14 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $A = \{a, b, c\}$, then relation $R = \{(a, c)\}$ on A is **[Conceptual Application]**

(*a*) reflexive only (*b*) symmetric and reflexive only

(*c*) transitive only (*d*) reflexive and transitive only

2. Let the function 'f' be defined by $f(x) = 5x + 2, \forall x \in R$. Then '$f$' is **[NCERT Part-I, Page 7]**

(*a*) onto function (*b*) one-one, onto function

(*c*) one-one (*d*) one-one but not onto

3. The principal value of $\sin^{-1}\left(\sin \frac{4\pi}{5}\right)$ is **[NCERT Part-I, Page 19]**

(*a*) $\frac{\pi}{6}$ (*b*) $\frac{4\pi}{5}$ (*c*) $\frac{\pi}{5}$ (*d*) π

4. The maximum value of xy subject to $x + y = 16$ is **[Conceptual Application]**

(*a*) 8 (*b*) 64 (*c*) 20 (*d*) 24

5. If $f(x) = x - \frac{1}{2}\log(x^2 + 1)$, then f is **[NCERT Part-I, Page 153]**

(*a*) increasing for $x > 0$ (*b*) decreasing for $x < 0$

(*c*) increasing for $x \in R$ (*d*) decreasing for $x \in R$

6. The function $f(x) = \sin x$ is **[NCERT Part-I, Page 105]**

(*a*) continuous for all real numbers x.
(*b*) discontinuous for all real numbers x.
(*c*) continuous when $x = 0$
(*d*) None of these

7. The function $[x]$ represents greatest integer function less than or equal to x. Then function $f(x) = x - [x]$ is discontinuous at **[NCERT Part-I, Page 105]**

(*a*) only one integral point
(*b*) two integral points
(*c*) all integral points
(*d*) None of these

8. Two matrices A and B of order $p \times q$ and $r \times s$ respectively, can be added only if **[NCERT Part-I, Page 44]**

(*a*) $p = r, q = s$
(*b*) $p = q, r = s$
(*c*) $p = q$
(*d*) None of these

9. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ and $A + B - D = O$ (zero-matrix), then D matrix will be **[NCERT Part-I, Page 46-47]**

(*a*) $\begin{bmatrix} 0 & 1 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$
(*b*) $\begin{bmatrix} 0 & -2 \\ -3 & -7 \\ -5 & -6 \end{bmatrix}$
(*c*) $\begin{bmatrix} 0 & 2 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$
(*d*) $\begin{bmatrix} 0 & 2 \\ 3 & 7 \\ 6 & 5 \end{bmatrix}$

10. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$, then $|AB|$ is equal to **[NCERT Part-I, Page 51-52, 89]**

(*a*) 32
(*b*) 16
(*c*) 8
(*d*) 4

11. The area bounded by the curve $y = x|x|$, the x-axis and the ordinates $x = -1$ and $x = 1$ is given by **[Conceptual Application]**

(*a*) 0
(*b*) $\frac{1}{3}$
(*c*) $\frac{2}{3}$
(*d*) $\frac{4}{3}$

12. Area of region bounded by ellipse $\frac{x^2}{49} + \frac{y^2}{4} = 1$ is **[Conceptual Application]**

(*a*) 49 sq units
(*b*) 4 sq units
(*c*) 14 sq units
(*d*) 14π sq units

13. $\int_0^{2\pi} \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx =$ **[NCERT Part-II, Page 268]**

(*a*) $-2\sqrt{2}$
(*b*) -2
(*c*) $\sqrt{2}$
(*d*) $2\sqrt{2}$

14. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx =$ **[Integrated Question]**

(*a*) $\frac{\pi}{2} - \log 2$
(*b*) π
(*c*) $\frac{\pi}{4}$
(*d*) $\frac{\pi}{2} + \log 2$

15. Solution of the differential equation $\frac{dy}{dx} - \frac{1}{x}y = 2x^2$ is **[NCERT Part-II, Page 322-323]**

(*a*) $y^3 = x + Cx^2$
(*b*) $C = xy^3$
(*c*) $y = x^3 + Cx$
(*d*) $x^3 = y + Cy^2$

16. A line makes angle α, β, γ with x-axis, y-axis and z-axis respectively then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to **[NCERT Part-II, Page 377-378]**

(*a*) 2
(*b*) 1
(*c*) -2
(*d*) -1

17. A die is rolled, the outcome is an even number. What is the probability that it is prime? **[NCERT Part-II, Page 408]**

(*a*) $\frac{2}{3}$
(*b*) $\frac{1}{2}$
(*c*) $\frac{1}{3}$
(*d*) $\frac{3}{4}$

18. E and F are two events, such that $P(E/F) = P(F/E)$. If $P(E) = \frac{1}{2}$, then $P(F) =$

[NCERT Part-II, Page 408-409]

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) 1 (*d*) 0

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. Assertion (*A*): $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx = e^{\tan^{-1} x} + C$ **[NCERT Part-II, Page 226-227]**

Reason (*R*): $\frac{d}{dx}(e^{\tan^{-1} x} + C) = \frac{e^{\tan^{-1} x}}{1+x^2}$

20. Assertion (*A*): Derivative of x^x with respect to x is $x^x\,(1 + \log x)$. **[NCERT Part-I, Page 130]**

Reason (*R*): Derivative of x^x with respect to x is $x\,x^{x-1}$.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Solve the differential equation $\frac{dy}{dx} = \frac{x \cdot e^x \log x + e^x}{x \cos y}$. **[NCERT Part-II, Page 306-307]**

