1 SAMPLE PAPER

[Easy Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If A_{ij} represents cofactor of an element a_{ij} of a determinant corresponding to a matrix $A = [a_{ij}]$, then for matrix $A = \begin{bmatrix} 1 & 4 & -2 \\ 0 & 1 & 3 \\ 1 & 2 & -2 \end{bmatrix}$, A_{32} is **[NCERT Part-I, Page 84]**

(*a*) 3 (*b*) –3 (*c*) 5 (*d*) –5

2. The matrix $\begin{bmatrix} 3 & -1 \\ 2 & k \end{bmatrix}$ is invertible for **[NCERT Part-I, Page 90]**

(*a*) $k = 0$ (*b*) $k \neq 3$ (*c*) $k = \frac{2}{3}$ (*d*) $k \neq \frac{-2}{3}$

3. If $A = [a_{ij}]$ is a matrix of order 2×3 such that $a_{ij} = 2j - i$, then a_{32} is **[Conceptual Application]**

(*a*) 4 (*b*) does not exist (*c*) 1 (*d*) –1

4. The values of x for which $\begin{vmatrix} 6 & -2 \\ 2 & 4 \end{vmatrix} = x^2 - 12x$ are **[NCERT Part-I, Page 77]**

(*a*) –2, 14 (*b*) 2, –14 (*c*) –2, –14 (*d*) 2, 14

5. The value(s) of p for which vector $p(\hat{i} - \hat{j} + \hat{k})$ represents a unit vector is **[NCERT Part-II, Page 341]**

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\pm\sqrt{3}$ (*c*) ± 1 (*d*) $\pm\frac{1}{\sqrt{3}}$

6. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 1$ then the range of $|\lambda\vec{a}|$ is **[Conceptual Application]**

(a) [0, 8] (b) [–12, 8] (c) [0, 12] (d) [8, 12]

7. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then value of x is **[NCERT Part-I, Page 90]**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) –2

8. Integrating factor for the solution of differential equation $\sin^2 x \frac{dy}{dx} + y = \cot x$ is **[NCERT Part-II, Page 322-323]**

(a) $e^{-\cot x}$ (b) $\cot x$ (c) $-\cot x$ (d) $e^{\cot x}$

9. $\int_{-1}^{1} |1 - x| dx$ is equal to **[NCERT Part-II, Page 273-274]**

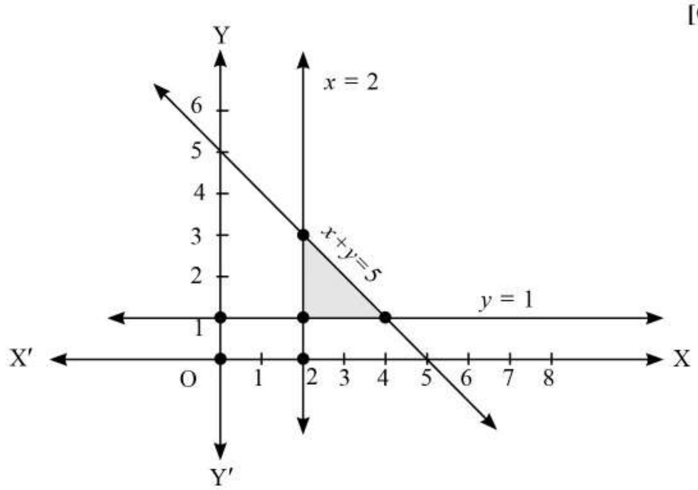
(a) 1 (b) 2 (c) 3 (d) –3

10. A function 'f' is said to be continuous for $x \in R$ if **[Conceptual Application]**

(a) 'f' is continuous at $x = 0$ (b) 'f' is differentiable at $x = 0$

(c) 'f' is continuous for $R - \{0\}$ (d) 'f' is differentiable for $x \in R$

11. The feasible region corresponding to the linear constraints of a linear programming problem is given as: **[Conceptual Application]**



Which of the following is not constraint to the given linear programming problem?

(a) $x \ge 2$ (b) $x + y \ge 5$ (c) $y \ge 1$ (d) $y \ge 0$

12. Cosine of an angle between the vectors $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ is **[NCERT Part-II, Page 355-356]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $\cos^{-1}\left(-\frac{1}{3}\right)$

13. The corner points of the bounded feasible region determined by the linear constraints are $A(1, 3)$, $B(0, 4)$, $C(2, 5)$ and $D(6, 2)$. Let $Z = px + qy$, the condition on p and q such that minimum Z occurs on any point of segment AD is **[NCERT Part-II, Page 398]**

(a) $p + 5q = 0$ (b) $p = 5q$ (c) $5p = q$ (d) $5p + q = 0$

14. The function $f(x) = |x| + |x - 1|$ is **[Integrated Question]**

(a) continuous at $x = 0, 1$ but not differentiable. (b) not continuous at $x = 0, 1$ but differentiable.

(c) not continuous and differentiable at $x = 0, 1$. (d) continuous as well as differentiable at $x = 0, 1$.

15. Direction cosines of a vector equally inclined to the coordinate axes are **[NCERT Part-II, Page 349]**

(a) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (b) 1, 1, 1 (c) 0, 0, 0 (d) None of these

16. The equations of y-axis are **[NCERT Part-II, Page 382]**

(a) $\frac{x}{0} = \frac{y}{1} = \frac{z}{1}$ (b) $\frac{x-1}{1} = \frac{y}{0} = \frac{z}{1}$ (c) $\frac{x-1}{0} = \frac{y}{1} = \frac{z+5}{0}$ (d) $\frac{x}{0} = \frac{y-1}{1} = \frac{z}{0}$

17. Three persons fire a target and their probabilities of hitting a target are 0.2, 0.3 and 0.5 respectively. The probability that target is hit is **[Conceptual Application]**

(*a*) 0.993 (*b*) 0.94 (*c*) 0.72 (*d*) 0.90

18. If p and q represent degree and order of a differential equation $\left(\frac{dy}{dx}\right)^3 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$, then value of $2p - 3q$ is **[NCERT Part-II, Page 301-302]**

(*a*) 1 (*b*) –4 (*c*) 8 (*d*) 3

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. **Assertion (*A*):** A reflexive relation in a given set A is an identity relation also. **[Integrated Question]**

Reason (*R*): The number of one-one functions from set $A = \{1, 2\}$ to set $A = \{1, 2\}$ are 2.

