted when none of the white cards is picked. **[Conceptual Application]**

(*i*) What is the probability that he pickup 3 white cards?

(*ii*) What is the probability that he will pick exactly 1 white card ?

(*iii*) What is the probability of getting maximum marks?

OR

(*iii*) Find the probability that he gets the least marks in an interaction.

Case Study - 2

37. Organisations innovate different methods to keep their employees in good mental health. For that they give them freedom to work from any place. A company for their employees developed an area with a rectangular space enclosed by circle for refreshing and working as shown. **[Conceptual Application]**

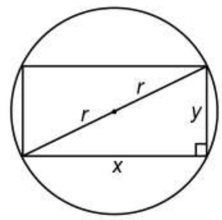
(*i*) What does rectangle enclosed by a circle means?

(*ii*) If a rectangle of sides x and y is inscribed in a circle of radius r, then establish relation between x, y and r.

(*iii*) Find the area of a rectangle in terms of x only.

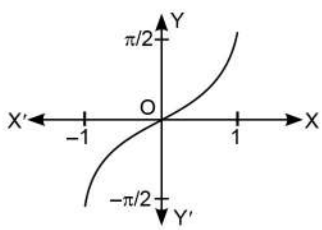
OR

(*iii*) Find the maximum area where employees can sit and work or enjoy.



Case Study - 3

38. Learning and evaluation go side by side and it helps the student to judge himself/herself where he/she stands regarding understanding of the subject. One day in Mathematics class, teacher asked the students to answer the questions with respect to the graph drawn on the board. **[NCERT Part-I, Page 20]**



(*i*) The figure represents graph of which inverse trigonometric function?

(*ii*) What is the value of the given function, where $x = \frac{-1}{2}$?

SOLUTIONS

1. (*c*) $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8 \Rightarrow 2x^2 + 6x - 6x = 8$

$$\Rightarrow 2x^2 = 8$$
$$\Rightarrow x^2 = 4$$
$$\Rightarrow x = 2 \text{ or } -2 \text{ (rejected)}$$
$$\therefore x = 2$$

2. (*d*), as $A^2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

3. (*b*), by definition.

4. (*c*) $y = A \cos 2x + B \sin 2x$

$$\Rightarrow \frac{dy}{dx} = -2A \sin 2x + 2B \cos 2x$$
$$\Rightarrow \frac{d^2y}{dx^2} = -4A \cos 2x - 4B \sin 2x$$
$$\Rightarrow \frac{d^2y}{dx^2} = -4(A \cos 2x + B \sin 2x)$$
$$\Rightarrow \frac{d^2y}{dx^2} = -4y$$

5. (*a*), as for $x = 5, f(5)$ is not defined.

6. (*c*)

7. (*b*), as if $f(x) = \frac{1}{4x^2 + 2x + 1}$ is maximum then

$4x^2 + 2x + 1$ should be minimum

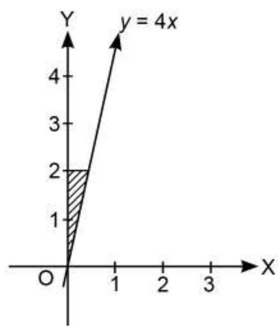
$$\Rightarrow 4\left[x^2 + \frac{1}{2}x + \frac{1}{4}\right] = 4\left[\left(x + \frac{1}{4}\right)^2 + \frac{1}{4} - \frac{1}{16}\right]$$
$$= 4\left[\left(x + \frac{1}{4}\right)^2 + \frac{3}{16}\right] = 4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4},$$

Minimum value of $(4x^2 + 2x + 1)$ is $\frac{3}{4}$.

So, maximum value of $f(x) = \frac{4}{3}$

8. (*d*), as

$$\text{area} = \int_0^2 x \, dy$$
$$= \int_0^2 \frac{y}{4} dy$$
$$= \left[\frac{1}{8}y^2\right]_0^2$$
$$= \frac{1}{8} \times (4 - 0)$$



$$= \frac{1}{2} \text{ sq units}$$

9. (*a*) **Hint:** Area $= \int_0^{\frac{\pi}{2}} \cos x\, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos x) dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x\, dx$

10. (*a*), $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2\int e^t dt = 2e^t + C = 2e^{\sqrt{x}} + C$ $\quad \Big|$ Let $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

11. (*b*), as $\int \tan^6 x \cdot \sec^2 x\, dx = \int t^6\, dt = \frac{t^7}{7} + C = \frac{\tan^7 x}{7} + C$

$\Big|$ Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

12. (*c*) as <cos α, cos α , cos α> will be dc's of the line.