22. There are three coins in which one is a fair coin with probability $\frac{1}{2}$ and two biased coins with probabilities $\frac{1}{3}$ and $\frac{2}{3}$ for a head. One of the coins is tossed twice. If head appears both times, what is the probability that the biased coin with probability $\frac{2}{3}$ for a head is chosen? **[NCERT Part-II, Page 408]**

OR

A fair die is rolled. Consider the events $E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$.
Find **[NCERT Part-II, Page 408]**

(*i*) $P(E/G)$ and $P(G/E)$ (*ii*) $P(E \cup F/G)$

23. For any vector $\vec{a}$, prove that $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2|\vec{a}|^2$.

[NCERT Part-II, Page 347-348, 363]

24. If $y = \cos^{-1} x$, then what are the values which x and y (principle value branch) can take?

[NCERT Part-I, Page 21]

25. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find the value of $|A^2 - 2A|$. **[Conceptual Application]**

OR

If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x. **[NCERT Part-I, Page 77]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Find the derivative of $\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$, with respect to x. **[Conceptual Application]**

27. Show that the semi-vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$. **[Conceptual Application]**

28. Evaluate $\int_0^4 |x^2 - 4|\, dx$. **[Integrated Question]**

OR

Show that the general solution of the differential equation $(x^2+x+1)dy + (y^2+y+1)dx = 0$, is given by $\frac{x+y+1}{1-x-y-2xy} = A$, where A is a parameter. **[NCERT Part-II, Page 304-305]**

29. Let N be the set of all natural numbers and R be a relation on $N \times N$ defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Examine whether R is an equivalence relation.

[NCERT Part-II, Page 2]

30. Find the area of region $\{(x, y) : x^2 + y^2 \leq 1, 2x + 2y \geq 1, x \geq 0, y \geq 0\}$ using integration.

[Conceptual Application]

OR

Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the ordinates $x = 0$ and $x = ae$, where $b^2 = a^2(1 - e^2)$ and $e < 1$. **[Conceptual Application]**

31. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

[NCERT Part-I, Page 105, 113, 119]

OR

If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$. **[NCERT Part-I, Page 137]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. A line passes through (2, –1, 3) and is perpendicular to the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain its equation in vector and Cartesian form.

[Conceptual Application]

OR

Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$.

[NCERT Part-II, Page 386-387]

33. If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2$; $\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=5$; $\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=-4$ **[NCERT Part-I, Page 94]**

OR

Find the product of given matrices and state how we can use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 2$. **[NCERT Part-I, Page 94]**

34. Find the particular solution of the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$, given that $y = 2$ when $x = \frac{\pi}{2}$. **[NCERT Part-II, Page 322-323]**

35. Solve the following *LPP* graphically: **[NCERT Part-II, Page 397-398]**

Maximise $Z = x + y$,

subject to the constraints

$x \geq 0, y \geq 0$

$3x + 2y \leq 45$

$2x + 3y \leq 42$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. An insurance company insured 9000 persons of which 2000 are scooter drivers, 4000 are truck drivers and rest are car drivers. The probability of a scooter, a truck and car driver meeting with an accident is 0.01, 0.04 and 0.02 respectively. With the above information, answer the following. **[NCERT Part-II, Page 408, 425]**

(*i*) What is the probability of being a car driver?

(*ii*) What is the conditional probability of accident of a car driver?

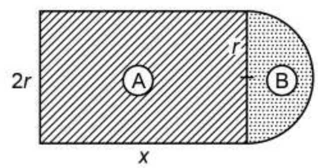
(*iii*) What is the probability that truck driver meets with an accident?

OR

(*iii*) What is the probability of meeting with an accident?

Case Study - 2

37. A company wants to construct a new complex and according to the plan it has to have a big rectangular hall for the employees and an attached semicircular reception cum office. The total perimeter of the plan is 400 m as shown. **[Conceptual Application]**



(*i*) Find the relation between x and r.

(*ii*) Express area of rectangular region A in terms of r.

(*iii*) Find the maximum area of region A.

OR

(*iii*) What is the maximum area occupied by the new complex?

Case Study - 3

38. The Indian Coast Guard (ICG) while patrolling, saw a suspicious boat with four men. They were nowhere looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of coast guard helicopter and boat are (2, 3, 5) and (1, 4, 2) respectively. **[NCERT Part-II, Page 381-382]**

Based on the above information, answer the following questions.

(*i*) Find the equation of line joining the positions of the helicopter and boat in Cartesian form.

(*ii*) If the soldier decides to shoot the boat at given instant of time, where the distance is measured in metres, then what is the distance that bullet has to travel?

SOLUTIONS

1. (c)

2. (b)

3. (c), as

$$\sin^{-1}\left(\sin\frac{4\pi}{5}\right) = \sin^{-1}\left\{\sin\left(\pi-\frac{\pi}{5}\right)\right\}$$
$$= \sin^{-1}\left\{\sin\frac{\pi}{5}\right\} = \frac{\pi}{5}.$$

4. (b) We have, $x+y = 16 \Rightarrow y = 16-x$ ...(i)

Now, let $P = xy$

$\Rightarrow$ $P = x(16-x)$ {(using (i)}

$\Rightarrow$ $P = 16x - x^2$

$\Rightarrow$ $\dfrac{dP}{dx} = 16-2x$

For maxima or minima, $\dfrac{dP}{dx} = 0 \Rightarrow 16-2x = 0 \Rightarrow x = 8$

$$\frac{d^2P}{dx^2} = -2 \Rightarrow \left(\frac{d^2P}{dx^2}\right)_{x=8} = -2 < 0$$

$\therefore P$ is maximum when $x = 8$

Now, from (i), when $x = 8$, then $y = 8$.

