17 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory.***

(*ii*) *This Question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each.***

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each.***

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each.***

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. The value $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is **[NCERT Part-I, Page 77-79]**

(*a*) –7 (*b*) 7 (*c*) 8 (*d*) 10

2. If $A = \text{diag}(3, -1)$, then matrix A is **[NCERT Part-I, Page 40]**

(*a*) $\begin{bmatrix} 0 & 3 \\ 0 & -1 \end{bmatrix}$ (*b*) $\begin{bmatrix} -1 & 0 \\ 3 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ (*d*) $\begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}$

3. Total number of possible matrices of order 2×3 with each entry 1 or 2 is **[NCERT Part-I, Page 36-37]**

(*a*) 6 (*b*) 36 (*c*) 32 (*d*) 64

4. If $x^2 + y^2 = 5$ then $\frac{dy}{dx}$ is **[NCERT Part-I, Page 122-123]**

(*a*) $-x^2$ (*b*) $-2x$ (*c*) $\frac{x^2}{y^2}$ (*d*) $-\frac{x}{y}$

5. Derivative of $\cot x°$ with respect to x is **[NCERT Part-I, Page 120-121]**

(*a*) $\text{cosec}\, x°$ (*b*) $\text{cosec}\, x° \cot x°$ (*c*) $-1° \text{cosec}^2 x°$ (*d*) $-1° \text{cosec}\, x° \cot x°$

6. $\int \frac{x^2}{1+x^3}\,dx$ is equal to **[NCERT Part-II, Page 235-236]**

(*a*) $\frac{2}{3x}+C$ (*b*) $2\log x + C$

(*c*) $\frac{1}{3}\log|1+x^3|+C$ (*d*) $3\log(1+x^3)+C$

7. $\int \frac{x^2+4x}{x^3+6x^2+5}\,dx$ is equal to **[NCERT Part-II, Page 235-236]**

(*a*) $|x^3+6x^2+5|+C$ (*b*) $\frac{1}{3}\log|x^3+6x^2+5|+C$

(*c*) $\log|x^2+6x|+C$ (*d*) $\frac{1}{2}\log|x^2+4x|+C$

8. If p and q are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+3\frac{dy}{dx}+\frac{d^3y}{dx^3}=4$, then the value of $2p-3q$ is **[NCERT Part-II, Page 301-302]**

(*a*) 7 (*b*) –7 (*c*) 3 (*d*) –3

9. $\int \cot^2 x\,dx$ equals to **[NCERT Part-II, Page 241]**

(*a*) $\cot x - x + C$ (*b*) $\cot x + x + C$ (*c*) $-\cot x + x + C$ (*d*) $-\cot x - x + C$

10. Area of the region bounded by the curve $y=\sqrt{49-x^2}$ and the x-axis is **[Conceptual Application]**

(*a*) $\frac{49}{2}\pi$ sq units (*b*) 98π sq units (*c*) 49π sq units (*d*) 240π sq units

11. If $|\vec{a}|=8, |\vec{b}|=3$ and $|\vec{a}\cdot\vec{b}|=12\sqrt{3}$ then the value of $|\vec{a}\times\vec{b}|$ is **[NCERT Part-II, Page 363]**

(*a*) 12 (*b*) $12\sqrt{3}$ (*c*) 6 (*d*) $4\sqrt{3}$

12. If vectors $(2\hat{i}+6\hat{j}+14\hat{k})$ and $(\hat{i}-\lambda\hat{j}+7\hat{k})$ are perpendicular then value of λ is **[NCERT Part-II, Page 355-356]**

(*a*) $\frac{50}{3}$ (*b*) $-\frac{50}{3}$

(*c*) 6 (*d*) 100

13. If $(2\hat{i}+6\hat{j}+14\hat{k})\times(\hat{i}-\lambda\hat{j}+7\hat{k})=\vec{0}$ then λ is **[NCERT Part-II, Page 366]**

(*a*) $\frac{27}{2}$ (*b*) $-\frac{27}{2}$ (*c*) 3 (*d*) –3

14. The direction cosines of the vector $3\hat{i}-2\hat{j}+6\hat{k}$ are **[NCERT Part-II, Page 349]**

(*a*) 3, –2, 6 (*b*) 6, –2, 3 (*c*) $\frac{3}{7},\frac{-2}{7},\frac{6}{7}$ (*d*) $\frac{1}{7},\frac{2}{7},\frac{3}{7}$

15. The distance of point (2, 5, 7) from the ***x***-axis is **[Conceptual Application]**

(*a*) 2 (*b*) $\sqrt{74}$ (*c*) $\sqrt{29}$ (*d*) $\sqrt{53}$

16. The value of p for which $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+p\hat{j}+3\hat{k}$ are parallel vectors is **[NCERT Part-II, Page 356]**

(*a*) 3 (*b*) $\frac{3}{2}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

17. Two events A and B are said to be independent if **[NCERT Part-II, Page 418]**

(a) $P(A \cup B) = P(A) \cdot P(B)$ (b) $P(A \cap B) = 0$

(c) $P(A \cap B) = P(A) \cdot P(B)$ (d) None of these

18. The probability that husband will be alive 10 years hence is $\frac{7}{15}$ and wife will be alive 10 years is $\frac{7}{10}$. The probability that at least one will be alive 10 years hence is **[Conceptual Application]**

(a) $\frac{21}{150}$ (b) $\frac{126}{150}$ (c) $\frac{77}{150}$ (d) $\frac{24}{150}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The order and degree of differential equation $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is 1 and 2 respectively. **[NCERT Part-I, Page 301-302]**

Reason (R): Order and degree (if defined) of a differential equation are always positive integer.

