16 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. Let $S = \{1, 2, 3, 4, 5\}$ and let $A = S \times S$. **[NCERT Part-I, Page 2]**

Define the relation R on A as follows $(a, b)\ R\ (c, d)$ if $ad = bc$. Then R is

(*a*) not reflexive (*b*) not symmetric

(*c*) not transitive (*d*) an equivalence relation

2. Which of these is an equivalence relation on $\{0, 1, 2\}$? **[NCERT Part-I, Page 2]**

(*a*) $\{(0, 0), (1, 1), (0, 1), (1, 0)\}$

(*b*) $\{(0, 0), (1, 1), (2, 2), (0, 1), (1, 2)\}$

(*c*) $\{(0, 0), (1, 1), (2, 2), (0, 1), (1, 2), (1, 0), (2, 1)\}$

(*d*) $\{(0, 0), (1, 1), (2, 2)\}$

3. If $x \in R$, then $f(x) = |x - 1|$ is **[NCERT Part-I, Page 105, 118-119]**

(*a*) differentiable at $x = 1$ (*b*) not differentiable at $x = 1$

(*c*) not continuous at $x = 1$ (*d*) None of these

4. The matrix $\begin{bmatrix} 4+3k & 3 \\ 1+2k & 2 \end{bmatrix}$ is singular matrix, for k equal to **[NCERT Part-I, Page 89]**

(*a*) 0 (*b*) –1 (*c*) 1 (*d*) no value of k

5. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then adj A is **[NCERT Part-I, Page 88]**

(*a*) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (*b*) $\begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ (*c*) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (*d*) $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

6. If $\begin{bmatrix} -3+x & 14y-p \\ z+3x & -16 \end{bmatrix} = \begin{bmatrix} 14 & 30 \\ y & p \end{bmatrix}$ then matrix $\begin{bmatrix} x & y \\ z & p \end{bmatrix}$ is **[NCERT Part-I, Page 41]**

(*a*) $\begin{bmatrix} 14 & 30 \\ 1 & 16 \end{bmatrix}$ (*b*) $\begin{bmatrix} 14 & 1 \\ 30 & 16 \end{bmatrix}$ (*c*) $\begin{bmatrix} 17 & 1 \\ -50 & -16 \end{bmatrix}$ (*d*) $\begin{bmatrix} 14 & 30 \\ -6 & 1 \end{bmatrix}$

7. The point on the curve $y = x^2$, where the rate of change of x-coordinate is equal to the rate of change of y-coordinate is **[NCERT Part-I, Page 147-148]**

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (*d*) (1, 1)

8. If $\sin^{-1}x + \sin^{-1}y = \pi$, then values of x and y are **[NCERT Part-I, Page 27]**

(*a*) 1, 2 (*b*) 1, 1 (*c*) 2, 1 (*d*) 2, 2

9. The function $f(x) = x^2$ is differentiable at $x = 1$, then value of $f'(1) =$ **[NCERT Part-I, Page 118-119]**

(*a*) –1 (*b*) 1 (*c*) 2 (*d*) – 2

10. The area bounded by the line $y = 2x - 2$, $y = -x$ and the x-axis is given by **[Conceptual Application]**

(*a*) $\frac{9}{2}$ sq units (*b*) $\frac{43}{6}$ sq units (*c*) $\frac{35}{6}$ sq units (*d*) None of these

11. The area cut off from the parabola $4y = 3x^2$ by the line $2y = 3x + 12$ is **[Conceptual Application]**

(*a*) 29 sq units (*b*) 28 sq units (*c*) 27 sq units (*d*) 26 sq units

12. If $\int \frac{2^x}{\sqrt{1-4^x}}\,dx = p.\sin^{-1}(2^x) + C$, then '$p$' is equal to **[NCERT Part-II, Page 235-236]**

(*a*) $\log_e 2$ (*b*) $\frac{1}{2}\log_e 2$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\log_e 2}$

13. $\int \frac{x + \cos 6x}{3x^2 + \sin 6x}\,dx$ is equal to **[NCERT Part-II, Page 235-236]**

(*a*) $3x^2 + \sin 6x + C$ (*b*) $1 - 6\sin 6x + C$

(c) $\log|3x^2 + \sin 6x| + C$ (*d*) $\frac{1}{6}\log|3x^2 + \sin 6x| + C$

14. $\int \frac{xe^x}{(1+x)^2}\,dx$ is equal to **[NCERT Part-II, Page 262]**

(*a*) $\frac{e^x}{x+1} + C$ (*b*) $e^x(x + 1) + C$ (*c*) $-\frac{e^x}{(x+1)^2} + C$ (*d*) $\frac{e^x}{1+x^2} + C$

15. If p and q are degree and order of a differential equation $\frac{dy}{dx} + \frac{1}{\frac{dy}{dx}} = 9$, then $2p + q$ is **[NCERT Part-II, Page 301-302]**

(*a*) 5 (*b*) 4 (*c*) 3 (*d*) 7

16. The integrating factor of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ is **[NCERT Part-II, Page 322-323]**

(*a*) x (*b*) $1 + x^2$ (*c*) x^2 (*d*) $x^2 + 2$

17. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals

[Conceptual Application]

(*a*) $\frac{4}{15}$ (*b*) $\frac{8}{45}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{2}{9}$

18. If for non zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b}$ is a unit vector and $|\vec{a}| = |\vec{b}| = \sqrt{2}$, then angle θ between vectors $\vec{a}$ and $\vec{b}$ is **[NCERT Part-II, Page 363]**

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{6}$ (*d*) $-\frac{\pi}{2}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.

(*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.

(*c*) *A* is true but *R* is false.

(*d*) *A* is false but *R* is true.

19. **Assertion (*A*):** If for the function $f(x) = \lambda x^2 - 5x + 5$, $f'(1) = 5$, then value of λ is 10.

Reason (*R*): If a function is differentiable at a point, then it is necessarily continuous at that point.

[NCERT Part-II, Page 118-120]

20. **Assertion (*A*):** Two dice are rolled and if it is given that the sum of the numbers on both dice is greater than 6, then probability of getting a doublet is $\frac{3}{16}$. **[NCERT Part-II, Page 408]**

Reason (*R*): Probability of an event E when F is given is $P(E/F) = \frac{P(E \cap F)}{P(F)}$

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the cartesian and vector equations of a line passing through the points (3, 4, 1) and (5, 1, 6).