So, maximum value of P(i.e. xy) $= 8 \times 8 = 64$

5. (c) We have, $f(x) = x - \dfrac{1}{2}\log(x^2+1)$

Now, $f'(x) = 1 - \dfrac{x}{x^2+1}$

$\Rightarrow$ $f'(x) = \dfrac{x^2+1-x}{x^2+1}$

$\Rightarrow$ $f'(x) = \dfrac{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}{x^2+1}$

Clearly, $f'(x) > 0 \ \forall\ x \in R$.

So, $f(x)$ is increasing for all $x \in R$.

6. (a), The function $f(x) = \sin x$ is defined for all real numbers.

Also $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \sin x = 0$

Let a be any real number

let $x = a+h$, if $x \to a$, then $h \to 0$

$$\lim_{x\to a} f(x) = \lim_{x\to a} \sin x$$

$$= \lim_{h\to 0} \sin(a+h)$$

$$= \lim_{h\to 0} [\sin a \cos h + \cos a \sin h]$$

$$= \lim_{h\to 0} \sin a \cos h + \lim_{h\to 0} \cos a \sin h$$

$$= \sin a \lim_{h\to 0} \cos h + \cos a \lim_{h\to 0} \sin h$$

$$= \sin a(1) + \cos a(0)$$

$$= \sin a$$

Also $f(a) = \sin a$

$\Rightarrow \lim_{x\to a} \sin x = f(a) = \sin a$

$\Rightarrow f(x)$ is continuous in the set of all real numbers.

7. (c), $[x]$ represents greatest integer function less than or equal to x

$\therefore x - [x]$ is defined at all integral points.

Now $f(x) = x - [x]$ is defined at all integral points

Consider 'n' be any integer.

$$\text{LHL} = \lim_{x\to n^-} f(x) = \lim_{x\to n^-} x - [x]$$

$$= \lim_{x\to n^-} x - \lim_{x\to n^-} [x] \qquad \left\{\begin{matrix}\text{as } x \to n^- \\ \Rightarrow [x] \to (n-1)\end{matrix}\right\}$$

$$= n - (n-1) = 1$$

$$\text{RHL} = \lim_{x\to n^+} f(x) = \lim_{x\to n^+} x - [x] \qquad (\text{as } x \to n^+ \Rightarrow [x] \to n)$$

$$= \lim_{x\to n^+} x - \lim_{x\to n^+} [x]$$

$$= n - n = 0$$

$$\lim_{x\to n^-} f(x) \neq \lim_{x\to n^+} f(x)$$

$\Rightarrow$ $f(x)$ is not continuous at $x = n$

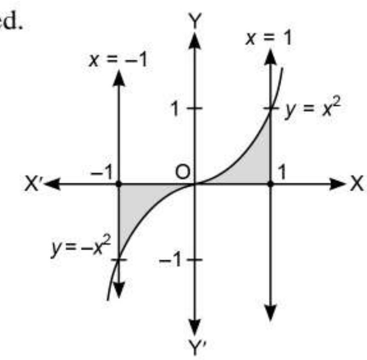
8. (a) Two matrices can be added it they are of same order.

9. (a) as, $D = A + B$

10. (b) as, $|AB| = |A|\,|B|$

11. (c) We have, $y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

Plotting the graph, area of the shaded region is to be calculated.



$$\text{Area} = -\int_{-1}^{0} -x^2 dx + \int_0^1 x^2 dx$$

$$= \frac{1}{3}[x^3]_{-1}^0 + \frac{1}{3}\int_0^1 [x^3]_0^1$$

$$= \frac{1}{3}[0+1] + \frac{1}{3}[1-0]$$

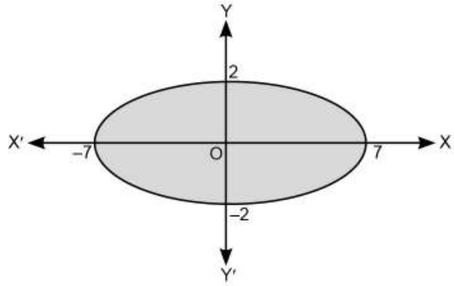
$$= \frac{2}{3}\text{sq units}$$

12. (*d*) Required area $= 4 \times \frac{2}{7}\int_0^7 \sqrt{49 - x^2}\, dx$

$$= 4 \times \frac{2}{7}\left[\frac{x}{2}\sqrt{49 - x^2} + \frac{49}{2}\sin^{-1}\left(\frac{x}{7}\right)\right]_0^7$$

$$= 4 \times \frac{2}{7}\left[\frac{49}{2} \times \frac{\pi}{2} - 0\right]$$

$$= 14\pi \text{ sq units}$$



13. (*d*) $\int_0^{2\pi} \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

$$= -2\left[\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]_0^{2\pi}$$

$$= -2\left[\cos\left(\pi + \frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)\right]$$

$$= -2\left[-\cos\left(\frac{\neq}{4}\right) - \cos\left(\frac{\neq}{4}\right)\right]$$

$$= (-2) \times -2 \times \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

14. (*a*) Let $I = \int_0^1 \sin^{-1}\left(\frac{2x}{1 + x^2}\right) dx$

Put $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

as $x \to 0, \theta \to 0$; as $x \to 1, \theta \to \frac{\pi}{4}$

$\therefore$ $I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1 + \tan^2\theta}\right) \cdot \sec^2\theta\, d\theta$

$$= \int_0^{\frac{\pi}{4}} \underset{①}{2\theta} \cdot \underset{②}{\sec^2\theta}\ d\theta$$

$$= \left[2\theta \cdot \tan\theta\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 2\tan\theta\ d\theta$$

$$= \frac{\pi}{2} + 2\left[\log|\cos\theta|\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{2} + 2\left[\log\frac{1}{\sqrt{2}}\right]$$

$$= \frac{\pi}{2} - 2\log 2^{1/2} = \frac{\pi}{2} - \log 2$$

15. (*c*) As, $\frac{dy}{dx} - \frac{1}{x} \cdot y = 2x^2$

Now, $P(x) = -\frac{1}{x};\ Q(x) = 2x^2$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int -\frac{1}{x} dx} = \frac{1}{x}$