Now, $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \quad \cos\alpha = \pm\frac{1}{\sqrt{3}}$

13. (*d*) as $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ are direction cosines of a line

$\therefore \quad \frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2} = 1 \Rightarrow \frac{3}{k^2} = 1$

$\Rightarrow \quad k^2 = 3$

$\Rightarrow \quad k = \pm\sqrt{3}$

14. (*b*), result related to $\vec{a}$ and $k\vec{a}$, k is scalar.

15. (*c*), as $\overrightarrow{AB}$ = Position vector of B – Position vector of A

16. (*a*), as $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{1 \times 3}{\sqrt{3} \times 2} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

17. (*a*) Consider the following events:

A : Sum obtained on both dice is 7

B : Second die exhibits an odd number

$\therefore A = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$B = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3),$
$(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)\}$

Now, $A \cap B = \{(6, 1), (4, 3), (2, 5)\}$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)} = \frac{3}{18} = \frac{1}{6}$$

18. (*d*), as probability $= \dfrac{\frac{3}{4} \cdot \frac{1}{6}}{\frac{3}{4} \cdot \frac{1}{6} + \frac{1}{4} \cdot \frac{5}{6}} = \frac{3}{8}$.

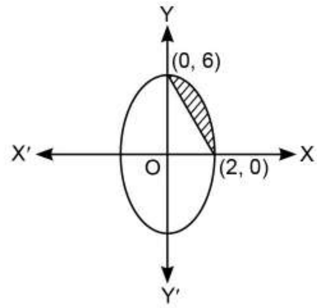
19. (*a*), equation of ellipse is $9x^2 + y^2 = 36$ ...(*i*)

$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{36} = 1$

equation of line is $\qquad 3x + y = 6 \qquad$ ...(*ii*)

$$\Rightarrow \qquad \frac{x}{2} + \frac{y}{6} = 1$$

So area is represented by the shaded region



$$\text{area} = \int_0^2 y_{\text{ellipse}} dx - \int_0^2 y_{\text{line}} dx$$

$$= \int_0^2 \sqrt{36 - 9x^2}\, dx - \int_0^2 (6 - 3x) dx$$

$$= 3\int_0^2 \sqrt{(4 - x^2)}\, dx - \int_0^2 (6 - 3x) dx$$

$$= 3\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[6x - \frac{3x^2}{2}\right]_0^2$$

$$= 3[0 + 2\sin^{-1} 1 - 0 - 0] - [(12 - 6) - 0]$$

$$= 3 \times 2 \times \frac{\pi}{2} - 6 = (3\pi - 6) \text{ sq units}$$

A is true, R is also true and the correct explanation of A

20. (*c*), Assertion is true but the Reason is false.

21. Consider the equation, $\qquad \frac{dy}{dx} = 1 + x + y + xy$

$$\Rightarrow \qquad \frac{dy}{dx} = 1(1 + x) + y(1 + x) = (1 + x)(1 + y)$$

$$\Rightarrow \qquad \frac{dy}{1 + y} = (1 + x)dx$$

Integrating both sides, we get

$$\int \frac{dy}{1 + y} = \int (1 + x)dx$$

$$\Rightarrow \qquad \log|1 + y| = x + \frac{x^2}{2} + C \qquad \text{...}(i)$$

Given $\qquad y = 0$ when $x = 1$

$$\Rightarrow \qquad \log 1 = 1 + \frac{1}{2} + C \Rightarrow C = \frac{-3}{2}$$

Substituting in (*i*)

$$\log|1 + y| = x + \frac{x^2}{2} - \frac{3}{2} \text{ is required solution.}$$

22. $\qquad P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$\Rightarrow \qquad \frac{7}{10} = \frac{3}{5} + \frac{3}{10} - P(E \cap F)$$

$$\Rightarrow \qquad P(E \cap F) = \frac{3}{5} + \frac{3}{10} - \frac{7}{10} = \frac{1}{5}$$

Also $\qquad P(E) \cdot P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50}$

As $P(E) \cdot P(F) \neq P(E \cap F)$

Hence, events E and F are not independent.

