15 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. The value of $2\left[\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)\right]$ is **[NCERT Part-I, Page 27]**

   (*a*) 2π (*b*) $\frac{\pi}{2}$ (*c*) π (*d*) $-\pi$

2. If $\theta = \sin^{-1}\{\sin(-600°)\}$, then one of the possible value of θ is **[NCERT Part-I, Page 19]**

   (*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{2}$ (*c*) $\frac{2\pi}{3}$ (*d*) $\frac{-2\pi}{3}$

3. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is **[NCERT Part-I, Page 2]**

   (*a*) not reflexive (*b*) only symmetric
   (*c*) reflexive neither symmetric nor transitive (*d*) only transitive

4. If $\begin{bmatrix} 15-x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix, then $x =$ **[NCERT Part-I, Page 89]**

   (*a*) $-\frac{62}{3}$ (*b*) $\frac{62}{3}$ (*c*) $\frac{29}{3}$ (*d*) $\frac{61}{5}$

5. If $A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$ then $A^{-1} =$ **[NCERT Part-I, Page 90]**

(*a*) $\begin{bmatrix} 1+bc & 1 \\ 1 & a \end{bmatrix}$ (*b*) $\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$ (*c*) $\begin{bmatrix} a & 1+bc \\ b & c \end{bmatrix}$ (*d*) $\begin{bmatrix} 1+bc & b \\ a & c \end{bmatrix}$

6. If $x\begin{bmatrix} 3 \\ 2 \end{bmatrix} + y\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ then values of x and y are **[NCERT Part-I, Page 94]**

(*a*) 3, 4 (*b*) 3, – 4 (*c*) –3, 4 (*d*) – 3, – 4

7. If $y = \log x$, then $\frac{d^2y}{dx^2}$ is **[NCERT Part-I, Page 137]**

(*a*) $-\frac{1}{x^2}$ (*b*) $\frac{1}{x}$ (*c*) 1 (*d*) x

8. The value of k for which the function **[NCERT Part-I, Page 105]**

$f(x) = \begin{cases} x^2, & \text{if } x > 0 \\ kx, & \text{if } x < 0 \end{cases}$, is continuous at $x = 0$ is

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) no values of k

9. If $x = t^2, y = t^3$, then $\frac{d^2y}{dx^2}$ is **[NCERT Part-I, Page 134-135]**

(*a*) $\frac{3}{2}$ (*b*) $\frac{3}{4t}$ (*c*) $\frac{3}{2t}$ (*d*) $\frac{3}{4}$

10. The possible number which exceeds its square by the greatest possible number is **[Conceptual Application]**

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{3}{2}$

11. The value of n for which the differential equation $\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + y^2x}$ is homogeneous is **[NCERT Part-I, Page 312]**

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

12. Area of the region bounded by the curve $y = \cos x$ between $= 0$ and $x = \pi$ is **[Conceptual Application]**

(*a*) 2 sq units (*b*) 4 sq units (*c*) 3 sq units (*d*) 1 sq unit

13. The value of $\int \frac{dx}{1+\cos x} =$ **[NCERT Part-II, Page 228, 241]**

(*a*) $\tan\frac{x}{2} + C$ (*b*) $\tan^3\frac{x}{2} + C$ (*c*) $3\tan\frac{x}{2} + C$ (*d*) $\frac{1}{4}\tan\frac{x}{2} + C$

14. The position vectors of points *A, B, C* and *D* are $\vec{a}, \vec{b}, 2\vec{a} + 3\vec{b}$ and $\vec{a} - 2\vec{b}$ respectively. Then vectors $\overrightarrow{DB}$ and $\overrightarrow{AC}$ are respectively **[NCERT Part-II, Page 339]**

(*a*) $3\vec{b} - \vec{a}, \vec{a} + 3\vec{b}$ (*b*) $3\vec{a} + \vec{b}, \vec{a} + 3\vec{b}$ (*c*) $3\vec{b} + \vec{a}, \vec{a} - 3\vec{b}$ (*d*) $3\vec{a} - \vec{b}, \vec{a} + 3\vec{b}$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \cos x \, dx =$ **[NCERT Part-II, Page 274]**

(*a*) 1 (*b*) 0 (*c*) $\frac{\pi}{2}$ (*d*) $-\frac{\pi}{2}$

16. The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is **[Conceptual Application]**

(*a*) $\frac{4}{3}$ sq units (*b*) 1 sq unit (*c*) $\frac{2}{3}$ sq units (*d*) $\frac{1}{3}$ sq units

17. If A and B are two events such that $P(A) \neq 0$ and $P(B \mid A) = 1$, then **[NCERT Part-II, Page 408]**

(*a*) $A \subset B$ (*b*) $B \subset A$ (*c*) $B = \phi$ (*d*) $A = \phi$

18. If for events A and B, if $P(A) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{2}, P\left(\frac{B}{A}\right) = \frac{2}{3}$, then $P(B)$ is **[NCERT Part-II, Page 408]**

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{6}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{2}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.
(*b*) Both A and R are true but R is not the correct explanation of A.
(*c*) A is true but R is false.
(*d*) A is false but R is true.

19. Assertion (A): $\begin{bmatrix} -9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -7 \end{bmatrix}$ is a diagonal matrix. **[NCERT Part-I, Page 40]**

Reason (R) : If the elements of the principal diagonal of a square matrix are equal, it is called a scalar matrix.