$\therefore$ Solution is, $y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx$

$$\Rightarrow \quad \frac{y}{x} = \int 2x^2 \times \frac{1}{x} dx$$

$$\Rightarrow \quad \frac{y}{x} = x^2 + C$$

$$\Rightarrow \quad y = x^3 + Cx$$

16. (*d*), as $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$$\Rightarrow \frac{1 + \cos 2\alpha}{2} + \frac{1 + \cos 2\beta}{2} + \frac{1 + \cos 2\gamma}{2} = 1$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

17. (*c*), *A* die is rolled $\therefore$ outcome S = {1, 2, 3, 4, 5, 6}

A : even number {2, 4, 6}

B : {2, 3, 5} and $A \cap B = \{2\}$

$$\therefore P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

18. (*a*), $P(E/F) = P(F/E)$

$$\Rightarrow \quad \frac{P(E \cap F)}{P(F)} = \frac{P(F \cap E)}{P(E)} \Rightarrow P(F) = P(E) = \frac{1}{2}$$

19. (*a*), Both A and R are true and R is the correct explanation of A.

Let $\quad \text{I} = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

So, $\quad \text{I} = \int dt = t + C = e^{\tan^{-1}x} + C$

Let $e^{\tan^{-1}x} = t$

$\Rightarrow \quad e^{\tan^{-1}x} \cdot \frac{1}{1+x^2} dx = dt$

A is true, R is also true and the correct explanation of A.

20. (*c*), Assertion is true but the Reason is false.

21.

$$\int \cos y \, dy = \int e^x \left(\log x + \frac{1}{x} \right) dx$$

$\sin y = e^x \log x + C$ is required solution.

[using $\int e^x \{f(x) + f'(x)\}\, dx = e^x \cdot f(x) + C$]

22. Consider the following events:

A: unbiased coin with probability of head as $\frac{1}{2}$ is selected.

B: biased coin with probability of head as $\frac{1}{3}$ is selected.

C: biased coin with probability of head as $\frac{2}{3}$ is selected.

Now, $\quad P(A) = P(B) = P(C) = \frac{1}{3}$

E: in both tosses head appears on the chosen coin

$$P(E/A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4};\ P(E/B) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \quad \text{and } P(E/C) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

Using Bayes' Theorem,

Probability that the biased coin with probability $\frac{2}{3}$ for head is chosen

$$P(C/E) = \frac{P(C)\cdot P(E/C)}{P(A)\cdot P(E/A)+P(B)\cdot P(E/B)+P(C)\cdot P(E/C)}$$

$$= \frac{\frac{1}{3}\times\frac{4}{9}}{\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{9}+\frac{1}{3}\times\frac{4}{9}} = \frac{16}{9+4+16} = \frac{16}{29}$$

OR

$$P(E\cap G) = \frac{2}{6}, P(E) = \frac{3}{6}, P(G) = \frac{4}{6}$$

Let $E \cup F = H = \{1, 2, 3, 5\}$

Now, $P((E\cup F)\cap G) = P(H\cap G) = \frac{3}{6}$

(*i*) $P(E/G) = \frac{P(E\cap G)}{P(G)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{1}{2}$ and $P(G/E) = \frac{P(E\cap G)}{P(E)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$

(*ii*) $P(E\cup F/G) = \frac{P[(E\cup F)\cap G)]}{P(G)} = \frac{\frac{3}{6}}{\frac{4}{6}} = \frac{3}{4}$

23. Let α, β, γ be the angles which vector $\vec{a}$ makes with unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$.

Consider $|\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$

$$= [|\vec{a}||\hat{i}|\sin\alpha]^2+[|\vec{a}||\hat{j}|\sin\beta]^2+[|\vec{a}||\hat{k}|\sin\gamma]^2$$

$$= |\vec{a}|^2|\hat{i}|^2\sin^2\alpha+|\vec{a}|^2|\hat{j}|^2\sin^2\beta+|\vec{a}|^2|\hat{k}|^2\sin^2\gamma$$

$$= |\vec{a}|^2[\sin^2\alpha+\sin^2\beta+\sin^2\gamma] \qquad [\because |\hat{i}|^2=|\hat{j}|^2=|\hat{k}|^2=1]$$

$$= |\vec{a}|^2[1-\cos^2\alpha+1-\cos^2\beta+1-\cos^2\gamma]$$

$$= |\vec{a}|^2[3-(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)]$$

$$= |\vec{a}|^2(3-1)=2|\vec{a}|^2 \qquad [\because \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1]$$

24. $x\in[-1, 1]$ and $y\in[0, \pi]$

25. $A^2 = A\cdot A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}$

Now, $A^2-2A = \begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}-\begin{bmatrix}2 & 4\\6 & 8\end{bmatrix} = \begin{bmatrix}5 & 6\\9 & 14\end{bmatrix}$

Now, $|A^2-2A| = \begin{vmatrix}5 & 6\\9 & 14\end{vmatrix} = 70-54 = 16$

OR

$$\begin{vmatrix}x+3 & -2\\-3x & 2x\end{vmatrix} = 8 \Rightarrow (x+3)2x-(-2)(-3x) = 8$$

