18 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*
(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*
(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*
(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*
(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*
(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*
(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*
(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*
(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then A^2 is equal to **[NCERT Part-I, Page 51]**

(*a*) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (*d*) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. If A is a 3×2 matrix, B is a 3×3 matrix and C is a 2×3 matrix, then the elements in A, B and C are respectively **[NCERT Part-I, Page 36]**

(*a*) 6, 9, 8 (*b*) 6, 9, 6 (*c*) 9, 6, 6 (*d*) 6, 6, 9

3. If A_{ij} denotes the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then value of $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ is **[NCERT Part-I, Page 84]**

(*a*) 0 (*b*) 5 (*c*) 10 (*d*) –5

4. Function $f(x) = \begin{cases} x-1, & \text{if } x < 2 \\ 2x-3, & \text{if } x \geq 2 \end{cases}$ is a continuous function **[NCERT Part-I, Page 105]**

(*a*) for $x = 2$ only. (*b*) for all real values of x such that $x \neq 2$.
(*c*) for all real values of x. (*d*) for all integral values of x only.

5. A line makes angles of 45° and 60° with the positive axes of x and y respectively. The angle made by the same line with the positive axis of z is, **[NCERT Part-II, Page 378]**

(*a*) 30° or 60° (*b*) 60° or 90° (*c*) 90° or 120° (*d*) 60° or 120°

6. Sum of order and degree of differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 = 0$ is **[NCERT Part-II, Page 301-302]**

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

7. The graph of inequations $x \le y$ and $y \le x + 3$ is located in **[Conceptual Application]**

(*a*) II quadrant (*b*) I, II quadrants

(*c*) I, II, III quadrants (*d*) II, III, IV quadrants

8. The magnitude of the vector $\left(6\hat{i} + 2\hat{j} + 3\hat{k}\right)$ is **[NCERT Part-II, Page 348]**

(*a*) 5 (*b*) 7 (*c*) 12 (*d*) 1

9. The value of integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}}\, dx$ is **[NCERT Part-II, Page 273-274]**

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{2}$ (*c*) 2 (*d*) 1

10. The solution set of equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is **[NCERT Part-I, Page 78-79]**

(*a*) {0, 1} (*b*) {1, 2} (*c*) {1, 5} (*d*) {2, – 1}

11. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is **[NCERT Part-II, Page 398]**

(*a*) $p = q$ (*b*) $p = 2q$ (*c*) $q = 2p$ (*d*) $q = 3p$

12. The value of λ for which the two vectors $\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ and $\left(3\hat{i} + \lambda\hat{j} + \hat{k}\right)$ are perpendicular is **[NCERT Part-II, Page 355-356]**

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

13. For any 2 × 2 matrix A, if A (adj. A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ is equal to **[NCERT Part-II, Page 88]**

(*a*) 0 (*b*) 10 (*c*) 20 (*d*) 100

14. If $P(A) = \frac{4}{5}$, and $P(A \cap B) = \frac{7}{10}$, then $P\left(\frac{B}{A}\right)$ is equal to **[NCERT Part-II, Page 408]**

(*a*) $\frac{1}{10}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{7}{8}$ (*d*) $\frac{17}{20}$

15. If the I.F. of the differential equation $\frac{dy}{dx} + 5y = \cos x$ is $e^{\int A\,dx}$ then A is equal to **[NCERT Part-II, Page 323]**

(*a*) 0 (*b*) 1 (*c*) 3 (*d*) 5

16. The area of the parallelogram whose adjacent sides are $\left(\hat{i} + \hat{k}\right)$ and $\left(2\hat{i} + \hat{j} + \hat{k}\right)$ is **[NCERT Part-II, Page 365]**

(*a*) $\sqrt{2}$ sq units (*b*) $\sqrt{3}$ sq units

(*c*) 3 sq units (*d*) 4 sq units

17. If $y = \sec x°$, then $\frac{dy}{dx}$ is equal to **[NCERT Part-I, Page 118-119]**

(*a*) $\sec x \tan x$ (*b*) $\sec x° \tan x°$

(*c*) $\frac{\pi}{180°} \sec x° \tan x°$ (*d*) None of these

18. The lines in a space which are neither intersecting nor parallel, are called **[Conceptual Application]**

(*a*) concurrent lines (*b*) intersecting lines

(*c*) skew lines (*d*) parallel lines

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.
(*b*) Both A and R are true but R is not the correct explanation of A.
(*c*) A is true but R is false.
(*d*) A is false but R is true.

19. Consider a function $f: R \to R$, defined by **[NCERT Part-I, Page 7]**

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

Assertion (A): f is not onto function.

Reason (R): $3 \in R$, (co-domain of f) has pre image in domain of f.

