19 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(*iii*) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions,*** *carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If the matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ and $C = [c_{ij}]$ are of the same order, say $m \times n$, satisfy associative law, then **[NCERT Part-I, Page 46-47]**

(*a*) $(A + B) + C = A + (B + C)$ (*b*) $A + B = B + C$

(*c*) $A + C = B + C$ (*d*) $A + B + C = A - B - C$

2. If A is a $m \times n$ matrix with entries a_{ij}, then the matrix A can be represented as **[NCERT Part-I, Page 37]**

(*a*) $A = [a_{ij}]_{m\times n}$ (*b*) $A = [a_{ji}]_{m\times n}$

(*c*) $A = [a_{ij}]_{n\times m}$ (*d*) $A = [a_{ji}]_{n\times m}$

3. If $[1 \quad x \quad 1]\begin{bmatrix}1 & 3 & 2\\0 & 5 & 1\\0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix} = O$ then $x =$ **[NCERT Part-I, Page 51]**

(*a*) $-\frac{1}{2}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) –1

4. The integrating factor of the differential equation: $x\frac{dy}{dx} + y = e^x$ is **[NCERT Part-II, Page 322-323]**

(*a*) x (*b*) y (*c*) $\log x$ (*d*) None of these

5. A line makes α, β and γ angles with the positive axes of x, y and z respectively. Then value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is **[NCERT Part-II, Page 377-378]**

(*a*) 1 (*b*) –1 (*c*) 2 (*d*) –2

6. Sum of order and degree of differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 3x - \frac{dy}{dx}$ is **[NCERT Part-II, Page 301-302]**

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

7. The maximum value of $Z = 5x + 2y$, subject to constraints: **[NCERT Part-II, Page 397-398]**
$x + y \leq 7, x + 2y \leq 10, x, y \geq 0$ is

(*a*) 10 (*b*) 26 (*c*) 35 (*d*) 70

8. If $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, then the projection of $\vec{a}$ on $\vec{b}$ is **[NCERT Part-II, Page 358]**

(*a*) $\sqrt{5}$ (*b*) $\sqrt{7}$ (*c*) $\sqrt{12}$ (*d*) $\sqrt{6}$

9. The value of integral $\int_{-1}^{1} |x|\, dx$ is **[NCERT Part-II, Page 274]**

(*a*) 1 (*b*) 3 (*c*) 2 (*d*) None of these

10. For what value of *k*, points $P(3, -2)$, $Q(8, 8)$ and $R(k, 2)$ are collinear? **[NCERT Part-I, Page 82]**

(*a*) $k = 5$ (*b*) $k = -5$ (*c*) $k = 15$ (*d*) None of these

11. Objective function of an LPP is **[Conceptual Application]**

(*a*) a constant (*b*) a function to be optimised

(*c*) a relation between the variables (*d*) None of these

12. $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$ is equal to **[NCERT Part-I, Page 363-364]**

(*a*) $(\vec{a} \times \vec{b})$ (*b*) $2(\vec{a} \times \vec{b})$ (*c*) $2(\vec{a} \cdot \vec{b})$ (*d*) None of these

13. If $A = \begin{bmatrix} 3 & 4 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 15 \\ 0 & -10 \end{bmatrix}$ then $|AB|$ is equal to : **[Integrated Question]**

(*a*) 80 (*b*) 100 (*c*) – 110 (*d*) None of these

14. If $P(A) = \frac{1}{5}$, and $P(B) = 0$, then $P(A/B)$ is equal to **[NCERT Part-II, Page 408-409]**

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) not defined

15. $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is equal to **[NCERT Part-II, Page 241]**

(*a*) $\tan x + \cot x + C$ (*b*) $\tan x - \cot x + C$

(*c*) $\tan x \cot x + C$ (*d*) $\tan x - \cot 2x + C$

16. For unit vector $\vec{a}$, $(\vec{x} + \vec{a}) \cdot (\vec{x} - \vec{a}) = 5$ then $|\vec{x}|$ is **[NCERT Part-II, Page 356]**

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$ (*c*) 3 (*d*) $\sqrt{6}$

17. If $y = e^{(1+\log x)}$, then $\frac{dy}{dx}$ is equal to: **[Conceptual Application]**

(*a*) e (*b*) 1 (*c*) 0 (*d*) $x \log_e x$

18. Under what condition do $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{2}, k \right\rangle$ represent direction cosines of a line? **[NCERT Part-II, Page 378]**

(*a*) $k = \frac{1}{2}$ (*b*) $k = -\frac{1}{2}$ (*c*) $k = \pm\frac{1}{2}$ (*d*) None of these

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.

(*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.

(*c*) *A* is true but *R* is false.

(*d*) *A* is false but *R* is true.

19. The relation $R = \{(1, 1) (2, 2), (3, 3)\}$ is defined on set $\{1, 2, 3\}$. **[NCERT Part-I, Page 2]**

Assertion (*A*): Relation is only reflexive but neither symmetric nor transitive.
Reason (*R*): A relation R on a set A is said to be reflexive, if $(a, a) \in R$ for all $a \in A$.