20. Assertion (A) : $f(x) = x^2, x \in R$ is neither increasing nor decreasing in R. **[NCERT Part-I, Page 160]**

Reason (R) : $f(x) = x^2$, is increasing for $x > 0$ and decreasing for $x < 0$ in R.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Solve the following differential equation: $\frac{dy}{dx} = x^3 \operatorname{cosec} y$, given that $y(0) = 0$. **[NCERT Part-II, Page 306-307]**

22. Given two in dependent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$ find P(neither A nor B) without using $P(A \cap B)$.

OR

Given that E and F are events such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$. Find $P(\overline{E}/\overline{F})$. **[NCERT Part-II, Page 408-409]**

23. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1,1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric nor transitive. **[NCERT Part-I, Page 2]**

24. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i} - \hat{j} + \hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively. **[Conceptual Application]**

25. If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$. **[Conceptual Application]**

OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Find the intervals in which the function f given by $f(x) = \tan x - 4x, x \in \left(0, \frac{\pi}{2}\right)$ is
   (*i*) strictly increasing **[NCERT Part-I, Page 153]**
   (*ii*) strictly decreasing

27. If $y = e^{x \sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$. **[NCERT Part-I, Page 130]**

28. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x = 1$.
[NCERT Part-I, Page 118-119]

OR

If $x = a \sec\theta, y = b \tan\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$. **[NCERT Part-I, Page 134-135]**

29. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b$ is "divisible by 2"$\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e. [0].
[NCERT Part-I, Page 2, 4]

30. Find the area of the region bounded by the curves $x^2 + y^2 = 4, y = \sqrt{3}x$ and x-axis in the first quadrant.
[Conceptual Application]

OR

Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration. **[Conceptual Application]**

31. Find $\int \frac{x^2+1}{(x^2+2)(x^2+3)}dx$. **[NCERT Part-II, Page 252-253]**

OR

Find the general solution of the following differential equation: $x\,dy - (y + 2x^2)dx = 0$
[NCERT Part-II, Page 322-323]

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Hence solve the system of equations:

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

[NCERT Part-I, Page 94-95]

OR

Evaluate the product AB, where **[NCERT Part-I, Page 94-95]**

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Hence solve the system of linear equations

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7$$

33. Find the particular solution of the differential equation $\tan x.\dfrac{dy}{dx} = 2x\tan x + x^2 - y$; $(\tan x \neq 0)$ given that $y = 0$ when $x = \dfrac{\pi}{2}$. **[NCERT Part-II, Page 322-323]**

34. Find the shortest distance between the lines **[NCERT Part-II, Page 386-387]**

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

and $$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

If the lines intersect, find their point of intersection.

OR

Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\,.\,\vec{a} = 21$. **[Conceptual Application]**

35. Solve the following linear programming problem (LPP) graphically. **[NCERT Part-I, Page 397-398]**

Maximise $Z = x + 2y$

subject to constraints:

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay processes 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

[NCERT Part-II, Page 408, 424-425]

Based on above information, answer the questions given below.

(*i*) What is the conditional probability that an error is committed in processing given that Sonia processed the form?

(*ii*) What is the probability that Sonia processed the form and committed an error?

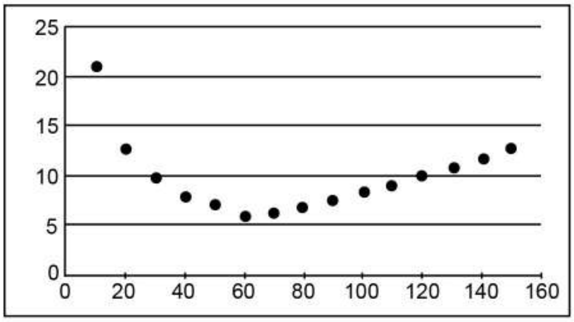
(*iii*) What is the total probability of committing an error in processing the form?

OR

(*iii*) If an error is found in the selected form, then what is the probability that it was processed by Vinay?

Case Study - 2

37. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h. **[Integrated Question]**



The relation between fuel consumption $F(l/100 \text{ km})$ and speed V (km/h) under some constraints is given as $F = \dfrac{V^2}{500} - \dfrac{V}{4} + 14$.

On the basis of the above information, answer the following questions:

(*i*) Find F, when $V = 40$ km/h.

(*ii*) Find $\dfrac{dF}{dV}$.

(*iii*) Find the speed V for which fuel consumption F is minimum.