20. **Assertion(A):** If $\frac{d}{dx}[f(x)] = (x-1)^3 (x-2)^8$, then $f(x)$ has neither maximum nor minimum at $x = 2$. **[NCERT Part-I, Page 164]**

Reason(R): $\frac{d}{dx}$ $[f(x)]$ changes sign from negative to positive at $x = 2$.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Show that : $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ **[Conceptual Application]**

OR

Find domain of $f(x) = \frac{1}{2}\sec^{-1}(5x-3)$. **[Conceptual Application]**

22. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 2 cm. **[NCERT Part-I, Page 147-148]**

23. Find the smallest value of the polynomial $x^3 - 18x^2 + 96x$. **[NCERT Part-I, Page 166]**

OR

Find the interval in which $y = x^2 e^{-x}$ is strictly increasing. **[NCERT Part-I, Page 153]**

24. Evaluate $\int_0^4 \frac{dx}{\sqrt{x^2+2x+3}}$. **[NCERT Part-II, Page 246]**

25. Prove that the function given by $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R. **[NCERT Part-I, Page 153]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int \frac{x^2+x+1}{x^2-1}\,dx$. **[NCERT Part-II, Page 252-253]**

27. In a company, the probability of selecting A, B, C as a directs is $\frac{4}{9}, \frac{2}{9}, \frac{1}{3}$ respectively. The probability hat bonus scheme will be introduced, if chosen, by A, B, C is $\frac{3}{10}, \frac{1}{2}, \frac{4}{5}$ respectively. What is the probability that bounds scheme will be introduced. **[NCERT Part-II, Page ???-???]**

28. Evaluate: $\int \frac{e^x (x-3)}{(x-1)^3} dx$. **[NCERT Part-II, Page 273-274]**

OR

Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

29. Find the particular solution of differential equation: $x(x^2 - 1) \frac{dy}{dx} = 1$, given that $y = 0$ when $x = 2$. **[NCERT Part-II, Page 306-307]**

OR

Manufacturer can sell x items at a price of ₹ $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit. **[NCERT Part-II, Page 166]**

30. Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397]**

Maximise $Z = 4x + 6y$, subject to the constraints $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$.

OR

Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397]**

Minimise $Z = 11x - 10y$, subject to the constraints $x + y \leq 20$, $3x + 2y \leq 48$, $x, y \geq 0$.

31. If $y = e^{ax} \sin bx$, show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right) y = 0$. **[NCERT Part-II, Page 137]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find the area (in sq units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$ and x-axis lying in the first quadrant. **[Conceptual Application]**

33. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b): b = a + 1\}$ is reflexive, symmetric or transitive. **[NCERT Part-I, Page 2]**

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer. **[NCERT Part-I, Page 7]**

34. If $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & -2 \\ 1 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & -7 & -1 \\ -5 & 10 & 5 \\ -2 & -1 & 7 \end{bmatrix}$, find AB and hence solve the system of equations:

$3x + 2y - z = 4$, $x + 3y - 2z = 2$, $x + y + 3z = 5$. **[NCERT Part-I, Page 94]**

35. Find the shortest distance between the lines: $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$. **[NCERT Part-II, Page 386-387]**

OR

Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **[Conceptual Application]**

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Suppose Annu throws a die. If she gets a prime number she tosses a coin three times and notes the number of heads. If she gets other than prime number, she tosses a coin once and notes whether a head or tail is obtained. Suppose E_1 is the event of getting a prime number and E_2 be event of getting a non prime number when a die is thrown and A is event of getting exactly one head with the coin. **[NCERT Part-II, Page 425]**

(*i*) Find $P(E_1)$.

(*ii*) Find $P(E_2)$.

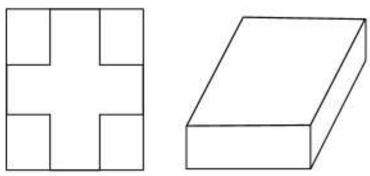
(*iii*) What is the probability that she threw a prime number with the die, if she obtained exactly one head?

OR

(*iii*) What is the probability that she threw a non-prime number with the die, if she gets exactly one head?

Case Study - 2

37. A student has a square sheet with side 18 cm and his teacher says to make a box which is open at the top by cutting a square from each corner of sheet and folding up the flaps. If x cm be the side of square which is cut from each corner, then answer the following: **[NCERT Part-I, Page 166]**



(*i*) Express the volume of box in terms of 'x'.

(*ii*) Find the value of x so that volume of box is maximum.

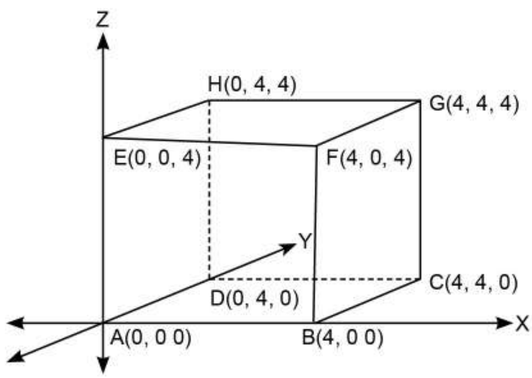
(*iii*) Find the maximum volume of box.

OR

(*iii*) Find the surface area of open box when volume is maximum.

Case Study - 3

38. There is a cube which is placed in three dimensional Cartesian systems as shown in figure with vertices $A(0, 0, 0)$, $B(4, 0, 0)$, $C(4, 4, 0)$, $D(0, 4, 0)$, $E(0, 0, 4)$, $F(4, 0, 4)$, $G(4, 4, 4)$ and $H(0, 4, 4)$.



Answer the following according to above information: **[NCERT Part-II, Page 339-340]**

(*i*) Find direction ratios of diagonals EC and AG.

(*ii*) Find the angle between diagonals HB and DF.