$\Rightarrow$ $2x^2+6x-6x = 8 \Rightarrow 2x^2 = 8 \Rightarrow x = 2\in N$

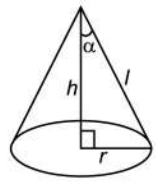
26. Let $$y = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

$$\frac{dy}{dx} = \frac{x}{2}\cdot\frac{1}{2\sqrt{a^2-x^2}}(-2x)+\frac{1}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\cdot\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\cdot\frac{1}{a}$$

$$= \frac{-x^2}{2\times\sqrt{a^2-x^2}}+\frac{1}{2}\sqrt{a^2-x^2}+\frac{1}{2}\frac{a^2}{\sqrt{a^2-x^2}}$$

$$= \frac{1}{2}\left[\frac{-x^2+a^2-x^2+a^2}{\sqrt{a^2-x^2}}\right]=\sqrt{a^2-x^2}.$$

27. Let r be radius of the base, h the height and l the slant height of a cone.



$\therefore$ $$l^2 = h^2+r^2 \quad ...(i)$$

Given, total surface area, $S = \pi rl + \pi r^2$ ...(*ii*)

Volume of the cone,

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\sqrt{l^2-r^2} \quad \text{[from } (i)]$$

$$= \frac{1}{3}\pi r^2\sqrt{\left(\frac{S-\pi r^2}{\pi r}\right)^2-r^2} \quad \text{[from } (ii)]$$

If V is maximum, then $V^2 = W$(say) is maximum.

$\therefore$ $$V^2 = W = \frac{1}{9}\pi^2 r^4\left[\frac{S^2+\pi^2r^4-2S\pi r^2-\pi^2r^4}{\pi^2r^2}\right]$$

$$= \frac{1}{9}r^2[S^2-2S\pi r^2]$$

$$W = \frac{S}{9}[Sr^2-2\pi r^4];\ \frac{dW}{dr} = \frac{S}{9}[2Sr-8\pi r^3]$$

For maximum volume, $\frac{dW}{dr} = 0$

$\Rightarrow$ $$8\pi r^3 = 2Sr$$

$\Rightarrow$ $$r^2 = \frac{S}{4\pi} \quad ...(iii)$$

$$\frac{d^2W}{dr^2} = \frac{S}{9}[2S-24\pi r^2]$$

$\Rightarrow$ $$\left.\frac{d^2W}{dr^2}\right]_{r=\sqrt{\frac{S}{4\pi}}} = \frac{S}{9}[2S-6S]<0$$

Hence, volume is maximum at $r = \sqrt{\frac{S}{4\pi}}$

$\therefore$ from (*iii*), we have

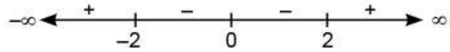
$$4\pi r^2 = S \Rightarrow 4\pi r^2 = \pi rl+\pi r^2 \quad \text{[from } (ii)]$$

$\Rightarrow$ $$3\pi r^2 = \pi rl \Rightarrow \frac{r}{l}=\frac{1}{3}$$

$\Rightarrow$ $$\sin\alpha = \frac{1}{3} \Rightarrow \alpha = \sin^{-1}\frac{1}{3}$$

28. Consider $\int_0^4 |x^2 - 4|\, dx$

Clearly $x^2 - 4 = (x + 2)(x - 2)$. The signs of $x^2 - 4$ for different values of x are shown.



$$\therefore \qquad |x^2 - 4| = \begin{cases} -(x^2-4), & 0 \le x < 2 \\ (x^2-4), & 2 \le x \le 4 \end{cases}$$

$$\therefore \qquad \int_0^4 |x^2 - 4|\, dx = -\int_0^2 (x^2-4)\,dx + \int_2^4 (x^2-4)\,dx$$

$$= -\left[\frac{x^3}{3} - 4x\right]_0^2 + \left[\frac{x^3}{3} - 4x\right]_2^4$$

$$= -\left(\frac{8}{3} - 8\right) + 0 + \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right) = \frac{16}{3} + \frac{16}{3} + \frac{16}{3} = 16$$

OR

Consider equation $(x^2 + x + 1)dy + (y^2 + y + 1)dx = 0$

$$\Rightarrow \qquad \frac{dy}{y^2+y+1} + \frac{dx}{x^2+x+1} = 0$$

Integrating both sides, we get

$$\int \frac{dy}{y^2+y+1} + \int \frac{dx}{x^2+x+1} = C, \text{ where } C \text{ is a constant of integration}$$

$$\Rightarrow \quad \int \frac{dy}{\left(y+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = C$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}} \tan^{-1} \frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + C_1 + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + C_2 = C, \; C_1 \text{ and } C_2 \text{ are constant of integration.}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{(2x+1)}{\sqrt{3}}\right) = \frac{\sqrt{3}}{2}\,(C - C_1 - C_2)$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1 - \frac{(2y+1)(2x+1)}{3}}\right] = \frac{\sqrt{3}k}{2} \qquad [\text{where } C - C_1 - C_2 = k]$$

$$\Rightarrow \quad \left(\frac{2y+1+2x+1}{\sqrt{3}}\right) \times \left(\frac{3}{3 - 4xy - 2x - 2y - 1}\right) = \tan \frac{\sqrt{3}k}{2}$$

$$\Rightarrow \quad \frac{2\sqrt{3}\,(x+y+1)}{2(1 - 2xy - x - y)} = \tan \frac{\sqrt{3}k}{2}$$

$$\Rightarrow \quad \frac{x+y+1}{1 - 2xy - x - y} = \frac{1}{\sqrt{3}} \tan \frac{\sqrt{3}k}{2}$$

$$\Rightarrow \quad \frac{x+y+1}{1 - x - y - 2xy} = A, \text{ where } A = \frac{1}{\sqrt{3}} \tan \frac{\sqrt{3}k}{2} \text{ is a parameter.}$$

29. Relation R on $N \times N$ is given by

$(a, b)\, R\, (c, d) \Leftrightarrow ad\,(b + c) = bc\,(a + d)$.