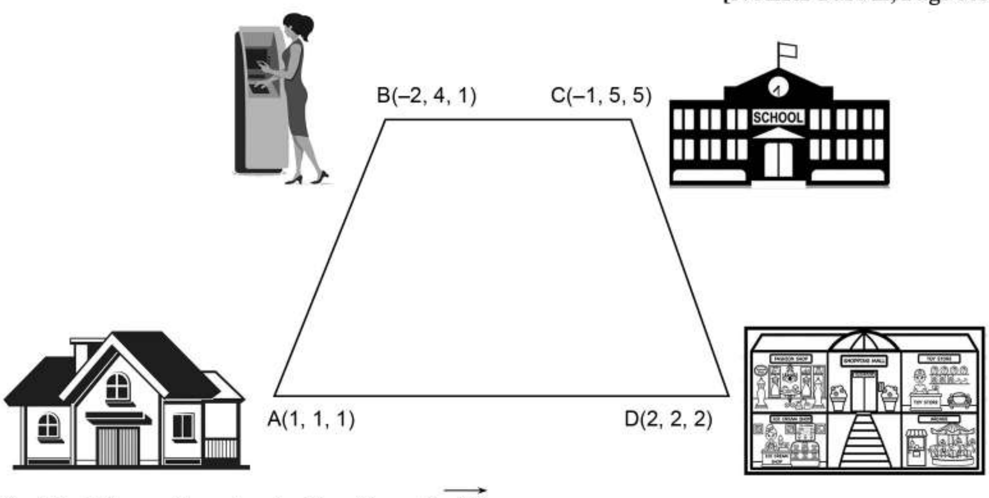
OR

(*iii*) Find the quantity of fuel required to travel 600 km at the speed V at which $\dfrac{dF}{dV} = -0{\cdot}01$.

Case Study - 3

38. Ritika starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to an ATM, from there to her daughter's school and then reaches the mall. In the diagram, A, B, C and D represent the coordinates of House, ATM, School and Mall respectively.

[NCERT Part-II, Page 339-340, 347]



(*i*) Find the unit vector in direction of $\overrightarrow{AB}$.

(*ii*) Find a vector of magnitude 6 in direction opposite to that of $\overrightarrow{CD}$.

SOLUTIONS

1. (*c*), We have $2\left[\sin^{-1}\left(\frac{-1}{2}\right)+\cos^{-1}\left(\frac{-1}{2}\right)\right] = 2[\theta_1 + \theta_2]$

Where $\theta_1 = \sin^{-1}\left(\frac{-1}{2}\right)$

$$\Rightarrow \quad \sin\theta_1 = \frac{-1}{2} = -\sin\frac{\pi}{6} = \left[\sin\left(-\frac{\pi}{6}\right)\right]$$

$$\Rightarrow \quad \theta_1 = \frac{-\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Also $\theta_2 = \cos^{-1}\left(-\frac{1}{2}\right)$

$$\Rightarrow \quad \cos\theta_2 = \frac{-1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$$

$$\Rightarrow \quad \theta_2 = \frac{2\pi}{3} \in [0, \pi]$$

$$\therefore \quad 2\left[\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)\right] = 2\left[-\frac{\pi}{6}+\frac{2\pi}{3}\right] = 2\left[\frac{-\pi+4\pi}{6}\right]$$

$$= 2\cdot\frac{\pi}{2} = \pi$$

2. (*a*), Here

$$\theta = \sin^{-1}\{\sin(-600°)\}$$
$$= \sin^{-1}\{-\sin(720° - 120°)\} \qquad \begin{bmatrix}\text{as } \sin(-600°) = -\sin 600° \\ \text{as } \sin(4\pi - \theta) = -\sin\theta\end{bmatrix}$$
$$= \sin^{-1}\{\sin 120°\} \neq 120°$$

So, $\sin^{-1}(\sin 120°) = \sin^{-1}[\sin(180° - 60°)] = \sin^{-1}(\sin 60°)$

$$= 60° = \frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \quad \theta = \frac{\pi}{3}$$

3. (*c*), *R* is reflexive only

as (1, 1), (2, 2), (3, 3) $\in R$.

but *R* is not symmetric

as (1, 2) $\in R$ but (2, 1) $\notin R$

R is not transitive.

As (1, 2) $\in R$ and (2, 3) $\in R$

but (1, 3) $\notin R$

$\Rightarrow$ *R* is reflexive, neither symmetric nor transitive.

4. (*c*), $\begin{bmatrix}15-x & x+1 \\ 2 & 4\end{bmatrix}$ is a singular matrix.

$$\therefore \quad \begin{vmatrix}15-x & x+1 \\ 2 & 4\end{vmatrix} = 0$$

$$\Rightarrow \quad 4(15-x) - 2(x+1) = 0$$

$$\Rightarrow \quad 60 - 4x - 2x - 2 = 0$$

$$\Rightarrow \quad 58 - 6x = 0$$

$$\Rightarrow \quad x = \frac{58}{6} = \frac{29}{3}$$

5. (b),
$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix} \Rightarrow |A| = \frac{a(1+bc)}{a} - bc$$
$$= 1 + bc - bc = 1$$

$\Rightarrow$ A^{-1} exists.

Now, $$\text{adj}\, A = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

$\Rightarrow$ $$A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = 1\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

6. (b), We have $$x\begin{bmatrix} 3 \\ 2 \end{bmatrix} + y\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 3x \\ 2x \end{bmatrix} + \begin{bmatrix} y \\ -y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$\Rightarrow$ $3x + y = 5$ ...(i)

and $2x - y = 10$ ...(ii)

On solving (i) and (ii), we get
$$3x + y = 5$$
$$2x - y = 10$$
$$5x = 15$$

$\Rightarrow$ $x = 3$

From (ii), $2x - 10 = y$

$\Rightarrow$ $6 - 10 = y$

$\Rightarrow$ $y = -4$

7. (a), as $y = \log x$

$\Rightarrow$ $$\frac{dy}{dx} = \frac{1}{x} \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

8. (d), as 'f' is not defined at $x = 0$. i.e. $f(0)$ does not exist.

