21 SAMPLE PAPER

[Pre-Board Paper based on CBSE pattern]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. Given matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $3A^3$ is **[NCERT Part-I, Page 51]**

(*a*) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ (*d*) $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

2. Given matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 4 & 1 \\ 2 & 1 & 1 \end{bmatrix}$, then $|A^{-1}|$ is **[Conceptual Application]**

(*a*) $\frac{1}{15}$ (*b*) $\frac{1}{21}$ (*c*) $-\frac{1}{21}$ (*d*) $-\frac{1}{15}$

3. If $\begin{vmatrix} x & 3 \\ 4 & 2x \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 4 & 0 \end{vmatrix}$, then value of x can be **[NCERT Part-I, Page 77]**

(*a*) $\pm 2\sqrt{2}$ (*b*) $2\sqrt{2}$ (*c*) 8 (*d*) ± 8

4. If $f(x) = \begin{cases} \dfrac{1-\cos x}{x^2}, & x \neq 0 \\ \dfrac{k}{3}, & x = 0 \end{cases}$ is continuous at $x = 0$, then value of k is **[NCERT Part-I, Page 105]**

(*a*) 6 (*b*) $\frac{3}{2}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{6}$

5. Direction ratios of the line joining the points (3, 1, –2) and (0, 2, 4) such that line makes acute angle with y-axis is **[NCERT Part-II, Page 379-380]**

(*a*) $<3, 1, -2>$ (*b*) $<0, 2, 4>$ (*c*) $<3, -1, -6>$ (*d*) $<-3, 1, 6>$

6. Degree of the differential equation $y - px = \sqrt{a^2p^2 + b^2}$, where $p = \frac{dy}{dx}$ is **[NCERT Part-II, Page 302]**

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) not defined

7. The corner points of the bounded feasible region determined by the system of linear constraints are (2, 0), (4, 2), (3, 7) and (0, 5). If the objective function is $Z = px + qy\ (p, q > 0)$, the condition on p and q such that value of Z at (4, 2) is half the value of Z at (0, 5) is **[NCERT Part-II, Page 397-398]**

(*a*) $4p = 13q$ (*b*) $p > 0, q > 0$ (*c*) $p + q > 0$ (*d*) $8p = q$

8. If for non zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then relation between $\vec{a}$ and $\vec{b}$ is **[Conceptual Application]**

(*a*) $\vec{a} = \vec{b}$ (*b*) $\vec{a} \parallel \vec{b}$ (*c*) $\vec{a} \perp \vec{b}$ (*d*) $|\vec{a}| > |\vec{b}|$

9. $\int_1^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}}dx$ is equal to **[NCERT Part-II, Page 274, 268]**

(*a*) 1 (*b*) $\frac{3}{2}$ (*c*) $\frac{1}{2}$ (*d*) 2

10. Given $|A| = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 5 & 6 \\ 0 & 2 & 1 \end{vmatrix}$, then value of $3M_{12} - 2A_{23}$, where M_{ij} and A_{ij} are minor and cofactor of a_{ij} respectively is **[NCERT Part-II, Page 84]**

(*a*) –5 (*b*) 11 (*c*) –11 (*d*) 5

11. For the given LPP, the point (3, 5) lies in feasible region for the constraint **[Conceptual Application]**

(*a*) $x - 3y \geq 0$ (*b*) $x - y \leq 0$ (*c*) $x + y \geq 12$ (*d*) $x - y \geq 0$

12. Vectors of magnitude 5 units in the direction of vector $2\hat{i} - \hat{j} + 2\hat{k}$ is (are) **[NCERT Part-II, Page 346-347]**

(*a*) $10\hat{i} - 5\hat{j} + 10\hat{k}$ (*b*) $\frac{10}{3}\hat{i} - \frac{5}{3}\hat{j} + \frac{20}{3}\hat{k}$

(*c*) $\frac{2}{5}\hat{i} - \frac{1}{5}\hat{j} + \frac{2}{5}\hat{k}$ (*d*) $\pm\left(\frac{10}{3}\hat{i} - \frac{5}{3}\hat{j} + \frac{10}{3}\hat{k}\right)$

13. For a given matrix A, $A^{-1} = \frac{1}{-4}$ Adj A. If A.Adj $A = KI$, the value of $16K$ is **[NCERT Part-I, Page 88, 90]**

(*a*) –4 (*b*) –64 (*c*) $-\frac{1}{4}$ (*d*) 4

14. If A and B are independent events of an experiment, then which of the following is not true? **[NCERT Part-II, Page 418]**

(*a*) $\overline{A}, B$ are independent (*b*) $A \cap B = \phi$

(*c*) $P(A \cap B) = P(A) \cdot P(B)$ (*d*) $\overline{A}, \overline{B}$ are independent

15. General solution of the differential equation. $\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$ is **[NCERT Part-II, Page 306-307]**

(*a*) $\tan^{-1} x + \tan^{-1} y = C$ (*b*) $\log\left|x + \sqrt{1-x^2}\right| + \log\left|y + \sqrt{1-y^2}\right| = C$

