20 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(*ii*) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(*iii*) *In* ***Section A****, Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each****.*

(*iv*) *In* ***Section B****, Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each****.*

(*v*) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each****.*

(*vi*) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each****.*

(*vii*) *In* ***Section E****, Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions****, carrying* ***4 marks each****.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If B is a non-singular matrix and A is a square matrix, then det $(B^{-1}AB)$ is equal to **[NCERT Part-I, Page 89]**

(*a*) det (A^{-1}) (*b*) det (B^{-1}) (*c*) det (A) (*d*) det (B)

2. If matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{k}$ (adj A) then $k =$ **[NCERT Part-I, Page 90]**

(*a*) 7 (*b*) – 7 (*c*) 15 (*d*) – 11

3. If $A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}$ then A. (adj. A) = **[NCERT Part-I, Page 88]**

(*a*) $3I$ (*b*) $7I$ (*c*) I (*d*) None of these

4. If $y = \log_3 3^{\cos x}$, then $\frac{dy}{dx} =$ **[NCERT Part-I, Page 130]**

(*a*) $\sin x$ (*b*) $-\sin x$ (*c*) $3^{\cos x}$ (*d*) None of these

5. The line through the points (1, –1, 2), (3, 4, –2) and the line through the points (0, 3, 2), (3, 5, 6) are **[Conceptual Application]**

(*a*) parallel (*b*) perpendicular (*c*) inclined at 30° (*d*) None of these

6. In particular solution of differential equation, arbitrary constant is/are **[Conceptual Application]**

(*a*) 2 (*b*) 1 (*c*) 3 (*d*) 0

7. The feasible region for an LPP is always **[Conceptual Application]**

(*a*) convex polygon (*b*) concave polygon

(*c*) circular polygon (*d*) None of these

8. If $\vec{a} = 2\hat{i} - m\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 4\hat{k}$, then value of m, if $\vec{a}$ and $\vec{b}$ are orthogonal, is **[NCERT Part-II, Page 356]**

(*a*) 4 (*b*) 3 (*c*) –2 (*d*) 1

9. The value of integral $\int_0^5 \log x \, dx$ is **[NCERT Part-II, Page 259-260, 268]**

(*a*) 0 (*b*) $\log 5 - 1$ (*c*) $5 \log 5 - 5$ (*d*) None of these

10. A is the matrix of order 3×3. If $|A| = 5$, then $|3A|$ is equal to **[NCERT Part-I, Page 80]**

(*a*) 120 (*b*) 27 (*c*) 81 (*d*) 135

11. In objective function of an LPP, $Z = ax + by$, 'x' and 'y' are called **[Conceptual Application]**

(*a*) constant (*b*) decision variables

(*c*) constraints (*d*) None of these

12. If projection of $\vec{a}$ on $\vec{b}$ is zero then $\vec{a}$ and $\vec{b}$ are **[NCERT Part-II, Page 358, 356]**

(*a*) parallel (*b*) collinear (*c*) perpendicular (*d*) None of these

13. If A is symmetric matrix then $B'AB$ is **[Conceptual Application]**

(*a*) symmetric matrix (*b*) skew symmetric matrix

(*c*) null matrix (*d*) None of these

14. If A and B are two independent events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, then $P(A/B)$ is equal to **[NCERT Part-II, Page 408]**

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) Not defined

15. $\int \frac{1}{x + \sqrt{x}} dx$ is equal to **[NCERT Part-II, Page 235-236]**

(*a*) $\log(\sqrt{x} + 1) + C$ (*b*) $-\log(\sqrt{x} + 2) + C$

(*c*) $2\log(\sqrt{x} + 1) + C$ (*d*) $-2\log(\sqrt{x} + x) + C$

16. The vector with initial point $P(2, -3, 5)$ and terminal point $Q(3, -4, 7)$ is **[NCERT Part-II, Page 339-340]**

(*a*) $\hat{i} - \hat{j} + 2\hat{k}$ (*b*) $5\hat{i} - 7\hat{j} + 12\hat{k}$ (*c*) $-\hat{i} - \hat{j} - 2\hat{k}$ (*d*) None of these

17. Derivative of $\sin x$ with respect to $\cos x$ is equal to: **[Conceptual Application]**

(*a*) $\sin x$ (*b*) $\tan x$ (*c*) $\cot x$ (*d*) $-\cot x$

18. Direction cosines of line joining two points $A(2, 1, -2)$ and $B(-1, 0, -3)$ are **[NCERT Part-II, Page 379-380]**

(*a*) $\frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{2}{\sqrt{11}}$ (*b*) $\frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{2}{\sqrt{11}}$

(*c*) $\frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}$ (*d*) None of these

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. If function $f : R \rightarrow R$ is defined by **[NCERT Part-I, Page 7]**

$$f(x) = \begin{cases} 2x, & x > 3 \\ x^2, & 1 < x \le 3, \text{ then} \\ 3x, & x \le 1 \end{cases}$$

Assertion (*A*): function is one-one.

Reason (*R*): For $x_1, x_2 \in R$,

if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, then f is one-one.

20. **Assertion(*A*):** The greatest integer function is continuous between two consecutive integral points.

Reason(*R*): The greatest integer function is not continuous at integral points. **[NCERT Part-I, Page 105]**

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Evaluate: $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{4}\right)$. **[Conceptual Application]**

OR

Find the value of $\cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\} + \cos\left(\tan^{-1}\sqrt{3}\right)$. **[Conceptual Application]**

22. Find the value of k so that function $f(x) = \begin{cases} \frac{3k\cos x}{\pi - 2x}, & \text{if } x \ne \frac{\pi}{2} \\ 4 \quad , & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

[NCERT Part-I, Page 105]

23. Find the maximum value of $\frac{\log x}{x}$ in $(2, \infty)$. **[NCERT Part-I, Page 166]**

OR

Find the interval for which the function $f(x) = \cos x$ is increasing in the interval $(0, 2\pi)$.