For reflexive:

For $(a, b) \in N \times N$

$(a, b)\, R(a, b) \Rightarrow ab(b + a) = ba(a + b)$, true in N

Hence, reflexive

For symmetric:

For $(a, b), (c, d) \in N \times N$

$(a, b)\, R(c, d) \;\Rightarrow\; ad(b + c) = bc(a + d)$

$\Rightarrow cb(d + a) = da(c + b)$ $\quad$ ($\because$ $\times$ and $+$ is commutative in N)

$\Rightarrow (c, d)\, R(a, b)$ for all $(a, b), (c, d) \in N \times N$.

Hence, symmetric

For transitive:

For $(a, b), (c, d), (e, f) \in N \times N$

Let $(a, b)\, R(c, d)$ and $(c, d)\, R(e, f)$

$$\Rightarrow \quad ad(b + c) = bc(a + d) \text{ and } cf(d + e) = de(c + f)$$

$$\Rightarrow \quad \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \quad \text{...}(i)$$

$$\text{and} \quad \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c} \quad \text{...}(ii)$$

Adding (i) and (ii)

$$\Rightarrow \quad \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$$\Rightarrow \quad \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$$

$$\Rightarrow \quad af(e + b) = be(f + a) \quad \Rightarrow af(b + e) = be(a + f)$$

$$\Rightarrow \quad (a, b)\, R(e, f)$$

As $(a, b)\, R(c, d)$ and $(c, d)\, R(e, f) \Rightarrow (a, b)\, R(e, f)$

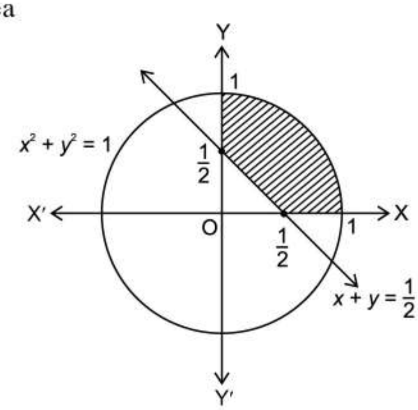
Hence, transitive.

As relation R is reflexive, symmetric and transitive, hence, R is an equivalence relation.

30. Given region is $\{(x, y) : x^2 + y^2 \le 1, 2x + 2y \ge 1, x \ge 0, y \ge 0\}$

Given curves corresponding to region are $x^2 + y^2 = 1$ and $x + y = \dfrac{1}{2}$,

Plotting the region we notice we have to find shaded region area



$$\text{Area} = \int_0^1 \sqrt{1 - x^2}\, dx - \int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right) dx$$

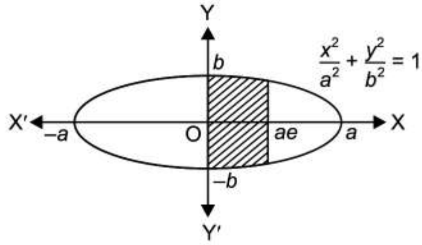
$$= \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x\right]_0^1 - \left[\frac{x}{2} - \frac{x^2}{2}\right]_0^{\frac{1}{2}}$$

$$= \left[\left(\frac{1}{2}\sqrt{0} + \frac{1}{2}\cdot \sin^{-1} 1\right) - (0 + 0)\right] - \left[\left(\frac{1}{4} - \frac{1}{8}\right) - 0\right]$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{8} = \left(\frac{\pi}{4} - \frac{1}{8}\right) \text{ sq units}$$

OR

We have to find the shaded area



As curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is symmetrical about the x-axis.

$$\therefore \quad \text{Required area} = 2\int_0^{ae} y\,dx$$

$$= 2\frac{b}{a}\int_0^{ae}\sqrt{a^2-x^2}\,dx$$

$$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$$

$$= \frac{2b}{a}\left[\left\{\frac{ae}{2}\sqrt{a^2-a^2e^2}+\frac{a^2}{2}\sin^{-1}e\right\}-\{0+0\}\right]$$

$$= \frac{2b}{a}\left[\frac{ae}{2}\sqrt{a^2(1-e^2)}+\frac{a^2}{2}\sin^{-1}e\right]$$

$$= \frac{2b}{a}\left[\frac{ae}{2}.b+\frac{a^2}{2}\sin^{-1}e\right] \quad [\because b^2 = a^2(1-e^2)]$$

$$= (b^2e + ab\sin^{-1}e) \text{ sq units}$$

31. Consider function $f(x) = 2x - |x|$

$$\Rightarrow \quad f(x) = \begin{cases} x, & x \geq 0 \\ 3x, & x < 0 \end{cases} \qquad \begin{bmatrix} \text{as } |x| = x, & \text{for } x \geq 0 \\ |x| = -x, & \text{for } x < 0 \end{bmatrix}$$

For continuity at $x = 0$,

$$\underset{x=0}{\text{LHL}} = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}(-3h) = 0$$

$$\underset{x=0}{\text{RHL}} = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}(h) = 0$$

$$f(0) = 0$$

As $\underset{x=0}{\text{LHL}} = \underset{x=0}{\text{RHL}} = f(0)$,

$\therefore$ function is continuous at $x = 0$

For derivability,

$$\underset{x=0}{\text{LHD}} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$= \lim_{h\to 0}\frac{-3h-0}{-h} = \lim_{h\to 0}(3) = 3$$

$$\underset{x=0}{\text{RHD}} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{h-0}{h} = \lim_{h\to 0}(1) = 1$$

