2 SAMPLE PAPER

[Easy Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $D(0, 6)$, $E(5, 4)$ and $F(3, 10)$ are the mid points of sides AB, BC and CA respectively of a ΔABC, then area of ΔABC is **[NCERT Part-I, Page 82]**

(*a*) 13 sq units (*b*) 26 sq units (*c*) 39 sq units (*d*) 52 sq units

2. If A is an invertible matrix of order 3 and $|A| = 3$, then if $|3A| = k|A|$, the value of k is **[NCERT Part-I, Page 80]**

(*a*) 243 (*b*) 27 (*c*) 81 (*d*) 9

3. If matrix $A = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$, then the value of $A^2 + 3I_2$ is **[Conceptual Application]**

(*a*) $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$ (*b*) $\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (*d*) I_2

4. If $A = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 4 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then value of $|A^{-1}|$ is **[NCERT Part-I, Page 90]**

(*a*) $\frac{1}{12}$ (*b*) 12 (*c*) $-\frac{1}{12}$ (*d*) –12

5. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, then the value of $|2\vec{a} + \vec{b} + \vec{c}|$ is **[Conceptual Application]**

(*a*) $-\sqrt{3}$ (*b*) $-\sqrt{6}$ (*c*) $\sqrt{6}$ (*d*) $\sqrt{3}$

6. The position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 3 : 5 is **[NCERT Part-II, Page 353]**

(*a*) $\frac{-\vec{a} - 13\vec{b}}{2}$ (*b*) $\frac{\vec{a} + 13\vec{b}}{2}$ (*c*) $\frac{11\vec{a} - 7\vec{b}}{8}$ (*d*) $\frac{13\vec{a} - \vec{b}}{8}$

7. If A and B are square matrices of order 3 such that $|A| = 4$ and $AB = 2I$, then value of $|B|$ is **[NCERT Part-II, Page 90]**

(*a*) $\frac{1}{2}$ (*b*) 2 (*c*) 4 (*d*) 1

8. The degree of differential equation $\left(1 + \frac{dy}{dx}\right)^2 + x + \sin\left(\frac{d^2y}{dx^2}\right) = 0$ is **[NCERT Part-II, Page 302]**

(*a*) 1 (*b*) 2 (*c*) 0 (*d*) not defined

9. Value of $\int_0^{\pi} |\cos x| dx$ is **[NCERT Part-II, Page 273-274]**

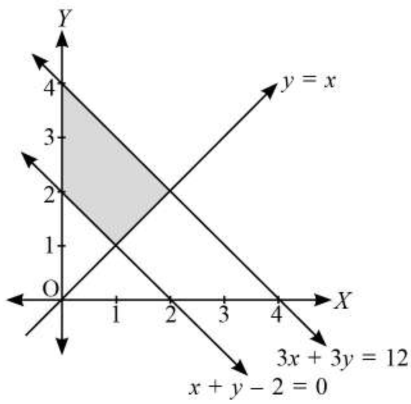
(*a*) 2 (*b*) 0 (*c*) 3 (*d*) 4

10. If $f(x) = \begin{cases} \frac{e^{x+\lambda} - e^{\lambda}}{x}, & x \neq 0 \\ e^4, & x = 0 \end{cases}$ is continuous at $x = 0$, then value of λ is **[NCERT Part-I, Page 105]**

(*a*) e^x (*b*) e^4 (*c*) 4 (*d*) e^{λ}

11. The feasible region corresponding to the linear constraints of linear programming problem is given in figure. Which of the following is not a constraint to the given linear programming problem?

[Conceptual Application]



(*a*) $x + y - 2 \leq 0$ (*b*) $x \geq 0$ (*c*) $y \geq x$ (*d*) $3x + 3y \leq 12$

12. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$ and $|\vec{a}| = 5$, then value of $|\vec{b}|$ is **[NCERT Part-II, Page 356, 363]**

(*a*) 25 (*b*) 5 (*c*) 9 (*d*) 3

13. The corner points of the bounded feasible region determined by system of linear constraints are (1, 3), (2, 5), (6, 3) and (0, 4). Let objective function be $Z = px + qy$, $p, q > 0$. If Z has same value at (2, 5) and (6, 3), then if $p = kq$, then value of k is **[NCERT Part-II, Page 398]**

(*a*) 2 (*b*) $\frac{1}{2}$ (*c*) –2 (*d*) $-\frac{1}{2}$

14. Which of the following function is continuous as well as differentiable for $x \in R$? **[Conceptual Application]**

(*a*) Absolute value function or Modulus function
(*b*) Polynomial function
(*c*) Signum function
(*d*) Greatest integer function