[NCERT Part-II, Page 381-382]

22. A pair of dice is thrown. Find the probability of getting 9 as the sum, if it is known that second dice always exhibits a prime number. **[NCERT Part-II, Page 408]**

OR

There are 2000 scooter drivers, 4000 car drivers and 6000 truck drivers all insured. The probabilities of an accident involving a scooter, a car, a truck are 0.01, 0.03, 0.15 respectively. One of the insured drivers meets with an accident. What is the probability that he is a truck driver? **[NCERT Part-II, Page 425]**

23. Solve the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$ **[NCERT Part-II, Page 312-314]**

24. Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ **[NCERT Part-I, Page 19]**

25. If A and B are symmetric matrices, show that $AB + BA$ is symmetric and $AB - BA$ is skew symmetric.

[NCERT Part-I, Page 63]

OR

Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$. **[NCERT Part-II, Page 46-47]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Show that $y = \log(1 + x) - \dfrac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

[NCERT Part-I, Page 153]

27. If $x^p y^q = (x + y)^{p+q}$, then show that (*i*) $\dfrac{dy}{dx}=\dfrac{y}{x}$ and (*ii*) $\dfrac{d^2y}{dx^2} = 0$. **[NCERT Part-I, Page 130, 137]**

28. Evaluate: $\displaystyle\int\frac{5x^2}{(x^2+4)(x^2+9)}dx$ **[NCERT Part-II, Page 252-253]**

OR

Evaluate $\displaystyle\int\frac{x^3}{x^4+3x^2+2}\,dx$ **[NCERT Part-II, Page 235-236, 246]**

29. Show that the relation R in R defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric. **[NCERT Part-I, Page 2]**

30. Evaluate: $\displaystyle\int_0^{\frac{3}{2}}|x\sin\pi x|\,dx$ **[Integrated Question]**

OR

Find the particular solution of the differential equation $\dfrac{dy}{dx} - 3y\cot x = 2\sin 2x$, given that $y = 2$ when $x = \dfrac{\pi}{2}$. **[NCERT Part-II, Page 322-323]**

31. If $f(x) = \begin{cases}\dfrac{1-\cos 4x}{x^2}, & \text{when } x<0\\ a, & \text{when } x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{when } x>0\end{cases}$ is continuous at $x = 0$, find the value of a.

[NCERT Part-I, Page 105]

OR

If $y = (\sin^{-1}x)^2$, then prove that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 2 = 0$. **[NCERT Part-II, Page 124, 137]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the system of linear equations:

$$x - y + z = -4$$
$$x - 2y - 2z = -9$$
$$2x + y + 3z = -1$$

[NCERT Part-I, Page 94]

OR

Express $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrix. **[NCERT Part-I, Page 63-65]**

33. Find the shortest distance between the following pair of lines and hence write whether the lines are intersecting or not: $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$ **[NCERT Part-II, Page 386-387]**

OR

If $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form of $\vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1} \parallel \vec{a}$ and $\vec{\beta_2} \perp \vec{a}$. **[Conceptual Application]**

34. Find $\int \sqrt{\tan x}\, dx$ **[NCERT Part-II, Page 235-236]**

35. Solve the following LPP graphically: **[NCERT Part-II, Page 397-398]**

Maximise $Z = 5x + 3y$

subject to the constraints:

$x \geq 0,\ y \geq 0$

$2x + y \leq 12$

$3x + 2y \leq 20$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. A husband and a wife appear for an interview, for two vacancies for the same post. Out of 7 male candidates, only one can be selected and out of 5 female candidates, only one can be selected. Assume both the events "husband's selection" and "wife's selection" as independent events.

(*i*) Find the probability of husband's selection. **[Conceptual Application]**

(*ii*) Find the probability of wife's selection.

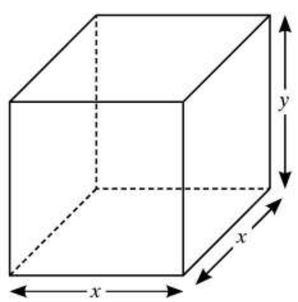
(*iii*) What is the probability that both will be selected?

OR

(*iii*) What is the probability that only one of them will be selected?

Case Study - 2

37. A given metal sheet is to be cast into an open tank with a square base and vertical sides as shown, where x be the side of base square and y be the vertical side. **[Conceptual Application]**



(*i*) If V represents the fixed volume of tank, then find the relation between V, x and y.

(*ii*) Express the total surface area of tank as function of x.

(*iii*) Find the value of x if total surface area of tank is minimum.

OR

(*iii*) Represent the relation in y and x if surface area is minimum.

Case Study - 3

38. Ginni purchased an air plant holder which is in the shape of a tetrahedron.

[NCERT Part-II, Page 339-340]

Let A, B, C and D are the coordinates of the air plant holder where $A \equiv (1, 1, 1)$, $B \equiv (2, 1, 3)$, $C \equiv (3, 2, 2)$ and $D \equiv (3, 3, 4)$.

(*i*) Find the vector $\overrightarrow{AB}$.

(*ii*) Find the vector $\overrightarrow{AC}$.

SOLUTIONS

1. (*d*), Let $a, b \in A$ s.t. $(a, b) \in R$

Now, $(a, b) \in R$

$\Rightarrow ab = ba$, true

So, $(a, b)\, R\, (a, b)$. So R is reflexive.

Let $(a, b), (c, d) \in S \times S$ s.t. $(a, b)\, R\, (c, d)$

Now, $(a, b)\, R\, (c, d)$

$$\Rightarrow \quad ad = bc$$
$$\Rightarrow \quad bc = ad$$
$$\Rightarrow \quad cb = da$$

$\Rightarrow (c, d)\, R\, (a, b)$

$\Rightarrow$ R is symmetric.

Let $(a, b), (c, d), (e, f) \in S \times S$ s.t. $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

Now, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$,

$$\Rightarrow \quad ad = bc \text{ and } cf = de$$
$$\Rightarrow \quad \frac{a}{b} = \frac{c}{d} \text{ and } \frac{c}{d} = \frac{e}{f}$$
$$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$$
$$\Rightarrow \quad af = be$$
$$\Rightarrow \quad (a, b)\, R\, (e, f)$$

So, R is transitive.

$\Rightarrow R$ is an equivalence relation.