SOLUTIONS

1. (*d*)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. (*b*) A is *a* 3×2 matrix,
No. of elements = 6
B is a 3×3 matrix
No. of elements = 9
C is *a* 2×3 matrix,
No. of elements = 6

3. (*a*)

$$a_{11} = 2;\ a_{12} = -3;\ a_{13} = 5$$

$$A_{31} = -12;\ A_{32} = 22;\ A_{33} = 18$$

$$\therefore \quad a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 2 \times (-12) + (-3) \times 22 + 5 \times 18 = -24 - 66 + 90 = 0$$

4. (*c*) $f(x) = \begin{cases} x-1, & \text{if } x < 2 \\ 2x-3, & \text{if } x \geq 2 \end{cases}$

Note: A polynomial function is always a continuous function. And the given question is a polynomial function with degree 1. Thus the continuity is for all real value of x.

5. (*d*)

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 45° + \cos^2 60° + \cos^2\gamma = 1$$

$$\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\cos^2\gamma = 1 - \frac{3}{4}$$

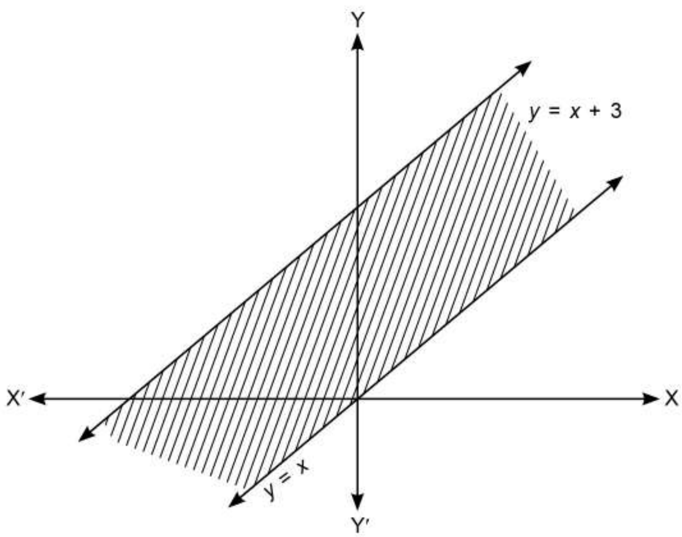
$$\cos^2\gamma = \frac{1}{4}$$

$$\cos\gamma = \pm\frac{1}{2}$$

Therefore required angle is 60° or 120°.

6. (*c*)

7. (*c*)



8. (*b*) Let
$$\vec{a} = \left(6\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$|\vec{a}| = \sqrt{6^2 + 2^2 + 3^2} = \sqrt{49} = 7$$

9. (*b*)
$$I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}}\, dx \qquad ...(i)$$
$$\Rightarrow \quad I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}}\, dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get
$$2I = \int_3^6 \frac{\sqrt{9-x} + \sqrt{x}}{\sqrt{9-x} + \sqrt{x}}\, dx$$
$$\Rightarrow \quad 2I = \int_3^6 1\, dx = \Big[x\Big]_3^6$$
$$\Rightarrow \quad I = \frac{3}{2}$$

10. (*d*) Equation is $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

On expanding : $1(-10x^2 - 10x) - 4(5x^2 - 5) + 20(2x + 2) = 0$
$$\Rightarrow \quad -10x^2 - 10x - 20x^2 + 20 + 40x + 40 = 0$$
$$\Rightarrow \quad -30x^2 + 30x + 60 = 0$$
$$\Rightarrow \quad 30(-x^2 + x + 2) = 0$$
$$\Rightarrow \quad (x - 2)\ (x + 1) = 0 \Rightarrow x = 2, x = -1$$

11. (*d*) $Z = px + qy$, where $p, q > 0$.

Z occurs at both the points (15, 15) and (0, 20)
$$\therefore \quad 15p + 15q = 20q$$
$$\Rightarrow \quad 15p = 5q$$
$$\Rightarrow \quad 3p = q$$

12. (*d*) Vectors $\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ and $\left(3\hat{i} + \lambda\hat{j} + \hat{k}\right)$ are perpendicular

therefore $\left(2\hat{i} - \hat{j} + 2\hat{k}\right) \cdot \left(3\hat{i} + \lambda\hat{j} + \hat{k}\right) = 0$
$$\Rightarrow \quad 6 - \lambda + 2 = 0$$
$$\Rightarrow \quad \lambda = 8$$

13. (*b*)
$$A(\text{adj. } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Also, $A(\text{adj. } A) = |A|\ I$

then $|A| = 10$

14. (*c*) We have $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$.

Now,
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{7}{10} \times \frac{5}{4} = \frac{7}{8}$$

15. (*d*) $A = 5$

16. (*b*) Let $\vec{a} = \hat{i} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$.

Now, $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

Now, $$\text{area} = |\vec{a} \times \vec{b}| = |-\hat{i} + \hat{j} + \hat{k}|$$
$$= \sqrt{3} \text{ sq units}$$

17. (*c*) We have, $$y = \sec x^\circ$$

Now, $$x^\circ = \frac{\pi}{180} \times x^c$$

So, $$y = \sec \frac{\pi}{180}x$$

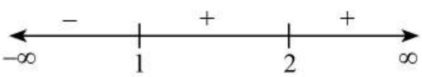
Now, $$\frac{dy}{dx} = \frac{\pi}{180^\circ} . \sec\left(\frac{\pi x}{180^\circ}\right) . \tan\left(\frac{\pi x}{180^\circ}\right)$$

$\Rightarrow$ $$\frac{dy}{dx} = \frac{\pi}{180^\circ} \sec x^\circ \tan x^\circ$$

18. (*c*) Skew lines

19. (*c*) (A) is true but (R) is false.