20. **Assertion (*A*):** Every differentiable function is continuous but converse is not necessarily true. **[NCERT Part-I, Page 120]**

Reason (*R*): Function $f(x) = |x|$ is continuous.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Evaluate : $\tan\left(\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5}\right)$. **[Conceptual Application]**

OR

Find the value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$. **[Conceptual Application]**

22. The radius of a cylinder is increasing uniformly at the rate of 2 cm/s and its altitude is decreasing at the rate of 3 cm/s. Find the rate at which the volume of the cylinder is increasing or decreasing when the radius is 3 cm and altitude is 4 cm. **[NCERT Part-I, Page 147-148]**

23. At $x = 1$, the function $f(x) = x^4 - 32x^2 + ax + 10$ attains its maximum value on interval $[0, 2]$, then find the value of 'a'. **[NCERT Part-I, Page 166]**

OR

Find the interval(s) in which $y = 2x^3 + 9x^2 + 12x - 1$ is strictly decreasing. **[NCERT Part-I, Page 153]**

24. Evaluate $\int_0^{\pi/4} |\sin x - \cos x|\, dx$. **[Conceptual Application]**

25. A ladder is resting against a wall at an angle of 30°. A man is ascending the ladder at the rate of 3 m/min. What is his rate of approaching the wall? **[NCERT Part-I, Page 147-148]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ **[NCERT Part-II, Page 273-274]**

27. The odds against solving a problems by A and B are 3:2 and 2:7 respectively. If both try to solve the problem independently what is the probability that problem is solved. **[Conceptual Application]**

28. Evaluate: $\int x(\log x)^2 dx$ **[NCERT Part-II, Page 259-260]**

OR

Evaluate: $\int \frac{1}{x(x^4 - 1)} dx$ **[NCERT Part-II, Page 252-253]**

29. Find the particular solution of differential equation: $2(y + 3) - xy\frac{dy}{dx} = 0$, given that $y(1) = -2$. **[NCERT Part-II, Page 306-307]**

OR

Find the minimum value of $(ax + by)$, where $xy = c^2$. **[Conceptual Application]**

30. Solve the following LPP graphically: **[NCERT Part-II, Page 397-398]**

Maximise $Z = 3x + 5y$,

Subject to the constraints: $x + 4y \leq 24$, $3x + y \leq 21$, $x + y \leq 9$, $x \geq 0$, $y \geq 0$.

OR

Solve the following LPP graphically: **[NCERT Part-II, Page 397-398]**

Minimize $Z = 9x - 11y$,

Subject to the constrains $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$

31. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$ **[NCERT Part-II, Page 130, 137]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$.

[Conceptual Application]

33. Show that the relation R on the set R of real numbers, defined as **[NCERT Part-II, Page 2]**

$R = \{(a, b): a \leq b^2\}$ is neither reflexive, nor symmetric, nor transitive.

OR

Let the function $f: R \to R$ be defined by $f(x) = \frac{2x - 7}{4}$. Is f one-one and onto? Justify your answer.

[NCERT Part-II, Page 7]

34. If $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations: $x + y + z = 6$, $x + 2z = 7$ and $3x + y + z = 12$.

[NCERT Part-II, Page 94-95]

35. Find the foot of the perpendicular drawn from the point $(-1, -1, 3)$ on the line

$\vec{r} = -2\hat{i} - 2\hat{j} - 8\hat{k} + \lambda(2\hat{i} - 4\hat{j} - 5\hat{k})$. **[Conceptual Application]**

OR

Show that lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersects. Also, find the point of intersection.

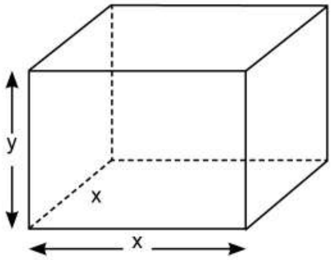
[Conceptual Application]

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. A farmer wants to make a square base open tank to collect the garbage in his field so as to hold given quantity of garbage. The length of each side of base is x units and depth is 'y' units and quantity of garbage is V cubic units. **[Conceptual Application]**



(*i*) Find the relation between V, x and y.

(*ii*) Express the surface area of tank in terms of 'x'.

(*iii*) Find the relation between 'x' and 'y' so that surface area of tank is minimum.

OR

(*iii*) If quantity of garbage is 8m^3 and depth is 2m, then find x and minimum surface area.

Case Study - 2

37. Priti visits three car showrooms A, B and C as she wants to purchase an electric car. The probabilities to visit showroom A is $\frac{1}{4}$ and B is $\frac{1}{2}$. The probability to purchase an electric car from showroom A is $\frac{1}{3}$, from showroom B is $\frac{1}{2}$ and from C is $\frac{1}{4}$. Let E_1, E_2 and E_3 are the events of visiting the showrooms A, B and C respectively and A is the event that she purchases an electric car. **[Conceptual Application]**

(*i*) What is the probability to visit showroom C?

(*ii*) What is the probability that she purchases an electric car?

(*iii*) If she purchases an electric car then find the probability that it was from showroom C.

OR

(*iii*) If she purchases an electric car, then find the probability that it was from showroom A.

Case Study - 3

38. In given figure, the coordinates of top three corners of a box are $A(1, -7, 7)$, $B(3, -2, 2)$, $C(-2, -3, -1)$ and CD is parallel to AB. **[NCERT Part-II, Page 339, 365]**

(*i*) Find vector perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(*ii*) Find the area of triangle ABC.