[NCERT Part-I, Page 153]

24. Evaluate $\int \frac{1}{1+\tan x} dx$ **[NCERT Part-II, Page 241]**

25. If side of an equilateral triangle is increasing at the rate of 4 cm/min, at what rate its area is increasing when side is equal to 10 cm? **[NCERT Part-I, Page 147-148]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int_1^4 \{|x-1| + |x-2|\} dx$. **[Integrated Question]**

27. The probability of solving a specific problem by three students A, B and C are $\frac{1}{3}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. Find the probability that problem is solved by exactly two students. **[Conceptual Application]**

28. Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+8}} dx$. **[NCERT Part-II, Page 246-247]**

OR

Evaluate: $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$. **[NCERT Part-II, Page 252-253]**

29. Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers. **[Conceptual Application]**

OR

Find the general solution of differential equation: **[NCERT Part-II, Page 306-307]**

$\frac{dy}{dx} = 1 - x + y - xy.$

30. Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397-398]**

Maximise $Z = 600x + 400y$, subject to the constraints $x + y \le 200$, $y \ge 4x$, $x \ge 20$, $x \ge 0$, $y \ge 0$.

OR

Solve the following linear programming problem graphically:

Minimise: $Z = 2x + y$ **[NCERT Part-II, Page 397-398]**

Subject to constraints: $3x + y \ge 9$, $x + y \ge 7$, $x + 2y \ge 8$, $x \ge 0$, $y \ge 0$

31. If $x = 3 \sin \theta - \sin 3\theta$, $y = 3 \cos \theta - \cos 3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$. **[NCERT Part-II, Page 134-135]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find the area of the region $\{(x, y) : 0 \le y \le x^2 + 2, 0 \le y \le x + 2, 0 \le x \le 4\}$. **[Conceptual Application]**

33. Show that the relation R defined by $(a, b)\, R(c, d) \Leftrightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$ for all $(a, b), (c, d) \in A \times A$, where $A = \{1, 2, 3, ..., 10\}$ is an equivalence relation. **[NCERT Part-I, Page 2]**

OR

Let $f : N \to N$ given by $f(x) = \begin{cases} \frac{x+1}{2} & \text{, if } x \text{ is odd} \\ \frac{x}{2} & \text{, if } x \text{ is even} \end{cases}$. **[NCERT Part-I, Page 7]**

State whether the function f is bijective or not.

34. A stationery shop has 3 types of pens 'A', 'B' and 'C'. Renu purchased 1 pen of each variety for a total of ₹ 37. Riya purchased 4 pens of 'A' type, 3 pens of 'B' type and 2 pens of 'C' type for ₹ 106, while Shikha purchased 6 pens of 'A' type, 2 pens of 'B' type and 3 pens of 'C' type for ₹ 129. Using matrix method, find cost of each type of pen. **[NCERT Part-I, Page 94-95]**

35. Find the vector and Cartesian equation of line which is parallel to line joining the points $A(2, 3, 4)$ and $B(6, 8, 7)$ and passing through the point $P(-1, -6, 3)$. Also, find the direction cosines of line joining the points A and P. **[Conceptual Application]**

OR

Find the shortest distance between the lines **[NCERT Part-II, Page 386-387]**

$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$.

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. The selling price of pen drive is given by the equation $x = \dfrac{600 - p}{8}$, where p is price per unit and 'x' is number of units produced. If total cost of pen drives is given by $x^2 + 78x + 2500$ and revenue function is equal to $p \cdot x$(selling price × number of units) then **[Conceptual Application]**

(*i*) find the value of 'p'.

(*ii*) express the profit function in terms of 'x'.

(*iii*) find the value of 'x' for which profit is maximum.

OR

(*iii*) find the intervals in which profit is increasing.

Case Study - 2

37. Two helicopters I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. **[Conceptual Application]**

(*i*) Find the probability that the target is hit by only one helicopter.

(*ii*) Find the probability that both hit the target.

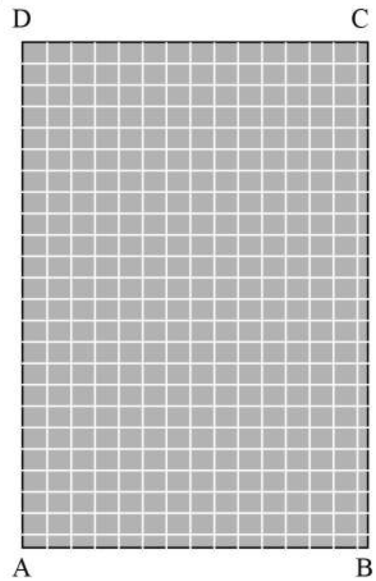
(*iii*) The second helicopter will bomb only if the first misses the target. Find the probability that the target is hit by the second helicopter.

OR

(*iii*) Find the probability that the target is not hit by helicopters.

Case Study - 3

38. In given figure, coordinates of corners of rectangular solar panel $ABCD$ are given by $A\left(-1, \frac{1}{2}, 4\right), B\left(1, \frac{1}{2}, 4\right)$, $C\left(1, -\frac{1}{2}, 4\right)$ and $D\left(-1, -\frac{1}{2}, 4\right)$. **[NCERT Part-II, Page 365, 355-356]**



(*i*) Find the area of solar panel $ABCD$.

(*ii*) Find the angle between two diagonals AC and BD of the panel.

SOLUTIONS

1. (*c*)
$$\det(B^{-1}AB) = |B^{-1}|\cdot|AB|$$
$$= |B^{-1}|\,|A|\cdot|B|$$
$$= \frac{1}{|B|}|A|\,|B| = |A| = \det A$$

2. (*c*)
$$A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$
$\Rightarrow$
$$|A| = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$
$$= 3(2+1) + 2(1-0) + 4(1-0)$$
$$= 9 + 2 + 4 = 15$$
$$A^{-1} = \frac{1}{|A|}\text{adj } A$$
$\therefore$ $k = 15$

3. (*b*)
$$A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}$$
$\Rightarrow$
$$|A| = \begin{vmatrix} 1 & 3 & 2 \\ -2 & 5 & 1 \\ 0 & 3 & 2 \end{vmatrix}$$
$$= 1(10-3) - 3(-4-0) + 2(-6-0)$$
$$= 7 + 12 - 12 = 7$$
$$(A \cdot \text{adj } A) = |A|I$$
$$= 7I$$

4. (*b*)
$$y = \log_3 3^{\cos x}$$
$\therefore$ $y = \cos x$ $\qquad (\log_a a^x = x)$

Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = -\sin x$$

5. (*b*) Direction ratios of line through the points (1, – 1, 2) and (3, 4, – 2) are (2, 5, – 4).