9. (b), $x = t^2 \Rightarrow \frac{dx}{dt} = 2t$

$y = t^3 \Rightarrow \frac{dy}{dt} = 3t^2$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3t^2}{2t} = \frac{3}{2}t$

Now, $\frac{d^2y}{dx^2} = \frac{3}{2} \times \frac{dt}{dx} = \frac{3}{2} \times \frac{1}{2t} = \frac{3}{4t}$

10. (b), Let number x exceeds its square by the greatest possible number y. Then,
$$y = x - x^2$$
$$y' = 1 - 2x,$$

For maximum y, $y' = 0 \Rightarrow 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$
$$y'' = -2$$

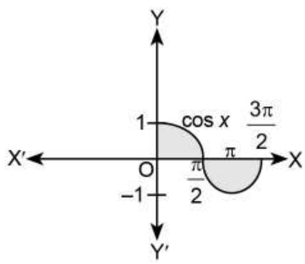
$\therefore \quad y''\Big]_{x=\frac{1}{2}} = -2 < 0$

$\therefore$ y is maximum for $x = \dfrac{1}{2}$.

Hence, the number is $\dfrac{1}{2}$.

11. (c)

12. (a), Required area $= \int_0^{\frac{\pi}{2}} \cos x\, dx + \int_{\frac{\pi}{2}}^{\pi} (-\cos x)\, dx$



$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}$

$= \left(\sin \dfrac{\pi}{2} - \sin 0\right) - \left(\sin \pi - \sin \dfrac{\pi}{2}\right)$

$= (1 - 0) - (0 - 1)$

$= 2$ sq units.

13. (a), $\displaystyle\int \frac{dx}{1 + \cos x}$

$= \displaystyle\int \left(\frac{1 - \cos x}{\sin^2 x}\right) dx$

$= \displaystyle\int [\text{cosec}^2 x - \cot x \,.\, \text{cosec}\, x]\, dx$

$= -\cot x + \text{cosec}\, x + C$

$= \dfrac{-\cos x}{\sin x} + \dfrac{1}{\sin x} + C$

$= \dfrac{2 \sin^2\left(\frac{x}{2}\right)}{2 \sin\left(\frac{x}{2}\right) . \cos\left(\frac{x}{2}\right)} + C = \tan\left(\dfrac{x}{2}\right) + C$

14. (a), $\overrightarrow{DB}$ = Position vector of B – position vector of $D = \vec{b} - (\vec{a} - 2\vec{b}) = 3\vec{b} - \vec{a}$

$\overrightarrow{AC}$ = Position vector of C – position vector of $A = 2\vec{a} + 3\vec{b} - \vec{a} = \vec{a} + 3\vec{b}$

15. (b), Let $f(x) = \sin^3 x \,.\, \cos x$

$\Rightarrow f(-x) = \sin^3(-x) \,.\, \cos(-x)$

$= -\sin^3 x \,.\, \cos x$

$= -f(x)$

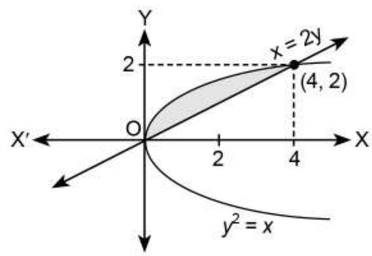
So, f is an odd function.

$\therefore \displaystyle\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)\, dx = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \,.\, \cos dx = 0$

16. (a), Eliminating 'x' from $y^2 = x$ and $x = 2y$, we get $y^2 - 2y = 0 \Rightarrow y = 0, 2$

When $y = 0, x = 0$ and when $y = 2, x = 4$

$\therefore$ Point of intersection are (0, 0) and (4, 2)



Required area $= \displaystyle\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx$

$= \left[\dfrac{2}{3} x^{3/2} - \dfrac{x^2}{4}\right]_0^4$

$= \left(\dfrac{2}{3} \times 8 - 4\right) - 0 = \dfrac{4}{3}$ sq. units

17. (a),

18. (*c*), $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1}{2}, P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{2}{3}$

$\Rightarrow \frac{1}{2}P(B) = \frac{2}{3}P(A)$

$\Rightarrow \quad P(B) = \frac{2}{3} \times \frac{1}{4} \times 2$

$= \frac{1}{3}$

19. (*c*), Assertion is true but the reason is false.

20. (*a*), Both the assertion and the reason are correct and the reason is the correct explanation of the assertion.

21.

$$\frac{dy}{dx} = x^3 \operatorname{cosec} y \,;\, y(0) = 0$$

$$\Rightarrow \quad \int \frac{dy}{\operatorname{cosec} y} = \int x^3 dx$$

$$\Rightarrow \quad \int \sin y \, dy = \int x^3 dx$$

$$\Rightarrow \quad -\cos y = \frac{x^4}{4} + C \qquad ...(i)$$

Given $y = 0$, when $x = 0$

$$\Rightarrow \quad -1 = C$$

$$\cos y = 1 - \frac{x^4}{4} \qquad \text{[from } (i)\text{]}$$

22. As A and B are independent events

$$\begin{aligned} \therefore \quad P(\text{neither } A \text{ nor } B) &= P(\overline{A} \cap \overline{B}) \\ &= P(\overline{A})P(\overline{B}) \\ &= [1 - P(A)][1 - P(B)] \\ &= [1 - 0.3][1 - 0.6] \\ &= 0.7 \times 0.4 = 0.28. \end{aligned}$$

OR

$$P(\overline{E}/\overline{F}) = \frac{P(\overline{E} \cap \overline{F})}{\mathrm{P}(\overline{F})} = \frac{1 - P(E \cup F)}{1 - P(F)} \qquad ...(i)$$

Now

$$\begin{aligned} P(E \cup F) &= P(E) + P(F) - P(E \cap F) \\ &= 0.8 + 0.7 - 0.6 = 0.9 \end{aligned}$$

Substituting value of $P(E \cup F)$ in (*i*), we get

$$P(\overline{E}/\overline{F}) = \frac{1 - 0.9}{1 - 0.7} = \frac{0.1}{0.3} = \frac{1}{3}$$

23. Given $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ defined on $R : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$

For reflexive: As $(1, 1), (2,2), (3, 3) \in R$.