20. (*c*)



Signs of $f'(x)$ for different values of x

It is clear from figure $\frac{d}{dx}f(x)$ has no sign change at $x = 2$. Hence, $f(x)$ has neither maximum nor minimum at $x = 2$.

So (A) is true but (R) is false.

21. LHS $= \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

Let $$\sin^{-1}\frac{3}{4} = y$$

therefore, $$\sin y = \frac{3}{4} \text{ and } \cos y = \frac{\sqrt{7}}{4}$$

$\Rightarrow$ $$\tan\frac{y}{2} = \sqrt{\frac{1 - \cos y}{1 + \cos y}}$$

$\Rightarrow$ $$\tan\frac{y}{2} = \sqrt{\frac{1 - \frac{\sqrt{7}}{4}}{1 + \frac{\sqrt{7}}{4}}}$$
$$= \sqrt{\frac{4 - \sqrt{7}}{4 + \sqrt{7}}}$$
$$= \frac{4 - \sqrt{7}}{3} = \text{RHS.}$$

Hence Proved

OR

$$5x - 3 \le -1 \text{ or } 5x - 3 \ge 1$$

$$\Rightarrow \qquad x \le \frac{2}{5} \text{ or } x \ge \frac{4}{5}$$

$\therefore$ Domain is $\left(-\infty, \frac{2}{5}\right] \cup \left[\frac{4}{5}, \infty\right)$.

22. Let r be radius of the circle and A be its area at any instant of time 't'.

We have $\frac{dr}{dt} = 3$ cm/s; $\frac{dA}{dt} = ?$

We know that $A = \pi r^2$

Differentiating both sides. with respect to 't' we get

$$\frac{dA}{dt} = 2r\pi \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 6\pi r$$

$$\Rightarrow \qquad \left(\frac{dA}{dt}\right)_{r=2} = 6\pi \times 2 = 12\pi \text{ cm}^2\text{/s}.$$

23. Let $f(x) = x^3 - 18x^2 + 96x$.

Differentiating both sides w.r.t. x, we get

$$f'(x) = 3x^2 - 36x + 96$$
$$= 3(x^2 - 12x + 32)$$
$$= 3(x - 4)(x - 8) \qquad ...(i)$$

For critical points, $f'(x) = 0$

$$\Rightarrow \qquad 3(x - 4)(x - 8) = 0$$

$$\Rightarrow \qquad x = 4, 8$$

On differentiating both sides of (i), w.r.t. x, we get

$$f''(x) = 3(x - 4)\,1 + 3(x - 8)\,1 = 6(x - 6)$$

At $x = 8$ sign of $f''(x)$ is positive.

Therefore $x = 8$ is point of minima.

Therefore minimum value $= 8^3 - 18 \times 8^2 + 96 \times 8$

$= 512 - 1152 + 768$

$= 128$

OR

We have, $y = x^2e^{-x}$

Now, $\frac{dy}{dx} = -x^2e^{-x} + 2xe^{-x}$

$= e^{-x}(2x - x^2)$

For 'y' to be strictly increasing

$$\frac{dy}{dx} > 0$$

$$\Rightarrow \qquad \frac{2x - x^2}{e^x} > 0$$

$\Rightarrow \quad 2x - x^2 > 0 \quad [\because e^{-x} > 0 \ \forall \ x \in R]$

$\Rightarrow \quad x(2 - x) > 0$

$\Rightarrow \quad x(x - 2) < 0$

$\Rightarrow \quad x \in (0, 2)$

So, $f(x)$ is strictly increasing on (0, 2).

24. $$I = \int_0^4 \frac{dx}{\sqrt{x^2 + 2x + 3}}$$

$$I = \int_0^4 \frac{dx}{\sqrt{(x+1)^2 + (\sqrt{2})^2}}$$

$$= \left[\log\left|x + 1 + \sqrt{(x+1)^2 + (\sqrt{2})^2}\right|\right]_0^4$$

$$= \log\left(5 + \sqrt{25+2}\right) - \log\left(1 + \sqrt{1+2}\right)$$

$$= \log\left(5 + \sqrt{27}\right) - \log\left(1 + \sqrt{3}\right) = \log\left(\frac{5 + \sqrt{27}}{1 + \sqrt{3}}\right)$$

25. $f(x) = x^3 - 3x^2 + 6x - 100$

Differentiating w.r.t. x, we get

$$\frac{d}{dx} f(x) = 3x^2 - 6x + 6$$

$$= 3(x^2 - 2x + 2)$$

$$= 3[(x - 1)^2 + 1]$$

But $(x - 1)^2 \geq 0 \ \forall \ x \in \text{R}$

Therefore $\frac{d}{dx} f(x) > 0$.

Therefore $f(x)$ is strictly increasing for all real values of x.