20. If 'f' be a function defined by $f(x) = x^2 - x + 1$ for $x \in R$, then

Assertion (*A*): the function 'f' is neither increasing nor decreasing on $[-1, 1]$. **[Integrated Question]**

Reason (*R*): Marginal cost related to cost function $C(x)$ is $C'(x)$, where x represents number of units produced.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Is the function $f(x) = \log(\cos x)$ strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ or not? **[NCERT Part-I, Page 153]**

OR

Find whether the function $y = x^3 + x^2 + x + 1$ has local maxima or local minima. **[NCERT Part-I, Page 166]**

22. If at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$, then find a. **[NCERT Part-I, Page 164]**

23. Represent $\sec\left[\tan^{-1}\left(\frac{y}{3}\right)\right]$ in terms of y. **[Conceptual Application]**

OR

Write the branch of $\tan^{-1} x$ other than principal value branch in which it becomes bijection.

24. If $\int \frac{1}{\sqrt{4-9x^2}}dx = \frac{1}{3}\sin^{-1}(ax) + C$, then find the value of a. **[NCERT Part-II, Page 244]**

25. Without using derivatives, find the maximum and minimum value for the function $f(x) = |3\sin x - 2|$. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $y = e^{a\sin^{-1}x}$, $-1 \le x \le 1$, then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. **[NCERT Part-I, Page 137]**

27. A card from a pack of 52 cards is lost. From the remaining cards, 2 cards are drawn without replacement and found to be both aces. Find the probability that lost card is an ace.

28. If $x = a(\cos\theta + \log\tan\frac{\theta}{2})$ and $y = a\sin\theta$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$. **[NCERT Part-II, Page 134-135]**

OR

Solve the differential equation:

$(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$, given that $y = 1$ when $x = 1$. **[NCERT Part-II, Page 306-307]**

29. Evaluate: $\int \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\}dx$. **[NCERT Part-II, Page 259-260]**

OR

Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$. **[NCERT Part-II, Page 273-274]**

30. Solve the following linear programming problem: **[NCERT Part-II, Page 397-398]**

Maximize $Z = x + 2y$

Subject to the constraints

$x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x \geq 0,\ y \geq 0.$

OR

Solve the following linear programming problem: **[NCERT Part-II, Page 397-398]**

Minimize $Z = 6x + 3y$

Subject to the constraints

$4x + y \geq 80,\ x + 5y \geq 115,\ 3x + 2y \leq 150,\ x \geq 0,\ y \geq 0.$

31. Evaluate: $\int \sin(\log x)dx$. **[NCERT Part-II, Page 259-260]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Using matrices, solve the following system of equations

$x + y - z = 3,\ 2x + 3y + z = 10,\ 3x - y - 7z = 1.$ **[NCERT Part-I, Page 94-95]**

33. Find the value of λ, so that the lines L_1: $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and L_2: $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find equations of a line passing through point $(3, -1, 7)$ and parallel to L_2.

[Conceptual Application]

OR

Find the vector and cartesian equations of a line passing through the point $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-20}{8} = \frac{z-5}{-5}$. **[Conceptual Application]**

34. Using integration, find the area of the region bounded by the following lines:

$5x - 2y - 10 = 0,\ x + y - 9 = 0,\ 2x - 5y - 4 = 0.$ **[Conceptual Application]**

35. Show that the relation S on the set R of real numbers defined as $S = \{(a, b) : a \leq b\}$ is reflexive and transitive but not symmetric. **[NCERT Part-I, Page 2]**

OR

Show that the function $f : N \rightarrow N$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto. **[NCERT Part-I, Page 7]**

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Two friends live in the houses A and B whose locations are represented by coordinates (3, 1, 2) and (–1, 4, 0) respectively. **[Conceptual Application]**

Using the above information, answer the following questions:

(*i*) If the points $A(3, 1, 2)$ and $B(-1, 4, 0)$ are joined by vector $\overrightarrow{AB}$, then find $\overrightarrow{AB}$.

(*ii*) Find the unit vector in the direction of $\overrightarrow{AB}$.

(*iii*) Write a vector of magnitude 5 units along the vector $\overrightarrow{AB}$.

OR

(*iii*) Write the direction cosines of $\overrightarrow{AB}$.

Case Study - 2

37. A coach is training three players for badminton competition. He observes that player A can win 4 out of 5 matches. Player B can win 3 out of 4 matches and player C can win 2 out of 3 matches played.

(*i*) What is the probability that all the three will win the matches in a competition? **[Conceptual Application]**

(*ii*) What is the probability that A loses but B and C win the game?

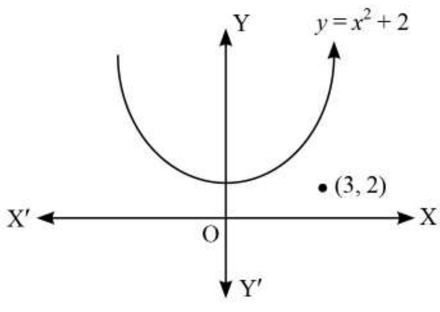
(*iii*) What is the probability that at least one out of A, B and C will win the game?

OR

(*iii*) What is the probability that none of them will win the game?

Case Study - 3

38. The jet of an enemy is flying along the curve $y = x^2 + 2$ and a soldier is placed at the point (3, 2). If at a given time, jet is at point (x, y), then **[Conceptual Application]**



(*i*) what is the distance D between the jet and the soldier in terms of x?

(*ii*) for what value of x jet is nearest to the soldier?