Hence, reflexive

For symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$.

Hence, not symmetric.

For transitive: $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$. Hence, not transitive.

24. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{d} = 4\hat{i} + 5\hat{k}$

$$\because \quad \vec{a} + \vec{b} = \vec{d}$$

$$\Rightarrow \quad \vec{b} = \vec{d} - \vec{a} = 4\hat{i} + 5\hat{k} - \hat{i} + \hat{j} - \hat{k} = 3\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = -5\hat{i} - \hat{j} + 4\hat{k}$$

Area of parallelogram = $|\vec{a} \times \vec{b}| = \sqrt{25+1+16} = \sqrt{42}$ sq units

25.

$$A^2 = 2A$$

$$\Rightarrow \quad |AA| = |2A|$$

$$\Rightarrow \quad |A||A| = 8|A| \qquad (\because |AB| = |A||B| \text{ and } |2A| = 2^3|A|)$$

$$\Rightarrow \quad |A|(|A| - 8) = 0$$

$$\Rightarrow \quad |A| = 0 \text{ or } 8$$

OR

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, \; 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow \quad A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$$\Rightarrow \quad A^{-1}(A^2 - 5A + 7I) = A^{-1}O$$

$$\Rightarrow \quad A - 5I + 7A^{-1} = O$$

$$\Rightarrow \quad 7A^{-1} = 5I - A$$

$$\Rightarrow \quad A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right) = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

26.

$$f(x) = \tan x - 4x$$

$$f'(x) = \sec^2 x - 4$$

(*i*) For $f(x)$ to be strictly increasing,

$$f'(x) > 0$$

$$\Rightarrow \quad \sec^2 x - 4 > 0$$

$$\Rightarrow \quad \sec^2 x > 4$$

$$\Rightarrow \quad \cos^2 x < \frac{1}{4} \Rightarrow \cos^2 x < \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad -\frac{1}{2} < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2} \qquad \left[\because \; x \in \left(0, \frac{\pi}{2}\right)\right]$$

So, f is strictly ↑ing on $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

(*ii*) For $f(x)$ to be strictly decreasing,

$$f'(x) < 0$$

$$\Rightarrow \quad \sec^2 x - 4 < 0$$

$$\Rightarrow \quad \sec^2 x < 4$$

$$\Rightarrow \quad \cos^2 x > \frac{1}{4}$$

$$\Rightarrow \quad \cos^2 x > \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \cos x > \frac{1}{2} \text{ or } \cos x < \frac{-1}{2}$$

Now, $$\cos x > \frac{1}{2} \qquad \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$$

$$\Rightarrow \quad 0 < x < \frac{\pi}{3}$$

So, f is strictly $\downarrow$ing on $\left(0, \frac{\pi}{3}\right)$.

27. Let $u = e^{x\sin^2 x}$ and $v = (\sin x)^x$

So that $$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(i)$$

Now, $$u = e^{x\sin^2 x}$$

$$\Rightarrow \quad \frac{du}{dx} = e^{x\sin^2 x}[x(2\sin x \cos x) + \sin^2 x \cdot 1]$$
$$= e^{x\sin^2 x}[x\sin 2x + \sin^2 x] \qquad ...(ii)$$

Also, $$v = (\sin x)^x$$

$$\Rightarrow \quad \log v = x \log(\sin x)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v}\frac{dv}{dx} = x\cot x + \log(\sin x)$$

$$\frac{dv}{dx} = (\sin x)^x[x\cot x + \log(\sin x)] \qquad ...(iii)$$

Substituting from (ii), (iii) in (i), we get

$$\frac{dy}{dx} = e^{x\sin^2 x}[x\sin 2x + \sin^2 x] + (\sin x)^x[x\cot x + \log(\sin x)]$$

28. $$\text{RHD} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{[1+h]-[1]}{h}$$

$$= \lim_{h \to 0} \frac{(1-1)}{h} = 0$$

$$\text{LHD} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{[1-h]-[1]}{-h} = \lim_{h \to 0} \frac{0-1}{-h}$$

$$= \lim_{h \to 0} \frac{1}{h} \to \infty$$

Since, $$\text{RHD} \neq \text{LHD}$$

Therefore $f(x)$ is not differentiable at $x = 1$.