2. (*d*), as (*a*) R is not reflexive as $(2, 2) \notin R$.

(*b*) R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$.

(*c*) R is not transitive as $(0, 1), (1, 2) \in R$ but $(0, 2) \notin R$.

(*d*) R is reflexive, symmetric and transitive. $\therefore$ R is an equivalence relation.

3. (*b*), Consider left hand derivative of $f(x)$ at $x = 1$

$$\text{LHD} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$
$$= \lim_{h \to 0} \frac{|(1-h) - 1| - |1 - 1|}{-h}$$
$$= \lim_{h \to 0} \frac{|-h| - |0|}{-h} = \lim_{h \to 0} \frac{+h}{-h} = -1$$

Consider right hand derivative of $f(x)$ at $x = 1$

$$\text{RHD} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0} \frac{|(1+h) - 1| - |1 - 1|}{h}$$
$$= \lim_{h \to 0} \frac{|h| - 0}{h} = \lim_{h \to 0} \frac{h}{h} = 1$$

LHD $\neq$ RHD $\Rightarrow f(x)$ is not differentiable at $x = 1$.

4. (*d*), if matrix is singular, then

$$\begin{vmatrix} 4+3k & 3 \\ 1+2k & 2 \end{vmatrix} = 0$$

$\Rightarrow$ $8 + 6k - 3 - 6k = 0$

$\Rightarrow$ $5 = 0$, false.

$\therefore$ matrix is not singular for any k.

5. (*c*), as matrix formed by cofactors of each element in $|A|$,

$$A_{11} = +(\cos\theta), A_{12} = -(\sin\theta)$$

$$A_{21} = -(-\sin\theta), A_{22} = +(\cos\theta)$$

$\therefore$ $$\text{adj}\, A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

6. (*c*), $$\begin{bmatrix} -3+x & 14y-p \\ z+3x & -16 \end{bmatrix} = \begin{bmatrix} 14 & 30 \\ y & p \end{bmatrix}$$

Using equality of matrices, we get

$$p = -16$$

Now, $-3 + x = 14$

$\Rightarrow$ $x = 14 + 3 = 17$

Now, $14y - p = 30$

$\Rightarrow$ $14y = 30 + p = 30 - 16$

$\Rightarrow$ $14y = 14 \Rightarrow y = 1$

Now, $z + 3x = y$

$\Rightarrow$ $z = y - 3x$

$= 1 - 3(17) = 1 - 51 = -50$

Hence, matrix is $\begin{bmatrix} 17 & 1 \\ -50 & -16 \end{bmatrix}$

7. (*c*) as $y = x^2$ ...(*i*)

$\Rightarrow$ $\dfrac{dy}{dt} = 2x\dfrac{dx}{dt}$

$\Rightarrow$ $x = \dfrac{1}{2}$ $\left[\because \dfrac{dx}{dt} = \dfrac{dy}{dt}\right]$

So, from (*i*), we get

$$y = \frac{1}{4}$$

$\therefore$ Point is $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$.

8. (*b*), as $\sin^{-1}x + \sin^{-1}y = \pi$

$\Rightarrow$ $\sin^{-1}x + \sin^{-1}y = \dfrac{\pi}{2} + \dfrac{\pi}{2}$

$\Rightarrow$ $\sin^{-1}x = \dfrac{\pi}{2}, \sin^{-1}y = \dfrac{\pi}{2}$ $\left[\text{as } \dfrac{-\pi}{2} \le \sin^{-1}x \le \dfrac{\pi}{2}\right]$

$\Rightarrow$ $x = \sin\dfrac{\pi}{2}, y = \sin\dfrac{\pi}{2} \Rightarrow x = 1, y = 1.$

9. (*c*), $f(x) = x^2$

$\Rightarrow$ $f'(x) = 2x$

$\Rightarrow$ $f'(1) = 2 \times 1 = 2$

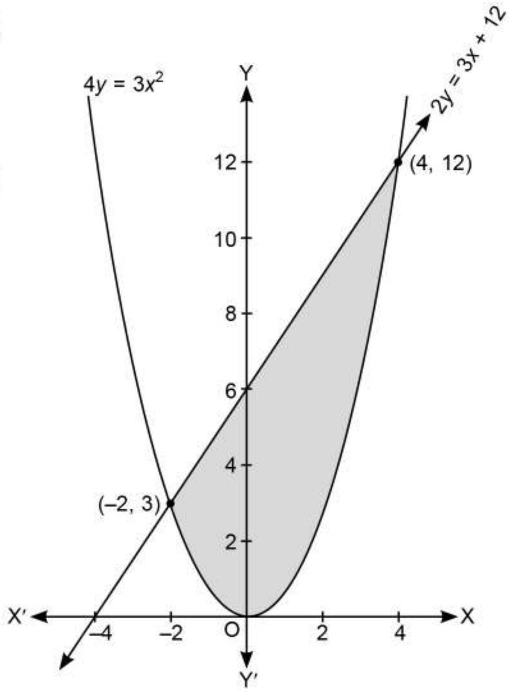
10. (*d*)

11. (*c*) Eliminating 'y' from $4y = 3x^2$ and $2y = 3x + 12$, we get

$$x = -2, 4$$

when $x = -2, y = 3$ and when $x = 4, y = 12$

So, points of intersection of the given line and parabola are (–2, 3) and (4, 12).



$$\text{Required area} = \int_{-2}^{4}\left(\frac{3x+12}{2}\right)dx - \int_{-2}^{4}\frac{3x^2}{4}dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4\times 3}\left[x^3\right]_{-2}^{4}$$

$$= \frac{1}{2}[(24 + 48) - (6 - 24)] - \frac{1}{4}[64 - (-8)]$$

$$= 45 - 18 = 27 \text{ sq units}$$

12. (*d*), let $2^x = t \Rightarrow 2^x \cdot \log_e 2\, dx = dt$

$$\therefore \int \frac{2^x}{\sqrt{1-4^x}}dx = \frac{1}{\log_e 2} \cdot \int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{\log_e 2} \cdot \sin^{-1}(2^x) + C \Rightarrow p = \frac{1}{\log_e 2}$$