As $\underset{x=0}{\text{LHD}} \neq \underset{x=0}{\text{RHD}}$,

$\therefore$ function is not derivable (differentiable) at $x = 0$

OR

Consider $\quad y = \sin(\sin x) \quad$ ...(*i*)

$$\Rightarrow \quad \frac{dy}{dx} = \cos(\sin x).\cos x \quad \text{...}(ii)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \cos x[-\sin(\sin x).\cos x] - \sin x.\cos(\sin x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -y \cdot \cos^2 x - \frac{\sin x}{\cos x} \cdot \cos x \cdot \cos(\sin x) \quad \text{[from } (i)\text{]}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -y \cdot \cos^2 x - \tan x\left(\frac{dy}{dx}\right) \quad \text{[from } (ii)\text{]}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y \cdot \cos^2 x = 0.$$

32. Vector equation of a line passing through (2, –1, 3) is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda'\vec{n} \quad ...(i)$$

As line (i) is perpendicular to the lines

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

$\therefore\ (2\hat{i} - 2\hat{j} + \hat{k}) \cdot \vec{n} = 0$ and $(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot \vec{n} = 0$

$$\Rightarrow \quad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= \hat{i}(-6) - \hat{j}(3) + \hat{k}(6) = -6\hat{i} - 3\hat{j} + 6\hat{k}$$

Substituting in (i), we get

equation of line in vector form as

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda'(-6\hat{i} - 3\hat{j} + 6\hat{k})$$

or $\quad \vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k}),$ [where $t = -3\lambda'$, t is a scalar]

General point on line is

$$\vec{r} = (2 + 2t)\hat{i} + (-1 + t)\hat{j} + (3 - 2t)\hat{k}$$

$$\Rightarrow \quad x = 2 + 2t,\ y = -1 + t \text{ and } z = 3 - 2t$$

$$\Rightarrow \quad \frac{x-2}{2} = t,\ \frac{y+1}{1} = t, \text{ and } \frac{z-3}{-2} = t$$

Equation of line in Cartesian form is

$$\frac{x-2}{2} = \frac{y+1}{1} = \frac{z-3}{-2}.$$

OR

Lines are $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$

and $\quad \vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$

The shortest distance $= \left|\dfrac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|.$

Here $\quad \vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k},\ \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k};$

$$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k},\ \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k} = \hat{j} - 4\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\Rightarrow \qquad |\vec{b}_1 \times \vec{b}_2| = \sqrt{4+16+9} = \sqrt{29}$$

The shortest distance $= \left| \dfrac{(\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{29}} \right| = \left| \dfrac{-4+12}{\sqrt{29}} \right| = \dfrac{8}{\sqrt{29}}$ units

33. Consider
$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$$

We have
$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$
$$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$
$$= 150 + 330 + 720 = 1200 \neq 0$$

Hence, A^{-1} exists.

Matrix formed by cofactors of each element in $|A|$.

$$\begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$$

$$\therefore \qquad \text{Adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}'$$
$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Consider equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$$
$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$$

Corresponding matrix equation is

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$AX = B$ is matrix equation.

Its solution is $X = A^{-1}B$

$$\Rightarrow \quad X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 150+750-300 \\ 220-500-120 \\ 144+0+96 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ -400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{3} \\ \frac{1}{5} \end{bmatrix} \Rightarrow \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = -\frac{1}{3}, \frac{1}{z} = \frac{1}{5}$$

$$\Rightarrow \quad x = 2; y = -3; z = 5.$$

OR

First we find the product

$$\text{Let } AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad AB = I$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \quad \text{...}(i)$$

Equations are,

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
$$3x - 2y + 4z = 2$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow AX = C \text{ is matrix equation}$$

Its solution is $X = A^{-1}C$ ...(*ii*)

Now from (*i*), we can write

$$A^{-1} = B$$

Now, we substitute in (*ii*) and solve for X to get x, y, z in the following manner.

$$X = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \Rightarrow x = 0; y = 5; z = 3$$

34. Consider equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$

Here, $P(x) = -3\cot x$ and $Q(x) = \sin 2x$

Integrating factor (I.F.) $= e^{\int P\,dx} = e^{\int -3\cot x\,dx} = e^{-3\int \cot x\,dx} = e^{-3\log|\sin x|} = e^{\log|\sin x|^{-3}} = \frac{1}{\sin^3 x}$

$$\text{Solution is (I.F.)}\, y = \int \{(\text{I.F.})\,Q(x)\}\,dx$$

$$\frac{1}{\sin^3 x}\cdot y = \int \frac{1}{\sin^3 x}\cdot \sin 2x\,dx$$

$$= \int \frac{2\sin x \cos x}{\sin^3 x}dx = 2\int \text{cosec}\, x \cot x\,dx$$

$$\Rightarrow \quad \frac{1}{\sin^3 x}\cdot y = -2\,\text{cosec}\, x + C$$

$$\Rightarrow \quad y = -2\sin^2 x + C\sin^3 x \quad \text{...}(i)$$

Given $y = 2$, when $x = \frac{\pi}{2}$

$$\therefore \quad 2 = -2\sin^2\frac{\pi}{2} + C\,.\sin^3\frac{\pi}{2}$$

$$\Rightarrow \quad 2 = -2 + C \Rightarrow C = 4$$

Substituting in (i), we get

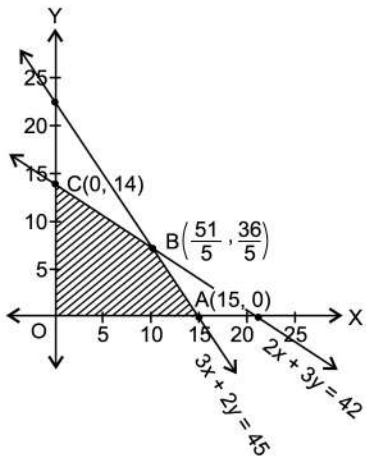
$y = -2\sin^2 x + 4\sin^3 x$ as the particular solution.