SOLUTIONS

1. (*b*)
$$A_{32} = \text{cofactor of } a_{32} \text{ in } |A|$$
$$= (-1)^5 \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} = -3$$

2. (*d*) If invertible, then $\begin{vmatrix} 3 & -1 \\ 2 & k \end{vmatrix} \neq 0$

$\Rightarrow \quad 3k + 2 \neq 0$

$\Rightarrow \quad k \neq \dfrac{-2}{3}$

3. (*b*)

4. (*a*)
$$\begin{vmatrix} 6 & -2 \\ 2 & 4 \end{vmatrix} = x^2 - 12x$$

$\Rightarrow \quad x^2 - 12x = 28$

$\Rightarrow \quad x^2 - 12x - 28 = 0$

$\Rightarrow \quad (x - 14)(x + 2) = 0$

$\Rightarrow \quad x = -2, 14$

5. (*d*) For unit vector $\sqrt{p^2 + p^2 + p^2} = 1$

$\Rightarrow \quad \sqrt{3p^2} = 1$

$\Rightarrow \quad p = \pm \dfrac{1}{\sqrt{3}}$

6. (*c*)
$$-3 \leq \lambda \leq 1$$

$\Rightarrow \quad 0 \leq |\lambda| \leq 3$

$\Rightarrow \quad 0|\vec{a}| \leq |\lambda||\vec{a}| \leq 3|\vec{a}|$

$\Rightarrow \quad 0 \leq |\lambda \vec{a}| \leq 12 \text{ or } |\lambda \vec{a}| \in [0, 12]$

7. (*b*)
$$\text{Adj}(A) = \begin{bmatrix} x & 0 \\ -x & 2x \end{bmatrix}, |A| = \begin{vmatrix} 2x & 0 \\ x & x \end{vmatrix} = 2x^2$$

Now, $\quad A^{-1} = \dfrac{\text{Adj}(A)}{|A|}$

$\Rightarrow \quad A^{-1} = \dfrac{1}{2x^2}\begin{bmatrix} x & 0 \\ -x & 2x \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2x} & 0 \\ -\dfrac{1}{2x} & \dfrac{1}{x} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow \quad \dfrac{1}{2x} = 1, \dfrac{-1}{2x} = -1, \dfrac{1}{x} = 2$

$\Rightarrow \quad x = \dfrac{1}{2}$

8. (*a*)
$$\sin^2 x \frac{dy}{dx} + y = \cot x$$

$\Rightarrow \quad \dfrac{dy}{dx} + (\text{cosec}^2 x)y = \cot x \cdot \text{cosec}^2 x$

Integrating factor $= e^{\int \text{cosec}^2 x \, dx} = e^{-\cot x}$

9. (*b*) $$\int_{-1}^{1} |1-x|dx = \int_{-1}^{1}(1-x)dx = \left[x - \frac{x^2}{2}\right]_{-1}^{1}$$
$$= \left(1-\frac{1}{2}\right) - \left(-1-\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = 2$$

10. (*d*) Differentiable function is continuous also.

11. (*b*) As for (0, 0), $0 \geq 5$ is a false statement.

12. (*c*) $$\cos\theta = \frac{1-1-1}{\sqrt{1+1+1}\sqrt{1+1+1}} = \frac{-1}{3}$$

13. (*c*) $$Z_A = Z_D \Rightarrow p + 3q = 6p + 2q$$
$$\Rightarrow \quad 5p = q$$

14. (*a*) $f(x) = |x - a|$, is continuous but not differentiable at $x = a$, $a \in R$

15. (*a*) As, $$l^2 + m^2 + n^2 = 1$$
$$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\Rightarrow 3\cos^2\alpha = 1 \qquad (\because \alpha = \beta = \gamma)$$
$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$
$\therefore$ *dc's* of the vector are : $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

16. (*d*)

17. (*c*) $$\text{P(target is hit)} = 1 - \text{P(none hit target)}$$
$$= 1 - (0.8)(0.7)(0.5)$$
$$= 1 - 0.28 = 0.72$$

18. (*b*) $$p = 1 \qquad q = 2$$
$$2p - 3q = 2 - 6 = -4$$

19. (*d*) (A) is false but (R) is true.

20. (*b*) Both (A) and (R) are true but (R) is not the correct explanation of (A).

21. $$f(x) = \log(\cos x)$$
$$\Rightarrow \quad f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$$
For $\left(0, \frac{\pi}{2}\right)$, $\tan x > 0$
$$\Rightarrow \quad f'(x) < 0 \Rightarrow f(x) \text{ decreases on } \left(0, \frac{\pi}{2}\right).$$

OR

$$y = x^3 + x^2 + x + 1$$
$$\Rightarrow \quad y' = 3x^2 + 2x + 1$$
For critical points, $y' = 0 \Rightarrow 3x^2 + 2x + 1 = 0$

Now, $D = b^2 - 4ac = 4 - 12 = -8 < 0$

As $D < 0$, no solution in real numbers for $f'(x) = 0$.

As there are no critical points, hence no local maximum or local minimum.

22. $$f(x) = x^4 - 62x^2 + ax + 9$$
$$f'(x) = 4x^3 - 124x + a$$
As, f attains maximum at $1 \in [0, 2]$, then $f'(1) = 0$

Now, $f'(1) = 0 \Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$.

23.

$$\sec\left\{\tan^{-1}\left(\frac{y}{3}\right)\right\} = \sqrt{1+\tan^2\left(\tan^{-1}\frac{y}{3}\right)}$$

$$= \sqrt{1+\frac{y^2}{9}} = \frac{\sqrt{9+y^2}}{3}$$

OR

To get another branch we can add π to principal branch i.e. $\left(\frac{-\pi}{2}+\pi, \frac{\pi}{2}+\pi\right)$ i.e. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

Other branch in which $\tan^{-1} x$ becomes bijection is $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

24.

$$\int \frac{1}{\sqrt{4-9x^2}}dx = \int \frac{1}{\sqrt{4-(3x)^2}}dx$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{4-t^2}}dt \qquad \left|\begin{array}{l}\text{Let } 3x = t \\ \Rightarrow 3dx = dt\end{array}\right.$$

$$= \frac{1}{3}\sin^{-1}\frac{t}{2} + C$$

$$= \frac{1}{3}\sin^{-1}\frac{3x}{2} + C$$

Comparing with $\frac{1}{3}\sin^{-1}(ax) + C$ we get, $a = \frac{3}{2}$.