20. **Assertion (A):** The function $f(x) = |x|$ is differentiable at $x = 0$. **[NCERT Part-I, Page 118-119]**

Reason (R): If a function is differentiable at any point, it is continuous at that point also.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find equation of the curve passing through (1, 1) and satisfying the differential equation $\frac{dy}{dx} = \frac{2y}{x}$.

[NCERT Part-II, Page 306-307]

22. A and B throw a pair of dice turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is $\frac{9}{17}$. **[Conceptual Application]**

OR

In the given table variate X and its corresponding probability is given, find p if $\sum P(X) = 1$.

[Conceptual Application]

X	$P(X)$
0	0
1	$2p$
2	$2p$
3	$3p$
4	p^2
5	$2p^2$
6	$7p^2$
7	$2p$

Find the value of p.

23. Find the value of $\tan^{-1}(1) + \tan^{-1}(-\sqrt{3})$. **[NCERT Part-I, Page 24]**

24. Write the Cartesian equation of the following line given in vector form: $\vec{r} = 2\hat{i} + \hat{j} - 4\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$.
[NCERT Part-II, Page 382]

25. Find a matrix X such that $2A + B + X = O$, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$.
[NCERT Part-I, Page 44]

OR

If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then find the value of k. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 5 m away from the wall? **[NCERT Part-I, Page 147-148]**

27. If $y = 3at^2$; $x = 5bt^4$, find $\dfrac{d^2y}{dx^2}$ at $t = 1$. **[NCERT Part-I, Page 134-135]**

28. Evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$ **[NCERT Part-II, Page 273-274]**

OR

Find the general solution of the differential equation $y - x\dfrac{dy}{dx} = x + y\dfrac{dy}{dx}$. **[NCERT Part-II, Page 313-314]**

29. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angled triangles T_1 with sides 3, 4, 5; T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related? **[NCERT Part-I, Page 2, 4]**

30. Find the values of a and b so that the function

$f(x) = \begin{cases} x^2 + 3x + a \ , & x \le 1 \\ bx + 2 \ , & x > 1 \end{cases}$ is differentiable for $x \in R$. **[NCERT Part-I, Page 118-119]**

OR

Find $\dfrac{dy}{dx}$, if $y = (\log x)^x + x^{\log x}$. **[NCERT Part-I, Page 130]**

31. Find the area of the region bounded by the lines **[Conceptual Application]**
$y = 4x + 5, x + y = 5$ and $x - 4y + 5 = 0$.

OR

Make a rough sketch of the region given below and find its area using integration
$\{(x, y): 0 \le y \le x^2 + 3; 0 \le y \le 2x + 3, 0 \le x \le 3)$. **[Conceptual Application]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ **[NCERT Part-II, Page 386-387]**

and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

OR

Find the value of p, so that the lines **[NCERT Part-II, Page 383]**

$l_1 : \frac{1-x}{3} = \frac{7y+14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equations of a line passing through a point (3, 2, –4) and parallel to line l_1.

33. If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. How we can use A^{-1} to solve the system of equations?

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2;\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5;\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4.$ **[NCERT Part-I, Page 94]**

OR

If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence show that how we can use A^{-1} to solve the system of equations?

$2x + y - 3z = 13;\ 3x + 2y + z = 4,\ x + 2y - z = 8.$ **[NCERT Part-I, Page 94]**

34. Find the area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum.

[Conceptual Application]

35. Solve the following linear programming problem (LPP) graphically. **[NCERT Part-II, Page 397]**

Maximise $Z = 20x + 40y$

subject to constraints:

$1.5x + 3y \le 42,\ 3x + y \le 24, x \ge 0, y \ge 0.$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals. **[NCERT Part-II, Page 424-425]**

Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions:

(*i*) Find the probability that the airplane will not crash.

(*ii*) Find $P(A|E_1) + P(A|E_2)$.

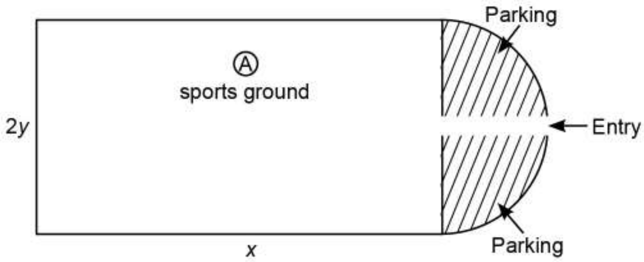
(*iii*) (*a*) Find $P(A)$.

OR

(*iii*) (*b*) Find $P(E_2|A)$.

Case Study - 2

37. The government of a state, which has mostly hilly area decided to have adventurous playground on the top of hill having plane area and space for 10000 persons to sit at a time. After survey it was decided to have rectangular play ground with a semicircular parking at one end of play ground only as space is less. The total perimeter of the field is measured as 1000 m as shown. **[NCERT Part-I, Page 166]**



(*i*) Looking at the figure (plan), find the relation between x and y.