26. $I = \int \frac{x^2 + x + 1}{x^2 - 1} dx$

$$\Rightarrow \quad \int \frac{x^2 + x + 1}{x^2 - 1} dx = \int 1\, dx + \int \frac{x + 2}{(x+1)(x-1)} dx \quad ...(i)$$

Let $$\frac{x + 2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\Rightarrow \quad x + 2 = A(x - 1) + B(x + 1)$$

On comparing the coefficients of x and constant term, we get $A + B = 1$ and $B - A = 2$

On solving, we get $B = \frac{3}{2}, A = -\frac{1}{2}$

$$\int \frac{x + 2}{(x+1)(x-1)} dx = \frac{-1}{2} \int \frac{1}{x+1} dx + \frac{3}{2} \int \frac{1}{x-1} dx = \frac{-1}{2} \log|x+1| + \frac{3}{2} \log|x-1| + C$$

Therefore from (i), $$I = \int \frac{x^2 + x + 1}{x^2 - 1} dx = x - \frac{1}{2} \log|x+1| + \frac{3}{2} \log|x-1| + C$$

27. $P(A) = \frac{4}{9}, P(B) = \frac{2}{4}, P(C) = \frac{1}{3}$

P(Bouns scheme is introduced)

$= P(A)P\left(\frac{E}{A}\right) + P(B)P\left(\frac{E}{B}\right) + P(C)\,P\left(\frac{E}{C}\right)$

$= \frac{4}{9} \times \frac{3}{10} + \frac{2}{9} \times \frac{1}{2} + \frac{1}{3} \times \frac{4}{5}$

$= \frac{12}{90} + \frac{1}{9} + \frac{4}{15} = \frac{12+10+24}{90} = \frac{46}{90} = \frac{23}{45}.$

28.
$$I = \int \frac{e^x (x-3)}{(x-1)^3}\,dx \quad \Rightarrow \quad I = \int \frac{e^x[(x-1)-2]}{(x-1)^3}\,dx$$

$$\Rightarrow \quad I = \int \frac{e^x (x-1)}{(x-1)^3}\,dx - \int \frac{2e^x}{(x-1)^3}\,dx$$

$$\Rightarrow \quad I = \int \frac{e^x}{(x-1)^2}\,dx - \int e^x . \frac{2}{(x-1)^3}\,dx$$

$$\Rightarrow \quad I = \int e^x \left[\frac{1}{(x-1)^2} + \frac{(-2)}{(x-1)^3}\right] dx$$

$$\Rightarrow \quad I = \frac{e^x}{(x-1)^2} + C$$

Note: $\int e^x [f(x) + f'(x)]dx = e^x \cdot f(x) + C$

OR

$$I = \int_0^\pi \frac{x \sin x}{1 + \cos^2 x}\,dx \qquad ...(i)$$

Now,
$$I = \int_0^\pi \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx \qquad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^\pi \frac{\pi \sin x}{1+\cos^2 x}\,dx$$

$2I = -\int_1^{-1} \frac{\pi\, dt}{1+t^2}$ [Let $\cos x = t \Rightarrow -\sin x\, dx = dt$, if $x = 0$ then $t = 1$ and $x = \pi$ then $t = -1$]

$$\Rightarrow \quad 2I = \int_{-1}^{1} \frac{\pi dt}{1+t^2}$$

$$\Rightarrow \quad 2I = \pi\,[\tan^{-1} t]_{-1}^{1}$$

$$\Rightarrow \quad I = \frac{\pi^2}{4}$$

29. $x(x^2-1)\frac{dy}{dx} = 1$

$\Rightarrow \quad dy = \frac{dx}{x(x^2-1)}$

Integrating both sides, we get

$\Rightarrow \quad y = \int \frac{1}{x^3\left(1 - \frac{1}{x^2}\right)}\,dx$

So, $y = \frac{1}{2}\int \frac{1}{t}\,dt$ [put $\left(1 - \frac{1}{x^2}\right) = t \Rightarrow \frac{2}{x^3}dx = dt$]

$\Rightarrow \quad y = \frac{1}{2}\log|t| + C \Rightarrow y = \frac{1}{2}\log\left|\left(1 - \frac{1}{x^2}\right)\right| + C$

On putting $y = 0$ and $x = 2$, in the above equation we get $C = -\frac{1}{2}\log\frac{3}{4}$

So, particular solution is $y = \frac{1}{2}\log\left|\left(1 - \frac{1}{x^2}\right)\right| - \frac{1}{2}\log\frac{3}{4}$.

OR

Profit = selling price – cost price

$\therefore \quad$ profit $(P) = x\left(5 - \frac{x}{100}\right) - \left(\frac{x}{5} + 500\right)$

$= 5x - \frac{x^2}{100} - \frac{x}{5} - 500 = -\frac{x^2}{100} + \frac{24x}{5} - 500$

$P' = \frac{-2x}{100} + \frac{24}{5};$

For maximum P, $P' = 0$

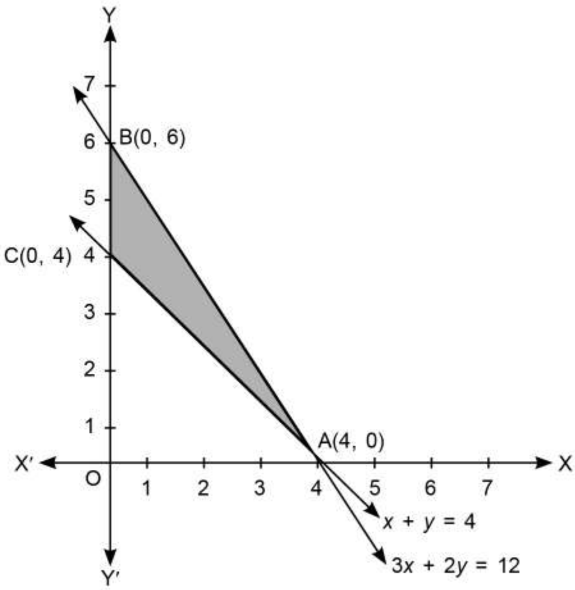
$\Rightarrow \quad \frac{x}{50} = \frac{24}{5} \Rightarrow x = 240$

$P'' = -\frac{1}{50};\ [P'']_{x=240} < 0$

Hence, the manufacturer can earn maximum profit if he sells 240 items.