25. For all $x \in R$, $\quad -1 \le \sin x \le 1$

$\Rightarrow \quad -3 \le 3\sin x \le 3$

$\Rightarrow \quad -5 \le 3\sin x - 2 \le 1$

$\Rightarrow \quad 0 \le |3\sin x - 2| \le 5$

Maximum value = 5, Minimum value = 0

26.

$$y = e^{a\sin^{-1}x}$$

$\Rightarrow$
$$\log y = a\sin^{-1}x$$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = a\cdot\frac{1}{\sqrt{1-x^2}}$$

$\Rightarrow$
$$\sqrt{1-x^2}\frac{dy}{dx} = ay$$

Squaring both sides, we get

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

Differentiating both sides w.r.t. x, we get

$$(1-x^2)\times 2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = a^2\times 2y\left(\frac{dy}{dx}\right)$$

Dividing by $2\left(\frac{dy}{dx}\right)$, we get

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = a^2y$$

$\Rightarrow$
$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

27. E_1: Lost card is an ace

E_2: Lost card is not an ace

E: Two card drawn are ace

$P(E_1): = \frac{4}{52} = \frac{1}{13}$ $P(E_2) = \frac{12}{13}$

$P\left(\frac{E}{E_1}\right) = \frac{^3C_2}{^{51}C_2}$ $P\left(\frac{E}{E_2}\right) = \frac{^4C_2}{^{51}C_2}$

Using Bayes' theorem

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\cdot\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)\,.\,P\left(\frac{E}{E_2}\right)} = \frac{\frac{1}{13}\times\frac{^2C_2}{^{51}C_2}}{\frac{1}{13}\,\frac{^3C_2}{^{51}C_2} + \frac{12}{13}\cdot\frac{^4C_2}{^{51}C_2}} = \frac{3}{3+72} = \frac{3}{75} = \frac{1}{25}$$

28. Consider, $x = a(\cos\theta + \log\tan\frac{\theta}{2})$; $y = a\sin\theta$

$$\frac{dy}{d\theta} = a\cos\theta \quad ...(i)$$

$$\Rightarrow \quad \frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\frac{\theta}{2}}\,.\,\sec^2\frac{\theta}{2}\,.\,\frac{1}{2}\right)$$

$$= a\left(-\sin\theta + \frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= a\left(-\sin\theta + \frac{1}{\sin\theta}\right) = a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$$

$$\Rightarrow \quad \frac{dx}{d\theta} = \frac{a\cos^2\theta}{\sin\theta} \quad ...(ii)$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx} = \frac{a\cos\theta\times\sin\theta}{a\cos^2\theta} \quad \text{[from } (i) \text{ and } (ii)]$$

$$= \tan\theta$$

$$\therefore \quad \left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = \tan\frac{\pi}{4} = 1$$

OR

$$\frac{dy}{dx} = \frac{-(y^2 + x^2y^2)}{x^2 - x^2y}$$

$$= \frac{-y^2(1+x^2)}{x^2(1-y)}$$

$$\Rightarrow \quad \frac{1-y}{y^2}dy = -\left(\frac{1+x^2}{x^2}\right)dx$$

$$\Rightarrow \quad \int\frac{1-y}{y^2}\,dy = \int\frac{-(1+x^2)}{x^2}\,dx \quad \text{[Integrating both sides]}$$

$$\Rightarrow \quad \int\left(\frac{1}{y^2} - \frac{1}{y}\right)dy = \int\left(-\frac{1}{x^2} - 1\right)dx.$$

$$\Rightarrow \quad -\frac{1}{y} - \log|y| = \frac{1}{x} - x + C \quad ...(i)$$

Given $y = 1$ when $x = 1$, then from (i),

$$-1 - \log 1 = 1 - 1 + C \Rightarrow -1 = C$$

$\therefore$ From (i), $\quad -\frac{1}{y} - \log|y| = \frac{1}{x} - x - 1$ is particular solution.

29. $$\int\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx = \int \underset{①}{\frac{1}{\log x}}\cdot \underset{②}{1}\,dx - \int\frac{1}{(\log x)^2}dx$$

$$= \frac{1}{\log x}\cdot x - \int -(\log x)^{-2}\cdot\frac{1}{x}\cdot x\,dx - \int\frac{1}{(\log x)^2}dx$$

$$= \frac{x}{\log x} + \int\frac{1}{(\log x)^2}dx - \int\frac{1}{(\log x)^2}dx$$

$$= \frac{x}{\log x} + C$$

Alternatively:

Let $\log x = t$
$\Rightarrow\ x = e^t$
$\Rightarrow\ dx = e^t\,dt$

$$\int\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx = \int e^t\left\{\frac{1}{t}-\frac{1}{t^2}\right\}dt$$

$$f(t) = \frac{1}{t} \text{ and } f'(t) = -\frac{1}{t^2}$$

Using result $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + C$

$$= e^t\cdot\frac{1}{t} + C$$

$$= e^{\log x}\cdot\frac{1}{\log x} + C = \frac{x}{\log x} + C$$

OR

$$I = \int_0^{\frac{\pi}{2}}(2\log\sin x - \log\sin 2x)dx$$

$$= \int_0^{\frac{\pi}{2}}\log\left(\frac{\sin^2 x}{2\sin x\cos x}\right)dx$$

$$= \int_0^{\frac{\pi}{2}}\log\left(\frac{\tan x}{2}\right)dx \quad ...(i)$$

$$I = \int_0^{\frac{\pi}{2}}\log\left(\frac{\cot x}{2}\right)dx \quad ...(ii)$$

[Using property : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$]

On adding (*i*) and (*ii*), we get

$$2I = \int_0^{\frac{\pi}{2}}\left[\log\left(\frac{\tan x}{2}\right)+\log\left(\frac{\cot x}{2}\right)\right]dx$$