SOLUTIONS

1. (*a*) $(A + B) + C = A + (B + C)$

2. (*a*) $A = [a_{ij}]_{m \times n}$

3. (*b*)
$$\begin{aligned}
[1 \quad x \quad 1]\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 1\\ 0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix} &= O\\
\Rightarrow \quad [1 \quad x \quad 1]\begin{bmatrix}x+3-4\\ 5-2\\ 3-4\end{bmatrix} &= O\\
\Rightarrow \quad [1 \quad x \quad 1]\begin{bmatrix}x-1\\ 3\\ -1\end{bmatrix} &= O\\
\Rightarrow \quad x - 1 + 3x - 1 &= 0\\
\Rightarrow \quad 4x &= 2\\
\Rightarrow \quad x &= \frac{1}{2}
\end{aligned}$$

4. (*a*)
$$x\frac{dy}{dx} + y = e^x$$
$$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{x}\cdot y = \frac{e^x}{x}$$

Comparing with $\frac{dy}{dx} + Py = Q,$

we get $P = \frac{1}{x}, Q = \frac{e^x}{x}$

$$IF = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

5. (*b*) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$= 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma - 3 = 2(1) - 3 = -1$

$\{\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = l^2 + m^2 + n^2 = 1\}$

6. (*a*)
$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 3x - \frac{dy}{dx}$$

On squaring both sides, we get
$$1 + \left(\frac{dy}{dx}\right)^2 = 9x^2 + \left(\frac{dy}{dx}\right)^2 - 6x\frac{dy}{dx} \quad \Rightarrow \quad 6x\left(\frac{dy}{dx}\right) + 1 = 9x^2$$

So, order = 1, degree = 1

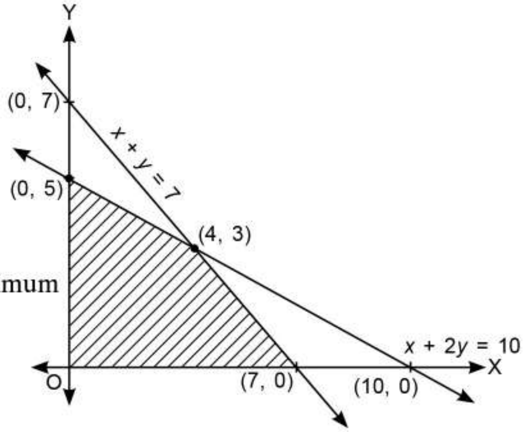
Sum = 1 + 1 = 2

7. (*c*) Solving equations $x + y = 7$ and $x + 2y = 10$, we get

$y = 3, x = 4$

Points	**Values of $Z = 5x + 2y$**
(0, 5)	10
(7, 0)	35
(0, 0)	0
(4, 3)	26

→ Maximum



8. (*d*)

$$\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{a}\cdot\vec{b} = 2 \times (-1) + (-3) \times 2 + 2 \times 1$$

$$= -2 - 6 + 2$$

$$= -6$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \left|\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right|$$

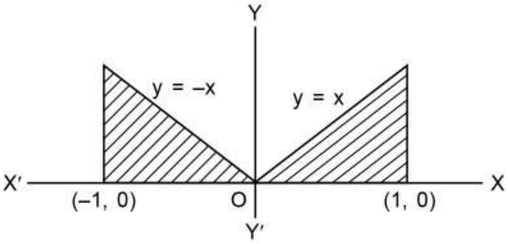
$$= \left|\frac{-6}{\sqrt{1+4+1}}\right| = \sqrt{6}$$

9. (*a*)

$$\int_{-1}^{1}|x|dx = \int_{-1}^{0}-x\,dx + \int_{0}^{1}x\,dx$$

$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= -\left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right) = 1$$



10. (*a*) Points $P(3, -2)$, $Q(8, 8)$, $R(k, 2)$ are collinear.

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 3(8-2) + 2(8-k) + 1(16-8k) = 0$$

$$\Rightarrow \quad 18 + 16 - 2k + 16 - 8k = 0$$

$$\Rightarrow \quad k = 5$$

11. (*b*) Objective function of a linear programming problem is a function to be optimised.

12. (*b*) $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$$

$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$$

$$= 2(\vec{a} \times \vec{b})$$

13. (*d*) We have,

$$A = \begin{bmatrix} 3 & 4 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 15 \\ 0 & -10 \end{bmatrix}$$

$$|AB| = |A||B|$$

$$= (-12) \times (-10) = 120$$

14. (*d*) We have,

$$P(A) = \frac{1}{5} \quad ; \quad P(B) = 0$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A \cap B)}{0} = \text{Not defined}$$

15. (*b*) $\int \frac{dx}{\cos^2 x \sin^2 x}$

$$= \int \left(\frac{\cos^2 x}{\cos^2 x \sin^2 x} + \frac{\sin^2 x}{\cos^2 x \sin^2 x} \right) dx$$

$$= \int \left(\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \right) dx$$

$$= \int \operatorname{cosec}^2 x\, dx + \int \sec^2 x\, dx$$

$$= -\cot x + \tan x + C$$

$$= \tan x - \cot x + C$$

16. (*d*) $(\vec{x} + \vec{a}) \cdot (\vec{x} - \vec{a}) = 5$

$\Rightarrow \quad |\vec{x}|^2 - \vec{x} \cdot \vec{a} + \vec{a} \cdot \vec{x} - |\vec{a}|^2 = 5$

$\Rightarrow \quad |\vec{x}|^2 = 6 \qquad [\because |\vec{a}| = 1]$

$\Rightarrow \quad |x| = \sqrt{6}$

17. (*a*) $y = e^{1 + \log x} = e^{\log e + \log x}$

$\Rightarrow \quad y = e^{\log(ex)}$

$\Rightarrow \quad y = ex$

Differentiating w.r.t x, we get

$$\frac{dy}{dx} = e$$

18. (*c*) We have, $\frac{1}{\sqrt{2}}, \frac{1}{2}, k$ are direction cosines of the given line.

Let $\quad l = \frac{1}{\sqrt{2}}, m = \frac{1}{2}, n = k$

$\therefore \quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} + k^2 = 1$

$\Rightarrow \quad k^2 = \frac{1}{4}$

$\Rightarrow \quad k = \pm\frac{1}{2}$

19. (*d*) *A* is false but *R* is true.

20. (*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.