OR

Bag contains: 5 white, 4 red balls

$$P(W) = \frac{5}{9}, \quad P(R) = \frac{4}{9},$$

Balls are drawn with replacement

$$\text{P(2 red, 1 white)} = \text{P(RRW)} + \text{P(RWR)} + \text{P(WRR)} = \frac{4}{9}\times\frac{4}{9}\times\frac{5}{9}+\frac{4}{9}\times\frac{5}{9}\times\frac{4}{9}+\frac{5}{9}\times\frac{4}{9}\times\frac{4}{9}$$

$$= 3\times\frac{4}{9}\times\frac{4}{9}\times\frac{5}{9}=\frac{240}{729} = \frac{80}{243}$$

23. Let $\vec{a}$ and $\vec{b}$ be two given vectors such that $|\vec{a}| = 1, |\vec{b}| = 1, |\vec{a} + \vec{b}| = 1$.

Consider $\quad |\vec{a} + \vec{b}| = 1 \Rightarrow |\vec{a} + \vec{b}|^2 = 1$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 1 \Rightarrow 1 + 1 + 2\vec{a}\cdot\vec{b} = 1 \Rightarrow \vec{a}\cdot\vec{b} = -\frac{1}{2}$ ...(*i*)

Consider $\quad |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2\times\left(-\frac{1}{2}\right)$ [From (*i*)]

$= 1 + 1 + 1 = 3$

$\Rightarrow \quad |\vec{a} - \vec{b}| = \sqrt{3}$

24. Let $\theta = \sec^{-1}(-2)$

$\Rightarrow \sec\theta = -2 = -\sec\frac{\pi}{3} = \sec(\pi - \frac{\pi}{3}) = \sec\frac{2\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{3} \in [0, \pi] - \{\frac{\pi}{2}\}$.

25. Given $\quad A + B = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ ...(*i*)

$2A + 3B = \begin{bmatrix}5 & 6\\7 & 8\end{bmatrix}$ ...(*ii*)

Multiplying (*i*) by 2 and subtracting from (*ii*), we get

$$2A + 3B - 2A - 2B = \begin{bmatrix}5 & 6\\7 & 8\end{bmatrix} - 2\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

$\Rightarrow \quad B = \begin{bmatrix}5 & 6\\7 & 8\end{bmatrix} - \begin{bmatrix}2 & 4\\6 & 8\end{bmatrix} = \begin{bmatrix}5-2 & 6-4\\7-6 & 8-8\end{bmatrix} \Rightarrow B = \begin{bmatrix}3 & 2\\1 & 0\end{bmatrix}$

Substituting in (*i*), we get

$$A + \begin{bmatrix}3 & 2\\1 & 0\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

$\Rightarrow \quad A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} - \begin{bmatrix}3 & 2\\1 & 0\end{bmatrix} = \begin{bmatrix}1-3 & 2-2\\3-1 & 4-0\end{bmatrix} \Rightarrow A = \begin{bmatrix}-2 & 0\\2 & 4\end{bmatrix}$

Hence, $A = \begin{bmatrix}-2 & 0\\2 & 4\end{bmatrix}, B = \begin{bmatrix}3 & 2\\1 & 0\end{bmatrix}$

OR

$f(A) = A^2 - 4A + I$, now $A^2 = \begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix} = \begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}$

Now, $4A = 4\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix} = \begin{bmatrix}8 & 12\\4 & 8\end{bmatrix}, I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

We get $f(A) = \begin{bmatrix}7 & 12\\4 & 7\end{bmatrix} - \begin{bmatrix}8 & 12\\4 & 8\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