Direction ratios of line through the points (0, 3, 2) and (3, 5, 6) are (3, 2, 4).

Now $a_1a_2 + b_1b_2 + c_1c_2 = 3 \times 2 + 2 \times 5 + 4 \times (-4) = 0$

$\therefore$ Lines are perpendicular.

6. (*d*) In particular solution, there is no arbitrary constant.

7. (*a*) convex polygon.

8. (*d*)
$$\vec{a} = 2\hat{i} - m\hat{j} + \hat{k}$$
$$\vec{b} = -\hat{i} + 2\hat{j} + 4\hat{k}$$

For orthogonal vectors, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow \quad 2 \times (-1) - m \times 2 + 1 \times 4 = 0$

$\Rightarrow \quad -2 - 2m + 4 = 0$

$\Rightarrow \quad -2m = -2$

$\Rightarrow \quad m = 1$

9. (*c*) Let $I = \int_0^5 \log \underset{①}{x} \; \underset{②}{1} \cdot dx$

$$= [x \cdot \log x]_0^5 - \int_0^5 \frac{1}{x} \times x \, dx$$

$$= 5 \log 5 - [x]_0^5$$

$$= 5 \log 5 - 5$$

10. (*d*) $|A| = 5$

$$|3A| = 3^3 \cdot |A|$$

$$= 27 \times 5 = 135$$

11. (*b*) Decision variables

12. (*c*) Projection of $\vec{a}$ on $\vec{b} = 0$

$\Rightarrow$ Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$\Rightarrow \quad 0 = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$\Rightarrow \quad \vec{a} \cdot \vec{b} = 0$

$\therefore$ $\vec{a}$ is perpendicular to $\vec{b}$.

13. (*a*) Given $A' = A$

$$(B'AB)' = (AB)'(B')'$$

$$= B'A'B$$

$$= B'AB$$

$\therefore$ $B'AB$ is symmetric matrix.

14. (*b*) For independent events, $P(A \cap B) = P(A) \times P(B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A) \times P(B)}{P(B)} = \frac{1}{2}$$

15. (*c*) Let $I = \int \dfrac{dx}{x + \sqrt{x}}$

$$= \int \frac{dx}{\sqrt{x}(\sqrt{x} + 1)}$$

Let $\sqrt{x} + 1 = t$

$\Rightarrow \dfrac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \dfrac{dx}{\sqrt{x}} = 2dt$

$$= 2\int \frac{dt}{t}$$

$$= 2 \log |t| + C$$

$$= 2 \log |\sqrt{x} + 1| + C$$

16. (*a*) Vector $\overrightarrow{PQ} = P.V. \text{ of } Q - P.V. \text{ of } P$

$$= 3\hat{i} - 4\hat{j} + 7\hat{k} - 2\hat{i} + 3\hat{j} - 5\hat{k} = \hat{i} - \hat{j} + 2\hat{k}$$

17. (*d*) Let $u = \sin x$ and $v = \cos x$

$\therefore$ $\frac{du}{dx} = \cos x$, and $\frac{dv}{dx} = -\sin x$

$$\frac{du}{dv} = \frac{du}{dx} \div \frac{dv}{dx}$$

$\therefore$ $\frac{du}{dv} = -\frac{\cos x}{\sin x}$

$$= -\cot x$$

18. (*c*) Direction ratios of the line $AB = \langle -1-2, 0-1, -3+2 \rangle$

$$= \langle -3, -1, -1 \rangle$$

$$|AB| = \sqrt{9+1+1} = \sqrt{11}$$

$$\text{DC's are} = \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}$$

19. (*d*) (A) is false but (R) is true.

20. (*b*) Both (A) and (R) are true but (R) is not the correct explanation of (A).

21. $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{4}\right) = \tan\frac{x}{2}$

Let $\cos^{-1}\frac{\sqrt{5}}{4} = x$

$\Rightarrow \cos x = \frac{\sqrt{5}}{4}$

$\therefore$ We have $\tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}$

$\Rightarrow$ $\tan\frac{x}{2} = \sqrt{\frac{1-\frac{\sqrt{5}}{4}}{1+\frac{\sqrt{5}}{4}}}$

$$= \sqrt{\frac{4-\sqrt{5}}{4+\sqrt{5}}}$$

$\Rightarrow$ $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{4}\right) = \frac{4-\sqrt{5}}{\sqrt{11}}$

OR

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) + \cos\left(\tan^{-1}\sqrt{3}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] + \cos\left[\tan^{-1}\left(\tan\frac{\pi}{3}\right)\right]$$

$$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{3}\right)$$

$$= \left(\frac{\pi}{6} + \frac{1}{2}\right) = \left(\frac{\pi+3}{6}\right)$$

22. $$f(x) = \begin{cases} \frac{3k\cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 4, & x = \frac{\pi}{2} \end{cases}$$

For continuous function

$$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{2}^-} \frac{3k\cos x}{\pi - 2x} = \lim_{x \to \frac{\pi}{2}^+} \frac{3k\cos x}{\pi - 2x} = 4$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{3k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{3k\cos\left(\frac{\pi}{2} + h)\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)} = 4$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{3k\sin h}{2h} = \lim_{h \to 0} \frac{-3k\sin h}{-2h} = 4$$

$$\Rightarrow \quad \frac{3k}{2} = \frac{3k}{2} = 4$$

$$\Rightarrow \quad 3k = 8$$

$$\Rightarrow \quad k = \frac{8}{3}$$

23. Let $$f(x) = \frac{\log x}{x} \qquad ...(i)$$

Differentiating w.r.t. x, we get

$$f'(x) = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2} \qquad ...(ii)$$