21. Let $\quad \cos^{-1}\frac{1}{2} = x \quad$ and $\quad \sin^{-1}\frac{3}{5} = y$

$\cos x = \frac{1}{2} \quad$ and $\quad \sin y = \frac{3}{5}$

$\therefore \quad \tan x = \sqrt{3} \quad$ and $\quad \tan y = \frac{3}{4}$

$\therefore \quad \tan\left(\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5}\right) = \tan(x + y)$

$$= \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$= \frac{\sqrt{3} + \frac{3}{4}}{1 - \sqrt{3} \times \frac{3}{4}}$$

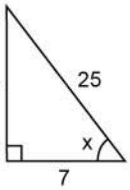
$$= \left(\frac{4\sqrt{3} + 3}{4 - 3\sqrt{3}}\right)$$

OR

$\cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right)$

Let $\cos^{-1}\frac{7}{25} = x$

$\Rightarrow$ $\cos x = \frac{7}{25}$



We have

$$\cot x = \frac{7}{24} \Rightarrow \cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right) = \frac{7}{24}$$

22. Let r be the base radius and h be the height of the cylinder at a particular instant of time 't'. Let V be its volume at that instant.

Given: $\frac{dr}{dt} = 2$ cm/s; $\frac{dh}{dt} = -3$ cm/s

We know that:

Volume of cylinder, $V = \pi r^2 h$

On differentiating both sides w.r.t 't', we get

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + h \times 2r\frac{dr}{dt}\right)$$

Now,

$$\left[\frac{dV}{dt}\right]_{r=3,\,h=4} = \pi[9 \times (-3) + 4 \times 2 \times 3 \times 2]$$

$$= \pi[-27 + 48]$$

$$= 21\pi \text{ cm}^3/\text{s}$$

So, volume is increasing at the rate of 21π cm^3/s.

23. We have, $f(x) = x^4 - 32x^2 + ax + 10$

Differentiating w.r.t. 'x', we get

$$f'(x) = 4x^3 - 64x + a$$

A.T.Q at $x = 1, f'(x) = 0$, so

$0 = 4(1)^3 - 64(1) + a$

$\Rightarrow$ $0 = 4 - 64 + a$

$\Rightarrow$ $a = 60$

OR

$$y = 2x^3 + 9x^2 + 12x - 1$$

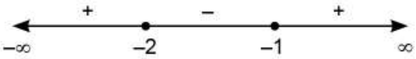
Differentiating w.r.t 'x', we get

$$\frac{dy}{dx} = 6x^2 + 18x + 12$$

$$= 6(x^2 + 3x + 2)$$

$$= 6(x + 2)(x + 1)$$

Intervals	Sign of $f'(x)$	Nature of f
$(-\infty, -2)$	+ve	Strictly increasing
$(-2, -1)$	–ve	Strictly decreasing
$(-1, \infty)$	+ve	Strictly increasing



$\therefore$ In $(-2, -1)$, 'f' is strictly decreasing.

24. If $0 < x < \frac{\pi}{4},$

then $\sin x < \cos x$

$\therefore$ $|\sin x - \cos x| = \cos x - \sin x$; when $0 < x < \frac{\pi}{4}$

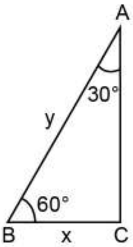
$$I = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$$

$$= \left[\sin x + \cos x\right]_0^{\frac{\pi}{4}}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0)$$

$$= \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

25. Let AB be ladder resting with wall at an angle of 30°. Let AC be the wall. Suppose at any instant of time t, man is at point B. Let BC $= x$ and AB $= y$.



Given: $\frac{dy}{dt} = 3$ m/min

Now, $\frac{x}{y} = \cos 60°$

$\Rightarrow$ $x = y\cos 60° \Rightarrow x = \frac{y}{2}$

Differentiating w.r.t. 't', we get

$$\frac{dx}{dt} = \frac{dy}{dt} \times \frac{1}{2}$$

$$= 3 \times \frac{1}{2} = \frac{3}{2} \text{ m/min}$$

26. Let $I = \int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$ ...(*i*)

$\Rightarrow$ $I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)}dx \quad \left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

$\Rightarrow$ $I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x}dx$

$\Rightarrow$ $I = \int_0^{\pi} \frac{\pi\tan x}{\sec x + \tan x}dx - \int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$2I = \pi\int_0^{\pi}\left(\frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x}\right)dx$$

$$= \pi\int_0^{\pi}(\tan x\sec x - \tan^2 x)dx = \pi\int_0^{\pi}(\sec x\tan x - \sec^2 x + 1)dx$$

$$= \pi[\sec x - \tan x + x]_0^{\pi}$$

$$= \pi[\sec\pi - \tan\pi + \pi - \sec 0 + \tan 0 - 0] = \pi(\pi - 2)$$

$\therefore$ $I = \frac{\pi}{2}(\pi - 2)$

27. Odds against A solving a problem are 3:2 $\Rightarrow P(A) = \frac{2}{5}, P(\overline{A}) = \frac{3}{5}$

Odds against B solving a problem are 2:7 $\Rightarrow P(B) = \frac{7}{9}, P(\overline{B}) = \frac{2}{9}$

P(problem is solved)

$= P$(at least one solved)

$= 1 - P$(none solves)