26. Given $e^y = y^x$

Taking log of both sides, we get

$$y \log e = x \log y \Rightarrow y = x \log y \quad \text{...}(i) \ [\log e = 1]$$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1$$

$$\left(1 - \frac{x}{y}\right)\frac{dy}{dx} = \log y \Rightarrow \left(\frac{y-x}{y}\right)\frac{dy}{dx} = \log y \Rightarrow \frac{dy}{dx} = \frac{y \log y}{y - x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x \log y \cdot \log y}{x \log y - x} = \frac{x(\log y)^2}{x(\log y - 1)} \quad [\text{using } (i)]$$

$$\therefore \quad \frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$$

27. Consider function

$$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$$

$$f'(x) = 6x^3 - 12x^2 - 90x$$

$$= 6x(x^2 - 2x - 15)$$

$$= 6x(x-5)(x+3) \quad \text{...}(i)$$

For critical/turning/stationary points,

$$f'(x) = 0$$

$$\Rightarrow \quad 6x(x-5)(x+3) = 0$$

$$\Rightarrow \quad x = 0, 5, -3$$

	$x < -3$	$-3 < x < 0$	$0 < x < 5$	$x > 5$
6	+	+	+	+
x	–	–	+	+
$x - 5$	–	–	–	+
$x + 3$	–	+	+	+
$f'(x)$	–	+	–	+
	↓	↑	↓	↑

(*i*) Function strictly increases for $(-3, 0) \cup (5, \infty)$

(*ii*) Function strictly decreases for $(-\infty, -3) \cup (0, 5)$

28. Consider $\displaystyle\int \frac{e^x}{(e^x-1)^2(e^x+2)}dx = \int \frac{1}{(t-1)^2(t+2)}dt$ $\quad$ | Let $e^x = t$ $\Rightarrow e^x\, dx = dt$

Let $$\frac{1}{(t-1)^2(t+2)} = \frac{A}{t-1} + \frac{B}{(t-1)^2} + \frac{C}{t+2} \quad \text{...}(i)$$

$$\Rightarrow \quad 1 = A(t-1)(t+2) + B(t+2) + C(t-1)^2$$
$$= A(t^2 + t - 2) + B(t+2) + C(t^2 - 2t + 1)$$
$$= t^2(A + C) + t(A + B - 2C) + (-2A + 2B + C)$$

Comparing the coefficients, we get

$$A + C = 0 \Rightarrow A = -C$$
$$A + B - 2C = 0 \Rightarrow -3A = B$$
$$-2A + 2B + C = 1 \Rightarrow -2A - 6A - A = 1 \Rightarrow A = -\frac{1}{9}$$
$$\Rightarrow \quad A = -\frac{1}{9}, B = \frac{1}{3}, C = \frac{1}{9}$$

Substituting in (*i*) and integrating, we get

$$\int \frac{1}{(t-1)^2(t+2)} dt = -\frac{1}{9}\int \frac{1}{t-1} dt + \frac{1}{3}\int \frac{1}{(t-1)^2} dt + \frac{1}{9}\int \frac{1}{t+2} dt$$
$$= -\frac{1}{9}\log|t-1| - \frac{1}{3(t-1)} + \frac{1}{9}\log|t+2| + C$$
$$\int \frac{e^x}{(e^x-1)^2(e^x+2)} dx = -\frac{1}{9}\log|e^x - 1| - \frac{1}{3(e^x-1)} + \frac{1}{9}\log|e^x + 2| + C$$

OR

Consider the equation, $\frac{dy}{dx} + 2y = xe^{4x}$

Here $P(x) = 2, Q(x) = xe^{4x}$

Integrating factor $(I.F.) = e^{\int 2\,dx} = e^{2x}$

Solution is
$$(I.F.)\,y = \int \{(I.F.)\,Q(x)\}\,dx$$
$$e^{2x}.y = \int e^{2x}\cdot xe^{4x}\,dx = \int \underset{①}{x} \cdot \underset{②}{e^{6x}}\,dx = x\cdot\frac{e^{6x}}{6} - \int 1\cdot\frac{e^{6x}}{6}\,dx$$
$$= \frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x} + C$$

$\Rightarrow \quad y = \frac{1}{6}xe^{4x} - \frac{1}{36}e^{4x} + Ce^{-2x}$ is the required solution.