OR

$$x = a\sec\theta \Rightarrow \frac{dx}{d\theta} = a\sec\theta\tan\theta \qquad ...(i)$$

$$y = b\tan\theta \Rightarrow \frac{dy}{d\theta} = b\sec^2\theta \qquad ...(ii)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} = \frac{b}{a}\text{cosec }\theta \qquad [\text{from}(i) \text{ and } (ii)]$$

Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-b}{a}\text{cosec}\,\theta\cot\theta \times \frac{d\theta}{dx}$$

$$= \frac{-b}{a}\text{cosec}\,\theta\cot\theta \times \frac{1}{a\sec\theta\tan\theta} \quad \text{[using } (i)\text{]}$$

$$= \frac{-b}{a^2}\cot^3\theta$$

$$\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{6}} = \frac{-b}{a^2}\left[\cot\frac{\pi}{6}\right]^3 = \frac{-b}{a^2}(\sqrt{3})^3 = -\frac{3\sqrt{3}b}{a^2}$$

29. For reflexive: Let $a \in Z$.

Since, $a + a = 2a$ which is even and hence divisible by 2.

$\therefore$ $(a, a) \in R\ \forall a \in Z$

Hence, R is reflexive.

For symmetric: Let $a, b \in Z$. If $(a, b) \in R$, then $a + b = 2\lambda \Rightarrow b + a = 2\lambda$

$\Rightarrow (b, a) \in R$. Hence, R is symmetric.

For transitive: Let $a, b, c \in Z$.

If $(a, b) \in R$ and $(b, c) \in R$

then $a + b = 2\lambda$ ...(i)

and $b + c = 2\mu$ ...(ii)

Adding (i) and (ii) we get

$$a + 2b + c = 2(\lambda + \mu)$$

$\Rightarrow$ $a + c = 2(\lambda + \mu - b)$

$\Rightarrow$ $a + c = 2k$, where $\lambda + \mu - b = k \Rightarrow (a, c) \in R$

Hence, R is transitive

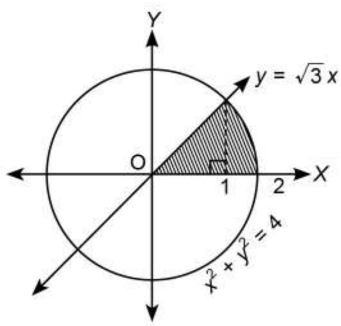
Equivalence class containing $0 = \{(a, 0) \in R : a \in Z\}$

$\therefore$ $[0] = \{... -4, -2, 0, 2, 4 ...\}$

30. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get $x^2 + 3x^2 = 4$

$\Rightarrow$ $x^2 = 1$

$\Rightarrow$ $x = \pm 1$



$$\text{Required area} = \sqrt{3}\int_0^1 x\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx$$

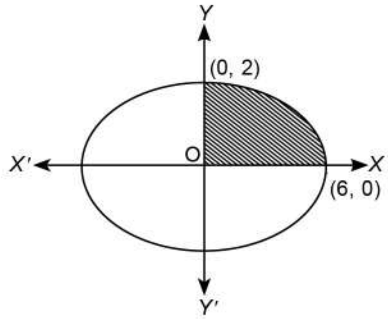
$$= \sqrt{3}\left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$$

$$= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6}\right]$$

$$= \frac{2\pi}{3} \text{ sq units.}$$

OR

Curve is symmetrical to both the axes.



$$\text{Required area} = 4\int_0^6 y\,dx = 4\times\frac{1}{3}\int_0^6 \sqrt{36-x^2}\,dx$$

$$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2-x^2}+18\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

$$= \frac{4}{3}\left[18\times\frac{\pi}{2}-0\right] = 12\pi \text{ sq units}$$

31. Putting $x^2 = y$ to make partial fractions.

$$\frac{x^2+1}{(x^2+2)(x^2+3)} = \frac{y+1}{(y+2)(y+3)} = \frac{A}{y+2}+\frac{B}{y+3}$$

$$\Rightarrow \quad y+1 = A(y+3)+B(y+2)$$

Comparing coefficients of y and constant terms on both sides we get

$$A + B = 1 \text{ and } 3A + 2B = 1$$

On solving, we get $A = -1,\ B = 2.$

$$\int\frac{x^2+1}{(x^2+2)(x^2+3)}dx = \int\frac{-1}{x^2+2}dx+2\int\frac{1}{x^2+3}dx$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+C$$

OR

The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y+2x^2}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}-\frac{1}{x}y = 2x$$

Here $$P = -\frac{1}{x},\ Q = 2x$$

$$\text{IF} = e^{\int P dx} = e^{-\int\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$$

The solution is:

$$\Rightarrow \quad y\times\frac{1}{x} = \int\left(2x\times\frac{1}{x}\right)dx$$

$$\Rightarrow \quad \frac{y}{x} = 2x + C$$

$$\Rightarrow \quad y = 2x^2 + Cx$$

32.

$$|A| = 1(-1-2)-2(-2-0)+0 = -3+4 = 1$$

$$|A| \neq 0. \text{ Hence, } A^{-1} \text{ exists}$$

Let A_{ij} be the cofactor of each element in $|A|$

$$A_{11} = -3,\ A_{12} = +2,\ A_{13} = 2$$

$$A_{21} = -2,\ A_{22} = 1,\quad A_{23} = +1$$

$$A_{31} = -4,\ A_{32} = +2,\ A_{33} = +3$$

$$\therefore \qquad \text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}'$$

$$\text{Adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{Adj } A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

The given equations can be written in matrix form as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix},$$

Which is of the form $A'X = B$

$$\Rightarrow \qquad X = (A')^{-1}\text{B} = (A^{-1})'B$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \qquad x = 0,\quad y = -5,\quad z = -3$$