$\Rightarrow$ $$2I = \int_0^{\frac{\pi}{2}}\log\left(\frac{\tan x}{2}\times\frac{\cot x}{2}\right)dx$$

$\Rightarrow$ $$2I = \int_0^{\frac{\pi}{2}}\log\left(\frac{1}{4}\right)dx \quad [\because \tan x\times\cot x = 1]$$

$\Rightarrow$ $$2I = \log\left(\frac{1}{4}\right)\int_0^{\frac{\pi}{2}}1\,dx = \log\left(\frac{1}{4}\right)[x]_0^{\frac{\pi}{2}}$$

$\Rightarrow$ $$2I = \frac{\pi}{2}\log\left(\frac{1}{4}\right)$$

$\Rightarrow$ $$I = \frac{\pi}{4}\log\left(\frac{1}{4}\right) = \frac{\pi}{4}(\log 1 - \log 4) = \frac{\pi}{4}(0 - 2\log 2) = -\frac{\pi}{2}\log 2$$

Alternatively:

$$I = \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx \qquad ...(i)$$

Using property: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{\frac{\pi}{2}} \left[2 \log\left\{\sin\left(\frac{\pi}{2} - x\right)\right\} - \log \sin 2\left\{\frac{\pi}{2} - x\right\}\right] dx$$

$$= \int_0^{\frac{\pi}{2}} [2 \log \cos x - \log \sin(\pi - 2x] dx$$

$$= \int_0^{\frac{\pi}{2}} [2 \log \cos x - \log \sin 2x] dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\frac{\pi}{2}} [2\{\log(\sin x) + \log(\cos x)\} - 2 \log \sin 2x] dx$$

$$= 2\int_0^{\frac{\pi}{2}} [\log (\sin x \cos x) - \log (\sin 2x)] dx$$

$$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin x \cos x}{2 \sin x \cos x}\right) dx = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\right) dx$$

$$= \log\left(\frac{1}{2}\right) \cdot \int_0^{\frac{\pi}{2}} 1 \cdot dx = \log\left(\frac{1}{2}\right) \cdot \Big[\, x \,\Big]_0^{\frac{\pi}{2}} = \log\left(\frac{1}{2}\right) \cdot \left(\frac{\pi}{2} - 0\right)$$

$$I = \frac{\pi}{2} \log \frac{1}{2} = -\frac{\pi}{2} \log 2$$

30. To maximise $Z = x + 2y$

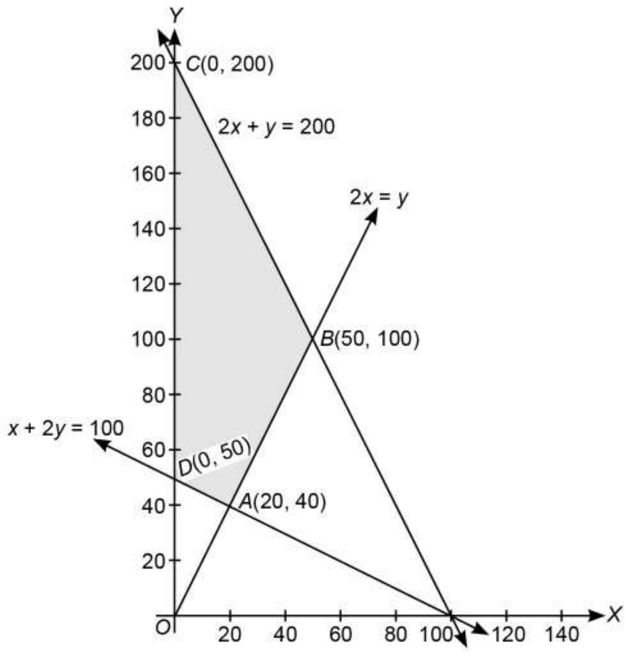
subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$

On plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for maximum Z are $A(20, 40)$, $B(50, 100)$, $C(0, 200)$ and $D(0, 50)$.

Corner Points	$Z = x + 2y$	Values	
$A(20, 40)$	20 + 80	100	
$B(50, 100)$	50 + 200	250	
$C(0, 200)$	0 + 400	400	← Maximum
$D(0, 50)$	0 + 100	100	

$\therefore$ Z is maximum for $C(0, 200)$, i.e. $x = 0$, $y = 200$.



OR

The given constraints are :