For critical point $f'(x) = 0$

$$\Rightarrow \quad \frac{1 - \log x}{x^2} = 0$$

$$\Rightarrow \quad \log x = 1$$

$$\Rightarrow \quad \log x = \log e$$

$$\therefore \quad x = e$$

Differentiating (*ii*) w.r.t. x, we get

$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)2x}{x^4}$$

$$\Rightarrow \quad [f''(x)]_{x=e} = \frac{-e^2 \times \frac{1}{e}}{e^4} < 0$$

$\therefore$ $x = e$ is the point of maxima.

$$\therefore \quad \text{Maximum value} = \frac{\log e}{e} = \frac{1}{e}$$

OR

$$f(x) = \cos x$$

Differentiating w.r.t. x, we get

$$f'(x) = -\sin x$$

$$f'(x) = 0$$

$$\Rightarrow \quad -\sin x = 0$$

$$\therefore \quad x = 0, \pi$$

Sign of $f'(x)$ in interval $(0, \pi)$:

Now, $f'(x) < 0$ when $x \in (0, \pi)$ $\quad [\because \sin x > 0 \text{ when } 0 < x < \pi]$

$\therefore f(x)$ is decreasing in the interval $(0, \pi)$.

Sign of $f'(x)$ in interval $(\pi, 2\pi)$:

Now, $f'(x) > 0$ when $x \in (\pi, 2\pi)$ $\quad [\because \sin x < 0 \text{ when } \pi < x < 2\pi]$

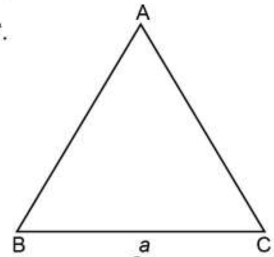
$\therefore f(x)$ is increasing in the interval $(\pi, 2\pi)$.

24. Let

$$I = \int \frac{dx}{1+\tan x}$$
$$= \int \frac{\cos x \, dx}{\sin x + \cos x}$$
$$= \frac{1}{2}\int \frac{2\cos x \, dx}{\sin x + \cos x}$$
$$= \frac{1}{2}\left[\int \frac{\cos x + \cos x + \sin x - \sin x}{\sin x + \cos x} dx\right]$$
$$= \frac{1}{2}\left[\int \frac{\cos x + \sin x}{\sin x + \cos x} dx + \int \frac{\cos x - \sin x}{\sin x + \cos x} dx\right]$$
$$= \frac{1}{2}\left[x + \int \frac{\cos x - \sin x}{\sin x + \cos x} dx\right]$$
$$= \frac{1}{2}\left[x + \int \frac{dt}{t}\right]$$
$$= \frac{1}{2}[x + \log|\cos x + \sin x|] + C$$

Let $\cos x + \sin x = t$
$\Rightarrow (-\sin x + \cos x)dx = dt$
$\Rightarrow (\cos x - \sin x)dx = dt$

25. Let side of an equilateral triangle be a and A be its area at any instant of time t.



$\therefore$ Area of triangle, $A = \frac{\sqrt{3}}{4}a^2$

Differentiating w.r.t. 't' we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \cdot 2a \cdot \frac{da}{dt}$$

$\Rightarrow$ $$\left.\frac{dA}{dt}\right]_{a=10} = \frac{\sqrt{3}}{4} \times 2 \times 10 \times 4 \qquad \left[\frac{da}{dt} = 4 \text{ cm/ min}, a = 10 \text{ cm}\right]$$
$$= 20\sqrt{3} \text{ cm}^2/\text{min}$$

26. $\int_1^4 (|x-1|) + |x-2|)dx$

When $x < 1$:

$\therefore$ $$|x-1| + |x-2| = -(x-1) - (x-2)$$
$$= -2x + 3$$ (Rejected no interval given for integrals)

When $1 \le x < 2$:

$\therefore$ $$|x-1| + |x-2| = x - 1 - x + 2$$
$$= 1$$

When $x \ge 2$:

$\therefore$ $$|x-1| + |x-2| = x - 1 + x - 2$$
$$= 2x - 3$$

$\therefore$ $$\int_1^4 (|x-1| + |x-2|)dx = \int_1^2 1dx + \int_2^4 (2x-3)dx$$
$$= [x]_1^2 + \left[\frac{2x^2}{2} - 3x\right]_2^4$$
$$= 2 - 1 + (16 - 12) - (4 - 6)$$
$$= 1 + 4 + 2$$
$$= 7$$

27. $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}, P(C) = \frac{1}{4}$

Problem is solved by exactly two students.

$\therefore$ P(Problem solved by exactly two students) $= P(AB\overline{C})$ or $P(A\overline{B}C)$ or $P(\overline{A}BC)$

$$= \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{4}$$

$$= \frac{1}{24}(3 + 1 + 2) = \frac{1}{4}$$

28. $\int \frac{x+2}{\sqrt{x^2+5x+8}} dx$

Let $\quad x + 2 = A\frac{d}{dx}(x^2 + 5x + 8) + B$

$$= A(2x + 5) + B$$

$\Rightarrow \quad x + 2 = 2Ax + (5A + B)$

Comparing coefficient of 'x' on both sides, we get

$$1 = 2A \quad \Rightarrow \quad A = \frac{1}{2}$$

Comparing the constant on both sides,

$$5A + B = 2$$

$\Rightarrow \quad B = 2 - \frac{5}{2} = -\frac{1}{2}$

$\therefore \quad \int \frac{x+2}{\sqrt{x^2+5x+8}} dx = \int \frac{\frac{1}{2}(2x+5) - \frac{1}{2}}{\sqrt{x^2+5x+8}} dx$

$$= \frac{1}{2}\int \frac{2x+5}{\underbrace{\sqrt{x^2+5x+8}}_{I_1}} dx - \frac{1}{2}\int \frac{dx}{\underbrace{\sqrt{x^2+5x+8}}_{I_2}}$$