30. We have to maximise $Z = 4x + 6y$, subject to $3x + 2y \le 12$, $x + y \ge 4$, $x, y \ge 0$

Plotting the inequations on the graph, we notice shaded portion is feasible solution.



Shaded portion ABC is the feasible region, where $A(4, 0)$, $C(0, 4)$ and $B(0, 6)$ are possible points for maximum Z.

Corner points	**Values of $Z = 4x + 6y$**	
$A(4, 0)$	16	
$B(0, 6)$	36	←Maximum
$C(0, 4)$	24	

Thus, Z is maximised at $B(0, 6)$ and its maximum value is 36.

OR

We have to minimise $Z = 11x - 10y$

Subject to the constraints:

$$x + y \leq 20 \quad ...(i)$$
$$3x + 2y \leq 48 \quad ...(ii)$$
$$x, y \geq 0 \quad ...(iii)$$

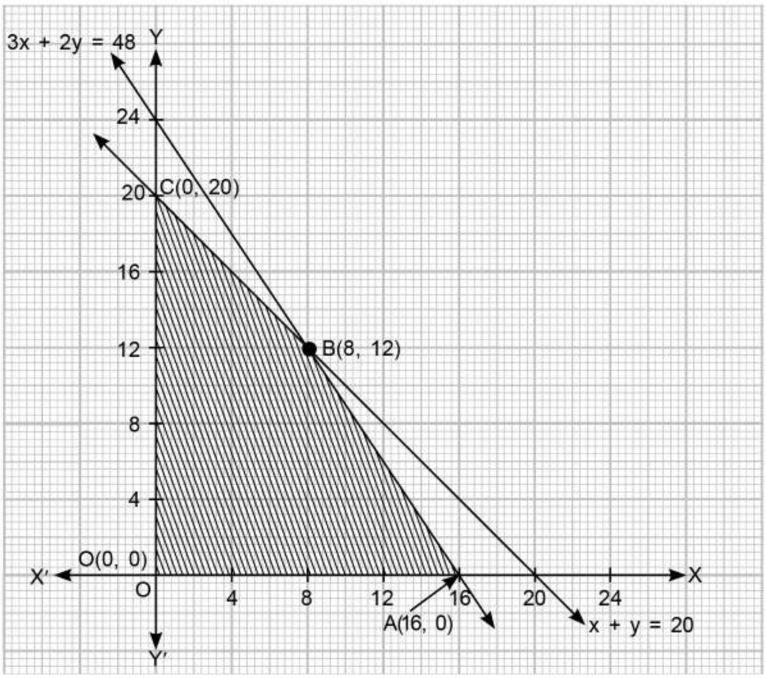
Converting (*i*) and (*ii*) to equations we get

$$x + y = 20 \quad ...(iv)$$
$$3x + 2y = 48 \quad ...(v)$$

Solving (*iv*) and (*v*), we get

$$x = 8, \quad y = 12$$

Let us graph the feasible region of the system of linear inequalities (*i*) to (*iii*). The feasible region is shaded, where $O(0, 0)$, $A(16, 0)$, $B(8, 12)$ and $C(0, 20)$ are possible points for minimum Z.



Corner points	Values of $Z = 11x - 10y$	
$O(0, 0)$	0	
$A(16, 0)$	176	
$B(8, 12)$	–32	
$C(0, 20)$	–200	←Minimum

$\therefore$ Minimum value = –200 at $x = 0$ and $y = 20$.

31. $y = e^{ax} \sin bx$

Differentiating w.r.t. 'x', we get

$$\frac{dy}{dx} = e^{ax} \cos bx \, . \, b + e^{ax} \, . \, a \sin bx$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{ax} \cos bx \, . \, b + ay$$

$\Rightarrow \qquad \dfrac{dy}{dx} - ay = e^{ax} \cos bx \,.\, b$

Differentiating again w.r.t. 'x', we get

$$\frac{d^2y}{dx^2} - a\frac{dy}{dx} = b(e^{ax} \cos bx \; a - e^{ax} \,.\, b \sin bx)$$

$\Rightarrow \qquad \dfrac{d^2y}{dx^2} - a\dfrac{dy}{dx} = b \; a \; e^{ax} \cos bx - b^2 \, e^{ax} \sin bx$

$\Rightarrow \qquad \dfrac{d^2y}{dx^2} - a\dfrac{dy}{dx} = a\left(\dfrac{dy}{dx} - ay\right) - b^2 \, y$

$\Rightarrow \qquad \dfrac{d^2y}{dx^2} - a\dfrac{dy}{dx} = a\dfrac{dy}{dx} - a^2 \, y - b^2 \, y$

$\Rightarrow \qquad \dfrac{d^2y}{dx^2} - 2a\dfrac{dy}{dx} + \left(a^2 + b^2\right) y = 0$ Hence Proved.