OR

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow \qquad AB = 6I$$

$$\Rightarrow \qquad A\left(\frac{1}{6}B\right) = I \Rightarrow A^{-1} = \frac{1}{6}(B)$$

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = C \text{ where } C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \qquad X = A^{-1}C = \frac{1}{6}BC$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$x = 2, \quad y = -1, \quad z = 4.$

33. Consider equation $\tan x . \frac{dy}{dx} = 2x \tan x + x^2 - y$

$$\Rightarrow \quad \tan x . \frac{dy}{dx} + y = 2x \tan x + x^2$$

$$\Rightarrow \quad \frac{dy}{dx} + \cot x \cdot y = (2x \tan x + x^2) \cot x$$

Here $P(x) = \cot x, Q(x) = (2x \tan x + x^2) \cot x$

$$\text{Integrating factor (I.F.)} = e^{\int \cot x \, dx} = e^{\log|\sin x|} = \sin x$$

$$\therefore \quad \text{Solution is (I.F.)}y = \int (\text{I.F.}) Q(x) dx$$

$$\Rightarrow \quad (\sin x.)y = \int \sin x (2x \tan x + x^2).\cot x \, dx$$

$$= \int (2x \sin x + x^2 \cos x) dx$$

$$= \int 2x \sin x \, dx + \int \underset{①}{x^2} \underset{②}{\cos x} \, dx$$

$$= \int 2x \sin x \, dx + x^2 \cdot \sin x - \int 2x \cdot \sin x \, dx$$

$$(\sin x.)y = x^2 \sin x + C \qquad \text{...}(i)$$

Given $y = 0$, when $x = \frac{\pi}{2}$

$$0 = \frac{\pi^2}{4} \cdot \sin \frac{\pi}{2} + C \Rightarrow C = -\frac{\pi^2}{4}$$

Substituting in (*i*), we get

$$y \sin x = x^2 \sin x - \frac{\pi^2}{4} \text{ is required solution.}$$

34. Line is $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

Here, $\vec{a_1} = 3\hat{i} + 2\hat{j} - 4\hat{k}$, and $\vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}$

and the other line is, $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Here, $\vec{a_2} = 5i - 2j$ and $\vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$\vec{a_2} - \vec{a_1} = 5\hat{i} - 2\hat{j} - 3\hat{i} - 2\hat{j} + 4\hat{k} = 2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{i}(12-4) - \hat{j}(6-6) + \hat{k}(2-6)$$

$$= 8\hat{i} - 4\hat{k}$$

So, $\left(\vec{b_1} \times \vec{b_2}\right).\left(\vec{a_2} - \vec{a_1}\right) = (8\hat{i} - 4\hat{k}) . (2\hat{i} - 4\hat{j} + 4\hat{k}) = 16 - 16 = 0$

$$S.D. = \left| \frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| = 0$$

So, the given lines are intersecting as the shortest distance between the lines is 0.

Now for point of intersection,

$$3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$\Rightarrow \quad 3 + \lambda = 5 + 3\mu \qquad \text{...}(i)$$

$$2 + 2\lambda = -2 + 2\mu \quad ...(ii)$$
$$-4 + 2\lambda = 6\mu \quad ...(iii)$$

Solving (i) and (ii), we get, $\mu = -2$ and $\lambda = -4$

Substituting in equation of line, we get position vector of point of intersection as,

$$\vec{r} = 5\hat{i} - 2\hat{j} + (-2)(3\hat{i} + 2\hat{j} + 6\hat{k})$$
$$= -\hat{i} - 6\hat{j} - 12\hat{k}$$

Point of intersection is $(-1, -6, -12)$.

OR

Given $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$

$\vec{d}$ is perpendicular to $\vec{c}$ and $\vec{b}$

$\Rightarrow \vec{d}$ is parallel to vector $\vec{c} \times \vec{b}$

$$\Rightarrow \quad \vec{d} = \lambda(\vec{c} \times \vec{b}) \quad ...(i)$$

$$\vec{c} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix} = \hat{i}(1) - \hat{j}(16) + \hat{k}(-13)$$
$$= \hat{i} - 16\hat{j} - 13\hat{k}$$

$$\therefore \quad \vec{d} = \lambda(\hat{i} - 16\hat{j} - 13\hat{k}) \quad \text{[from } (i)] ...(ii)$$

Also $\quad \vec{d} \cdot \vec{a} = 21 \Rightarrow 4(\lambda) + 5(-16\lambda) - 1(-13\lambda) = 21$

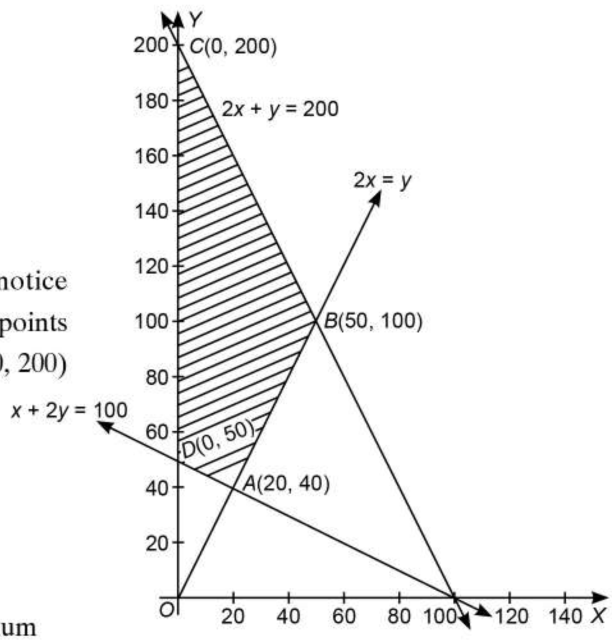
$$\Rightarrow \quad 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow -63\lambda = 21 \Rightarrow \lambda = -\frac{1}{3}$$

From (ii), we get $\vec{d} = \left(-\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}\right)$.