13. (*d*), as $\int \frac{x+\cos 6x}{3x^2+\sin 6x}dx = \frac{1}{6}\int \frac{1}{t}dt = \frac{1}{6}\log|t| + C$

$$= \frac{1}{6}\log|3x^2 + \sin 6x| + C$$

Let $3x^2 + \sin 6x = t$

$\Rightarrow (6x + 6\cos 6x)dx = dt$

$\Rightarrow (x + \cos 6x)dx = \frac{1}{6}dt$

14. (*a*), proceed as $\int e^x\left\{\frac{1}{1+x} - \frac{1}{(1+x)^2}\right\}dx$ and $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C$

15. (*a*), as given equation is $\frac{dy}{dx} + \frac{1}{\frac{dy}{dx}} = 9$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 - 9\left(\frac{dy}{dx}\right) + 1 = 0$$

order $q = 1$, degree $p = 2$

$$\therefore \quad 2p + q = 4 + 1 = 5$$

16. (*b*), $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$$

Now, $P(x) = \frac{2x}{1+x^2}; Q(x) = \frac{4x^2}{1+x^2}$

Now, $\text{I.F.} = e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log|1+x^2|} = 1 + x^2$

17. (*d*), If A and B are independent events, then A' and B' are also independent events.

Now,
$$\begin{aligned} P(A' \cap B') &= P(A') \cdot P(B') \\ &= [1 - P(A)]\,[1 - P(B)] \\ &= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right) \\ &= \frac{2}{5} \times \frac{5}{9} = \frac{2}{9} \end{aligned}$$

18. (*c*), as $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \dfrac{1}{\sqrt{2}\cdot\sqrt{2}} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{6}$

19. (*d*)
$$f(x) = \lambda x^2 - 5x + 5 \;\Rightarrow\; f'(x) = 2\lambda x - 5$$

Now,
$$f'(1) = 5 \;\Rightarrow\; 2\lambda - 5 = 5 \;\Rightarrow\; \lambda = 5$$

So, A is false.

But, R is true.

20. (*d*) Total outcomes $= 36$

Let E = getting a doublet,

$\therefore E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

Let F = getting the numbers whose sum is greater than 6

$F = \{(1, 6), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$$E \cap F = \{(4, 4), (5, 5), (6, 6)\}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{3/36}{21/36} = \frac{3}{21} = \frac{1}{7}$$

A is false.

R is true.

21. Let $\vec{a}$ and $\vec{b}$ are the position vectors of the points (3, 4, 1) and (5, 1, 6)

Now,
$$\vec{a} = 3\hat{i} + 4\hat{j} + \hat{k}\,;\; \vec{b} = 5\hat{i} + \hat{j} + 6\hat{k}$$

Now, vector equation of the line passing through the points with position vectors $\vec{a}$ and $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$\Rightarrow \quad \vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda(2\hat{i} - 3\hat{j} + 5\hat{k}) \qquad ...(i)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in (*i*), we get

$$x\hat{i} + y\hat{j} + z\hat{k} = (3 + 2\lambda)\hat{i} + (4 - 3\lambda)\hat{j} + (1 + 5\lambda)\hat{k}$$

$$\Rightarrow \quad x = 3 + 2\lambda;\, y = 4 - 3\lambda;\, z = 1 + 5\lambda$$

$$\Rightarrow \quad \frac{x-3}{2} = \lambda;\; \frac{y-4}{-3} = \lambda;\; \frac{z-1}{5} = \lambda$$

So,
$$\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} \qquad ...(ii)$$

So, equation (*i*) and (*ii*) represent the vector and cartesian equation respectively of the required line.

22. Let A be the event, when sum of 9 appears on both dice

$A = \{(3, 6), (4, 5), (5, 4)\ (6, 3)\}$

B : second die exhibits prime number

$B = \{(1, 2), (2, 2), (2, 3), (2, 5), (1, 3), (1, 5)$
$(3, 2), (3, 3), (3, 5), (4, 2), (4, 5), (5, 2)$
$(4, 3), (5, 3), (5, 5), (6, 2), (6, 3), (6, 5)\}$

$A \cap B = \{(4, 5), (6, 3)\}$

$\therefore$ Probability of getting 9 as the sum when second die exhibits prime number

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}.$$

OR

Consider the following events:

S : Selecting an insured scooter driver.

C : Selecting an insured car driver.

T : Selecting an insured truck driver.

E : Selected insured driver meets with an accident.

$P(S) = \frac{2000}{12000} = \frac{2}{12}; P(C) = \frac{4000}{12000} = \frac{4}{12}; P(T) = \frac{6000}{12000} = \frac{6}{12};$

$P(E/S) = 0.01; P(E/C) = 0.03; P(E/T) = 0.15$

Using Bayes' Theorem, the probability of accident of a truck driver is

$$P(T/E) = \frac{P(T)\cdot P(E/T)}{P(S)\cdot P(E/S)+P(C)\cdot P(E/C)+P(T)\cdot P(E/T)}$$

$$= \frac{\frac{6}{12}\times 0.15}{\frac{2}{12}\times 0.01+\frac{4}{12}\times 0.03+\frac{6}{12}\times 0.15}$$

$$= \frac{0.90}{0.02+0.12+0.90} = \frac{90}{104} = \frac{45}{52}$$

23. The given equation is

$$x\frac{dy}{dx} = y(\log y - \log x + 1)$$

$$\frac{dy}{dx} = \frac{y}{x}(\log\frac{y}{x} + 1) \qquad ...(i)$$

Put $\qquad y = vx$

$\Rightarrow \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ equation (i) becomes

$$v + x\frac{dv}{dx} = v(\log v + 1)$$

$\Rightarrow \qquad x\frac{dv}{dx} = v\log v$

$\Rightarrow \qquad \frac{1}{v\log v}dv = \frac{dx}{x}$

Integrating both sides, we get

$$\int\frac{1}{v\log v}\,dv = \int\frac{1}{x}.\,dx$$

Put $\log v = t$

$\Rightarrow \quad \frac{1}{v}dv = dt$

$\Rightarrow \qquad \int\frac{1}{t}.\,dt = \int\frac{1}{x}.\,dx$

$$\Rightarrow \quad \log|t| = \log|x| + \log C$$

$$\Rightarrow \quad \log|\log v| = \log|x| + \log C$$

$$\Rightarrow \quad \log\left|\log\frac{y}{x}\right| = \log|x| + \log C$$

$$\log\left|\log\frac{y}{x}\right| = \log|xC|$$

$$\Rightarrow \quad \log\frac{y}{x} = xC \text{ is required solution.}$$

24. We have $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$

$$\Rightarrow \quad \sin\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \sin\theta = -\sin\left(\frac{\pi}{6}\right) \qquad [\text{as } \sin(-x) = -\sin x]$$

$$\Rightarrow \quad \sin\theta = \sin\left(-\frac{\pi}{6}\right)$$

$$\Rightarrow \quad \theta = -\frac{\pi}{6} \in \left[-\frac{\neq}{2}, \frac{\neq}{2}\right]$$

Now, $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right] = \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$

$$= \sin\left[\frac{2\pi + \pi}{6}\right] = \sin\left[\frac{\pi}{2}\right] = 1$$

25. A and B are symmetric matrices.

$\therefore \quad A' = A$ and $B' = B$

Consider $(AB + BA)' = (AB)' + (BA)'$

$= B'A' + A'B'$ (But $B' = B$ and $A' = A$)

$= BA + AB = AB + BA$

Hence $(AB + BA)$ is symmetric.