(*c*) $\sin^{-1} x + \sin^{-1} y = C$ (*d*) $\sec^{-1} x + \sec^{-1} y = C$

16. The value of λ for which vectors $2\hat{i} - \hat{j} + 3\hat{k}$ and $4\hat{i} - 5\lambda\hat{j} + 6\hat{k}$ are parallel is **[Conceptual Application]**

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{5}{2}$ (*d*) 2

17. The function $f(x) = x - \cos x, x \in R$ is decreasing for **[NCERT Part-I, Page 164]**

(*a*) $\left[0, \frac{\pi}{2}\right]$ (*b*) $[0, \pi]$ (*c*) No value of R (*d*) $x \in R$

18. Direction ratios of the line $\frac{2-x}{2} = \frac{y+3}{3}; z = 5$ are **[NCERT Part-II, Page 382]**

(*a*) 2, 3, 1 (*b*) 2, 3, 0 (*c*) –2, 3, 5 (*d*) –2, 3, 0

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.
(*b*) Both A and R are true but R is not the correct explanation of A.
(*c*) A is true but R is false.
(*d*) A is false but R is true.

19. Let $f(x) = x^2 - x + 5, x \in [0, 5]$ be a given function. **[NCERT Part-I, Page 164]**

Assertion (*A*): 'f' increases in the interval [0, 5].

Reason (*R*): If in a given interval 'f' increases for some values of the interval and decreases for other values, then function is said to be neither increasing nor decreasing in the given interval.

20. **Assertion (*A*):** The function $f : R \to R$ defined as $f(x) = \frac{x}{x-1}$ is a bijective function.

[Conceptual Application]

Reason (*R*): For function to be defined each value of domain has an image in the co-domain.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then find the value of $3x - y + 2z$. **[Conceptual Application]**

OR

Find the domain of the function $\cos^{-1}(2x^2 - 5)$. **[NCERT Part-I, Page 21]**

22. Show that the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ increases in the interval $\left[\frac{3\pi}{8}, \frac{5\pi}{8}\right]$. **[NCERT Part-I, Page 164]**

23. Find the maximum value of $f(x) = x\sqrt{1-x}, x < 1$. **[NCERT Part-I, Page 166]**

OR

A particle moves along the curve $3y = ax^3 + 1$, such that for a point with x-coordinate 1, y-coordinate is changing twice as fast as x-coordinate. Find the value of a. **[NCERT Part-I, Page 147-148]**

24. Evaluate: $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$. **[NCERT Part-II, Page 235-236]**

25. Show that the function $f(x) = x^3 + x^2 + x + 1$, does not attain maximum or minimum value.

[NCERT Part-I, Page 166]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int \frac{x}{x^3 + x^2 + x + 1} dx.$ **[NCERT Part-II, Page 252-253]**

27. The probability of finding a green signal on a busy crossing X is 30% for the day. What is the probability of finding a green signal on crossing X in two consecutive days out of three? **[Conceptual Application]**

28. Evaluate: $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ **[NCERT Part-II, Page 241]**

OR

Evaluate: $\int_0^1 x \log(1 + 2x) dx.$ **[NCERT Part-II, Page 259-260, 268]**

29. Of all the rectangles each of which has perimeter 40 metres, find one which has maximum area. Find the area also. **[NCERT Part-I, Page 166]**

OR

Find the particular solution of the differential equation $x\frac{dy}{dx} + x\cos^2\left(\frac{y}{x}\right) = y$, given that when $x = 1, y = \frac{\pi}{4}$. **[NCERT Part-II, Page 313-314]**

30. Solve the following LPP graphically: **[NCERT Part-II, Page 397-398]**

Maximise $Z = 15x + 30y$, subject to the constraints $3x + y \le 12, x + 2y \le 10, x \ge 0, y \ge 0$.

OR

Solve the following LPP graphically: **[NCERT Part-II, Page 397-398]**

Minimise $Z = 2x + y$, subject to the constraints $x \ge 3, x \le 9, y \ge 0, x - y \ge 0, x + y \le 14$.

31. If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$ **[NCERT Part-I, Page 137]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find the area of the region bounded by the curves $x^2 = y, y = x + 2$ and x-axis, using integration. **[Conceptual Application]**

33. Show that the relation R in the set N of natural numbers given by $R = \{(a, b) : a \text{ is divisor of } b\}$ is reflexive and transitive but not symmetric relation. **[NCERT Part-I, Page 2]**

OR

Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}, x \in R$ is neither one-one nor onto. **[NCERT Part-I, Page 7]**

34. If matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations **[NCERT Part-I, Page 94-95]**

$x - 2y = 10, 2x - y - z = 8, -2y + z = 7.$

35. A line passes through the point $(2, -1, 3)$ and is perpendicular to the lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain the equation in vector and Cartesian form. **[Conceptual Application]**

OR

Find the distance between the lines given by $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$. **[NCERT Part-II, Page 386-387]**

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. A shopkeeper sells three types of flower seed as A_1, A_2, A_3. They are sold in the form of a mixture, the proportion of these seeds are 2 : 2 : 1 respectively. The germination rate of three types of seeds are 45%, 60% and 35% respectively. **[Conceptual Application]**

(*i*) Find the probability that chosen seed A_1 will germinate.

(*ii*) Find the probability that randomly chosen seed will germinate.

(*iii*) If chosen seed germinates, what is the probability it is of type A_2?