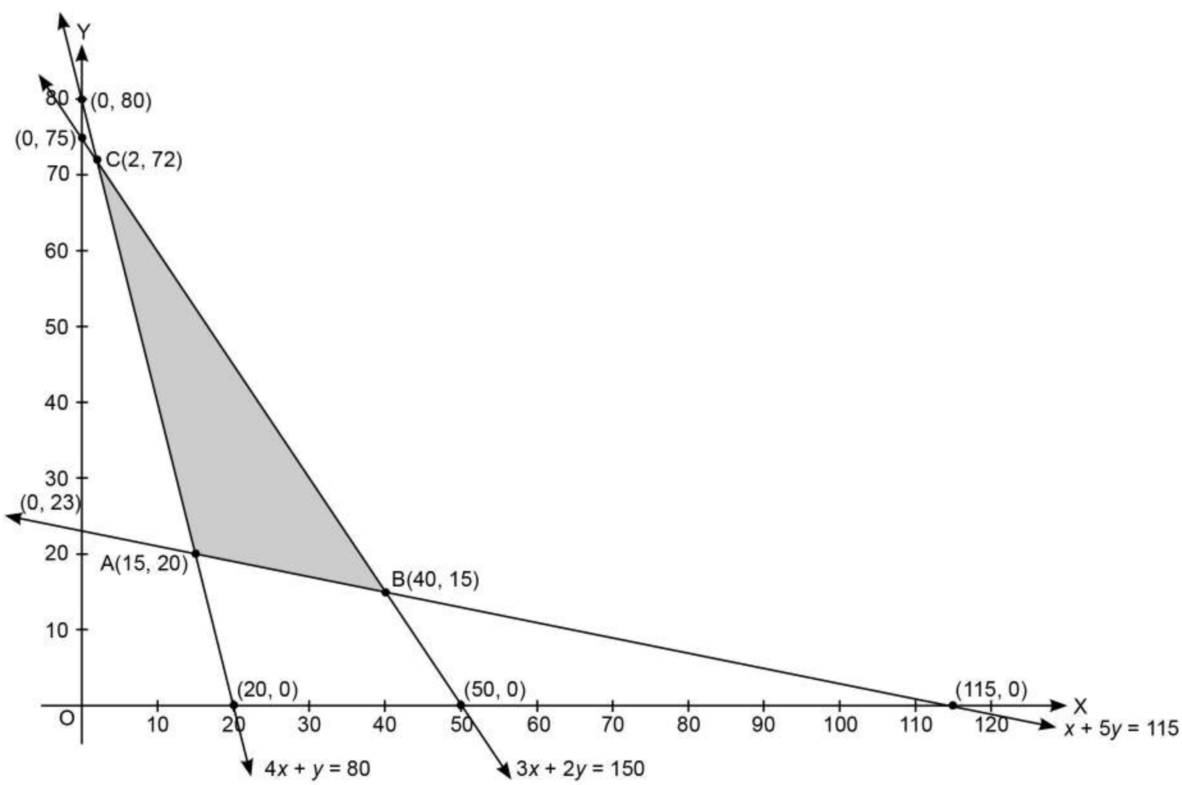
$4x + y \geq 80$

$x + 5y \geq 115$

$3x + 2y \leq 150$

$x \geq 0, y \geq 0$

On plotting the graph of inequations, we notice that shaded region is feasible solution.



Possible points for minimum Z are $A(15, 20)$, $B(40, 15)$, $C(2, 72)$.

Corner Points	$Z = 6x + 3y$	Values
$A(15, 20)$	90 + 60	150
$B(40, 15)$	240 + 45	285
$C(2, 72)$	12 + 216	228

← Minimum

Z is minimum at $A(15, 20)$ i.e. $x = 15$, $y = 20$, Minimum value of Z is 150.

31.

$$I = \int \sin(\log x)\,dx$$

$$= \int \underset{①}{\sin(\log x)} \cdot \underset{②}{1}\,dx$$

$$= \sin(\log x)\cdot x - \int \cos(\log x)\cdot \frac{1}{x}\cdot x\,dx$$

$$= x\sin(\log x) - \int \underset{①}{\cos(\log x)}\cdot \underset{②}{1}\,dx$$

$$= x\sin(\log x) - \left[\cos(\log x)\cdot x - \int -\sin(\log x)\cdot\frac{1}{x}\cdot x\,dx\right]$$

$$= x\cdot\sin(\log x) - x\cos(\log x) - I$$

$$\Rightarrow \quad 2I = x[\sin(\log x) - \cos(\log x)]$$

$$\Rightarrow \quad I = \frac{x}{2}[\sin(\log x) - \cos(\log x)] + C$$

32. Consider given system of equations :

$$x + y - z = 3$$

$$2x + 3y + z = 10$$

$$3x - y - 7z = 1$$

The above system of equations can be written in matrix form as :

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

i.e. $AX = B$, where $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

Its solution is $X = A^{-1}B$ and $A^{-1} = \frac{1}{|A|}AdjA$ ...(*i*)

$$|A| = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{vmatrix}$$

$$= 1(-20) - 1(-17) - 1(-11)$$

$$= -20 + 17 + 11 = 8 \neq 0. \text{ So } A^{-1} \text{ exists.}$$

Let A_{ij} be the cofactors of a_{ij} in $|A|$

Now, $A_{11} = (-1)^2(-20) = -20$, $A_{12} = (-1)^3(-17) = 17$, $A_{13} = (-1)^4(-11) = -11$

$$A_{21} = (-1)^3(-8) = 8, A_{22} = (-1)^4(-4) = -4, A_{23} = (-1)^5(-4) = 4$$

$$A_{31} = (-1)^4(4) = 4, A_{32} = (-1)^5(3) = -3, A_{33} = (-1)^6(1) = 1$$

$$\text{Adj } A = \begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

$\therefore$
$$A^{-1} = \frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

Now,
$$X = \frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix} \quad \text{[using } (i)]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -60+80+4 \\ 51-40-3 \\ -33+40+1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow$ $x = 3, y = 1, z = 1$ is the solution

33. Given lines are

$$L_1 : \frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$$

$$\Rightarrow \quad \frac{x-1}{-3} = \frac{7(y-2)}{\lambda} = \frac{z-3}{2}$$

$$\Rightarrow \quad \frac{x-1}{-21} = \frac{y-2}{\lambda} = \frac{z-3}{14}$$

and
$$L_2 : \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$$

$$\Rightarrow \quad \frac{-7(x-1)}{3\lambda} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\Rightarrow \quad \frac{x-1}{3\lambda} = \frac{y-1}{-7} = \frac{z-6}{35}$$

If lines L_1 and L_2 are perpendicular, then

$$(-21)(3\lambda) + \lambda(-7) + 14(35) = 0$$

$$\Rightarrow \quad -63\lambda - 7\lambda + 490 = 0$$

$$\Rightarrow \quad -70\lambda = -490$$

$$\Rightarrow \quad \lambda = 7$$

DR's of $L_2 = <21, -7, 35>$ i.e. $<3, -1, 5>$

DR's of line parallel to $L_2 = <3, -1, 5>$

$\therefore$ Equation of a line passing through point $(3, -1, 7)$ and parallel to L_2 are

$$\frac{x-3}{3} = \frac{y+1}{-1} = \frac{z-7}{5}$$

OR

Cartesian equation of line passing through point (1, 2, – 4) is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \quad ...(i)$$

Here, $\langle a, b, c \rangle$ are the dr's of the required line.