$= 1 - P(\overline{A})\ P(\overline{B})$

$= 1 - \frac{3}{5} \times \frac{2}{9} = 1 - \frac{2}{15} = \frac{13}{15}$

28. Let $I = \int \underset{\textcircled{2}}{x}(\log \underset{\textcircled{1}}{x})^2 dx$

Integrating by parts, we get

$$I = (\log x)^2 \cdot \frac{x^2}{2} - \int 2\log x \times \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2}(\log x)^2 - \int \underset{\textcircled{2}}{x} \log \underset{\textcircled{1}}{x}\ dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2} dx\right]$$

$$I = \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$

OR

$$I = \int \frac{dx}{x(x^4 - 1)}$$

$$= \int \frac{dx}{x^5\left(1 - \frac{1}{x^4}\right)}$$

Let $1 - \frac{1}{x^4} = y$

$\Rightarrow \frac{4}{x^5}dx = dy$

$\therefore$

$$I = \frac{1}{4}\int \frac{dy}{y}$$

$$= \frac{1}{4}\log|y| + C$$

$$= \frac{1}{4}\log\left|1 - \frac{1}{x^4}\right| + C$$

$$I = \frac{1}{4}\log\left|\frac{x^4 - 1}{x^4}\right| + C$$

29.

$$2(y + 3) - xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad xy\frac{dy}{dx} = 2(y + 3)$$

$$\Rightarrow \quad \frac{y}{y + 3}dy = \frac{2}{x}dx$$

$$\Rightarrow \quad \left(\frac{y + 3}{y + 3} - \frac{3}{y + 3}\right)dy = \frac{2}{x}dx$$

$\Rightarrow$ $$\left(1-\frac{3}{y+3}\right)dy = \frac{2}{x}dx$$

Integrating both sides, we get

$$\int\left(1-\frac{3}{y+3}\right)dy = \int\frac{2}{x}dx$$

$\Rightarrow$ $$y - 3\log|y+3| = 2\log|x| + C$$

Putting $x = 1, y = -2$, we get

$$-2 - 3\log 1 = 2\log 1 + C$$

$\therefore$ $$C = -2$$

$\therefore$ Particular solution is : $y - 3\log|y+3| = 2\log|x| - 2$

OR

Given $\quad xy = c^2 \quad$ ...(*i*)

Let $\quad Z = ax + by$

$\Rightarrow \quad Z = ax + \dfrac{bc^2}{x} \quad$ ...(*ii*)

Differentiating both sides w.r.t. *x*, we get

$$\frac{dZ}{dx} = a - \frac{bc^2}{x^2}$$

For minimum Z, $\dfrac{dZ}{dx} = 0 \Rightarrow a - \dfrac{bc^2}{x^2} = 0$

$\Rightarrow \quad x^2 = \dfrac{bc^2}{a} \Rightarrow x = \sqrt{\dfrac{bc^2}{a}}$

Now, $\quad \dfrac{d^2Z}{dx^2} = 0 + \dfrac{2bc^2}{x^3} = \dfrac{2bc^2}{x^3}$

$$\left.\frac{d^2Z}{dx^2}\right]_{x=\sqrt{\frac{bc^2}{a}}} = \frac{2bc^2}{\left(\frac{bc^2}{a}\right)^{3/2}} > 0$$

Hence, for $x = \sqrt{\dfrac{bc^2}{a}}$, Z is minimum.

Substituting in (*ii*), we get

$$\text{Minimum } Z = a\cdot\sqrt{\frac{bc^2}{a}} + \frac{bc^2}{\sqrt{\frac{bc^2}{a}}}$$

$$= 2\times\sqrt{abc^2}$$

30. We have to maximise, $\quad Z = 3x + 5y$

The given constraints: $\quad x + 4y \le 24 \quad$ ...(*i*)

$3x + y \le 21 \quad$ ...(*ii*)

$x + y \le 9 \quad$ ...(*iii*)

$x \ge 0, \quad y \ge 0 \quad$ ...(*iv*)

Converting (*i*) and (*iii*) inequations to equations and solving, we get

$$\begin{array}{r} x + 4y = 24 \\ x + y = 9 \\ - \quad - \quad\quad - \\ \hline 3y = 15 \\ \hline \end{array}$$