(*ii*) Find the area of sports ground in terms of x.

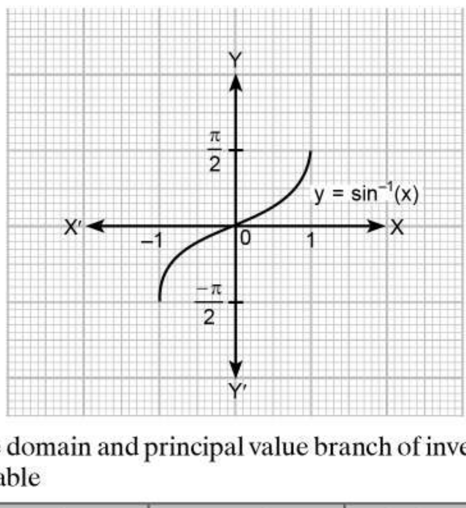
(*iii*) Find the value of x for which sports ground has the maximum area.

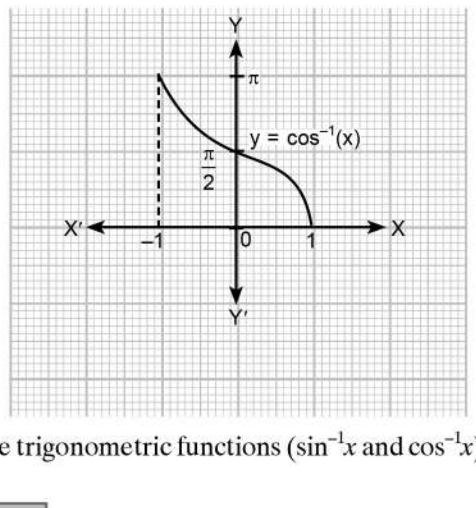
OR

(*iii*) Find the maximum area of the sports field.

Case Study - 3

38. Graphs of $\sin^{-1} x$ and $\cos^{-1}x$ are given below. **[NCERT Part-I, Page 20-21]**





The domain and principal value branch of inverse trigonometric functions ($\sin^{-1}x$ and $\cos^{-1}x$) are given in table

Functions	Domain	Principal value
$y = \sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$

(*i*) Find the domain of $\sin^{-1}(2x - 1)$.

(*ii*) Find the domain of $\cos^{-1}(2x - 1)$.

SOLUTIONS

1. (*a*), $\Delta = 6(-1) - 1(1) = -7$.

2. (*c*), as diag $(3, -1)$ is a diagonal matrix. Its order is 2×2 with diagonal elements 3 and (-1).

3. (*d*), as total elements are 6 and each entry can be done in 2 ways. Hence, total possibilities $= 2^6 = 64$.

4. (*d*), as $x^2 + y^2 = 5$

Differentiating both sides w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2y\frac{dy}{dx} = -2x$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}.$$

5. (*c*), as

$$x^\circ = \frac{\pi}{180}x^c$$

$$\therefore \quad \frac{d}{dx}(\cot x^\circ) = \frac{d}{dx}\left(\cot\frac{\pi}{180}x\right)$$

$$= -\frac{\pi}{180}\operatorname{cosec}^2\frac{\pi}{180}x = -1^\circ \operatorname{cosec}^2\frac{\pi}{180}x = -1^\circ \operatorname{cosec}^2 x^\circ$$

6. (*c*), as

$$\int\frac{x^2}{1+x^3}dx = \frac{1}{3}\int\frac{1}{t}\,dt = \frac{1}{3}\log|t| + C$$

$$= \frac{1}{3}\log|1 + x^3| + C$$

Let $1 + x^3 = t$
$\Rightarrow 3x^2dx = dt$
$\Rightarrow x^2dx = \frac{1}{3}dt$

7. (*b*), as

$$\int\frac{x^2+4x}{x^3+6x^2+5}dx = \frac{1}{3}\int\frac{1}{t}dt$$

$$= \frac{1}{3}\log|t| + C = \frac{1}{3}\log|x^3 + 6x^2 + 5| + C$$

Let $x^3 + 6x^2 + 5 = t$
$\Rightarrow (3x^2 + 12x)dx = dt$
$\Rightarrow (x^2 + 4x)dx = \frac{1}{3}dt$

8. (*b*), as degree $p = 1$ and order $q = 3$

$\therefore 2p - 3q = 2 - 9 = -7$

9. (*d*), $\int(\operatorname{cosec}^2 x - 1)dx = -\cot x - x + C$

10. (*a*), as area is above the x-axis

$$\therefore \quad \text{area} = 2\int_0^7\sqrt{49-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{49-x^2} + \frac{49}{2}\sin^{-1}\frac{x}{7}\right]_0^7$$

$$= 2\left[\left(\frac{7}{2}\times 0 + \frac{49}{2}\sin^{-1}1\right) - (0)\right] = \frac{49}{2}\pi \text{ sq units}$$

11. (*a*), as

$$|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|^2 = 64\times 9 - 144\times 3 = 576 - 432 = 144$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}| = 12$$