32. Given curves are $y = \sqrt{x}$ .... (i) and $2y - x + 3 = 0$ .... (ii)

On solving equations (i) and (ii), we get

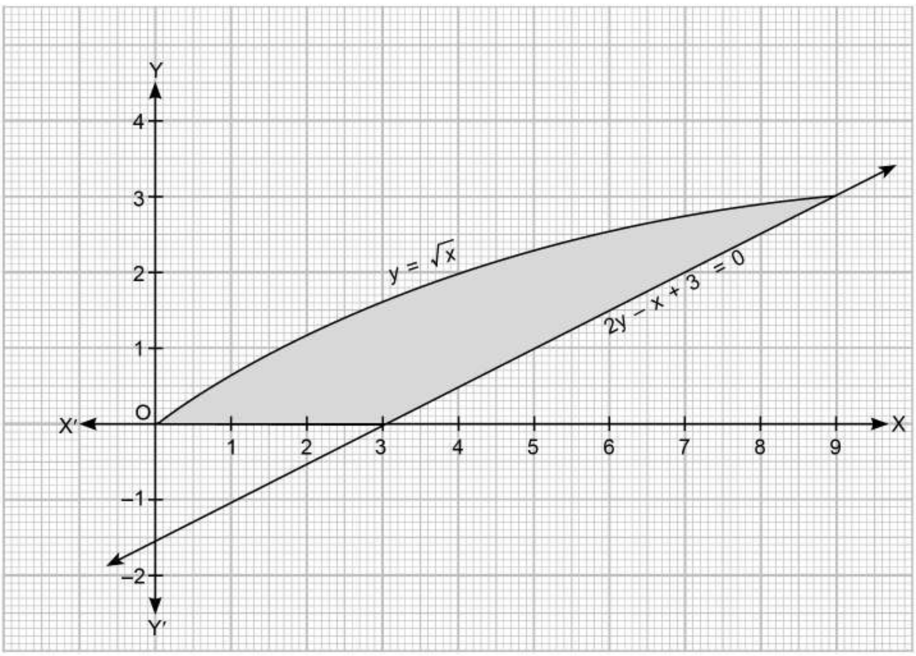
$$2y - y^2 + 3 = 0$$

$\Rightarrow \qquad y^2 - 2y - 3 = 0$

$\Rightarrow \qquad (y - 3)\,(y + 1) = 0$

$\Rightarrow \quad y = 3$ and $y = -1$(rejected because $\sqrt{x} = -1$ not possible)

$\Rightarrow \quad y = 3$



$\therefore \qquad$ Required area $= \displaystyle\int_0^3 (2y + 3)dy - \int_0^3 y^2 \, dy$

$$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3$$

$= 9 + 9 - 9$

$= 9$ sq units

33. Given relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$

$R = \{(1,2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

Reflexive:

Let $a \in A$, we have, $a \neq a + 1 \Rightarrow (a, a) \notin R$ as $(1, 1)$, $(2, 2)$, ... $(6, 6) \notin R$.

$\therefore$ It is not reflexive.

Symmetric: Let $a = 1$ and $b = 2$ i.e. $a, b \in A$, $\therefore b = a + 1 \Rightarrow 2 = 1 + 1 \Rightarrow (a, b) \in$ R but $a \neq b + 1$ as $1 \neq 2 + 1 \Rightarrow (b, a) \notin R$. So, $(1, 2) \in R$ but $(2, 1) \notin R$.

$\therefore$ It is not symmetric.

Transitive: Let $a, b, c \in A$. Now, if $(a, b) \in R \Rightarrow b = a + 1$ ...(*i*) and $(b, c) \in R \Rightarrow c = b + 1$ ...(*ii*)

From (*i*) and (*ii*), we have $c = (a + 1) + 1 = a + 2 \Rightarrow c = a + 2 \Rightarrow (a, c) \notin R$

For example, $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$.

$\therefore$ It is not transitive.

Hence, relation R is neither reflexive, nor symmetric, nor transitive.

OR

Given that, $A = R - \{3\}$, $B = R - \{1\}$. $f : A \to B$ is defined by $f(x) = \dfrac{x-2}{x-3}$

For injectivity : Let $x_1, x_2 \in$ A. Then,

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 3x_2 - 2x_1 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$$

So, $f(x)$ is an injective function.

For surjectivity:

Let $$y = \frac{x-2}{x-3}$$

$$\Rightarrow xy - 3y = x - 2$$

$$\Rightarrow xy - x = 3y - 2$$

$$\Rightarrow x(y - 1) = 3y - 2$$

$$\Rightarrow x = \frac{3y-2}{y-1} \in A \text{ for every } y \in B.$$

So, $f(x)$ is a surjective function. Hence, $f(x)$ is a bijective function i.e. one-one and onto.

34.

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & -2 \\ 1 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 & -7 & -1 \\ -5 & 10 & 5 \\ -2 & -1 & 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & -2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 11 & -7 & -1 \\ -5 & 10 & 5 \\ -2 & -1 & 7 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 33-10+2 & -21+20+1 & -3+10-7 \\ 11-15+4 & -7+30+2 & -1+15-14 \\ 11-5-6 & -7+10-3 & -1+5+21 \end{bmatrix}$$

$$\Rightarrow AB = 25I \quad ...(i)$$

The given system of equations is:

$$3x + 2y - z = 4, \quad x + 3y - 2z = 2, \quad x + y + 3z = 5$$

Matrix equation is $\begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & -2 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$

We have $\quad CX = D$ and $X = C^{-1}D$

Here $\quad C = A \Rightarrow C^{-1} = A^{-1}$

We have $\quad AB = 25I$

Therefore $\quad A^{-1} = \frac{1}{25}B \Rightarrow C^{-1} = \frac{1}{25}B$

Then $\quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 11 & -7 & -1 \\ -5 & 10 & 5 \\ -2 & -1 & 7 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 44 - 14 - 5 \\ -20 + 20 + 25 \\ -8 - 2 + 35 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 25 \\ 25 \\ 25 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ therefore $x = 1, y = 1, z = 1$.