35. To maximise $Z = x + 2y$

subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$



On plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for maximum Z are $A(20, 40)$, $B(50, 100)$, $C(0, 200)$ and $D(0, 50)$.

Points	$Z = x + 2y$	Values	
$A(20, 40)$	$20 + 80$	100	
$B(50, 100)$	$50 + 200$	250	
$C(0, 200)$	$0 + 400$	400	← Maximum
$D(0, 50)$	$0 + 100$	100	

$\therefore$ Z is maximum for $C(0, 200)$, i.e. $x = 0, y = 200$.

36. Consider the following events:

V : form is processed by Vinay

S : form is processed by Sonia

I : form is processed by Iqbal

E : form has an error.

(*i*) $P(E/S) = 0.04$

(*ii*) Required probability $= P(S) \times P\left(\frac{E}{S}\right) = \frac{20}{100} \times 0.04 = 0.008$

(*iii*)
$$P(E) = P(S) \times P\left(\frac{E}{S}\right) + P(V) \times P\left(\frac{E}{V}\right) + P(I) \times P\left(\frac{E}{I}\right)$$
$$= \frac{20}{100} \times 0.04 + \frac{50}{100} \times 0.06 + \frac{30}{100} \times 0.03$$
$$= 0.008 + 0.03 + 0.009 = 0.047$$

OR

(*iii*) $$P\left(\frac{V}{E}\right) = \frac{P(V) \times P\left(\frac{E}{V}\right)}{P(E)} = \frac{\frac{50}{100} \times 0.06}{0.047} = \frac{0.03}{0.047} = \frac{30}{47}$$

37. (*i*)
$$F = \frac{V^2}{500} - \frac{V}{4} + 14$$

Put $V = 40$, we get

$$F = \frac{(40)^2}{500} - \frac{40}{4} + 14$$

$\Rightarrow$ $$F = 7.2\ l/100 \text{ km}$$

(*ii*)
$$\frac{dF}{dV} = \frac{2V}{500} - \frac{1}{4} + 0$$

$\Rightarrow$ $$\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}$$

(*iii*) For minimum fuel consumption,

$$\frac{dF}{dV} = 0$$

$\Rightarrow$ $$\frac{V}{250} - \frac{1}{4} = 0$$

$\Rightarrow$ $$V = \frac{250}{4} = 62.5 \text{ km/h}$$

$$\frac{d^2F}{dV^2} = \frac{1}{250}$$

$\Rightarrow$ $$\left[\frac{d^2F}{dV^2}\right]_{V=62.5} = \frac{1}{250} > 0$$

$\therefore$ F is minimum at $V = 62.5$ km/h

OR

(*iii*) $$\frac{dF}{dV} = -0.01$$

$$\Rightarrow \quad \frac{V}{250} - \frac{1}{4} = \frac{-1}{100}$$

$$\Rightarrow \quad V = 60 \text{ km/h}$$

Now, $$F = \frac{60 \times 60}{500} - \frac{60}{4} + 14$$

$$\Rightarrow \quad F = 7.2 - 15 + 14$$

$$\Rightarrow \quad F = 6.2\, l/100 \text{ km}$$

$\therefore$ Quantity of fuel consumed to travel 100 km at 60 km/h = $6.2\, l$

So, quantity of fuel consumed to travel 600 km at 60 km/h = $6.2 \times 6 = 37.2\, l$

38. (*i*) Position vector of $B = -2\hat{i} + 4\hat{j} + \hat{k}$

Position vector of $A = \hat{i} + \hat{j} + \hat{k}$

Now, $$\overrightarrow{AB} = P.V. \text{ of } B - P.V. \text{ of } A = -2\hat{i} + 4\hat{j} + \hat{k} - \hat{i} - \hat{j} - \hat{k} = -3\hat{i} + 3\hat{j}$$

Unit vector in direction of $\overrightarrow{AB} = \widehat{AB} = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \dfrac{-3\hat{i} + 3\hat{j}}{\sqrt{9+9}} = \dfrac{-3}{3\sqrt{2}}(\hat{i} - \hat{j}) = \dfrac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$

(*ii*) $$\overrightarrow{CD} = P.V. \text{ of } D - P.V. \text{ of } C$$

$$= (2\hat{i} + 2\hat{j} + 2\hat{k}) - (-\hat{i} + 5\hat{j} + 5\hat{k}) = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\overrightarrow{DC} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

Now, $\widehat{DC} = \dfrac{\overrightarrow{DC}}{|\overrightarrow{DC}|} = \dfrac{3(-\hat{i} + \hat{j} + \hat{k})}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}(-\hat{i} + \hat{j} + \hat{k})$

Required vector = $6\,\widehat{DC} = 2\sqrt{3}\,(-\hat{i} + \hat{j} + \hat{k})$