12. (*a*), Since vectors are perpendicular, then

$$(2\hat{i}+6\hat{j}+14\hat{k})\cdot(\hat{i}-\lambda\hat{j}+7\hat{k}) = 0 \Rightarrow 2-6\lambda+98 = 0 \Rightarrow \lambda = \frac{50}{3}$$

13. (*d*), as $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 6 & 14\\ 1 & -\lambda & 7\end{vmatrix} = \vec{0}$

$$\Rightarrow \hat{i}(42+14\lambda)-\hat{j}(14-14)+\hat{k}(-2\lambda-6) = \vec{0} \quad \Rightarrow \quad \lambda = -3$$

14. (*c*), Let $\vec{r} = a\hat{i}+b\hat{j}+c\hat{k} = 3\hat{i}-2\hat{j}+6\hat{k}$. So, $\langle a, b, c\rangle = \langle 3, -2, 6\rangle$

Now, $|\vec{r}| = \sqrt{(3)^2+(-2)^2+(6)^2} = 7$

dc's of given vector $= \left\langle \frac{a}{|\vec{r}|}, \frac{b}{|\vec{r}|}, \frac{c}{|\vec{r}|}\right\rangle = \left\langle \frac{3}{7}, \frac{-2}{7}, \frac{6}{7}\right\rangle$

15. (*b*), as distance of point (2, 5, 7) from the *x*-axis is

$$\sqrt{5^2+7^2} = \sqrt{25+49} = \sqrt{74}$$

16. (*c*), as $\frac{3}{1}=\frac{2}{p}=\frac{9}{3} \Rightarrow p=\frac{2}{3}$

17. (*c*),

18. (*b*), Let *A* and *B* the event of husband and wife will alive 10 years respectively.

$$P(A) = \frac{7}{15}, P(B)=\frac{7}{10}$$

$$P(\bar{A}) = \frac{8}{15}, P(\bar{B})=\frac{3}{10}$$

P(at least 1 alive) $= 1 - P$ (none alive)

$$= 1 - P(\bar{A})P(\bar{B})$$

$$= 1-\frac{8}{15}\times\frac{3}{10} = 1-\frac{24}{150} = \frac{126}{150}$$

19. (*a*) Both A and R are true and R is the correct explanation of A.

$$y = x\frac{dy}{dx}+a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \quad \left(y-x\frac{dy}{dx}\right)^2 = a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$$

$$\Rightarrow \quad y^2+x^2\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx} = a^2+a^2\left(\frac{dy}{dx}\right)^2$$

So order = 1, degree = 2.

20. (*d*), A is false, but R is true.

21. Consider $\quad \frac{dy}{dx} = \frac{2y}{x}$

$$\Rightarrow \quad \int\frac{dy}{y} = 2\int\frac{dx}{x}$$

$$\Rightarrow \quad \log|y| = 2\log|x| + \log C = \log|x^2C|$$

$$\Rightarrow \quad y = Cx^2 \qquad \text{...}(i)$$

Given (*i*) passes through (1, 1)

$$\Rightarrow \quad 1 = C\cdot(1)^2 \Rightarrow C = 1$$

$\therefore$ from (*i*) $\quad y = x^2$ is the required curve.

22. S : getting a total of 9 = {(3, 6), (4, 5), (5, 4), (6, 3)}

$$P(S) = \frac{4}{36} = \frac{1}{9}.\ P(\bar{S}) = \frac{8}{9}$$

A can win in 1st, 3rd, 5th, 7th, ... throws

$$P(A \text{ wins}) = P(S) + [P(\bar{S})]^2 P(S) + [P(\bar{S})]^4 P(S) + \cdots$$

$$= \frac{1}{9} + \left(\frac{8}{9}\right)^2 \cdot \frac{1}{9} + \left(\frac{8}{9}\right)^4 \cdot \frac{1}{9} + \ldots\ldots$$

$$= \frac{\frac{1}{9}}{1 - \frac{64}{81}} = \frac{9}{17} \qquad \left[\begin{array}{l}\text{sum of infinite GP} \\ a + ar + ar^2 + \ldots = \frac{a}{1-r}\end{array}\right]$$

OR

Given, $\Sigma P(X) = 1 \Rightarrow 0 + 2p + 2p + 3p + p^2 + 2p^2 + 7p^2 + 2p = 1$

$\Rightarrow$ $10p^2 + 9p - 1 = 0 \Rightarrow 10p^2 + 10p - p - 1 = 0$

$\Rightarrow$ $10p(p + 1) - 1\,(p + 1) = 0 \Rightarrow (10p - 1)\,(p + 1) = 0$

$\Rightarrow$ $10p - 1 = 0$ or $p + 1 = 0$

$\Rightarrow$ $p = \frac{1}{10}$ or $p = -1$ (rejected)

$\therefore$ $p = \frac{1}{10}$.

23. The range of principle value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Now, $\tan^{-1}(1) = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\tan^{-1}(-\sqrt{3}) = \tan^{-1}\left\{-\tan\left(\frac{\pi}{3}\right)\right\} = \tan^{-1}\left\{\tan\left(\frac{-\pi}{3}\right)\right\} = \frac{-\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$\therefore$ $\tan^{-1}1 + \tan^{-1}(-\sqrt{3}) = \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$.