For $\quad I_1 = \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+8}} dx \quad$ | Let $x^2 + 5x + 8 = t$ $\Rightarrow (2x + 5)dx = dt$

$$I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2} \times 2\sqrt{t} + C_1$$

$$= \sqrt{x^2 + 5x + 8} + C_1$$

$$I_2 = \int \frac{dx}{\sqrt{x^2+5x+8}}$$

$$= \int \frac{dx}{\sqrt{x^2 + 5x + \frac{25}{4} - \frac{25}{4} + 8}}$$

$$= \int \frac{dx}{\sqrt{\left(x + \frac{5}{2}\right)^2 + \frac{7}{4}}}$$

$$I_2 = \int \frac{dx}{\sqrt{\left(x + \frac{5}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 8}\right| + C_2$$

$\therefore \quad I = \sqrt{x^2 + 5x + 8} - \frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 8}\right| + C$, where C (constant) $= C_1 - C_2$

OR

Let $I = \int \frac{x^2}{(x^2+1)(x^2+4)} dx$

put $x^2 = y$

$$\Rightarrow \quad \frac{x^2}{(x^2+1)(x^2+4)} = \frac{y}{(y+1)(y+4)}$$

Let $$\frac{y}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{(y+4)}$$

$$\Rightarrow \quad \frac{y}{(y+1)(y+4)} = \frac{(y+4)A+(y+1)B}{(y+1)(y+4)}$$

$$\Rightarrow \quad y = y(A+B)+(4A+B)$$

Comparing equal degree terms, we get

$\Rightarrow$ $A + B = 1$ ... (*i*) and $4A + B = 0$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$A = -\frac{1}{3}, B = \frac{4}{3}$

$$\therefore \quad \int \frac{x^2}{(x^2+1)(x^2+4)} dx = -\frac{1}{3}\int \frac{dx}{x^2+1} + \frac{4}{3}\int \frac{dx}{x^2+4}$$

$$= -\frac{1}{3} \times \tan^{-1}x + \frac{4}{3} \times \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$= -\frac{1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\frac{x}{2} + C$$

29. Let the numbers be x and $5 - x$ (as sum of two numbers is 5).

According to question

We have to minimise the sum of cubes of these numbers

$$x^3 + (5-x)^3 = y \text{ (say)}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 3x^2 + 3(5-x)^2(-1)$$

$$= 3x^2 - 3(25 + x^2 - 10x)$$

$$= 3x^2 - 75 - 3x^2 + 30x$$

For maximum or minimum

$$\Rightarrow \quad \frac{dy}{dx} = 0 \Rightarrow -75 + 30x = 0$$

$$\Rightarrow \quad 30x = 75$$

$$x = \frac{75}{30} = \frac{5}{2}$$

$$\frac{d^2y}{dx^2} = 30$$

$30 > 0$ then the minima occurs at $x = \frac{5}{2}$ and the other number is also $\frac{5}{2}$.

$$\text{Sum of squares} = \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{25}{4} + \frac{25}{4}$$

$$= \frac{50}{4} = \frac{25}{2}$$

OR

$$\frac{dy}{dx} = 1 - x + y - xy$$
$$= (1-x) + y(1-x)$$
$$= (1-x)(1+y)$$

$$\Rightarrow \quad \frac{dy}{1+y} = (1-x)dx$$

Integrating both sides, we get

$$\int \frac{dy}{1+y} = \int (1-x)\,dx$$

$$\log|1+y| = x - \frac{x^2}{2} + C$$

30. $Z = 600x + 400y$

Given inequations are

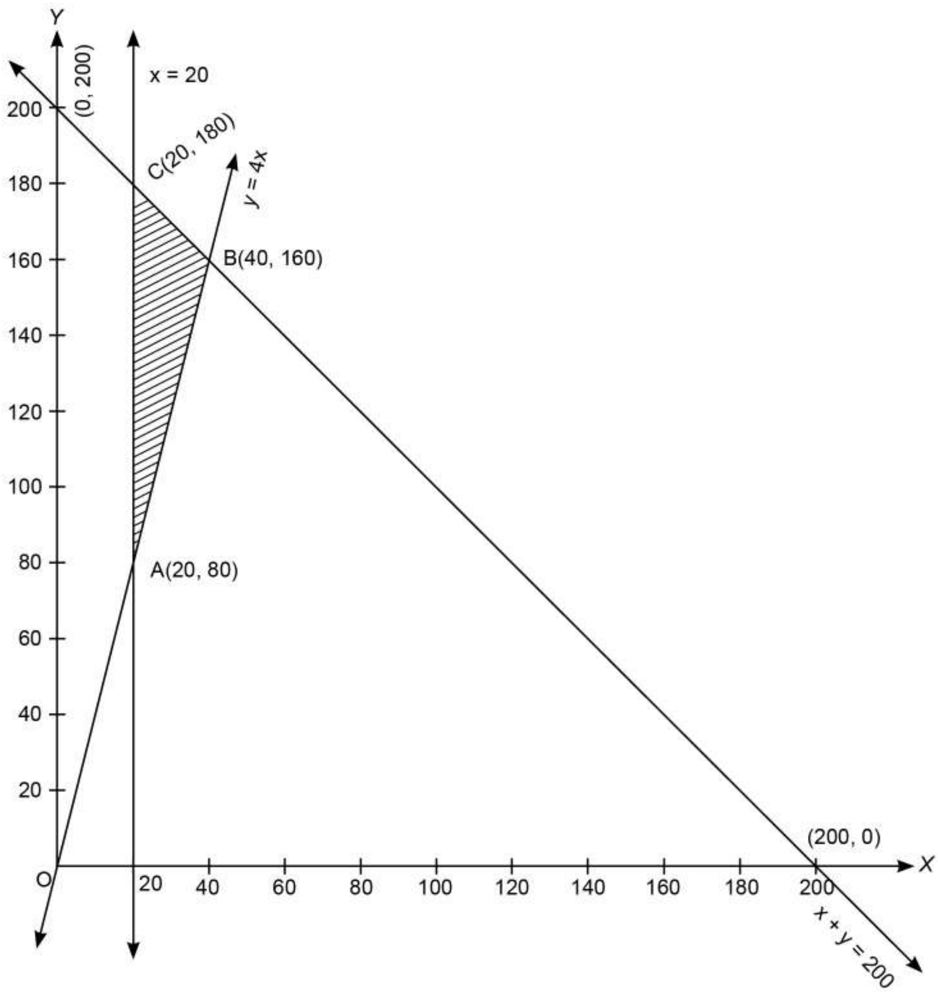
$x + y \leq 200$

$y \geq 4x$

$x \geq 20$

$x, y \geq 0$

Plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for maximum Z are $A(20, 80)$, $B(40, 160)$ and $C(20, 180)$.