If line (*i*) is perpendicular to lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

and $$\frac{x-15}{3} = \frac{y-20}{8} = \frac{z-5}{-5}$$

Then, $$3a - 16b + 7c = 0$$

$$3a + 8b - 5c = 0$$

$$\Rightarrow \quad \frac{a}{80-56} = \frac{-b}{-15-21} = \frac{c}{24+48}$$

$$\Rightarrow \quad \frac{a}{24} = \frac{b}{36} = \frac{c}{72} \text{ or } \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$$

From (*i*) Cartesian equations of line are :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

In vector form :

$$\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$\therefore$ Vector equation of line is:

$$\vec{r} = a + \lambda\vec{b}$$

$$\Rightarrow \quad \vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

34. The given equations are :

$$5x - 2y - 10 = 0 \quad ...(i)$$

$$x + y - 9 = 0 \quad ...(ii)$$

$$2x - 5y - 4 = 0 \quad ...(iii)$$

Now, equation (*i*) intersects x-axis at (2, 0) and y-axis at (0, –5).

Also, equation (*ii*) intersects x-axis at (9, 0) and y-axis at (0, 9).

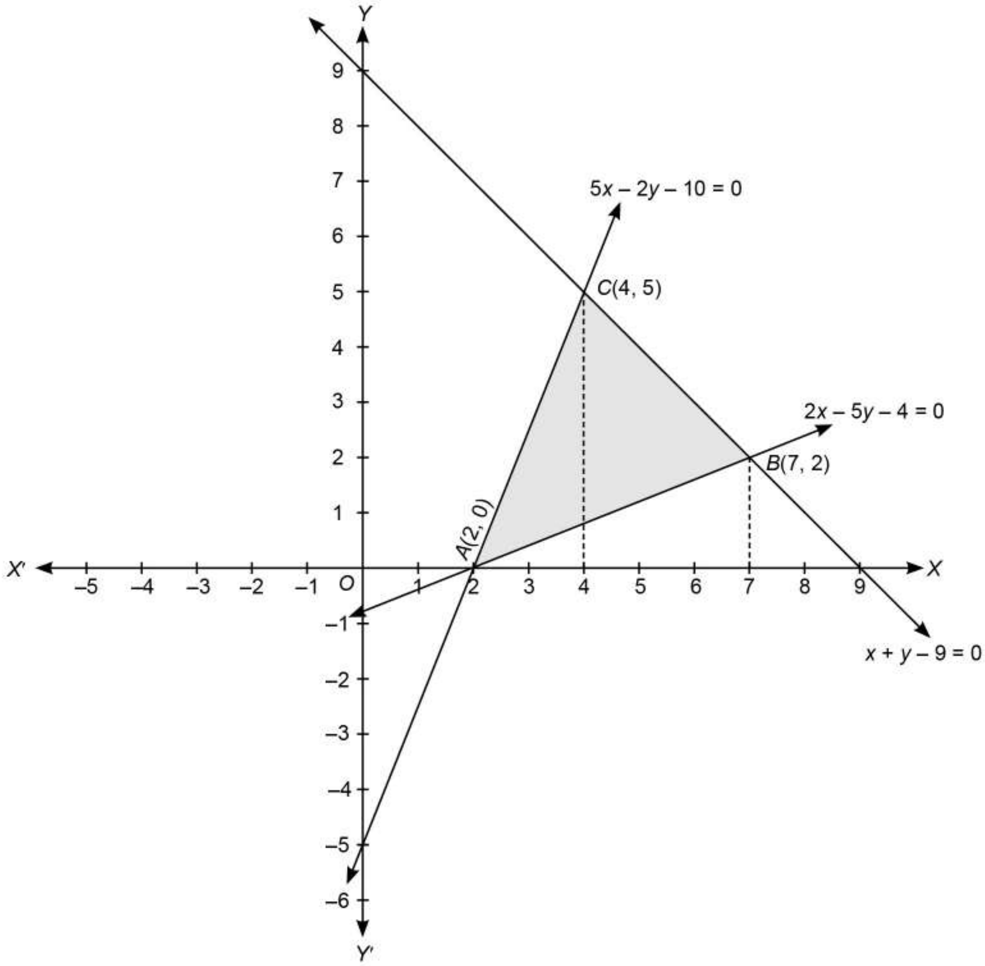
And equation (*iii*) intersects x-axis at (2, 0) and y-axis at $\left(0, \frac{-4}{5}\right)$.

Points of intersection:

The lines (*i*) and (*ii*) intersect at (4, 5).

The lines (*ii*) and (*iii*) intersect at (7, 2).

The lines (*i*) and (*iii*) intersect at (2, 0).



$$\text{Area} = \int_2^4 y_{AC}\,dx + \int_4^7 y_{BC} - \int_2^7 y_{AB}\,dx$$

$$= \int_2^4 \frac{(5x-10)}{2}dx + \int_4^7 (9-x)\,dx - \int_2^7 \frac{(2x-4)}{5}dx$$

$$= \frac{1}{2}\left[\frac{5x^2}{2} - 10x\right]_2^4 + \left[9x - \frac{x^2}{2}\right]_4^7 - \frac{1}{5}[x^2 - 4x]_2^7$$

$$= \frac{1}{2}[(40-40)-(10-20)] + \left[\left(63 - \frac{49}{2}\right) - (36-8)\right] - \frac{1}{5}[(49-28)-(4-8)]$$

$$= \frac{1}{2}[10] + \left[\frac{21}{2}\right] - \frac{1}{5}[25]$$

$$= 5 + \frac{21}{2} - 5 = \frac{21}{2} \text{ sq units.}$$

35. Given relation $S = \{(a, b) \in R \times R : a \leq b\}$

Reflexive: Let a be any arbitrary element of set R.

Now, $a \in R \Rightarrow (a, a) \in S \;\forall\; a \in R$ $[\because a \leq a \;\forall\; a \in R]$

Hence, S is reflexive.

Symmetric: Let $a, b \in R$.

Now, $(a, b) \in S \Rightarrow a \leq b \nRightarrow b \leq a$ i.e. $(b, a) \in S$

$\therefore$ $(a, b) \in S \nRightarrow (b, a) \in S$ for $a, b \in R$

Hence, S is not symmetric.