29. For one-one: For $x_1, x_2 \in R$

$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1^2 + 1} = \frac{x_2}{x_2^2 + 1}$

$\Rightarrow x_1x_2^2 + x_1 = x_1^2x_2 + x_2 \Rightarrow x_1x_2(x_2 - x_1) + (x_1 - x_2) = 0$

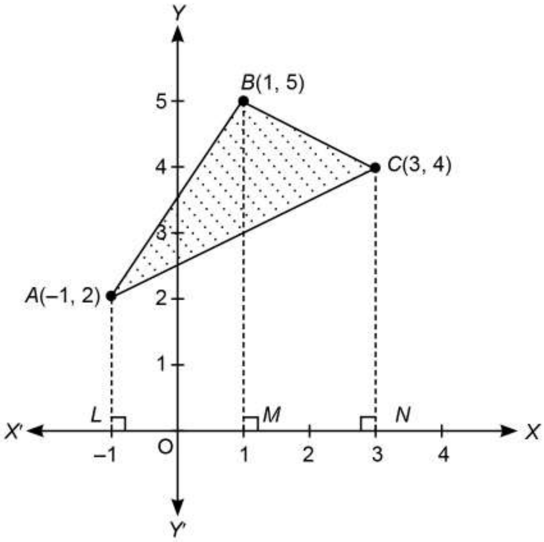
$\Rightarrow (x_2 - x_1)(x_1x_2 - 1) = 0 \Rightarrow x_2 - x_1 = 0$ or $x_1x_2 = 1$

$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$

Let $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we notice $f(x_1) = f(x_2)$ but $2 \neq \frac{1}{2}$. Hence, not one-one

30. Let vertices of the triangle are $A(-1, 2)$, $B(1, 5)$ and $C(3, 4)$.

Area (ABC) = area$(LABM)$ + area$(MBCN)$ – area$(LACN)$ ...(*i*)



Equation of AB:

$$y - 5 = \left(\frac{2-5}{-1-1}\right)(x-1) \Rightarrow y - 5 = \frac{3}{2}(x-1)$$

$$\Rightarrow \quad y = \frac{3}{2}x + \frac{7}{2} \qquad ...(ii)$$

Equation of BC:

$$y - 4 = \left(\frac{5-4}{1-3}\right)(x-3) \Rightarrow y - 4 = -\frac{1}{2}(x-3)$$

$$\Rightarrow \quad y = -\frac{1}{2}x + \frac{11}{2} \qquad ...(iii)$$

Equation of AC :

$$y - 4 = \left(\frac{2-4}{-1-3}\right)(x-3) \Rightarrow y - 4 = \frac{1}{2}(x-3)$$

$$\Rightarrow \quad y = \frac{1}{2}x + \frac{5}{2} \qquad ...(iv)$$

From (*i*), (*ii*), (*iii*) and (*iv*), we get

$$\text{Area}(ABC) = \int_{-1}^{1}\left(\frac{3}{2}x + \frac{7}{2}\right)dx + \int_{1}^{3}\left(-\frac{1}{2}x + \frac{11}{2}\right)dx - \int_{-1}^{3}\left(\frac{1}{2}x + \frac{5}{2}\right)dx$$

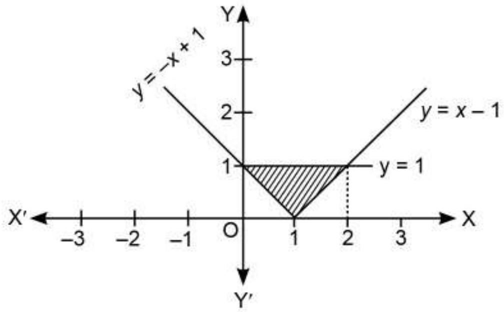
$$= \frac{1}{2}\left[\frac{3x^2}{2} + 7x\right]_{-1}^{1} + \frac{1}{2}\left[-\frac{x^2}{2} + 11x\right]_{1}^{3} - \frac{1}{2}\left[\frac{x^2}{2} + 5x\right]_{-1}^{3}$$

$$= \frac{1}{2}\left[\left(\frac{3}{2}+7\right) - \left(\frac{3}{2}-7\right) + \left(-\frac{9}{2}+33\right) - \left(-\frac{1}{2}+11\right) - \left(\frac{9}{2}+15\right) + \left(\frac{1}{2}-5\right)\right]$$

$$= \frac{1}{2}\left[\frac{17}{2} + \frac{11}{2} + \frac{57}{2} - \frac{21}{2} - \frac{39}{2} - \frac{9}{2}\right] = 4 \text{ sq units.}$$