Now, consider $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

$\Rightarrow (AB - BA)$ is skew symmetric.

OR

We have $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta \\ \cos\theta & \sin\theta\end{bmatrix}$

$$= \begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix} + \begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

26. We have

$$y = \log(1 + x) - \frac{2x}{2 + x}$$

Differentiating w.r.t. x on both sides,

$$\frac{dy}{dx} = \frac{1}{1+x} - \left[\frac{(2+x)\cdot 2 - (2x)(1)}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \left[\frac{4+2x-2x}{(2+x)^2}\right] = \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$= \frac{4+x^2+4x-4-4x}{(1+x)(2+x)^2} = \frac{x^2}{(1+x)(2+x)^2}$$

It is given that $x > -1$

$\Rightarrow$ $1 + x > 0$

Also, x^2 and $(2 + x)^2$ are always positive for $x > -1$

Hence, $\dfrac{x^2}{(1+x)(2+x)^2} > 0$ for $x > -1$

$\Rightarrow$ $\dfrac{dy}{dx} > 0$ for $x > -1$

$\Rightarrow$ function is increasing for $x > -1$

27. Given that

(*i*) $x^p y^q = (x + y)^{p+q}$

Taking log on both sides, we get

$$p \log x + q \log y = (p + q) \log (x + y)$$

Differentiating both sides, w.r.t. x, we get

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{(p+q)}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \quad \frac{p}{x} - \frac{p+q}{x+y} = \left[\frac{p+q}{x+y} - \frac{q}{y}\right]\left[\frac{dy}{dx}\right]$$

$$\Rightarrow \quad \frac{px+py-px-qx}{x(x+y)} = \left[\frac{py+qy-qx-qy}{y(x+y)}\right]\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{py-qx}{x} = \left[\frac{py-qx}{y}\right]\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{y}{x} \cdot \left[\frac{(py-qx)}{(py-qx)}\right] = \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$

(*ii*) $$\frac{d^2y}{dx^2} = \frac{x \cdot \frac{dy}{dx} - y}{x^2}$$

$\Rightarrow$ $$\frac{d^2y}{dx^2} = \frac{x}{x^2} \cdot \frac{dy}{dx} - \frac{y}{x^2}$$

$$= \left[\frac{dy}{dx} - \frac{dy}{dx}\right]\frac{1}{x} \qquad \left(\text{as } \frac{dy}{dx} = \frac{y}{x}\right)$$

$\Rightarrow$ $$\frac{d^2y}{dx^2} = 0$$

28. $\int \frac{5x^2}{(x^2+4)(x^2+9)} dx = 5\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

Put $x^2 = t$ in $\frac{x^2}{(x^2+4)(x^2+9)}$, we get

$$\frac{x^2}{(x^2+4)(x^2+9)} = \frac{t}{(t+4)(t+9)}$$

Let $$\frac{t}{(t+4)(t+9)} = \frac{A}{(t+4)} + \frac{B}{(t+9)}$$

$\Rightarrow$ $$t = A(t+9) + B(t+4)$$

$$= (A+B)t + 9A + 4B$$

Equating like coefficients, we get

$$A + B = 1 \text{ and } 9A + 4B = 0$$

Solving for A and B, we get

$$-5A = 4$$

$\Rightarrow$ $$A = -\frac{4}{5} \Rightarrow B = \frac{9}{5}$$

So, $$\frac{x^2}{(x^2+4)(x^2+9)} = \frac{-4}{5(x^2+4)} + \frac{9}{5(x^2+9)}$$

$\Rightarrow$ $$5\int \frac{x^2}{(x^2+4)(x^2+9)} \cdot dx = -4\int \frac{1}{x^2+4} dx + 9\int \frac{1}{x^2+9} dx$$

$$= -4\int \frac{1}{x^2+2^2} dx + 9\int \frac{1}{x^2+3^2} dx$$

$$= -4 \cdot \frac{1}{2} \cdot \tan^{-1}\frac{x}{2} + 9 \cdot \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$= -2\tan^{-1}\frac{x}{2} + 3\tan^{-1}\left(\frac{x}{3}\right) + C$$

OR

$$\int \frac{x^3}{x^4+3x^2+2}dx \qquad \left| \begin{array}{l} \text{Let } x^2 = t \\ \Rightarrow 2x\,dx = dt \end{array} \right.$$

$$= \frac{1}{2}\int \frac{t}{t^2+3t+2}\,dt$$

$$= \frac{1}{4}\int \frac{(2t+3)-3}{t^2+3t+2}dt$$

$$= \frac{1}{4}\left[\int \frac{2t+3}{t^2+3t+2}dt - 3\int \frac{1}{t^2+3t+2}dt\right]$$

[For 1st integral $\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$]

$$= \frac{1}{4}\left[\log|t^2+3t+2| - 3\int \frac{1}{\left(t+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dt\right]$$

$$= \frac{1}{4}\left[\log|t^2+3t+2| - 3.\frac{1}{2\times\frac{1}{2}}\log\left|\frac{t+\frac{3}{2}-\frac{1}{2}}{t+\frac{3}{2}+\frac{1}{2}}\right|\right] + C$$

$$= \frac{1}{4}\left[\log|x^4+3x^2+2| - 3\log\left|\frac{x^2+1}{x^2+2}\right|\right] + C$$