Transitive: Let $a, b, c \in R$ such that $(a, b) \in S$ and $(b, c) \in S$

Now, $(a, b) \in S \Rightarrow a \leq b$ and $(b, c) \in S \Rightarrow b \leq c$

$\Rightarrow$ $a \leq b, b \leq c \Rightarrow a \leq c$

$\Rightarrow$ $(a, c) \in S$

So, $(a, b) \in S$ and $(b, c) \in S \Rightarrow (a, c) \in S$ for all $a, b, c \in R$

Hence, relation S is transitive.

OR

Given function $f : N \rightarrow N$ defined as $f(x) = x^2 + x + 1$

For one-one: Let $x_1, x_2 \in N$.

Now, $f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow$ $x_1^2 - x_2^2 + x_1 - x_2 = 0$

$\Rightarrow$ $(x_1 - x_2)(x_1 + x_2) + (x_1 - x_2) = 0$

$\Rightarrow$ $(x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow$ $x_1 - x_2 = 0$ or $x_1 + x_2 + 1 = 0$ (not true as $x_1, x_2 \in N$)

$\Rightarrow$ $x_1 = x_2$

As $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for $x_1, x_2 \in N$

Hence one-one.

For onto: Let for $y \in N$ (codomain), there exists $x \in N$ (domain) s.t. $y = f(x)$

$\Rightarrow$ $y = x^2 + x + 1 \Rightarrow x^2 + x + (1 - y) = 0$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1 - 4(1 - y)}}{2} = \frac{-1 \pm \sqrt{4y - 3}}{2}$$

$\Rightarrow x$ may not belong to N. For e.g. when $y = 16$ there is no $x \in N$ such that $y = f(x)$.

Hence, f is not onto.

36. The given points are : $A(3, 1, 2)$ and $B(-1, 4, 0)$.

(*i*)
$$\overrightarrow{AB} = \left(-\hat{i} + 4\hat{j}\right) - \left(3\hat{i} + \hat{j} + 2\hat{k}\right) = -4\hat{i} + 3\hat{j} - 2\hat{k}$$

(*ii*) Unit vector in the direction of $\overrightarrow{AB} = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}\ \dfrac{-4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{16 + 9 + 4}}$

$$= \frac{-4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$$

(*iii*) Vector of magnitude 5 units along $\overrightarrow{AB}$

$$= 5\left[\frac{-4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}\right]$$

$$= \frac{-20}{\sqrt{29}}\hat{i} + \frac{15}{\sqrt{29}}\hat{j} - \frac{10}{\sqrt{29}}\hat{k}$$

OR

(*iii*) Direction cosines of $\overrightarrow{AB}$ are components of unit vector along $\overrightarrow{AB}$, *i.e.*

$$\left\langle \frac{-4}{\sqrt{29}}, \frac{3}{\sqrt{29}}, -\frac{2}{\sqrt{29}} \right\rangle$$

37. $$P(A) = \frac{4}{5}, P(B) = \frac{3}{4}, P(C) = \frac{2}{3}$$

(*i*) $$P(\text{All three winning}) = P(A \cap B \cap C) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$$

(*ii*) $$P(\bar{A} \cap B \cap C) = \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{10}$$

(*iii*) $$P(\text{at least one wins}) = 1 - P(\text{none wins}) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{59}{60}$$

OR

(*iii*) $$P(\text{none wins}) = P(\bar{A} \cap \bar{B} \cap \bar{C}) = \left(1-\frac{4}{5}\right)\left(1-\frac{3}{4}\right)\left(1-\frac{2}{3}\right) = \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$$

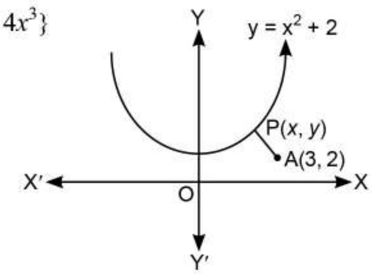
38. (*i*) Soldier is at $A(3, 2)$ and position of jet is $P(x, y)$.

$\therefore$ $$\text{Distance } (D) = \sqrt{(x-3)^2 + (y-2)^2}$$

Also $$y = x^2 + 2$$

$\therefore$ $$D = \sqrt{(x-3)^2 + x^4}$$

(*ii*) $$\frac{dD}{dx} = \frac{1}{2\sqrt{(x-3)^2 + x^4}} \times \{2(x-3) + 4x^3\} = \frac{4x^3 + 2x - 6}{2\sqrt{(x-3)^2 + x^4}} = \frac{2x^3 + x - 3}{\sqrt{(x-3)^2 + x^4}}$$



For critical points, $$\frac{dD}{dx} = 0$$

$\Rightarrow$ $$2x^3 + x - 3 = 0$$

$\Rightarrow$ $$(x-1)(2x^2 + 2x + 3) = 0$$

$\Rightarrow$ $$x - 1 = 0 \text{ or } 2x^2 + 2x + 3 = 0 \quad (\text{no real value of } x)$$

$\Rightarrow$ $$x = 1$$

$$\begin{array}{r} 2x^2 + 2x + 3 \\ x-1 \overline{) \; 2x^3 + x - 3} \\ 2x^3 - 2x^2 \\ (-) \quad (+) \\ \hline 2x^2 + x \\ 2x^2 - 2x \\ (-) \quad (+) \\ \hline 3x - 3 \\ 3x - 3 \\ (-)(+) \\ \hline 0 \\ \hline \end{array}$$

Now, $$\frac{d^2D}{dx^2} = \frac{\sqrt{(x-3)^2 + x^4} \times (6x^2 + 1) - (2x^3 + x - 3) \times \dfrac{(2x^3 + x - 3)}{\sqrt{(x-3)^2 + x^4}}}{(x-3)^2 + x^4}$$

$\Rightarrow$ $$\left[\frac{d^2D}{dx^2}\right]_{x=1} = \frac{\sqrt{5} \times (7) - 0}{5} = \frac{7\sqrt{5}}{5} > 0$$

So, D is minimum at $x = 1$.

$\therefore$ for $x = 1$, jet is nearest to the soldier.