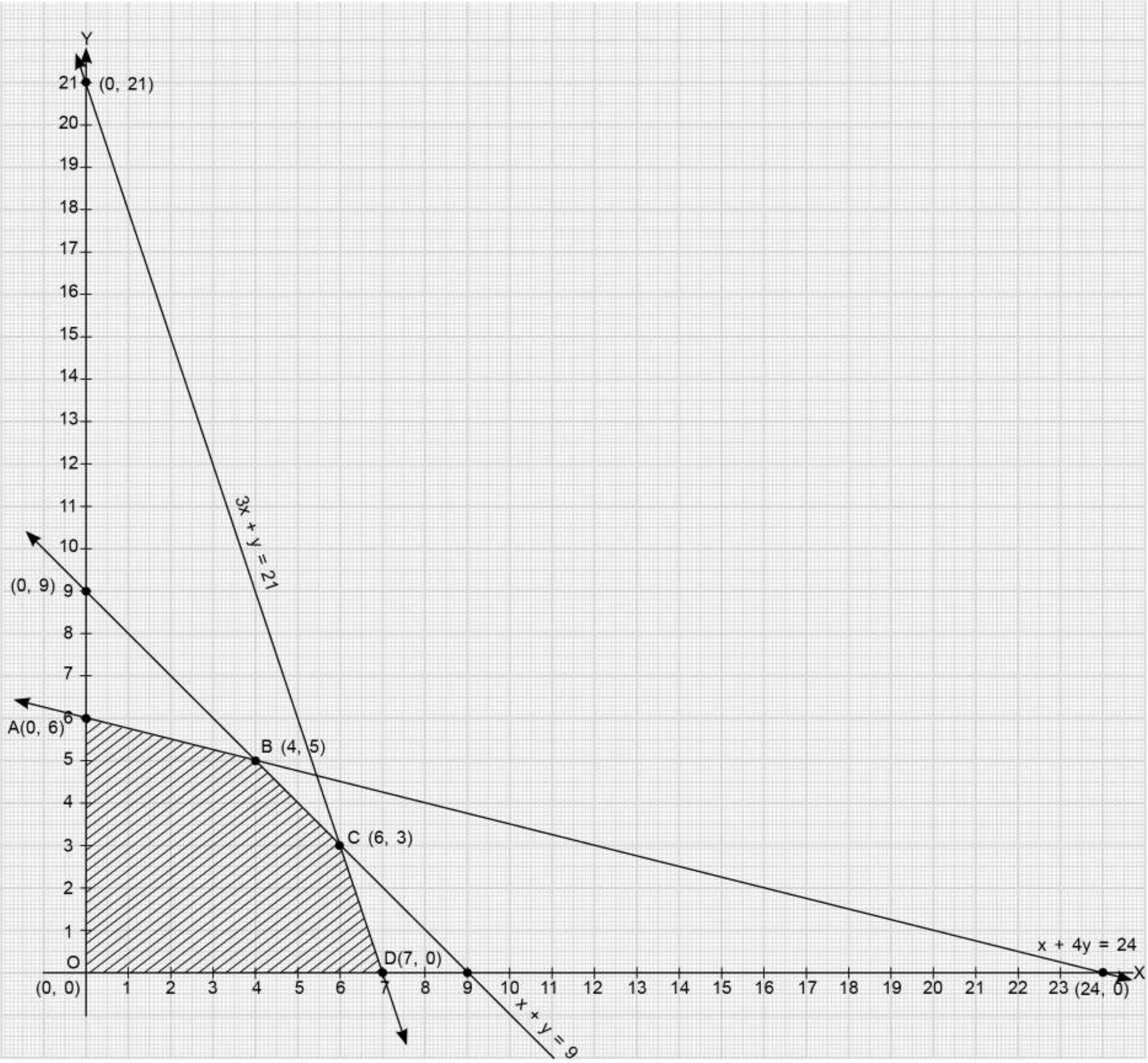
$\Rightarrow \quad y = 5$, and $x = 4$

Converting (*ii*) and (*iii*) inequations to equations and solving, we get

$$\begin{array}{r} x + y = 9 \\ 3x + y = 21 \\ - \quad - \quad - \\ \hline -2x = -12 \\ \hline \end{array}$$

$\Rightarrow$ $x = 6$, and $y = 3$

Converting (*i*) and (*ii*) inequations to equations and solving we get $x = \frac{60}{11}$ and $y = \frac{51}{11}$.



Corner Points	Values of $Z = 3x + 5y$	
$O(0, 0)$	0	
$A(0, 6)$	30	
$B(4, 5)$	37	← Maximum
$C(6, 3)$	33	
$D(7, 0)$	21	

$\therefore$ Maximum value = 37 at $x = 4, y = 5$

OR

We have to minimise, $Z = 9x - 11y$

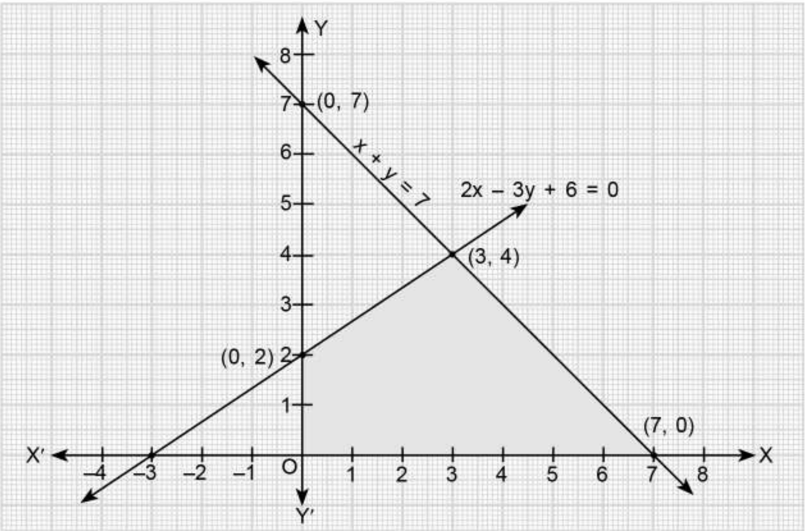
The given constraints are:

$$x + y \le 7 \quad ...(i)$$

$$2x - 3y + 6 \ge 0 \quad ...(ii)$$

$$x, y \ge 0 \quad ...(iii)$$

Let us graph the feasible region of the system of linear inequalities (*i*) to (*iii*). The shaded region is the feasible region.



Corner Points	Values of $Z = 3x + 5y$	
(0, 0)	0	
(7, 0)	63	
(3, 4)	–17	
(0, 2)	–22	← Minimum

$\therefore$ Minimum value = –22 at $x = 0, y = 2$.

31.
$$y = x^x \quad ...(i)$$

Taking log on both sides, we get

$$\log y = x \log x \quad ...(ii)$$

Differentiating w.r.t x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = x\frac{1}{x} + \log x$$

$$\Rightarrow \quad \frac{dy}{dx} = y + y \log x \quad ...(iii)$$

Differentiating (*iii*) w.r.t x, we get

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + y\frac{1}{x} + \log x\frac{dy}{dx} \quad ...(iv)$$

From (*iii*), we get
$$\log x = \frac{\frac{dy}{dx} - y}{y} \quad ...(v)$$

Putting this value in (*iv*), we get

$$\frac{d^2y}{dx^2} = \left[\frac{\frac{dy}{dx} - y}{y}\right] \times \frac{dy}{dx} + \frac{dy}{dx} + \frac{y}{x}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx} + \frac{dy}{dx} + \frac{y}{x}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 + \frac{y}{x}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