OR

(*iii*) If chosen seed germinates, what is the probability it is of type A_3?

Case Study - 2

37. A cylinder base in the form of a circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2. **[Conceptual Application]**

(*i*) If the radius of base of the cylinder is r cm and height is h cm, then write the volume V of the cylinder in terms of r.

(*ii*) Find r, for which $\frac{dV}{dr} = 0$.

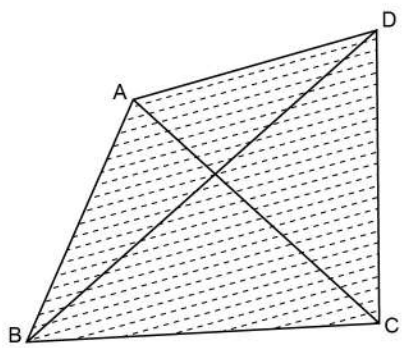
(*iii*) Find the radius of the cylinder when volume is maximum.

OR

(*iii*) Find the maximum volume. For maximum volume, is $h = r$? Verify.

Case Study - 3

38. A student purchased a pyramid with triangular base as shown and allotted the coordinates as $A(3, 1, 2)$, $B(0, 4, 1)$, $C(3, 2, 1)$ and $D(1, 1, 1)$. **[Conceptual Application]**



(*i*) Find ar(ΔABC).

(*ii*) Find $|\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BD}|$.

SOLUTIONS

1. (*b*)
$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$
$$3A^3 = 3IA = 3A = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

2. (*d*)
$$|A| = 0 - 1(-3) + 2(-9) = -15$$
$$|A^{-1}| = \frac{1}{|A|} = -\frac{1}{15}$$

3. (*a*)
$$2x^2 - 12 = 4$$
$$\Rightarrow \quad x^2 = 8$$
$$\Rightarrow \quad x = \pm 2\sqrt{2}$$

4. (*b*)
$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{k}{3}$$
$$\Rightarrow \quad \lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{4 \cdot \frac{x^2}{4}} = \frac{k}{3}$$
$$\Rightarrow \quad \frac{1}{2} = \frac{k}{3}$$
$$\Rightarrow \quad k = \frac{3}{2}$$

5. (*d*) Line through the points (3, 1, –2) and (0, 2, 4) is
$$\frac{x - 0}{-3} = \frac{y - 2}{1} = \frac{z - 4}{6}$$

DR's are –3, 1, 6

If line with DR's a, b, c makes acute angle with y-axis then $b > 0$

$\therefore$ DR's are $<-3, 1, 6>$

6. (*b*),
$$(y - px)^2 = a^2p^2 + b^2$$
$$\Rightarrow \quad (x^2 - a^2)p^2 - 2xyp - b^2 + y^2 = 0$$
$$\text{Degree} = 2, \text{ as } p^2 = \left(\frac{dy}{dx}\right)^2$$

7. (*d*),
$$2Z_{(4,\,2)} = Z_{(0,\,5)}$$
$$\Rightarrow \quad 2(4p + 2q) = 5q$$
$$\Rightarrow \quad 8p = q$$

8. (*c*)
$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$
$$\Rightarrow \quad (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$
$$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$
$$\Rightarrow \quad 4\vec{a} \cdot \vec{b} = 0 \quad \Rightarrow \quad \vec{a} \perp \vec{b}$$

9. (*a*) $$I = \int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx \quad ...(i)$$

Using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\,dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_1^3 1\cdot dx = [x]_1^3 = 2$$

$$I = 1$$

10. (*b*) $$M_{12} = 1, A_{23} = -(4) = -4$$

$$3M_{12} - 2A_{23} = 3 + 8 = 11$$

11. (*b*) For (3, 5), $x - y \le 0$ is true

12. (*d*) $$\text{Vectors} = \pm 5\left(\frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{4+1+4}}\right)$$

$$= \pm\left(\frac{10}{3}\hat{i} - \frac{5}{3}\hat{j} + \frac{10}{3}\hat{k}\right)$$

13. (*b*) $$A\cdot\text{Adj } A = |A|\cdot I \text{ and } A^{-1} = \frac{1}{|A|}\text{Adj } A$$

$\Rightarrow$ $|A| = -4$

$\Rightarrow$ $K = -4$

$\Rightarrow$ $16K = -64$

14. (*b*)

15. (*c*) $$\int \frac{dx}{\sqrt{1-x^2}} + \int \frac{dy}{\sqrt{1-y^2}} = \int 0dx$$

$\Rightarrow$ $\sin^{-1} x + \sin^{-1} y = C$

16. (*b*) $2\hat{i} - \hat{j} + 3\hat{k} = t(4\hat{i} - 5\lambda\hat{j} + 6\hat{k})$, where t is a scalar.

$\Rightarrow$ $2 = 4t, -1 = -5\lambda t, 3 = 6t$

$\Rightarrow$ $t = \frac{1}{2}$

$\Rightarrow$ $-1 = \frac{-5\lambda}{2}$

$\Rightarrow$ $\lambda = \frac{2}{5}$

17. (*c*) $f'(x) = 1 + \sin x > 0$ for $x \in R$. $\left\{1 + \sin x = \left(\cos\frac{x}{2} + \frac{\sin x}{2}\right)^2 \ge 0\right\}$

As $0 \le 1 + \sin x \le 2$

$\therefore$ Always increasing.