24. Point through which line passes is (2, 1, – 4) and dr's: 1, – 1, – 1.

$\therefore$ Cartesian equation of line is

$$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$$

25.

$$2A + B + X = O \Rightarrow X = -2A - B = -2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}.$$

OR

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix};\quad kA = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

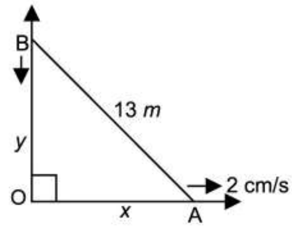
$$A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$A^2 = kA \Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

$\Rightarrow$ $k = 2$.

26. Let A be foot of ladder which is x m away from the wall and let y m be height of ladder on wall at any instant t.



Now, $y = \sqrt{169 - x^2} \Rightarrow \dfrac{dy}{dt} = \dfrac{-x}{\sqrt{169 - x^2}} . \dfrac{dx}{dt} = \dfrac{-x}{\sqrt{169 - x^2}} \times 2$

$\left.\dfrac{dy}{dt}\right]_{x=5} = -\dfrac{5}{12} \times 2 = -\dfrac{5}{6}$ cm/s $\qquad \left[\dfrac{dx}{dt} = 2 \text{ cm/s}\right]$

Hence, height of the wall is decreasing at the rate of $\dfrac{5}{6}$ cm/s.

27. Given $\quad y = 3at^2 \quad$ and $\quad x = 5bt^4$

$$\frac{dy}{dt} = 6at \quad \text{and} \quad \frac{dx}{dt} = 20bt^3 \qquad ...(i)$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{6at}{20bt^3} = \frac{3a}{10bt^2}$$

Now
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{3a}{10bt^2}\right) = \frac{3a}{10b} \cdot \frac{d}{dx}(t^{-2}) = \frac{3a}{10b} \cdot \left(-2t^{-3} \cdot \frac{dt}{dx}\right)$$

$$= -\frac{3a}{5b} \cdot \frac{1}{t^3} \cdot \frac{1}{20bt^3} = -\frac{3a}{100b^2t^6} \qquad \text{[from } (i)\text{]}$$

$$\therefore \quad \left.\frac{d^2y}{dx^2}\right]_{t=1} = \frac{-3a}{100b^2}.$$

28. Consider
$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\cot x}} \qquad ...(i)$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\cot\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\cot\left(\frac{\pi}{2} - x\right)}}$$

[using property $\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$]

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \frac{1}{\sqrt{\cot x}}}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + 1}\,dx \qquad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1 + \sqrt{\cot x}}{1 + \sqrt{\cot x}}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1.dx = \Big[x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\Rightarrow \quad I = \frac{\pi}{12}$$

Alternatively:

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}} dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx \quad ...(i)$$

Using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}} dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\cdot dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3}-\frac{\pi}{6}$$

$$2I = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$$

OR

Consider the equation

$$y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$$

$$\Rightarrow \quad y - x = (x+y)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y-x}{x+y} \quad ...(i) \text{ (homogenous)}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (i), $v + x\frac{dv}{dx} = \frac{vx - x}{x + xv} = \frac{v-1}{1+v}$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v-1}{v+1} - v = \frac{v-1-v^2-v}{v+1} = \frac{-(1+v^2)}{1+v}$$

$$\Rightarrow \quad \int \frac{1+v}{1+v^2} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{1}{1+v^2} dv + \int \frac{v}{1+v^2} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \tan^{-1} v + \frac{1}{2}\log|1+v^2| = -\log|x| + C \quad \left[\because \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C\right]$$

$$\Rightarrow \quad \tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right| = -\log|x| + C$$

$$\Rightarrow \quad \tan^{-1}\frac{y}{x} + \frac{1}{2}\log|x^2+y^2| - \frac{1}{2}\log|x|^2 = -\log|x| + C$$

$$\Rightarrow \quad \tan^{-1}\frac{y}{x} + \frac{1}{2}\log|x^2+y^2| = C$$ is required solution, where C is constant of integration.

29. A : set of all triangles and relation R is

$R = \{(T_1, T_2) \in A \times A : T_1 \sim T_2\}$.

For reflexive: For $T_1 \in A$,

$(T_1, T_1) \in R \Rightarrow T_1 \sim T_1$

which is true as every triangle is similar to itself.

Hence, R is reflexive.

For symmetric: For $T_1, T_2 \in A$,

$(T_1, T_2) \in R$

$\Rightarrow T_1 \sim T_2 \Rightarrow T_2 \sim T_1$

$\Rightarrow (T_2, T_1) \in R$ (from geometry)

Hence, R is symmetric.

For transitive: For $T_1, T_2, T_3 \in A$

Let $(T_1, T_2) \in R \Rightarrow T_1 \sim T_2$

and $(T_2, T_3) \in R \Rightarrow T_2 \sim T_3$

From geometry, we notice

$T_1 \sim T_2$ and $T_2 \sim T_3 \Rightarrow T_1 \sim T_3$

$\Rightarrow (T_1, T_3) \in R$

Hence, relation R is transitive.

As the relation R is reflexive, symmetric and transitive.

Hence, relation R is an equivalence relation.

In triangles T_1, T_2, T_3, triangles T_1 and T_3 are related as sides 3, 4, 5 and 6, 8, 10 are proportional.