32. The given curves are: $x = y + 4$ and $y^2 = 2x$

$\Rightarrow \qquad y^2 = 2y + 8$

$\Rightarrow \qquad y^2 - 2y - 8 = 0$

$\Rightarrow \qquad (y - 4)(y + 2) = 0$

$\Rightarrow \qquad y = 4, y = -2$

For $y = 4, x = 8$

For $y = -2, x = 2$

Area of shaded region = ar(*QACPQ*) – ar(*QAMOCP*)

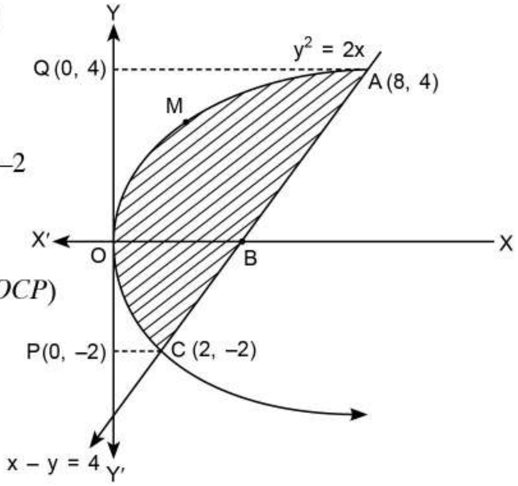
$$= \int_{-2}^{4} x_L dy - \int_{-2}^{4} x_P dy$$

$$= \int_{-2}^{4} (4 + y)dy - \int_{-2}^{4} \frac{y^2}{2} dy$$

$$= \left[4y + \frac{y^2}{2}\right]_{-2}^{4} - \frac{1}{2 \times 3}\left[y^3\right]_{-2}^{4}$$

$$= [(16 + 8) - (-8 + 2)] - \frac{1}{6}[64 + 8]$$

$$= 30 - 12 = 18 \text{ sq units}$$



33. $R = \{(a, b) : a \leq b^2\}$

For reflexive:

Let $\left(\frac{1}{2}, \frac{1}{2}\right) \in R$

$\Rightarrow \qquad \frac{1}{2} \leq \frac{1}{4}$, which is false

$\therefore$ *R* is not reflexive

For symmetric:

We have, $(1, 2) \in R$ as $1 \leq 4$

But $2 \leq 1^2$, is false

i.e. $(2, 1) \notin R$

$\therefore$ *R* is not symmetric

For transitive:

We have, $(7, 3) \in R$ as $7 \leq 9$

$(3, 2) \in R$ as $3 \leq 4$

Now, $7 \leq 2^2$ is false

i.e. $(7, 2) \notin R$

$\therefore$ R is not transitive.

$\therefore$ Given relation is neither reflexive, nor symmetric, nor transitive.

OR

We have, $$f(x) = \frac{2x-7}{4}$$

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{2x_1 - 7}{4} = \frac{2x_2 - 7}{4}$$

$$\Rightarrow \quad 2x_1 = 2x_2$$

$$\Rightarrow \quad x_1 = x_2$$

$\therefore$ f is one-one.

Let $f(x) = y$ {Where $y \in$R co-domain}

$$\Rightarrow \quad \frac{2x-7}{4} = y$$

$$\Rightarrow \quad 2x - 7 = 4y$$

$$\Rightarrow \quad x = \frac{4y+7}{2} \in R$$

Thus for all $y \in R$ (co-domain), there exists $x = \frac{4y+7}{2} \in R$ (domain).

$$f(x) = \frac{\frac{2(4y+7)}{2} - 7}{4}$$

$$= \frac{4y + 7 - 7}{4}$$

$$= y$$

$\therefore$ f is onto.

$\therefore$ f is one-one and onto function.

34. $$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$|A| = 1(0 - 2) - 1(0) + 3 \times (2 - 0) = 4 \neq 0$, $\therefore$ A^{-1} exists

$C_{11} = -2$, $C_{12} = 0$, $C_{13} = 2$

$C_{21} = 5$, $C_{22} = -2$, $C_{23} = -1$

$C_{31} = 1$, $C_{32} = 2$, $C_{33} = -1$

$$\text{adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$= \frac{1}{4}\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$$

The given system of equations is,

$$x + y + z = 6, \qquad x + 2z = 7, \qquad 3x + y + z = 12$$

Matrix equation is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$BX = C$$

$$X = B^{-1}C$$

Here, $B = A^T$

$\therefore$ $B^{-1} = (A^{-1})^T$

$\therefore$ $$B^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Now $X = B^{-1}C$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -12 + 24 \\ 30 - 14 - 12 \\ 6 + 14 - 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ $x = 3, \qquad y = 1, \qquad z = 2$

35. Cartesian form a line is

$$\frac{x+2}{2} = \frac{y+2}{-4} = \frac{z+8}{-5} \qquad \text{...}(i)$$

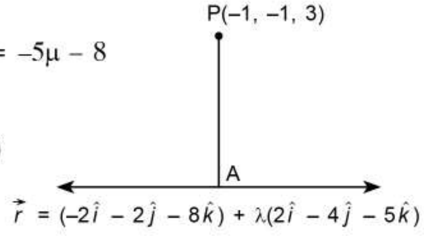
Let A be foot of perpendicular drawn from point P on the given line.