18. (*d*) Line is $\frac{x-2}{-2} = \frac{y+3}{3} = \frac{z-5}{0}$

DR's: –2, 3, 0.

19. (*d*) Assertion is false, as function neither increases nor decreases in [0, 5].
Reason is true.
Hence, (A) is false but (R) is true.

20. (*d*) Assertion is false as 'f' is not a function on R.
Reason is true.
Hence, (A) is false but (R) is true.

21. Principal value of $\sin^{-1} x$ lies between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \quad \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1} x = \frac{\pi}{2}, \sin^{-1} y = \frac{\pi}{2}, \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow \quad x = \sin\frac{\pi}{2}, y = \sin\frac{\pi}{2}, z = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad x = 1, y = 1, z = 1$$

$$3x - y + 2z = 3 - 1 + 2 = 4$$

OR

For domain $-1 \leq 2x^2 - 5 \leq 1$

$$\Rightarrow \quad 4 \leq 2x^2 \leq 6$$

$$\Rightarrow \quad 2 \leq x^2 \leq 3$$

$$\Rightarrow \quad \sqrt{2} \leq |x| \leq \sqrt{3}$$

$$\Rightarrow \quad -\sqrt{3} \leq x \leq -\sqrt{2} \text{ or } \sqrt{2} \leq x \leq \sqrt{3}.$$

22.

$$f'(x) = -2\sin\left(2x + \frac{\pi}{4}\right) \qquad ...(i)$$

Given $\quad \frac{3\pi}{8} \leq x \leq \frac{5\pi}{8}$

$$\Rightarrow \quad \frac{3\pi}{4} \leq 2x \leq \frac{5\pi}{4}$$

$$\Rightarrow \quad \pi \leq 2x + \frac{\pi}{4} \leq \frac{3\pi}{2}$$

$$\Rightarrow \quad 2x + \frac{\pi}{4} \in \text{3rd quadrant}$$

$$\sin\left(2x + \frac{\pi}{4}\right) < 0$$

From (*i*), $\quad f'(x) > 0 \;\Rightarrow\;$ function increases.

23.

$$f'(x) = x \cdot \frac{1}{2\sqrt{1-x}}(-1) + \sqrt{1-x} = \frac{2-3x}{2\sqrt{1-x}}$$

For a point of local maximum or minimum

$$f'(x) = 0 \Rightarrow x = \frac{2}{3}$$

$$f''(x) = \frac{2\sqrt{1-x}\,(-3) + (2-3x)\left(1-x\right)^{\frac{-1}{2}}}{(2\sqrt{1-x})^2}$$

$f''\left(\frac{2}{3}\right) < 0.$

$\therefore$ Maximum of f occurs at $x = \frac{2}{3}$ and $f\left(\frac{2}{3}\right) = \frac{2\sqrt{3}}{9}$.

OR

$$3y = ax^3 + 1$$

$$3\frac{dy}{dt} = 3ax^2\frac{dx}{dt} \qquad ...(i)$$

Also $\quad \dfrac{dy}{dt} = 2\dfrac{dx}{dt}$

So, $\quad 6\dfrac{dx}{dt} = 3ax^2\dfrac{dx}{dt} \qquad$ [from (*i*)]

$\Rightarrow \quad 2 = ax^2$

For $x = 1$, $\quad 2 = a(1)^2 \Rightarrow a = 2$

24. $$\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}dx = 2\int \sec^2 t\, dt$$

$$= 2\tan t + C = 2\tan\sqrt{x} + C$$

Let $\sqrt{x} = t$

$\Rightarrow \dfrac{1}{2\sqrt{x}}dx = dt$

25. $$f'(x) = 3x^2 + 2x + 1$$

For critical points, $\quad 3x^2 + 2x + 1 = 0$

$$D = 4 - 12 < 0$$

No solution, as $f'(x) \neq 0$ for any x, no maximum or minimum value.

26. $$\int \frac{x}{x^3+x^2+x+1}dx = \int \frac{x}{(x+1)(x^2+1)}dx$$

$$\frac{x}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \qquad ...(i)$$

$\Rightarrow$ $$x = A(x^2+1) + (Bx+C)(x+1)$$

$$= Ax^2 + A + Bx^2 + Bx + Cx + C = x^2(A+B) + x(B+C) + (A+C)$$

Comparing the coefficients of x^2, x and constant terms, we get

$$A + B = 0, B + C = 1, A + C = 0 \Rightarrow A = -B = -C = -\frac{1}{2}$$

$\therefore$ Substituting the values of A, B and C in (*i*) and then integrating both sides, we get

$$\int \frac{x}{x^3+x^2+x+1}dx = -\frac{1}{2}\int \frac{1}{x+1}dx + \int \frac{\frac{1}{2}x+\frac{1}{2}}{x^2+1}dx$$

$$= -\frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

27. Let green signals be on D_1, D_2 and D_3.

$$P(D_1) = P(D_2) = P(D_3) = \frac{30}{100} = \frac{3}{10}$$

$\therefore$ Probability of green signal on two consecutive days $= P(D_1\, D_2\, \overline{D}_3) + P(\overline{D}_1\, D_2\, D_3)$

$$= \frac{3}{10}\times\frac{3}{10}\times\frac{7}{10} + \frac{7}{10}\times\frac{3}{10}\times\frac{3}{10} = \frac{126}{1000} = 0.126$$