30. Given function $f(x) = \begin{cases} x^2+3x+a, & x \le 1 \\ bx+2, & x > 1 \end{cases}$

Since 'f' is differentiable for all $x \in R$, it should be differentiable at $x = 1$.

$$\underset{x=1}{\text{LHD}} = \text{L}f'(1) = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{\{(1-h)^2+3(1-h)+a\}-\{1+3+a\}}{-h}$$

$$= \lim_{h\to 0}\frac{1+h^2-2h+3-3h+a-4-a}{-h} = \lim_{h\to 0}\frac{(h^2-5h)}{-h} = \lim_{h\to 0}(-h+5) = 5$$

$$\underset{x=1}{\text{RHD}} = \text{R}f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{\{b(1+h)+2\}-\{1+3+a\}}{h}$$

$$= \lim_{h\to 0}\frac{b+bh+2-4-a}{h} = \lim_{h\to 0}\frac{bh+b-a-2}{h} \quad \text{...}(i)$$

We know if 'f' is differentiable at $x = 1$, then it is continuous at $x = 1$ also.

i.e., $\underset{x=1}{\text{LHL}} = \underset{x=1}{\text{RHL}} = f(1)$

$$\lim_{x\to 1^-}(x^2+3x+a) = \lim_{x\to 1^+}(bx+2) = 1+3+a$$

$\Rightarrow \quad 1+3+a = b+2 = 4+a$

$\Rightarrow \quad b-a-2 = 0 \quad \text{...}(ii)$

Substituting in (*i*), we get

$$\underset{x=1}{\text{RHD}} = \lim_{h \to 0} \frac{bh + 0}{h} = \lim_{h \to 0} b = b \qquad \text{[from (ii)]}$$

For differentiability at $x = 1$,

$$\underset{x=1}{\text{LHD}} = \underset{x=1}{\text{RHD}}$$

$$\Rightarrow \qquad 5 = b$$

Substituting in (*ii*), we get

$$5 - a - 2 = 0 \Rightarrow a = 3$$

Hence, $a = 3, b = 5$ for function to be differentiable for $x \in R$.

OR

Consider, $$y = (\log x)^x + x^{\log x}$$

$$= e^{x \log(\log x)} + e^{\log x \cdot (\log x)} [\because x^a = e^{a \log x}]$$

Now differentiating both sides, w.r.t. x, we get

$$\frac{dy}{dx} = e^{x \log(\log x)} \left\{\log(\log x) \cdot 1 + x \cdot \frac{1}{\log x} \cdot \frac{1}{x}\right\} + e^{\log x \,.\, \log x} \left\{\frac{1}{x} \cdot \log x + \frac{1}{x} \cdot \log x\right\}$$

$$= (\log x)^x \left\{\log(\log x) + \frac{1}{\log x}\right\} + x^{\log x} . \left(\frac{2}{x} . \log x\right)$$

Alternatively:

Consider $$y = (\log x)^x + x^{\log x}$$

Let $$y = u + v$$

$$\therefore \qquad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(i)$$

Consider $$u = (\log x)^x$$

Taking log of both sides, we get

$$\log u = x \,.\, \log(\log x)$$

Differentiating with respect to x, we get

$$\frac{1}{u} . \frac{du}{dx} = x . \frac{1}{\log x} . \frac{1}{x} + \log(\log x) \cdot 1$$

$$\frac{du}{dx} = u\left[\frac{1}{\log x} + \log(\log x)\right]$$

$$= (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right] \qquad ...(ii)$$

Consider $v = x^{\log x}$

Taking log of both sides, we get $\log v = \log x \,.\, \log x = (\log x)^2$

Differentiating with respect to x, we get $\frac{1}{v}\frac{dv}{dx} = 2(\log x) . \frac{1}{x}$

$$\Rightarrow \qquad \frac{dv}{dx} = v \cdot \frac{2}{x} \log x$$

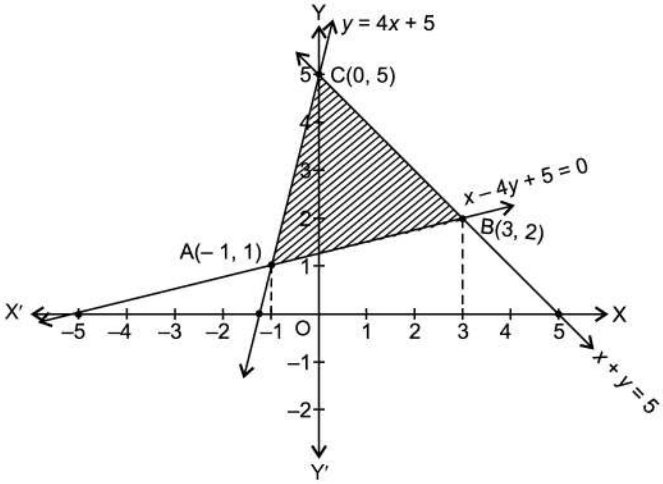
$$= x^{\log x} . \frac{2}{x} \log x \qquad ...(ii)$$

Substituting from (*ii*) and (*iii*) in (*i*), we get

$$\frac{dy}{dx} = (\log x)^x [\frac{1}{\log x} + \log(\log x)] + x^{\log x} \left(\frac{2}{x} \log x\right)$$