OR

Curve is $y = |x-1|$ and line $y = 1$. Plotting the curves, we get



We have to find shaded area.

$$\text{Area} = \int_0^2 1\,.\,dx - \left[\int_0^1 (1-x)dx + \int_1^2 (x-1)\,dx\right]$$

$$= [x]_0^2 - \left[x - \frac{x^2}{2}\right]_0^1 - \left[\frac{x^2}{2} - x\right]_1^2$$

$$= 2 - \left[\left(1-\frac{1}{2}\right) - 0 + (2-2) - \left(\frac{1}{2}-1\right)\right]$$

$$= 2 - \left[\frac{1}{2}+\frac{1}{2}\right] = 1 \text{ sq unit}$$

31. Given, $f(x) = x|x| = \begin{cases} x^2 & , x \ge 0 \\ -x^2 & , x < 0 \end{cases}$

$$\underset{x=0}{\text{LHD}} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{-(-h)^2-0}{-h} = \lim_{h\to 0} \frac{-h^2}{-h} = \lim_{h\to 0}(h) = 0$$

$$\underset{x=0}{\text{RHD}} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2-0}{h} = \lim_{h\to 0} h = 0$$

As $\underset{x=0}{\text{LHD}} = \underset{x=0}{\text{RHD}}$

$\therefore$ f is differentiable at $x = 0$.

OR

Consider, $x = a(\cos\theta + \log\tan\frac{\theta}{2});$

$y = a\sin\theta$

$$\frac{dy}{d\theta} = a\cos\theta \qquad ...(i)$$

$$\Rightarrow \quad \frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\frac{\theta}{2}}\,.\,\sec^2\frac{\theta}{2}\,.\,\frac{1}{2}\right);$$

$$= a\left(-\sin\theta + \frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= a\left(-\sin\theta + \frac{1}{\sin\theta}\right) = a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$$

$$\Rightarrow \qquad \frac{dx}{d\theta} = \frac{a\cos^2\theta}{\sin\theta} \qquad ...(ii)$$

$$\therefore \qquad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{a\cos\theta \times \sin\theta}{a\cos^2\theta} \qquad \text{[from } (i) \text{ and } (ii)]$$

$$= \tan\theta$$

$$\therefore \qquad \left.\frac{dy}{dx}\right]_{\theta = \frac{\pi}{4}} = \tan\frac{\pi}{4} = 1$$

32. Direction ratios of line through the points (1, – 1, 2) and (3, 4, – 2) are 3 – 1, 4 + 1, – 2 – 2, i.e. 2, 5, – 4 ...(*i*)

Direction ratios of line through the points (0, 3, 2) and (3, 5, 6) are 3 – 0, 5 – 3, 6 – 2, i.e. 3, 2, 4 ...(*ii*)

From (*i*) and (*ii*), as $3 \times 2 + 5 \times 2 - 4 \times 4 = 0$. Hence, lines are perpendicular.

OR

Given lines are

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

and

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Here $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k},\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

As, lines are parallel, then shortest distance = $\left|\dfrac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{|\vec{b}|}\right|$

Now, $\vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

So, $$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\left|(\vec{a_2} - \vec{a_1}) \times \vec{b}\right| = \sqrt{81 + 196 + 16} = \sqrt{293}$$

$$|\vec{b}| = \sqrt{4 + 9 + 36} = 7$$

$$\therefore \qquad \text{the shortest distance} = \left|\frac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{|\vec{b}|}\right| = \frac{\sqrt{293}}{7} \text{ units}$$

33. $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A');$ ...(*i*)

where $\frac{1}{2}(A + A')$ is symmetric and $\frac{1}{2}(A - A')$ is skew symmetric.