Alternatively:

$$\int \frac{x^3}{x^4+3x^2+2}dx \qquad \left| \begin{array}{l} \text{Let } x^2 = t \\ \Rightarrow 2x\,dx = dt \end{array} \right.$$

$$= \frac{1}{2}\int \frac{t}{t^2+3t+2}dt$$

Let $\quad t = A\cdot\frac{d}{dt}(t^2+3t+2) + B$

$\Rightarrow \quad t = A\cdot(2t+3) + B$

$\quad = 2A\,t + (3A + B)$

Comparing the coefficients, we get

$2A = 1,\ 3A + B = 0$

$\Rightarrow \quad A = \frac{1}{2}, B = -\frac{3}{2}$

$$\frac{1}{2}\int \frac{t}{t^2+3t+2}dt = \frac{1}{2}\int \frac{\frac{1}{2}(2t+3)-\frac{3}{2}}{t^2+3t+2}dt$$

$$= \frac{1}{4}\left[\int \frac{2t+3}{t^2+3t+2}dt - 3\int \frac{1}{t^2+3t+2}dt\right]$$

Now refer side solution.

29. It is given that

$$R = \{(a, b), a \le b\}$$

It is clear that

as $\qquad a = a$

$\Rightarrow \qquad (a, a) \in R$

$\Rightarrow R$ is reflexive

Also, as $(3, 4) \in R$ because $3 \le 4$

but $(4, 3) \notin R$ as 4 is not less than equal to 3

$\Rightarrow R$ is not symmetric.

Now, consider $(a, b), (b, c) \in R$

where $\qquad a \le b$ and $b \le c$

$\Rightarrow \qquad a \le c$

$\Rightarrow \qquad (a, c) \in R$

$\therefore R$ is transitive.

30. When $0 < x < 1$ then $x \sin \pi x > 0$

When $1 < x < \frac{3}{2}$ then $x \sin \pi x < 0$

$$\Rightarrow \quad \int_0^{\frac{3}{2}} |x \sin \pi x|dx = \int_0^1 x \sin \pi x\,dx + \int_1^{\frac{3}{2}} -x \sin \pi x \cdot dx \qquad ...(i)$$

Consider $\int \underset{①}{x} \underset{②}{\sin \pi x}\, dx = x\int \sin \pi x \cdot dx - \int\left[\frac{d(x)}{dx}\int \sin \pi x \cdot dx\right]dx$

$$= x\left[\frac{-\cos \pi x}{\pi}\right] - \int 1 \cdot \frac{(-\cos \pi x)}{\pi} \cdot dx = -\frac{x}{\pi}\cos \pi x + \int \frac{1}{\pi} \cdot \cos \pi x \cdot dx$$

$$= -\frac{x}{\pi}\cos \pi x + \frac{1}{\pi} \cdot \frac{\sin \pi x}{\pi} = -\frac{x}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x$$

Put this in equation (i), we get

$$\int_0^{\frac{3}{2}} |x \sin \pi x| dx = \left[\left(\frac{-x}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x\right)\right]_0^1 - \left[-\frac{x}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x\right]_1^{\frac{3}{2}}$$

$$= \left[-\frac{\cos \pi}{\pi} + \frac{1}{\pi^2}\sin \pi\right] - \left[0 + \frac{1}{\pi^2}\sin 0\right] - \left[-\frac{3}{2\pi}\cos\frac{3\pi}{2} + \frac{1}{\pi^2}\sin\frac{3\pi}{2} + \frac{\cos \pi}{\pi} - \frac{\sin \pi}{\pi^2}\right]$$

$$= \left[-\frac{(-1)}{\pi} + 0\right] - 0 - \left[-\frac{3}{2\pi}(0) + \frac{1}{\pi^2}(-1) + \frac{(-1)}{\pi} - 0\right]$$

$$= \frac{1}{\pi} + 0 - 0 - 0 + \frac{1}{\pi^2} + \frac{1}{\pi} = \frac{2}{\pi} + \frac{1}{\pi^2}$$

OR

Consider equation $\frac{dy}{dx} - 3y \cot x = 2\sin 2x$

Here, $P(x) = -3\cot x$ and $Q(x) = 2\sin 2x$

$$\text{Integrating factor (I.F.)} = e^{\int -3 \cot x\, dx} = e^{-3\int \cot x\, dx}$$

$$= e^{-3\log|\sin x|} = e^{\log|\sin x|^{-3}} = \frac{1}{\sin^3 x}$$

$$\text{Solution is (I.F.)}\, y = \int\{(\text{I.F.})Q(x)\}dx$$

$$\frac{1}{\sin^3 x} \cdot y = \int \frac{1}{\sin^3 x} \cdot 2\sin 2x\, dx$$

$$= \quad 2\int \frac{2\sin x \cos x}{\sin^3 x}dx = 4\int \text{cosec}\, x \cot x\, dx$$

$$\Rightarrow \quad \frac{1}{\sin^3 x} \cdot y = -4\,\text{cosec}\, x + C$$

$$\Rightarrow \quad y = -4\sin^2 x + C\sin^3 x \qquad ...(i)$$

Given $\quad y = 2$, when $x = \frac{\pi}{2}$

$$\therefore \quad 2 = -4\sin^2\frac{\pi}{2} + C\,.\sin^3\frac{\pi}{2}$$

$$\Rightarrow \quad 2 = -4 + C \Rightarrow C = 6$$

Substituting in (i), we get

$y = -4\sin^2 x + 6\sin^3 x$ as the particular solution.

31. Since, f is continuous at $x = 0$, then

$$\underset{(x=0)}{\text{LHL}} = \underset{(x=0)}{\text{RHL}} = f(0)$$

$$\Rightarrow \quad \underset{(x=0)}{\text{LHL}} = \underset{(x=0)}{\text{RHL}} = a \qquad ...(i)$$

$$\underset{(x=0)}{\text{LHL}} = \lim_{x \to 0^-} \frac{1 - \cos 4x}{x^2}$$

$$= \lim_{h \to 0} \frac{1 - \cos 4h}{h^2}$$

$$= \lim_{h \to 0} \frac{2\sin^2 2h}{h^2}$$

$$= 8 \lim_{h \to 0} \frac{\sin 2h}{2h} \cdot \lim_{h \to 0} \frac{\sin 2h}{2h}$$

$$= 8 \times 1 \times 1 = 8$$

$$\underset{(x=0)}{\text{RHL}} = \lim_{x \to 0^+} \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}$$