31. Given lines are $y = 4x + 5, x + y = 5$ and $x - 4y + 5 = 0$

Plotting these on graph, we notice we have to find shaded area.



$$\text{Area} = \int_{-1}^{0}(4x+5)\,dx + \int_{0}^{3}(5-x)\,dx - \int_{-1}^{3}\frac{x+5}{4}\,dx$$

$$= [2x^2+5x]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_0^3 - \frac{1}{4}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$$

$$= (0) - (2-5) + \left(15 - \frac{9}{2}\right) - (0) - \frac{1}{4}\left(\frac{9}{2}+15\right) + \frac{1}{4}\left(\frac{1}{2}-5\right)$$

$$= 3 + \frac{21}{2} - \frac{39}{8} - \frac{9}{8} = -3 + \frac{21}{2} = \frac{15}{2} \text{ sq units}$$

OR

Region = $\{(x, y): 0 \le y \le x^2 + 3, 0 \le y \le 2x + 3, 0 \le x \le 3\}$

On plotting the inequations we have to find the area of the shaded portion.

Eliminating y from corresponding equations, we get

$$x^2 + 3 = 2x + 3$$

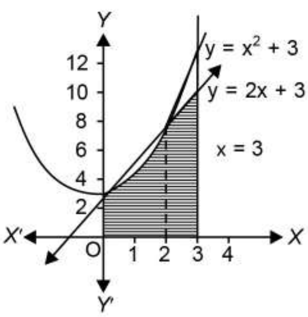
$$\Rightarrow \quad x = 0, 2$$

$$\therefore \quad \text{area} = \int_0^2 (x^2+3)\,dx + \int_2^3 (2x+3)\,dx.$$

$$= \left[\frac{x^3}{3} + 3x\right]_0^2 + [x^2+3x]_2^3$$

$$= \left(\frac{8}{3}+6\right) - (0) + (9+9) - (4+6)$$

$$= \left(\frac{8}{3}+6+18-10\right) \text{ sq units} = \frac{50}{3} \text{ sq units}$$



32. Lines are $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$

and $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$

The shortest distance = $\left|\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$.

Here

$$\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k},\ \vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k};$$

$$\vec{a_2} = \hat{i} - \hat{j} - \hat{k},\ \vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a_2} - \vec{a_1} = \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k} = \hat{j} - 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$\Rightarrow$ $$|\vec{b_1} \times \vec{b_2}| = \sqrt{4+16+9} = \sqrt{29}$$

The shortest distance

$$= \left| \frac{(\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{29}} \right| = \left| \frac{-4+12}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}} \text{ units}$$

OR

Given lines are

$$l_1 : \frac{1-x}{3} = \frac{7y+14}{p} = \frac{z-3}{2},$$

i.e. $$\frac{x-1}{-3} = \frac{y+2}{\frac{p}{7}} = \frac{z-3}{2}$$

and $$l_2 : \frac{7 - 7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5},$$

i.e. $$\frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

If lines are perpendicular then

$$(-3) \times \left(\frac{-3p}{7}\right) + \frac{p}{7} \times 1 + 2 \times (-5) = 0$$

$$\Rightarrow \quad \frac{9p}{7} + \frac{p}{7} = 10 \quad \Rightarrow \quad \frac{10p}{7} = 10$$

$$\Rightarrow \quad p = 7$$

Direction ratios of l_1 are – 3, 1, 2

Equations of the line passing through the point (3, 2,– 4) and parallel to l_1 is

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}.$$

33. Consider $$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$$

We have $$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 2(120-45) - 3(-80-30) + 10(36+36)$$

$$= 150 + 330 + 720 = 1200 \neq 0$$

Hence, A^{-1} exists.

Matrix formed by cofactors of each element in $|A|$ is given by,

$$\begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$$

$$\therefore \quad \text{Adj}\, A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Consider equations

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 2$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 5$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z} = -4$$

Corresponding matrix equation is

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

i.e. $AX = B$ is matrix equation.

Its solution is $X = A^{-1}B$ ...(*ii*)

Now A^{-1} is already known to us. So we can substitute and get matrix X and then x, y, z.

From (*ii*), we get

$$X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/3 \\ 1/5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/3 \\ 1/5 \end{bmatrix}$$

$\Rightarrow \quad x = 2; \quad y = -3; \quad z = 5$

OR

Consider, $$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix} \quad ...(i)$$

We have $$A^{-1} = \frac{1}{|A|}\,\text{adj}A$$

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{vmatrix} = 2(-2-2)-3(-1+6)+1(1+6)$$

$$= -8-15+7 = -16 \neq 0$$

Hence, A^{-1} exists.