35. Plotting the inequations we notice shaded portion is optimum solution,

Possible points for maximum Z are

$A(15, 0), B\left(\frac{51}{5}, \frac{36}{5}\right)$ and $C(0, 14)$

Points	$Z = x + y$	Values
$A(15, 0)$	15 + 0	15
$B\left(\frac{51}{5}, \frac{36}{5}\right)$	$\frac{51+36}{5}$	$\frac{87}{5} = 17.4$ ← maximum
$C(0, 14)$	0 + 14	14



$\therefore Z$ is maximum for $B\left(\frac{51}{5}, \frac{36}{5}\right)$, i.e. $x = \frac{51}{5}, y = \frac{36}{5}$.

36. Let E_1 : scooter driver is selected

E_2 : truck driver is selected

E_3 : car driver is selected

E : insured person meets with an accident

(i) $P(E_3) = \frac{9000 - 2000 - 4000}{9000} = \frac{1}{3}$

(ii) $P(E/E_3) = 0.02$

(iii) P(truck driver meeting with an accident) $= P(E_2)\,P(E/E_2)$

$$= \frac{4}{9} \times 0.04 = \frac{4}{225}$$

OR

(*iii*) $P(E) = P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2) + P(E_3)\,P(E/E_3)$

$$= \frac{2}{9} \times 0.01 + \frac{4}{9} \times 0.04 + \frac{3}{9} \times 0.02$$

$$= \frac{2 + 16 + 6}{900} = \frac{24}{900}$$

$$= \frac{2}{75}$$

37. (*i*) Perimeter = 400 m

$\Rightarrow\ 2r + x + \pi r + x = 400$

$\Rightarrow\ 2x + 2r + \pi r = 400$

(*ii*) Area $(A) = (2r)x = 2x \cdot r$

$= [400 - 2r - \pi r]r$ [using (*i*)]

$= r[400 - (2 + \pi)r]$

(*iii*) For maximum A, $\frac{dA}{dr} = 0$ and

$\frac{d^2A}{dr^2} < 0$ for r obtained from $\frac{dA}{dr} = 0$

Now $A = 400r - (2 + \pi)r^2$

$$\frac{dA}{dr} = 400 - 2(2 + \pi)r$$

For maximum A, $\frac{dA}{dr} = 0$

$\Rightarrow\ r = \frac{200}{2 + \pi}$

Now, $\frac{d^2A}{dr^2} = -4 - 2\pi$

$$\frac{d^2A}{dr^2} = -4 - 2\pi < 0 \text{ for } r = \frac{200}{\pi + 2}$$

So, A is maximum at $r = \left(\frac{200}{\pi + 2}\right)$ m

$\therefore\ A_{max} = \frac{200}{2 + \pi}[400 - 200] = \frac{40000}{2 + \pi}$ m^2

OR

(*iii*) Let C denotes the area of complex.

So, $C = A + B$

$\Rightarrow\ C = r[400 - (2 + \pi)r] + \frac{\pi r^2}{2}$

$\Rightarrow\ C = 400r - 2r^2 - \pi r^2 + \frac{\pi r^2}{2}$

$\Rightarrow\ C = 400r - 2r^2 - \frac{\pi r^2}{2}$

$\Rightarrow\ \frac{dC}{dr} = 400 - 4r - \pi r$

For maxima or minima, $\frac{dC}{dr} = 0 \Rightarrow r = \frac{400}{\pi + 4}$

$$\Rightarrow \quad \frac{d^2C}{dr^2} = -4 - \pi \Rightarrow \left[\frac{d^2C}{dr^2}\right]_{r=\frac{400}{\pi+4}} = -4 - \pi < 0$$

So, C is maximum at $r = \dfrac{400}{\pi + 4}$

$$C_{\max} = 400 \times \frac{400}{4+\pi} - (2+\pi)\frac{(400)^2}{(4+\pi)^2} + \frac{1}{2}\pi\frac{(400)^2}{(4+\pi)^2}$$

$$= \frac{(400)^2}{4+\pi}\left[1 - \frac{2+\pi}{4+\pi} + \frac{\pi}{2(4+\pi)}\right]$$

$$= \frac{(400)^2}{4+\pi}\left[\frac{8+2\pi-4-2\pi+\pi}{2(4+\pi)}\right]$$

$$= \frac{80000}{4+\pi}\ \text{m}^2$$

38. (*i*) Let $P(2, 3, 5)$ and Q(1, 4, 2) be the positions of helicopter and boat respectively.

Now, direction ratios of PQ are proportional to 1–2, 4–3, 2–5, *i.e.*, –1, 1, – 3 or 1, –1, 3

Equation of line is $\dfrac{x-2}{1} = \dfrac{y-3}{-1} = \dfrac{z-5}{3}$ or $\dfrac{x-1}{1} = \dfrac{y-4}{-1} = \dfrac{z-2}{3}$

(*ii*) Required distance = Distance between P and Q

$$= \sqrt{(1-2)^2 + (4-3)^2 + (2-5)^2}$$

$$= \sqrt{1+1+9} = \sqrt{11}\ \text{m}$$