Points	$Z = 600x + 400y$	Values
A(20, 80)	600 × 20 + 400 × 80	44000
B(40, 160)	600 × 40 + 400 × 160	88000
C(20, 180)	600 × 20 + 400 × 180	84000

← Maximum

∴ Maximum value of Z = 88000 at $x = 40$, $y = 160$

OR

For point B:

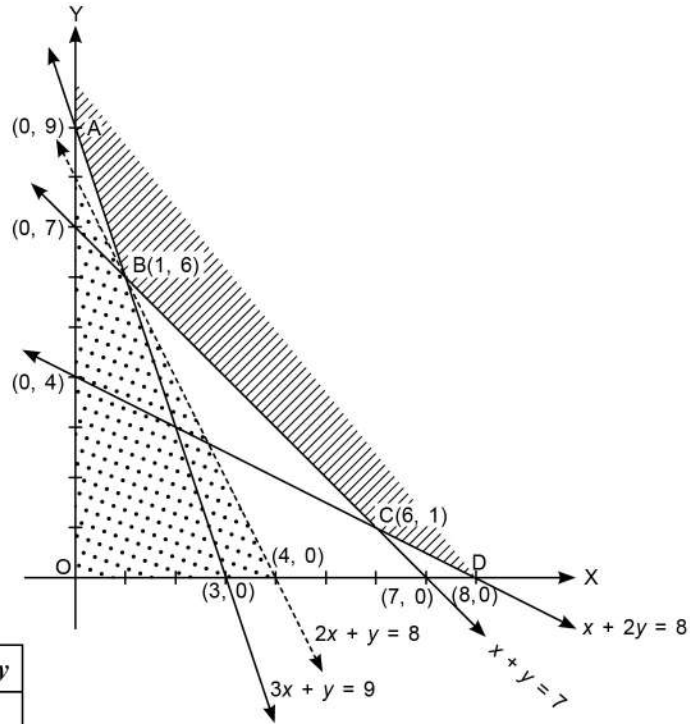
$$x + y = 7$$
$$3x + y = 9$$
$$- \quad - \quad -$$
$$-2x = -2$$
$$x = 1$$
$$y = 6$$

Coordinates of B are $B(1, 6)$

For point C:

$$x + 2y = 8$$
$$x + y = 7$$
$$- \quad - \quad -$$
$$y = 1$$
$$x = 6$$

Coordinates of C are $C(6, 1)$



Corner points	Values of $Z = 2x + y$
$A(0, 9)$	9
$B(1, 6)$	8
$C(6, 1)$	13
$D(8, 0)$	16

←Minimum

To check another value if any, consider $2x + y < 8$.

If resulting open half plane represented by $2x + y < 8$ has points common with feasible region then there will be no minimum value.

But open half plane represented by $2x + y < 8$ does not have points common with feasible region.

∴ Minimum value of $Z = 8$ at $x = 1$; $y = 6$

31. $x = 3\sin\theta - \sin 3\theta$ ...(*i*)

Differentiating both sides, w.r.t. θ, we get

$$\frac{dx}{d\theta} = 3\cos\theta - \cos 3\theta \times 3$$
$$= 3(\cos\theta - \cos 3\theta)$$
$$= 3 \times 2\sin 2\theta \cdot \sin\theta$$
$$\frac{dx}{d\theta} = 6.\sin\theta \sin 2\theta \quad ...(iii)$$

$y = 3\cos\theta - \cos 3\theta$ ...(*ii*)

Differentiating both sides, w.r.t. θ, we get

$$\frac{dy}{d\theta} = -3\sin\theta + \sin 3\theta \times 3$$
$$= 3(\sin 3\theta - \sin\theta)$$
$$= 3 \cdot 2\cos 2\theta \sin\theta$$
$$= 6\cos 2\theta \cdot \sin\theta \quad ...(iv)$$

From (*iii*) and (*iv*), we get

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$= \frac{6\cos 2\theta \cdot \sin\theta}{6\sin\theta \cdot \sin 2\theta}$$

$$\frac{dy}{dx} = \cot 2\theta \qquad ...(v)$$

Differentiating (*v*) w.r.t. *x*, we get

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 2\theta \times 2 \cdot \frac{d\theta}{dx}$$

$$= -\operatorname{cosec}^2 2\theta \times 2 \times \frac{1}{6\sin\theta\sin 2\theta}$$

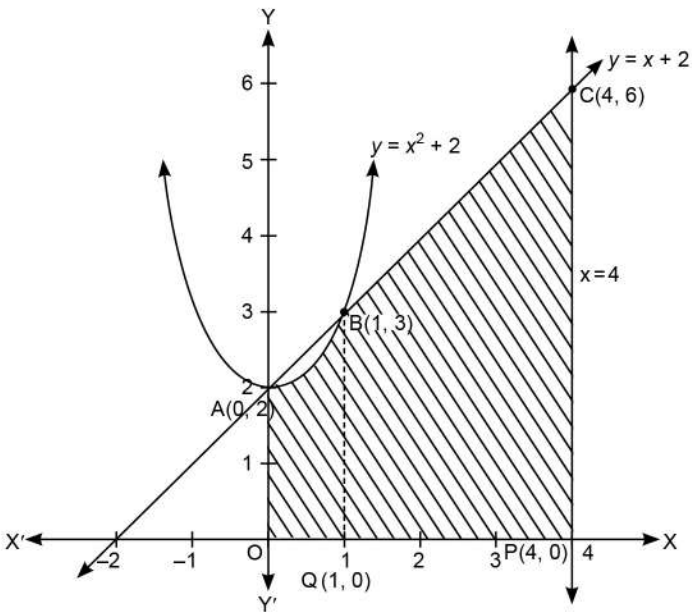
$$= -\frac{1}{3}\operatorname{cosec}^3 2\theta \cdot \operatorname{cosec}\theta$$