28. $$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx = \int \frac{2\cos^2 x - 2\cos^2\alpha}{\cos x - \cos\alpha}dx \qquad \{\text{As } \cos 2x = 2\cos^2 x - 1\}$$

$$= 2\int(\cos x + \cos\alpha)dx = 2\sin x + 2x\cdot\cos\alpha + C.$$

OR

$$\int \underset{②}{x} \cdot \underset{①}{\log(1+2x)}\, dx = \log(1+2x) \cdot \frac{x^2}{2} - \int \frac{2}{1+2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2} \log(1+2x) - \int \frac{x^2}{2x+1} dx$$

$$= \frac{x^2}{2} \log(2x+1) - \frac{1}{4} \int \frac{(4x^2-1)+1}{2x+1} dx$$

$$= \frac{x^2}{2} \log(2x+1) - \frac{1}{4} \int \left(2x - 1 + \frac{1}{2x+1}\right) dx$$

$$= \frac{x^2}{2} \log(2x+1) - \frac{1}{4} x^2 + \frac{x}{4} - \frac{1}{8} \log|2x+1|$$

$$\int_0^1 x \log(1+2x)\, dx = \left[\frac{x^2}{2} \log(2x+1) - \frac{x^2}{4} + \frac{x}{4} - \frac{1}{8} \log|2x+1|\right]_0^1$$

$$= \left[\frac{1}{2} \log 3 - \frac{1}{4} + \frac{1}{4} - \frac{1}{8} \log 3\right] - [0 - 0 + 0 - 0]$$

$$= \frac{3}{8} \log 3$$

29. Let x and y be the sides of a rectangle.

Given, $2(x + y) = 40$

$\Rightarrow$ $x + y = 20$ ...(*i*)

Area, $A = xy = x(20 - x) = 20x - x^2$

$\Rightarrow$ $A' = 20 - 2x$

For maximum area, $A' = 0$

$\Rightarrow$ $x = 10$

$A'' = -2$

$\Rightarrow$ $A'']_{x=10} < 0$

$\therefore$ area is maximum for $x = 10$, $y = 10$ [from (*i*)]

Area is maximum, when rectangle is a square.

Maximum area = 10 × 10 = 100 sq m.

OR

$$x\frac{dy}{dx} + x\cos^2\frac{y}{x} = y$$

$\Rightarrow$ $\dfrac{dy}{dx} + \cos^2\dfrac{y}{x} = \dfrac{y}{x}$ ...(*i*)

Let $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

From (*i*), we get

$$v + x\frac{dv}{dx} + \cos^2 v = v$$

$\Rightarrow$ $x\dfrac{dv}{dx} = -\cos^2 v$

$\Rightarrow$ $\displaystyle\int \sec^2 v\, dv = -\int \frac{dx}{x}$

$$\Rightarrow \qquad \tan v = -\log|x| + C$$

$$\Rightarrow \qquad \tan\frac{y}{x} = -\log|x| + C \qquad ...(ii)$$

Given when $x = 1$, $y = \frac{\pi}{4}$

$$\Rightarrow \qquad \tan\frac{\pi}{4} = -\log 1 + C \quad \Rightarrow \quad C = 1$$

$\therefore$ From (*ii*), we get

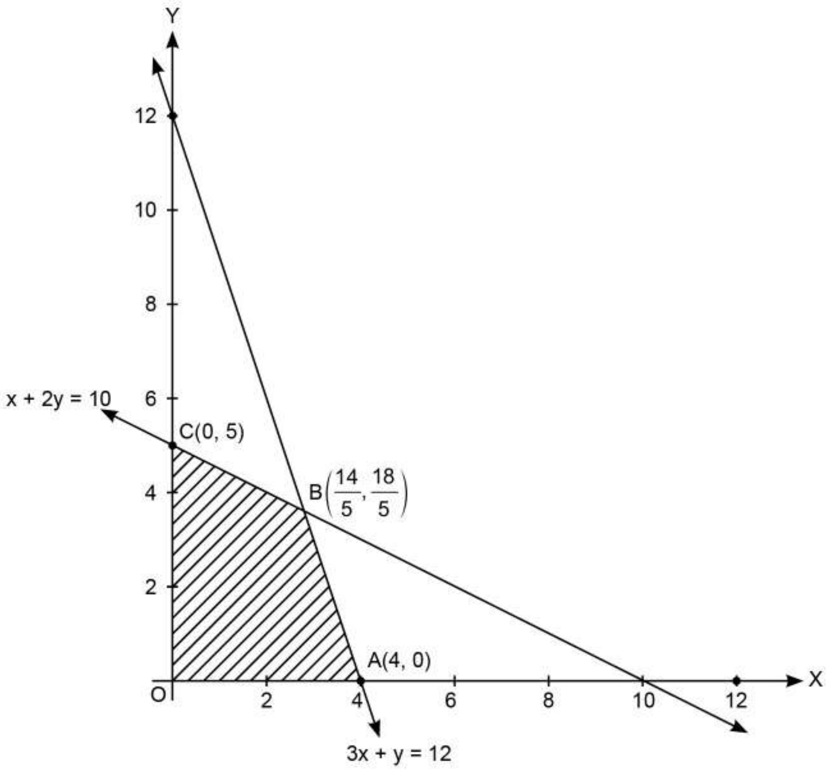
$$\tan\frac{y}{x} = -\log|x| + 1 \text{ is the required solution.}$$

30. To maximise $\quad Z = 15x + 30y$

Subject to the constraints:

$$3x + y \leq 12, x + 2y \leq 10, x \geq 0, y \geq 0$$

On plotting inequations, we get shaded portion as feasible solution.