Let $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ then $A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Consider $$\frac{1}{2}(A + A') = \frac{1}{2}\left\{\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & -2+3 & -4-1 \\ 3-2 & -2-2 & -5+1 \\ -1-4 & 1-5 & 2+2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} \qquad ...(ii)$$

Consider $\frac{1}{2}(A - A') = \frac{1}{2}\left\{\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right\}$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & -2-3 & -4+1 \\ 3+2 & -2+2 & -5-1 \\ -1+4 & 1+5 & 2-2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} \qquad ...(iii)$$

Adding (*ii*) and (*iii*), we get

$$= \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A \qquad \{\text{from } (i)\}$$

OR

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= 1\,(1-4) - 2\,(2-4) + 2\,(4-2)$$

$$= -3 + 4 + 4 = 5 \neq 0.$$

Let A_{ij} be the cofactor of each element in $|A|$.

$$A_{11} = +(1-4) = -3$$
$$A_{12} = -(2-4) = 2$$
$$A_{13} = +(4-2) = 2$$
$$A_{21} = -(2-4) = 2$$
$$A_{22} = +(1-4) = -3$$
$$A_{23} = -(2-4) = 2$$
$$A_{31} = +(4-2) = 2$$
$$A_{32} = -(2-4) = 2$$

$$A_{33} = +(1-4) = -3$$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj}\, A = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \quad \text{...}(i)$$

Consider $A^2 - 4A - 5I = O$

Multiplying both sides by A^{-1},

$$A^{-1}(AA) - 4A^{-1}A - 5A^{-1}I = A^{-1}O$$

$$\Rightarrow \quad (A^{-1}A)A - 4I - 5A^{-1} = O \Rightarrow IA - 4I - 5A^{-1} = O$$

$$\Rightarrow \quad IA - 4I = 5A^{-1} \Rightarrow A - 4I = 5A^{-1}$$

$$\text{LHS} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-0 & 2-0 \\ 2-0 & 1-4 & 2-0 \\ 2-0 & 2-0 & 1-4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = 5 \times \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$= 5A^{-1} = \text{RHS}$$

Hence $A^2 - 4A - 5I = O$

34. Consider equation $\frac{dy}{dx} - y = \sin x$

Here $P(x) = -1$, $Q(x) = \sin x$

Integrating factor (I.F.) $= e^{-\int dx} = e^{-x}$

$\therefore$ Solution is (I.F.)$y = \int \{(\text{I.F.})\, Q(x)\} dx$

$$e^{-x}.y = \int e^{-x} \sin x \, dx \quad \text{...}(i)$$

Consider,

$$I = \int \underset{①}{e^{-x}} \underset{②}{\sin x} \, dx$$

$$= e^{-x}.(-\cos x) - \int \{-e^{-x} \cdot (-\cos x)\} dx$$

$$\Rightarrow \quad I = -e^{-x} \cos x - \int \underset{①}{e^{-x}} \underset{②}{\cos x} \, dx$$

$$= -e^{-x} \cos x - \left[e^{-x} \sin x - \int (-e^{-x}).\sin x \, dx\right]$$

$$I = -e^{-x} \cos x - e^{-x} \sin x - I$$

$$\Rightarrow \quad 2I = -e^{-x}(\cos x + \sin x)$$

$$\Rightarrow \quad I = \frac{-e^{-x}}{2}(\cos x + \sin x)$$

Substituting in (*i*), we get

$$e^{-x}.y = -\frac{e^{-x}}{2}(\cos x + \sin x) + C$$

$\Rightarrow$ $$y = -\frac{1}{2}(\cos x + \sin x) + Ce^{x} \text{ is required solution.}$$