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{3}} = -\frac{1}{3}\left(\frac{2}{\sqrt{3}}\right)^3 \cdot \frac{2}{\sqrt{3}} = -\frac{1}{3} \times \frac{16}{9} = -\frac{16}{27}$$

32. $y \geq 0, y \leq x^2 + 2$

$y \leq x + 2$

$x \geq 0, x \leq 4$



Since $y = x + 2$ and $y = x^2 + 2$

$\Rightarrow \quad x^2 + 2 = x + 2$

$\Rightarrow \quad x(x-1) = 0 \Rightarrow x = 0$ or $x = 1$

Hence point of intersection are (1, 3) and (0, 2).

Area of shaded region $= \text{ar}(OABQO) + \text{ar}(BCPQB)$

$$= \int_0^1 y_1\,dx + \int_1^4 y_2\,dx$$

$$= \int_0^1 (x^2+2)\,dx + \int_1^4 (x+2)\,dx$$

$$= \left[\frac{x^3}{3}+2x\right]_0^1+\left[\frac{x^2}{2}+2x\right]_1^4$$

$$= \left[\left(\frac{1}{3}+2\right)-0\right]+\left[(8+8)-\left(\frac{1}{2}+2\right)\right]$$

$$= \frac{7}{3}+\frac{27}{2}=\frac{14+81}{6}=\frac{95}{6} \text{ sq units}$$

33. $(a, b)R\,(c, d) \Rightarrow \frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$

For reflexive: Let $(a, b)\ R\ (a, b)$

$$\Rightarrow \qquad \frac{1}{a}+\frac{1}{b} = \frac{1}{b}+\frac{1}{a}$$

Which is true because addition is commutative.

R is reflexive.

For symmetric:

Let $(a, b)\ R\ (c, d)$

$\Rightarrow \quad \frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c} \quad ...(i) \qquad \Rightarrow \quad \frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a} \quad ...(ii)$

From (i) and (ii), we get

$$(a, b)\ R\ (c, d) \Rightarrow (c, d)\ R\ (a, b)$$

$\therefore$ R is symmetric relation.

For transitive:

Let $(a, b)\ R\ (c, d)$

$$\Rightarrow \qquad \frac{1}{a}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c} \qquad ...(iii)$$

Let $(c, d)\ R\ (e, f)$

$$\Rightarrow \qquad \frac{1}{c}+\frac{1}{f} = \frac{1}{d}+\frac{1}{e} \qquad ...(iv)$$

From (iii) and (iv), we get

$$\frac{1}{a}+\frac{1}{c}+\frac{1}{f}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}$$

$$\Rightarrow \qquad \frac{1}{a}+\frac{1}{f} = \frac{1}{b}+\frac{1}{e}$$

$$\Rightarrow \qquad (a, b)\ R\ (e, f)$$

$\therefore$ R is transitive relation.

Hence, given relation is an equivalence relation.

OR

$$f(x) = \begin{cases} \frac{x+1}{2}, \text{ if } x \text{ is odd} \\ \frac{x}{2}, \text{ if } x \text{ is even} \end{cases}$$

For one-one:

We observe that,

$$f(1) = \frac{1+1}{2} = 1; f(2) = \frac{2}{2} = 1$$

So, we get $f(1) = f(2)$, but $1 \neq 2$.

Also $1, 2 \in N$.

Hence, f is not one-one.

For onto:

Let $f(x) = y$, such that $y \in N$.

When x is odd

$$y = \frac{x+1}{2}$$

$\Rightarrow$ $x = 2y - 1$

So, for each $y \in N$, $x \in N$.

When x is even

$$y = \frac{x}{2}$$

$\Rightarrow$ $x = 2y$

So for each $y \in N$, $x \in N$

So, for every $y \in N$, there exists $x \in N$ such that $f(x) = y$

Hence f is onto.

$\therefore$ f is not bijective function.

34. Let cost of type A pen be ₹ x, cost of type B pen be ₹ y and cost of type C pen be ₹ z.

According to the question,

$$x + y + z = 37$$
$$4x + 3y + 2z = 106$$
$$6x + 2y + 3z = 129$$

Matrix form is

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37 \\ 106 \\ 129 \end{bmatrix}$$

which is of the form, $AX = B \Rightarrow X = A^{-1}B$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 37 \\ 106 \\ 129 \end{bmatrix}$

Now,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix}$$

$$\Rightarrow \quad |A| = 1 \times 5 - 1 \times 0 + 1 \times (-10)$$
$$= -5 \neq 0$$

$\therefore$ A^{-1} exists.

Let C_{ij} be the cofactors of elements in $|A|$.