Possible points for maximum Z are $A(4, 0)$, $B\left(\frac{14}{5}, \frac{18}{5}\right)$, $C(0, 5)$.

Points	Z = 15*x* + 30*y*	Values
$A(4, 0)$	60 + 0	60
$B\left(\frac{14}{5}, \frac{18}{5}\right)$	42 + 108	150
$C(0, 5)$	0 + 150	150

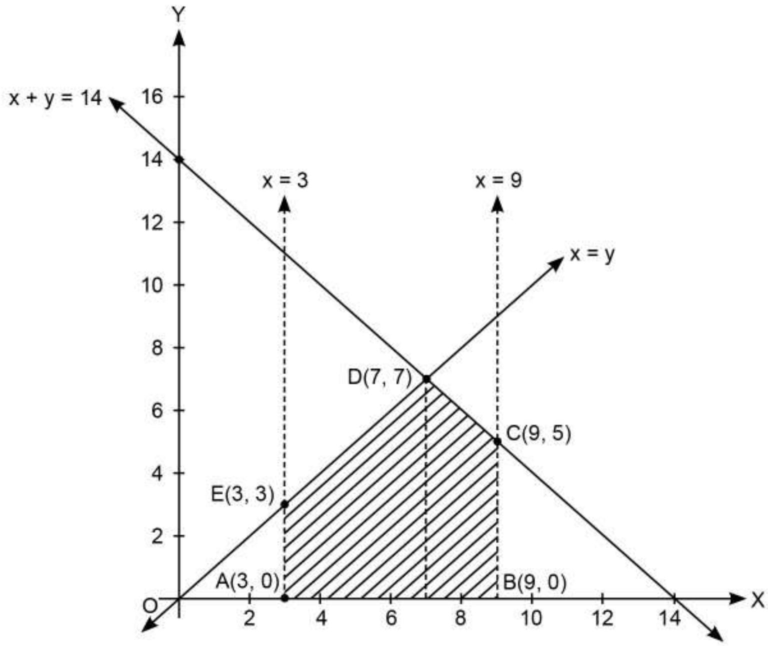
Maximum value of Z is 150 at $C(0, 5)$ or $B\left(\frac{14}{5}, \frac{18}{5}\right)$. So, maximum of Z occurs at all the points on the line segment joining the points $B\left(\frac{14}{5}, \frac{18}{5}\right)$ and $C(0, 5)$.

OR

To minimise $Z = 2x + y$

Subject to the constraints

$x \geq 3, x \leq 9, y \geq 0, x - y \geq 0, x + y \leq 14$



On plotting inequations we notice shaded portion is feasible solution.

Possible points for minimum Z are $A(3, 0)$, $B(9, 0)$, $C(9, 5)$, $D(7, 7)$, $E(3, 3)$.

Points	**$Z = 2x + y$**	**Values**	
$A(3, 0)$	$6 + 0$	6	← Minimum
$B(9, 0)$	$18 + 0$	18	
$C(9, 5)$	$18 + 5$	23	
$D(7, 7)$	$14 + 7$	21	
$E(3, 3)$	$6 + 3$	9	

Z is minimum at $A(3, 0)$ i.e. $x = 3, y = 0$.

Minimum value $Z = 6$

31. Given $$y = e^{a\cos^{-1}x} \quad ...(i)$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{a\cos^{-1}x} \cdot \frac{-a}{\sqrt{1-x^2}} = \frac{-ay}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2} \cdot \frac{dy}{dx} = -ay \quad \text{[From (i)]}$$

On squaring both sides, we get

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

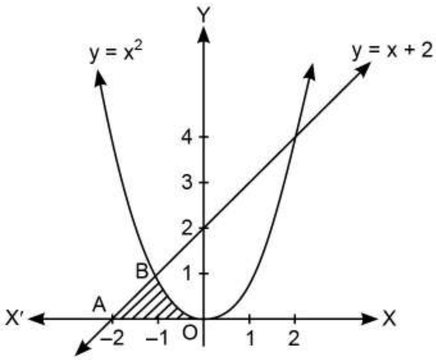
Differentiating both sides w.r.t. x, we get

$$(1-x^2)\left(2\frac{dy}{dx} \cdot \frac{d^2y}{dx^2}\right) - 2x \cdot \left(\frac{dy}{dx}\right)^2 = a^2\left(2y \cdot \frac{dy}{dx}\right)$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0 \quad \left(\text{On dividing by } 2\frac{dy}{dx}\right)$$

32. Given curves are $x^2 = y$ and $y = x + 2$.

Plotting the curves we notice we have to find the shaded area.