Suppose, $$\frac{x+2}{2} = \frac{y+2}{-4} = \frac{z+8}{-5} = \mu \text{ (say)}$$

$\Rightarrow$ $x = 2\mu - 2, \quad y = -4\mu - 2, \quad z = -5\mu - 8$

$\therefore$ Co-ordinates of A be $(2\mu - 2, -4\mu - 2, -5\mu - 8)$

DR's of $PA = (2\mu - 1, -4\mu - 1, -5\mu - 11)$



PA is perpendicular to given line (i),

$\therefore$ $2(2\mu - 1) - 4(-4\mu - 1) - 5(-5\mu - 11) = 0$

$$\Rightarrow \quad 4\mu - 2 + 16\mu + 4 + 25\mu + 55 = 0$$

$$\Rightarrow \quad 45\mu + 57 = 0$$

$$\Rightarrow \quad \mu = \frac{-57}{45}$$

$$\mu = \frac{-19}{15}$$

$\therefore$ Co-ordinates of foot of perpendicular are:

$$\left(2\times\left(\frac{-19}{15}\right)-2, -4\times\left(\frac{-19}{15}\right)-2, -5\left(\frac{-19}{15}\right)-8\right) \text{ i.e., } \left(\frac{-68}{15}, \frac{46}{15}, \frac{-5}{3}\right)$$

OR

The given lines are:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad ...(i)$$

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} \quad ...(ii)$$

Now, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ (say)

$$\Rightarrow \quad x = 2\lambda + 1, \quad y = 3\lambda + 2, \quad z = 4\lambda + 3$$

So, coordinates of any general point on line (*i*) are $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

Now, $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$ (say), then coordinates of any general point on line (*ii*) are $(5\mu + 4, 2\mu + 1, \mu)$. If lines intersect, they must have a common point. So, for some values of λ and μ we must have,

$$2\lambda + 1 = 5\mu + 4 \quad ...(iii)$$

$$3\lambda + 2 = 2\mu + 1 \quad ...(iv)$$

$$4\lambda + 3 = \mu \quad ...(v)$$

Solving (*iii*) and (*iv*), we get

$$\lambda = \mu = -1$$

Also, $\lambda = \mu = -1$ satisfies (*v*).

$\therefore$ given lines intersect each other.

Coordinates of point of intersection = $(2 \times (-1) + 1, 3 \times (-1) + 2, 4 \times (-1) + 3)$

i.e., $(-1, -1, -1)$

36. (*i*) Volume of tank $= x^2y$

$$V = x^2y$$

(*ii*)

$$y = \frac{V}{x^2} \quad [\because V = x^2y]$$

Surface area of tank $= x^2 + 4xy$

$$= x^2 + 4x\cdot\frac{V}{x^2}$$

$$= x^2 + \frac{4V}{x}$$

(*iii*) We have, $S = x^2 + \frac{4V}{x}$

Differentiating w.r.t. x, we get

$$\frac{dS}{dx} = 2x - \frac{4V}{x^2}$$

For maximum or minimum surface area,

$$\frac{dS}{dx} = 0 \quad \Rightarrow \quad 2x = \frac{4V}{x^2}$$

$$\Rightarrow \quad x^3 = 2V$$

$$\Rightarrow \quad x = (2V)^{\frac{1}{3}}$$

Now, $\frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$

$$\left(\frac{d^2S}{dx^2}\right)_{x=(2V)^{1/3}} = 2 + \frac{8V}{2V} > 0$$

$\therefore$ Surface area is minimum at $x = (2V)^{1/3}$

Now, $x^3 = 2V \Rightarrow V = \frac{x^3}{2}$

$\Rightarrow$ $\frac{x^3}{2} = x^2y$

$\Rightarrow$ $x = 2y$

OR

(*iii*) We have, $x^2y = V$

$\Rightarrow$ $8 = x^2 \times 2$

$\Rightarrow$ $x^2 = 4$

$\Rightarrow$ $x = 2$ (–2 rejected)

$$S = x^2 + \frac{4V}{x} = 4 + \frac{4 \times 8}{2} = 20 \text{ m}^2$$

37. (*i*) $P(E_3) = 1 - \left(\frac{1}{4} + \frac{1}{2}\right)$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

(*ii*) $P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right) + P(E_3) \times P\left(\frac{A}{E_3}\right)$

$$= \frac{1}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{1}{12} + \frac{1}{4} + \frac{1}{16}$$

$$= \left(\frac{4 + 12 + 3}{48}\right)$$

$$= \frac{19}{48}$$

(*iii*)

$$P(E_3/A) = \frac{P(E_3)\times P\left(\frac{A}{E_3}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)+P(E_3)\times P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{4}\times\frac{1}{4}}{\frac{19}{48}} = \frac{1}{16}\times\frac{48}{19}$$

$$= \frac{3}{19}$$

OR

(*iii*)

$$P(E_1/A) = \frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)+P(E_3)\times P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{4}\times\frac{1}{3}}{\frac{1}{4}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{2}+\frac{1}{4}\times\frac{1}{4}} = \frac{4}{19}$$

38. (*i*)

$$\overrightarrow{AB} = 2\hat{i}+5\hat{j}-5\hat{k}$$

$$\overrightarrow{AC} = -3\hat{i}+4\hat{j}-8\hat{k}$$

Vector perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{AC} = \overrightarrow{AB}\times\overrightarrow{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -5 \\ -3 & 4 & -8 \end{vmatrix}$$

$$= \hat{i}(-40+20) - \hat{j}(-16-15) + \hat{k}(8+15)$$

$$= -20\hat{i} + 31\hat{j} + 23\hat{k}$$

(*ii*)

$$\text{Ar }(\Delta ABC) = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$$

$$= \frac{1}{2}|-20\hat{i}+31\hat{j}+23\hat{k}|$$

$$= \frac{1}{2}\sqrt{400+961+529}$$

$$= \frac{1}{2}\sqrt{1890}$$

$$= \frac{3}{2}\sqrt{210} \text{ sq units.}$$