$$= \lim_{h \to 0} \frac{\sqrt{h}}{\sqrt{16 + \sqrt{h}} - 4}$$

$$= \lim_{h \to 0} \frac{\sqrt{h}\left(\sqrt{16 + \sqrt{h}} + 4\right)}{\left(16 + \sqrt{h} - 16\right)}$$

$$= \lim_{h \to 0} \left(\sqrt{16 + \sqrt{h}} + 4\right) = 8$$

$\therefore$ from (i), $8 = 8 = a \Rightarrow a = 8$

OR

We have $y = (\sin^{-1} x)^2$

Differentiating both sides, w.r.t. x, we get

$$\frac{dy}{dx} = 2(\sin^{-1} x) \cdot \frac{1}{\sqrt{1 - x^2}}$$

$$\Rightarrow \quad \sqrt{1 - x^2}\frac{dy}{dx} = 2(\sin^{-1} x)$$

Differentiating again w.r.t. x, on both sides,

$$\sqrt{1 - x^2}\,\frac{d^2y}{dx^2} + \frac{1}{2}\frac{(-2x)}{\sqrt{1 - x^2}} \cdot \frac{dy}{dx} = \frac{2}{\sqrt{1 - x^2}}$$

$$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} - x \cdot \frac{dy}{dx} = 2$$

$$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

32. Product of matrices is

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

If we consider, $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

and $$B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, \text{ then } AB = 8I$$

$$\Rightarrow \quad \left(\frac{1}{8}A\right)B = I$$

$$\Rightarrow \quad \frac{1}{8}A = B^{-1}$$

Consider the given equations

$$x - y + z = -4$$
$$x - 2y - 2z = -9$$
$$2x + y + 3z = -1$$

In matrix form, these equations are written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -9 \\ -1 \end{bmatrix}$$

Now, matrix equation is

$$BX = C$$

$$\Rightarrow \quad X = B^{-1} C \text{ is the required solution}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}AC$$

$$= \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ -9 \\ -1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 16 - 36 - 4 \\ 28 - 9 - 3 \\ -20 + 27 + 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -24 \\ 16 \\ 8 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$\Rightarrow x = -3, y = 2, z = 1$

OR

Consider $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$$\Rightarrow \quad A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A + A' = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

Consider, $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$

$$P' = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$

Here, $P = P'$

$\Rightarrow P$ is a symmetric matrix

Now, $$A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

Let $$Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix}$$

$$\Rightarrow \quad Q' = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix} = -Q$$

$\Rightarrow$ $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Now, representing A as sum of $P + Q$

Now, $$P + Q = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A$$

33. Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$

In vector form, these equations are written as

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

and $$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})$$

Here $$\vec{a_1} = \hat{i} - \hat{j} \text{ and } \vec{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{a_2} = -\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b_2} = 5\hat{i} + \hat{j}$$

$$\vec{a_2} - \vec{a_1} = -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j} = -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(0-5) + \hat{k}(2-15) = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$\therefore \quad (\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (-2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 5\hat{j} - 13\hat{k})$$

$$= (2 + 15 - 26) = 17 - 26 = -9$$

$$\text{Shortest distance} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| = \left| \frac{-9}{\sqrt{1+25+169}} \right| = \left| \frac{-9}{\sqrt{195}} \right| = \frac{9}{\sqrt{195}} \text{ units}$$

Here, shortest distance is not zero, so lines are not intersecting.

OR

$$\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k} \text{ and } \vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$$

Also, $\vec{b} = \vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1} \parallel \vec{a}$ and $\vec{\beta_2} \perp \vec{a}$

As $\vec{\beta_1} \parallel \vec{a} \Rightarrow \vec{\beta_1} = \lambda \vec{a}$

$$\vec{\beta_1} = \lambda(2\hat{i} - \hat{j} - 2\hat{k}) \quad ...(i)$$

Also, $\vec{b} = \vec{\beta_1} + \vec{\beta_2}$

$$(7\hat{i} + 2\hat{j} - 3\hat{k}) = \lambda(2\hat{i} - \hat{j} - 2\hat{k}) + \vec{\beta_2}$$

$$\Rightarrow \quad \vec{\beta_2} = (7 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (-3 + 2\lambda)\hat{k} \quad ...(ii)$$

We have $\vec{\beta_2} \perp \vec{a}$

$\Rightarrow \quad \vec{\beta_2} \cdot \vec{a} = 0$

$\Rightarrow \quad [(7 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (-3 + 2\lambda)\hat{k}] \cdot [2\hat{i} - \hat{j} - 2\hat{k}] = 0$

$\Rightarrow \quad 2(7 - 2\lambda) + (-1)(2 + \lambda) + (-2)(-3 + 2\lambda) = 0$

$\Rightarrow \quad 14 - 4\lambda - 2 - \lambda + 6 - 4\lambda = 0$

$\Rightarrow \quad 18 - 9\lambda = 0$

$\Rightarrow \quad \lambda = 2$

Putting value of λ in equation (*i*) and (*ii*) we get

$$\vec{\beta_1} = 4\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\vec{\beta_2} = 3\hat{i} + 4\hat{j} + \hat{k}$$

34. $$\int \sqrt{\tan x}\, dx = 2\int \frac{t^2}{1+t^4} dt = \int \frac{(t^2+1) + (t^2-1)}{t^4+1} dt$$

Let $\tan x = t^2$
$\Rightarrow x = \tan^{-1}(t^2)$
$\Rightarrow \quad dx = \frac{2t}{1+t^4} dt$

$$= \int \frac{t^2+1}{t^4+1} dt + \int \frac{t^2-1}{t^4+1} dt = \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt + \int \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$$

$$= \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2} dt + \int \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)^2-2} dt$$

In first integral substituting $t - \frac{1}{t} = y$, and in second integral substituting $t + \frac{1}{t} = z$.

$$\Rightarrow \quad \left(1+\frac{1}{t^2}\right)dt = dy$$

$$\int \frac{1}{y^2+2}dy = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{t-\frac{1}{t}}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{t^2-1}{\sqrt{2}t}\right]$$

$$\left(1-\frac{1}{t^2}\right)dt = dz$$

So, $$\int \frac{1}{z^2-2}dz = \frac{1}{2\sqrt{2}}\log\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right|$$

$$= \frac{1}{2\sqrt{2}}\log\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|$$

$$= \frac{1}{2\sqrt{2}}\left|\frac{t^2+1-\sqrt{2}t}{t^2+1+\sqrt{2}t}\right|$$

$$\therefore \quad \int\sqrt{\tan x}\,dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right| + C$$