$C_{11} = 5$, $C_{12} = 0$, $C_{13} = -10$

$C_{21} = -1$, $C_{22} = -3$ $C_{23} = 4$

$C_{31} = -1$, $C_{32} = 2$ $C_{33} = -1$

So,

$$\text{adj } A = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix} \qquad \left(\because A^{-1} = \frac{1}{|A|}\text{adj } A\right)$$

Now, $X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 37 \\ 106 \\ 129 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} 185 - 106 - 129 \\ 0 - 318 + 258 \\ -370 + 424 - 129 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -50 \\ -60 \\ -75 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 15 \end{bmatrix}$$

$\therefore$ Cost of type A pen = ₹ 10, cost of type B pen = ₹ 12, cost of type C pen = ₹ 15

35. Direction ratios of required line would be same as direction ratios of line AB.

Direction ratios of $\overrightarrow{AB}$ = <4, 5, 3>

Equation of line passing through point (x_1, y_1, z_1) having direction ratios a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Cartesian equation of line passing through (–1, – 6, 3) having direction ratios 4, 5, 3 is

$$\frac{x+1}{4} = \frac{y+6}{5} = \frac{z-3}{3}$$

$$\text{Vector equation is, } \vec{r} = (-\hat{i} - 6\hat{j} + 3\hat{k}) + \lambda(4\hat{i} + 5\hat{j} + 3\hat{k})$$

$$\text{Direction ratios of } \overrightarrow{AP} = \langle -3, -9, -1 \rangle$$

$$|\overrightarrow{AP}| = \sqrt{9 + 81 + 1} = \sqrt{91}$$

$$\therefore \quad \text{Direction cosines} = \left\langle \frac{-3}{\sqrt{91}}, \frac{-9}{\sqrt{91}}, \frac{-1}{\sqrt{91}} \right\rangle$$

OR

Lines are $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$

Here, $\vec{a_1} = 5\hat{i} + 7\hat{j} - 2\hat{k}$, $\vec{b_1} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{a_2} = -3\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{b_2} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

$$\vec{a_2} - \vec{a_1} = -8\hat{i} - 4\hat{j} + 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}(-4 - 2) - \hat{j}(12 + 3) + \hat{k}(6 - 3)$$

$$= -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{36 + 225 + 9}$$

$$= \sqrt{270} = 3\sqrt{30}$$

$$\text{Shortest distance between the lines} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{(-8\hat{i} - 4\hat{j} + 8\hat{k}) \cdot (-6\hat{i} - 15\hat{j} + 3\hat{k})}{\sqrt{270}} \right|$$

$$= \left|\frac{48+60+24}{\sqrt{270}}\right| = \frac{132}{\sqrt{270}}$$

$$= \frac{132}{3\sqrt{30}} \text{ units} = \frac{44}{\sqrt{30}} \text{ units}$$

36. Given $x = \frac{600-p}{8}$

$\Rightarrow \quad 8x = 600 - p$

$\Rightarrow \quad p = 600 - 8x$

$R(x) = p.x$

$= 600x - 8x^2$

Cost function, $C(x) = x^2 + 78x + 2500$

(*i*) $p = 600 - 8x$

(*ii*) $P(x) = R(x) - C(x)$

$= 600x - 8x^2 - x^2 - 78x - 2500$

$= -9x^2 + 522x - 2500$

(*iii*) $P(x) = -9x^2 + 522x - 2500$

Differentiating both sides w.r.t. x, we get

$P'(x) = -18x + 522$

For maximum or minimum profit

put $P'(x) = 0$

$\Rightarrow \quad 18x = 522$

$\Rightarrow \quad x = \frac{522}{18} = 29$

$P''(x) = -18 < 0$

$\therefore$ $P(x)$ is maximum when $x = 29$.

OR

(*iii*) $P(x) = -9x^2 + 522x - 2500$

Differentiating both sides w.r.t. x, we get

$P'(x) = -18x + 522$

Put $P'(x) = 0$, for critical point

$\therefore \quad -18x = -522 \Rightarrow x = 29$

Interval	sign of $P'(x)$
(0, 29)	+ve
(29, ∞)	–ve

$\therefore$ $P(x)$ is increasing in the interval (0, 29)

37. A: Event that helicopter I hits correctly.

B: Event that helicopter II hits correctly.

$P(A) = 0.3, P(B) = 0.2$

(*i*) $P(\text{Target hit by only one helicopter}) = P(A)P(\overline{B}) + P(\overline{A})\,P(B)$

$= 0.3 \times 0.8 + 0.7 \times 0.2$

$= 0.24 + 0.14$

$= 0.38$

(*ii*) $P(\text{Both hit target}) = P(A) \times P(B)$

$= 0.3 \times 0.2$

$= 0.06$

(*iii*) $P(\overline{A} \cap B) = P(\overline{A}) \times P(B)$

$= 0.7 \times 0.2$

$= 0.14$

OR

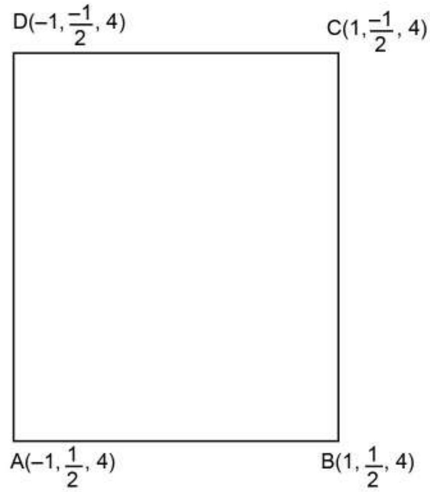
(*iii*) $\text{P(target not hit)} = \text{P}(\overline{A}) \times \text{P}(\overline{B})$

$= 0.7 \times 0.8$

$= 0.56$

38. (*i*) $\overrightarrow{AB} = 2\hat{i}$

$\overrightarrow{AD} = -\hat{j}$



Area of rectangle $= |\overrightarrow{AB} \times \overrightarrow{AD}|$

$$= \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{matrix} \right\|$$

$= |\hat{i}(0) - \hat{j}(0) + \hat{k}(-2)|$

$= |-2\hat{k}|$

$= 2$ sq units

(*ii*) $\overrightarrow{AC} = 2\hat{i} - \hat{j}$

$\overrightarrow{BD} = -2\hat{i} - \hat{j}$

$$\cos\theta = \frac{\overrightarrow{AC} \cdot \overrightarrow{BD}}{|\overrightarrow{AC}| \cdot |\overrightarrow{BD}|}$$

$$= \frac{-4+1}{\sqrt{5} \cdot \sqrt{5}}$$

$$= \frac{-3}{5}$$

$\Rightarrow$ $\theta = \cos^{-1}\left(\frac{-3}{5}\right)$