35. Lines are : $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$ and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

In vector form:

$$\vec{r} = (-2\hat{j} + \hat{k}) + \lambda(6\hat{i} + 6\hat{j} + \hat{k})$$

and $\quad \vec{r} = (-\hat{i} + 0\hat{j} + 0\hat{k}) + \mu(12\hat{i} + 6\hat{j} - \hat{k})$

Comparing with

$$\vec{r} = \vec{a_1} + \lambda\vec{b_1} \text{ and } \vec{r} = \vec{a_2} + \mu\vec{b_2}$$

we get

$$\vec{a_1} = -2\hat{j} + \hat{k}, \quad \vec{a_2} = -\hat{i} + 0\hat{j} + 0\hat{k}, \ \vec{b_1} = 6\hat{i} + 6\hat{j} + \hat{k},$$

$$\vec{b_2} = 12\hat{i} + 6\hat{j} - \hat{k}$$

$$\vec{a_2} - \vec{a_1} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-12)^2 + (18)^2 + (-36)^2} = \sqrt{1764} = 42$$

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = 12 + 36 + 36 = 84$$

Shortest distance $= \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$= \frac{84}{42} = 2$ units

OR

Equation of line passing through (1, 2, –4) having DR's <a, b, c> is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \qquad ...(i)$$

Line (i) is perpendicular to lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$$\therefore \quad 3a - 16b + 7c = 0 \qquad ...(ii)$$

$$3a + 8b - 5c = 0 \qquad ...(iii)$$

Solving (ii) and (iii) by cross-multiplication,

$$\frac{a}{80-56} = \frac{-b}{-15-21} = \frac{c}{24+48}$$

$$\frac{a}{24} = \frac{-b}{-36} = \frac{c}{72}$$

So, $\langle a, b, c\rangle \equiv \langle 2, 3, 6\rangle$

Therefore equation of line $= \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

$\therefore$ Vector equation of the line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

36. Let E_1 : Event that Annu gets a prime number when a die is thrown.

A : Event that she gets exact one head.

E_2 : Event that Annu gets non prime number when a die is thrown.

(i) $P(E_1) = \frac{1}{2}$

(ii) $P(E_2) = \frac{1}{2}$

(iii) $P\left(\frac{A}{E_1}\right) = \frac{3}{8}$ and $P\left(\frac{A}{E_2}\right) = \frac{1}{2}$

$$\text{Now, } P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right) + P(E_2)\times P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{3}{8}}{\frac{1}{2}\times\frac{3}{8} + \frac{1}{2}\times\frac{1}{2}} = \frac{3}{7}$$

OR

(iii)

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\times P\left(\frac{A}{E_2}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right) + P(E_2)\times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{3}{8} + \frac{1}{2}\times\frac{1}{2}} = \frac{4}{7}$$

37. (i)

Length of box $= (18 - 2x)$ cm

Breadth of box $= (18 - 2x)$ cm

Height $= x$ cm

Volume of the box $= x(18 - 2x)^2$ cm^3

(*ii*) $$\text{Volume} = x(18 - 2x)^2$$

Differentiating both sides w.r.t. x, we get

$$\frac{dV}{dx} = (18 - 2x)^2 + x \times 2(18 - 2x)(-2)$$

$$= (18 - 2x)(18 - 6x) \qquad ...(i)$$

For maximum or minimum volume, $\frac{dV}{dx} = 0$

$\Rightarrow 18 - 2x = 0$ or $18 - 6x = 0 \Rightarrow x = 9$ (not possible), $x = 3$

$\therefore x = 3$

Again differentiating (*i*), w.r.t. x, we get

$$\therefore \quad \frac{d}{dx}\left(\frac{dV}{dx}\right) = (18 - 2x)(-6) + (18 - 6x)(-2)$$

$$\Rightarrow \quad \left(\frac{d^2V}{dx^2}\right)_{x=3} = -72 < 0$$

At $x = 3$, volume is maximum.

(*iii*) Maximum volume $= 12 \times 12 \times 3$

$= 432 \text{ cm}^3$

OR

(*iii*) For maximum volume

$$l = 12 \text{ cm}, b = 12 \text{ cm}, h = 3 \text{ cm}$$

$$\text{Surface area} = lb + 2hl + 2hb$$

$$= 12 \times 12 + 2 \times 3 \times 12 + 2 \times 3 \times 12$$

$$= 144 + 72 + 72$$

$$= 288 \text{ cm}^2$$

38. $A(0, 0, 0)$, $B(4, 0, 0)$, $C(4, 4, 0)$, $D(0, 4, 0)$, $E(0, 0, 4)$, $F(4, 0, 4)$, $G(4, 4, 4)$ and $H(0, 4, 4)$.

(*i*) DR's of EC $= \langle 4, 4, -4 \rangle$

DR's of AG $= \langle 4, 4, 4 \rangle$

(*ii*) DR's of HB $= \langle 4, -4, -4 \rangle$

DR's of DF $= \langle 4, -4, 4 \rangle$

Let 'θ' be the angle between HB and DF

$$\cos\theta = \frac{4 \times 4 + \{(-4) \times (-4)\} + 4 \times \{-4\}}{\sqrt{4^2 + (-4)^2 + (-4)^2}\sqrt{4^2 + (-4)^2 + 4^2}} = \frac{1}{3}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$