35. Maximise, $Z = 5x + 3y$

Subject to constraints

$$x \geq 0, y \geq 0$$
$$2x + y \leq 12$$
$$3x + 2y \leq 20$$

Table for $2x + y = 12$

x	0	6	4
y	12	0	4

Table for $3x + 2y = 20$

x	0	6	4
y	10	1	4

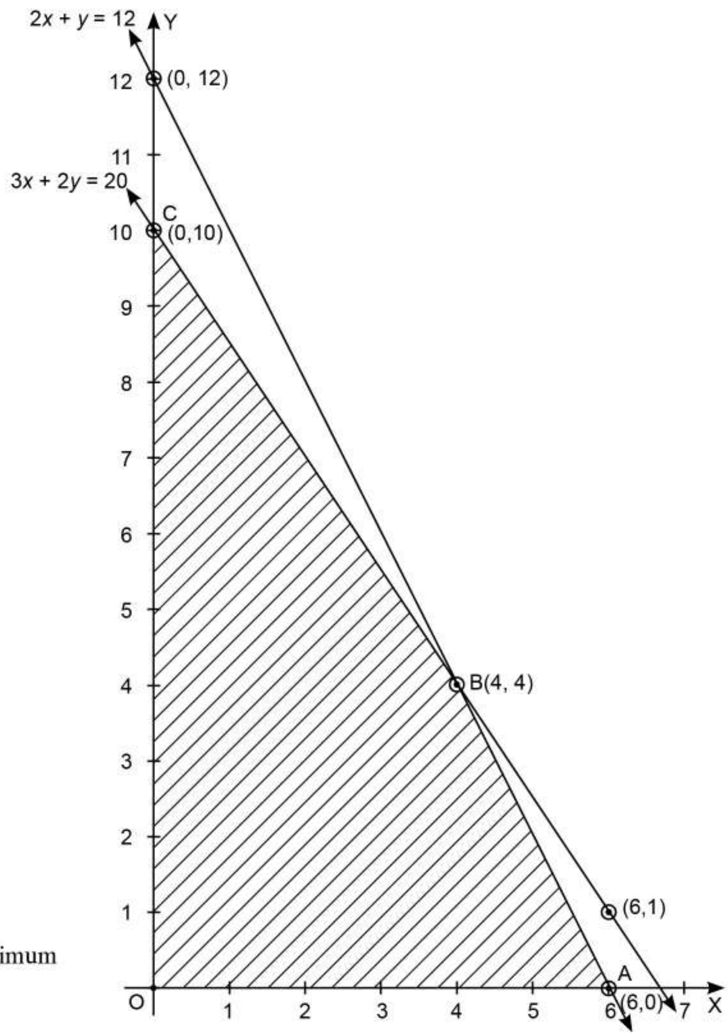
Plotting the points on graph,

From the graph, we observe, shaded region is the optimum solution

$\therefore$ For maximum Z, possible points are

$A(6, 0)$, $B(4, 4)$ and $C(0, 10)$

Points	$Z = 5x + 3y$	**value**	
$A(6, 0)$	$5(6) + 3(0)$	30	
$B(4, 4)$	$5(4) + 3(4)$	32	→ Maximum
$C(0, 10)$	$5(0) + 3(10)$	30	



Here, Z is maximum at $x = 4, y = 4$ and maximum value is 32.

36. (*i*) Out of 7 male candidates only one can be selected.

$$\therefore \quad P(\text{husband's selection}) = \frac{1}{7}$$

$$P(H) = \frac{1}{7}$$

(*ii*) Out of 5 female candidates only one can be selected

$$\therefore \quad P(\text{wife's selection}) = \frac{1}{5}$$

$$P(W) = \frac{1}{5}$$

(*iii*) $$P(\text{both are selected}) = P(H) \cdot P(W)$$

$$= \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$$

OR

(*iii*) $$P(\text{only one is selected}) = P(H\overline{W} \text{ or } \overline{H}W)$$

$$= P(H) \cdot P(\overline{W}) + P(\overline{H}) \cdot P(W)$$

$$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5}$$

$$= \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$$

37. (*i*) Here x be side of square base and y be length of vertical side, then volume V is represented as

$$V = x \cdot x \cdot y = x^2y \qquad ...(i)$$

(*ii*) Let S be the total surface area

$$S = (\text{area of base}) + (\text{area of four walls})$$

$$S = x^2 + 4xy \qquad [\because \text{Area of 4 walls} = 2y(x + x) = 4xy]$$

From equation (*i*) we get

$$y = \frac{V}{x^2}$$

Now, $$S = x^2 + 4x \cdot \frac{V}{x^2}$$

$$= x^2 + \frac{4V}{x}$$

(*iii*) We have $$S = x^2 + \frac{4V}{x}$$

$$\frac{dS}{dx} = 2x - \frac{4V}{x^2}$$

For minimum surface area,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad 2x - \frac{4V}{x^2} = 0$$

$$\Rightarrow \quad 2x = \frac{4V}{x^2}$$

$$\Rightarrow \quad 2x^3 = 4V$$

$$\Rightarrow \quad x^3 = 2V \qquad ...(iii)$$

$$\Rightarrow \quad x = \left(2V\right)^{\frac{1}{3}}$$

Now, $$\frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$$

$$\left[\frac{d^2S}{dx^2}\right]_{x=(2V)^{1/3}} = 2 + \frac{8V}{2V} = 6 > 0$$

$\therefore$ Surface area is minimum at $x = (2V)^{\frac{1}{3}}$.

OR

(*iii*) From equation (*iii*), we get

$$x^3 = 2V$$

Put $V = x^2y$ from equation (*i*)

$$x^3 = 2 \cdot x^2 y$$

$$x = 2y$$

38. (*i*) $\overrightarrow{AB}$ = position vector of B – position vector of A = $(2-1)\hat{i} + (1-1)\hat{j} + (3-1)\hat{k} = \hat{i} + 2\hat{k}$

(*ii*) $\overrightarrow{AC}$ = position vector of C – position vector of A = $(3-1)\hat{i} + (2-1)\hat{j} + (2-1)\hat{k} = 2\hat{i} + \hat{j} + \hat{k}$