Eliminating y, we get

$$x^2 = x + 2$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad (x-2)(x+1) = 0$$

$$\Rightarrow \quad x = 2 \text{ or } x = -1$$

$$\therefore \quad \text{Area} = \int_{-2}^{-1} y_{AB}\, dx + \int_{-1}^{0} y_{OB}\, dx$$

$$= \int_{-2}^{-1} (x+2)dx + \int_{-1}^{0} x^2 dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$$

$$= \left(\frac{1}{2} - 2\right) - \left(\frac{4}{2} - 4\right) + \left[0 - \left(\frac{-1}{3}\right)\right]$$

$$= \frac{-3}{2} + 2 + \frac{1}{3} = \frac{-9+12+2}{6} = \frac{5}{6} \text{ sq units}$$

33. Given relation $R = \{(a, b) \in N \times N : a \text{ is divisor of } b\}$

Reflexive: Let $a \in N$, s.t. $(a, a) \in R$. Now, $(a, a) \in R \Rightarrow a$ is divisor of a, which is true.

Hence, reflexive

Symmetric: Let for $a, b \in N$, $(a, b) \in R \Rightarrow a$ is divisor of b.

This may not imply b is divisor of a. *e.g.,* 3 is divisor of 15 but 15 is not divisor of 3.

$\therefore (a, b) \in R$ may not imply $(b, a) \in R$.

Hence, not symmetric.

Transitive: Let for $a, b, c \in N$,

$$(a, b) \in R \text{ and } (b, c) \in R$$

$\Rightarrow a$ is divisor of $b \Rightarrow b = \lambda a, \lambda \in N$

and b is divisor of $c \Rightarrow c = \mu b, \mu \in N$

$$c = \mu(\lambda a) = (\lambda\mu)a, \text{ where } \lambda, \mu \in N$$

$\Rightarrow a$ is divisor of $c \Rightarrow (a, c) \in R$

as $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$

Hence, R is transitive relation.

OR

Given $f : R \to R$, defined as $f(x) = \dfrac{x}{x^2+1}$.

Let for $x_1, x_2 \in R$,

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow \quad x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$$

$$\Rightarrow \quad x_1 x_2^2 - x_2 x_1^2 + x_1 - x_2 = 0$$

$$\Rightarrow \quad x_1 x_2 (x_2 - x_1) + 1(x_1 - x_2) = 0$$

$$\Rightarrow \quad (x_1 - x_2)(1 - x_1 x_2) = 0$$

$$\Rightarrow \qquad x_1 = x_2 \quad \text{or} \quad x_1 x_2 = 1$$

So $f(x_1) = f(x_2)$ is possible if $x_1 \cdot x_2 = 1$

e.g., let $x_1 = 2$ and $x_2 = \frac{1}{2}$

$$f(x_1) = \frac{2}{5}, f(x_2) = \frac{\frac{1}{2}}{\frac{1}{4}+1} = \frac{2}{5}$$

$\therefore f(x_1) = f(x_2)$ does not imply $x_1 = x_2$

Hence, not one-one.

Let for $y \in R$ (co-domain), there exists $x \in R$ (domain) such that $y = f(x)$.

$$\Rightarrow \qquad y = \frac{x}{x^2+1} \Rightarrow x^2y + y = x$$

$$\Rightarrow \qquad yx^2 - x + y = 0$$

$$\Rightarrow \qquad x = \frac{1 \pm \sqrt{1-4y^2}}{2y},$$

Here for $y = 0 \in R$ (co-domain), there is no value of $x \in R$ (domain) *i.e.,* for 0 from co-domain there is no preimage in domain. Hence, not onto.

34. Given matrix is
$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj } A$$

$$|A| = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} = 1(-3) - 2(-2) + 0 = 1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}' = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \qquad ...(i)$$

Given equations are

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

Matrix equation is

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$A'X = B$$

$$\text{Solution is } X = (A')^{-1}B$$
$$= (A^{-1})'B$$
$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}' \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 0, y = -5, z = -3$ is the solution.

35. Let the line passing through the point $(2, -1, 3)$ is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda'(a\hat{i} + b\hat{j} + c\hat{k}) \quad ...(i)$$

If line (i) is perpendicular to the lines

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$$

and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

then $(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k}) = 0$

$\Rightarrow$ $2a - 2b + c = 0$ ...(ii)

and $(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) = 0$

$\Rightarrow$ $a + 2b + 2c = 0$ ...(iii)

Solving (ii) and (iii), $\dfrac{a}{-4-2} = \dfrac{-b}{4-1} = \dfrac{c}{4+2}$, i.e. $\dfrac{a}{-6} = \dfrac{b}{-3} = \dfrac{c}{6}$

$\Rightarrow$ $\langle a, b, c\rangle \equiv \langle -6, -3, 6\rangle$ or $\langle 2, 1, -2\rangle$

From (i), line is $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda'(2\hat{i} + \hat{j} - 2\hat{k})$

Point through which line passes is $(2, -1, 3)$ and DR's of the line are $2, 1, -2$.