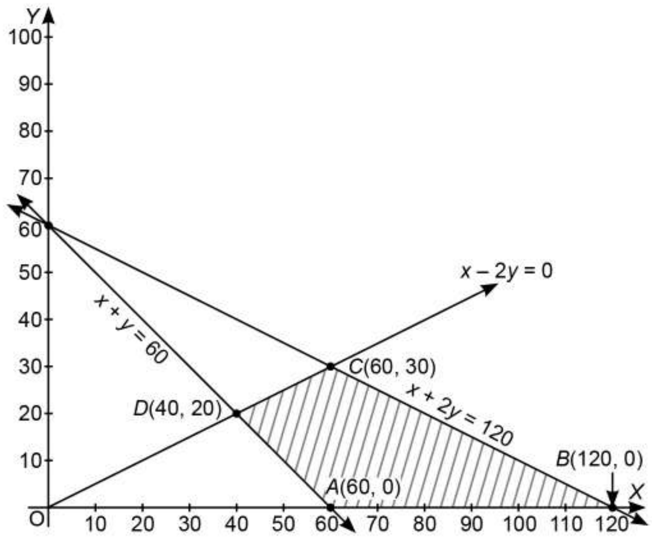
35. Minimise

$$Z = 5x + 10y$$

subject to the constraints

$$x \geq 0, y \geq 0$$
$$x - 2y \geq 0$$
$$x + y \geq 60$$
$$x + 2y \leq 120$$

Plotting the graph of inequations, we notice shaded portion is feasible solution.



Possible points for minimum Z are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$

Points	$Z = 5x + 10y$	Values	
$A(60, 0)$	300 + 0	300	← Minimum
$B(120, 0)$	600 + 0	600	
$C(60, 30)$	300 + 300	600	
$D(40, 20)$	200 + 200	400	

Z in minimum for $A(60, 0)$. Hence, for $x = 60$ and $y = 0$, Z is minimum.

36. (*i*) P(3 white cards) = 0, as it is impressible event.

(*ii*) $P(\text{1 white card}) = \dfrac{{}^{8}C_2 \times {}^{2}C_1}{{}^{10}C_3} = \dfrac{\frac{8 \times 7}{2} \times 2}{\frac{10 \times 9 \times 8}{6}} = \dfrac{8 \times 7 \times 2 \times 6}{2 \times 10 \times 9 \times 8} = \dfrac{7}{15}$

(*iii*) $P(\text{maximum marks}) = \dfrac{{}^{8}C_1 \times {}^{2}C_2}{{}^{10}C_3} = \dfrac{8 \times 1 \times 6}{10 \times 9 \times 8} = \dfrac{1}{15}$

OR

(*iii*) $P(\text{least marks}) = \frac{{}^8C_3 \times {}^2C_0}{{}^{10}C_3} = \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$

37. (*i*) Rectangle is inside the circle and its vertices lie on the circle

(*ii*) $x^2 + y^2 = (2r)^2 \Rightarrow x^2 + y^2 = 4r^2$

(*iii*) $A = xy = x\sqrt{4r^2 - x^2}$ [calculating y from (*ii*)]

OR

(*iii*) $\frac{dA}{dx} = \frac{x(-2x)}{2\sqrt{4r^2 - x^2}} + \sqrt{4r^2 - x^2} = \frac{4r^2 - 2x^2}{\sqrt{4r^2 - x^2}}$

For maximum area, $\frac{dA}{dx} = 0 \Rightarrow 4r^2 - 2x^2 = 0 \Rightarrow x = \sqrt{2}\,r$

$$\frac{d^2A}{dx^2} = \frac{\sqrt{4r^2 - x^2}\,(-4x) - (4r^2 - 2x^2)\cdot\left(\frac{-x}{\sqrt{4r^2 - x^2}}\right)}{(4r^2 - x^2)}$$

$$\left[\frac{d^2A}{dx^2}\right]_{x=\sqrt{2}r} = \frac{\sqrt{4r^2 - 2r^2} \times (-4 \times \sqrt{2}\,r) - 0}{2r^2} = -4$$

$\therefore \frac{d^2A}{dx^2} < 0$ for $x = \sqrt{2}\,r$

So, A is maximum at $x = \sqrt{2}\,r$

$\therefore$ $\text{Max}\,A = x\sqrt{4r^2 - x^2} = \sqrt{2}\,r\sqrt{4r^2 - (\sqrt{2}\,r)^2} = \sqrt{2}\,r\,\sqrt{2}\,r = 2r^2$ sq units.

38. (*i*) $y = \sin^{-1} x$ (*ii*) $\sin^{-1}\left(\frac{-1}{2}\right) = \frac{-\pi}{6}$