Matrix formed by cofactors of each element in $|A|$ is

$$\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}$$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}' = \begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = -\frac{1}{16}\begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix} \qquad ...(ii)$$

Consider equations

$$2x + y - 3z = 13$$
$$3x + 2y + z = 4$$
$$x + 2y - z = 8$$

Matrix equation is

$$\begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$$

$$\Rightarrow \qquad A^T X = B \qquad \text{[from } (i)]$$

$$\Rightarrow \qquad X = (A^T)^{-1}B \text{ is its solution}$$

$$\Rightarrow \qquad X = (A^{-1})^T B \qquad ..(iii)$$

Now we have A^{-1} [from (i)] and use $(A^T)^{-1} = (A^{-1})^T$ i.e. we take transpose of A^{-1} obtained and substitute in (iii) to get X and then x, y, z.

$$X = \frac{-1}{16}\begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}^T \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-16}\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}\begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

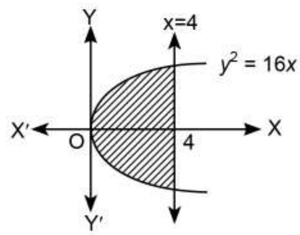
$$\therefore \qquad x = 1; \quad y = 2, \quad z = -3 \text{ is the solution.}$$

34. Given parabola is $y^2 = 16x \Rightarrow 4a = 16 \Rightarrow a = 4$

$\therefore$ latus rectum is represented by $x = 4$

We have to find shaded area.

Curve is symmetrical to the x-axis.



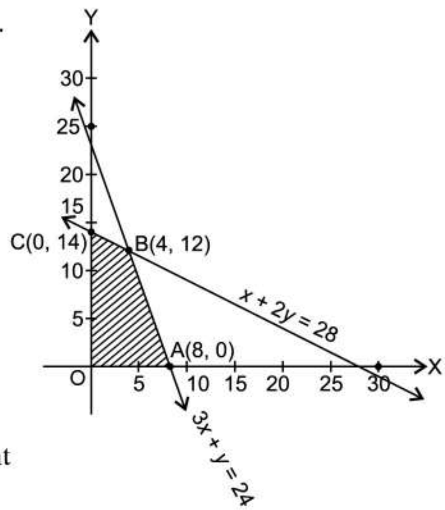
$$\therefore \qquad \text{Area} = 2\int_0^4 y\,dx = 2\int_0^4 4\sqrt{x}\,dx = \frac{8\times 2}{3}\left[x^{3/2}\right]_0^4$$

$$= \frac{16}{3}[8 - 0] = \frac{128}{3} \text{ sq units}$$

35. Plotting the inequations on graph, we notice shaded area is feasible solution.

Possible points for maximum Z are $A(8, 0)$, $B(4,12)$, $C(0, 14)$.

Points	$Z = 20x + 40y$	Values	
$A(8, 0)$	160 + 0	160	
$B(4, 12)$	80 + 480	560	← Maximum
$C(0, 14)$	0 + 560	560	← Maximum



Z is maximum for $B(4, 12)$ or $C(0, 14)$, i.e. $x = 4, y = 12$

or $\qquad x = 0,\ y = 14$

So, Maximum of Z occurs at all the points on the line segment joining points B and C.

36. (*i*) Required probability $= P(E_2) = 1 - P(E_1) = 1 - \frac{0.00001}{100} = 0.9999999 = 99.99999\%$

(*ii*) $P(A/E_1) = \frac{95}{100} = 0.95$

$P(A/E_2) = 1$

$\therefore\ P(A/E_1) + P(A/E_2) = 0.95 + 1 = 1.95$

(*iii*)
$$\begin{aligned} P(A) &= P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) \\ &= \frac{0.00001}{100} \times 0.95 + 0.9999999 \times 1 \\ &= 0.0000001 \times 0.95 + 0.9999999 \\ &= 0.000000095 + 0.9999999 \\ &= 0.999999995 \end{aligned}$$

OR

(*iii*) $P(E_2/A) = \frac{P(E_2)\times P(A/E_2)}{P(E_1)\times P(A/E_1) + P(E_2)\times P(A/E_2)} = \frac{0.9999999}{0.999999995} = \frac{999999900}{999999995} = \frac{199999980}{199999999}$

37. (*i*) Perimeter = 1000 m ...(*i*)

$\Rightarrow\quad 2y + 2x + \pi y = 1000$

(*ii*) Area of sports ground

$\therefore\quad A = 2xy = 2x\left[\frac{1000 - 2x}{2+\pi}\right]$ [from (*i*)]

$= \frac{2}{2+\pi}(1000x - 2x^2)\text{m}^2$

(*iii*) Now, $\frac{dA}{dx} = \frac{2}{2+\pi}(1000 - 4x)$

For maximum A, $\frac{dA}{dx} = 0 \Rightarrow \frac{2}{2+\pi}[1000 - 4x] = 0 \Rightarrow x = 250$

Now, $\frac{d^2A}{dx^2} = \frac{-8}{2+\pi}$

$\Rightarrow\quad \frac{d^2A}{dx^2} < 0$ for $x = 250$ m

OR

(*iii*) $A_{max} = \frac{2}{2+\pi}[1000 \times 250 - 2(250)^2]$

$= \frac{2\times 250}{2+\pi}(1000 - 500) = \frac{250000}{2+\pi}\text{m}^2$

38. (*i*) Let $y = \sin^{-1}(2x - 1)$

$\therefore\quad -1 \le (2x - 1) \le 1$

$\Rightarrow\quad 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$

$\Rightarrow\quad x \in [0, 1]$

(*ii*) Let $y = \cos^{-1}(2x - 1)$

$\therefore\quad -1 \le (2x - 1) \le 1$

$\Rightarrow\quad 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$

$\Rightarrow\quad x \in [0, 1]$