$\therefore$ Cartesian equation is $\dfrac{x-2}{2} = \dfrac{y+1}{1} = \dfrac{z-3}{-2}$

OR

Given lines are

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$

Here,

$$\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

and $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k},\ \vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$

We notice $\vec{b_1}$ and $\vec{b_2}$ are parallel vectors as $\dfrac{2}{4} = \dfrac{3}{6} = \dfrac{6}{12}$ or $\vec{b_1} \times \vec{b_2} = \vec{0}$

$\therefore$ lines are parallel.

$$\vec{a_2} - \vec{a_1} = 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$

$$= 2\hat{i} + \hat{j} - \hat{k}$$

$$\text{The shortest distance} = \frac{\left|\vec{b} \times (\vec{a_2} - \vec{a_1})\right|}{|\vec{b}|}$$

$$\vec{b} \times (\vec{a_2} - \vec{a_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix} = -9\hat{i} + 14\hat{j} - 4\hat{k}$$

$\therefore$ $$\text{The shortest distance} = \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{|2\hat{i} + 3\hat{j} + 6\hat{k}|}$$

$$= \frac{\sqrt{81+196+16}}{\sqrt{4+9+36}} = \frac{\sqrt{293}}{7} \text{ units}$$

36. $P(A_1) = \frac{2}{5}, P(A_2) = \frac{2}{5}, P(A_3) = \frac{1}{5}$

E : Seed germinates

$P(E/A_1) = \frac{45}{100}, P(E/A_2) = \frac{60}{100}, P(E/A_3) = \frac{35}{100}$

(*i*) Required probability = $P(A_1)\, P(E/A_1) = \frac{2}{5} \times \frac{45}{100} = 0.18$

(*ii*) P(randomly chosen seed germinate) = $P(A_1)\, P(E/A_1) + P(A_2)\, P(E/A_2) + P(A_3)(E/A_3)$

$$= \frac{2}{5} \times \frac{45}{100} + \frac{2}{5} \times \frac{60}{100} + \frac{1}{5} \times \frac{35}{100}$$

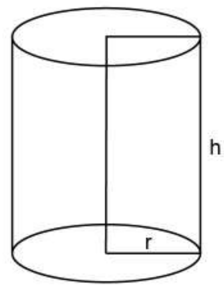
$$= 0.18 + 0.24 + 0.07 = 0.49$$

(*iii*) $P(A_2/E) = \frac{P(A_2)\, P(E/A_2)}{P(\text{Seed germinates})} = \frac{0.24}{0.49} = \frac{24}{49}$

OR

(*iii*) $P(A_3/E) = \frac{P(A_3)\, P(E/A_3)}{P(\text{Seed germinates})} = \frac{0.07}{0.49} = \frac{1}{7}$

37. (*i*) Volume, $V = \pi r^2 h$



Surface area, $S = 2\pi rh + \pi r^2$

$\Rightarrow \quad 75\pi = 2\pi rh + \pi r^2$

$\Rightarrow \quad 75 = 2rh + r^2 \quad \Rightarrow \quad h = \frac{75 - r^2}{2r}$

Now, $V = \pi r^2 \left[\frac{75 - r^2}{2r}\right] = \frac{\pi}{2}[75r - r^3]$

(*ii*) $\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$

Now, $\frac{dV}{dr} = 0$

$\Rightarrow \quad 3r^2 = 75$

$\Rightarrow \quad r^2 = 25$

$\Rightarrow \quad r = 5$

(*iii*) $\frac{d^2V}{dr^2} = \frac{\pi}{2}(-6r) = -3\pi r$

$$\left.\frac{d^2V}{dr^2}\right]_{r=5} = -15\pi < 0$$

$\therefore$ Volume is maximum for $r = 5$.

OR

(*iii*) $V_{max} = \frac{\pi}{2}[75 \times 5 - (5)^3] = \frac{\pi}{2}(375 - 125)$

$$= \frac{\pi}{2} \times 250 = 125\pi \text{ cm}^3$$

Also $h = \frac{75 - r^2}{2r} = \frac{75 - 25}{10} = \frac{50}{10} = 5 = r$

$\Rightarrow h = r$ for maximum volume.

38. (*i*) $A(3, 1, 2), B(0, 4, 1), C(3, 2, 1), D(1, 1, 1)$

$$\overrightarrow{AB} = -3\hat{i} + 3\hat{j} - \hat{k}$$

$$\overrightarrow{AC} = \hat{j} - \hat{k}$$

$$\therefore \quad \text{Area } (\Delta ABC) = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & -1 \\ 0 & 1 & -1 \end{vmatrix} = -2\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\therefore \quad \text{Area} = \frac{1}{2}|-2\hat{i} - 3\hat{j} - 3\hat{k}|$$

$$= \frac{1}{2}\sqrt{4+9+9} = \frac{1}{2}\sqrt{22} \text{ sq units}$$

(*ii*)

$$\overrightarrow{AB} = -3\hat{i} + 3\hat{j} - \hat{k}$$

$$\overrightarrow{BC} = 3\hat{i} - 2\hat{j}$$

$$\overrightarrow{BD} = \hat{i} - 3\hat{j}$$

$$\therefore \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BD} = \hat{i} - 2\hat{j} - \hat{k}$$

$$|\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BD}| = |\hat{i} - 2\hat{j} - \hat{k}|$$

$$= \sqrt{1+4+1} = \sqrt{6}$$