15. Direction cosines of the line $\frac{2x-1}{3} = \frac{2-y}{5}, z = 4$ are **[NCERT Part-II, Page 379-380]**

(*a*) $\frac{3}{\sqrt{109}}, \frac{-10}{\sqrt{109}}, 0$
(*b*) $\frac{3}{\sqrt{109}}, \frac{10}{\sqrt{109}}, 0$
(*c*) $\frac{3}{\sqrt{109}}, \frac{-10}{\sqrt{109}}, \frac{1}{\sqrt{109}}$
(*d*) $\frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}}, 0$

16. If a line makes angles α, β, γ with x, y and z-axes respectively, then value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is **[NCERT Part-II, Page 377-378]**

(*a*) –2 (*b*) 1 (*c*) 0 (*d*) 2

17. If A and B are two independent events such that $P(A) = 0.4$, $P(B) = 0.3$, then $P(A \cup B)$ is **[NCERT Part-II, Page 418]**

(*a*) 0.27 (*b*) 0.58 (*c*) 0.60 (*d*) 0.72

18. General solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2 - y}$ is **[NCERT Part-II, Page 306-307]**

(*a*) $e^{x^2 - y} = C$ (*b*) $e^y + e^{x^2} = C$ (*c*) $e^{-y} + e^{x^2} = C$ (*d*) $e^y = e^{x^2} + C$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. **Assertion (*A*):** The function 'f' defined by $f : R - \{0\} \to R$ as $f(x) = \frac{1}{x}$ is a bijective function. **[NCERT Part-I, Page 7]**

Reason (*R*): A function is said to be bijective if it is injective as well as surjective.

20. If the total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in ₹, then

Assertion (*A*): the marginal revenue when 5 units are sold is ₹ 66. **[Integrated Question]**

Reason (*R*): If for a function 'f', $f''(x_1) < 0$, where 'x_1' is a solution of $f'(x) = 0$, then function 'f' attains local maximum at 'x_1'.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Differentiate $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$ with respect to x. **[NCERT Part-I, Page 124]**

OR

If $x = at^2$, $y = 2bt$, then find (*i*) $\frac{dy}{dx}$ (*ii*) $\frac{d^2y}{dx^2}$ **[NCERT Part-I, Page 134-135]**

22. Find the critical points for the function $f(x) = x^3 - 3x^2 + 6x - 100$, if any. **[NCERT Part-I, Page 164]**

23. Evaluate $\sin\left[\cos^{-1}\left(\cos\frac{7\pi}{4}\right)\right]$. **[Conceptual Application]**

OR

Find the domain of $\cos^{-1}(3x + 4)$. **[NCERT Part-I, Page 21]**

24. Evaluate: $\int \cos x(\operatorname{cosec} x + 1)dx$. **[NCERT Part-II, Page 241]**

25. Find the interval for which the function $f(x) = \sin 3x$ is increasing in $\left(0, \frac{\pi}{2}\right)$. **[NCERT Part-I, Page 153]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $e^{x+y} = e^x + e^y$, then prove that $\frac{dy}{dx} = -e^{y-x}$. **[NCERT Part-I, Page 122-123]**

27. *A* speaks truth in 80% cases and *B* speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact? **[Conceptual Application]**

28. Find $\frac{dy}{dx}$, if $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$. **[NCERT Part-I, Page 130]**

OR

Find the particular solution of the differential equation

$(1 - y^2)(1 + \log x)dx + 2xydy = 0$, given that $y = 0$ when $x = 1$. **[NCERT Part-II, Page 306-307]**

29. Evaluate: $\int \frac{\sin\theta}{(5 - \sin^2\theta)(2 - \cos^2\theta)} d\theta$. **[NCERT Part-II, Page 253]**

OR

Evaluate: $\int_0^{3/2} |x \sin \pi x| dx$. **[NCERT Part-II, Page 273-274]**

30. Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397-398]**

Maximize $Z = 1000x + 600y$

Subject to the constraints

$x + y \le 200, x \ge 20, y - 4x \ge 0, x, y \ge 0$.

OR

Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397-398]**

Minimize $Z = 3x + 4y + 370$

Subject to the constraints

$x, y \ge 0, x + y \le 60, x + y \ge 10, x \le 40, y \le 50$.

31. Evaluate: $\int \sqrt{5 - 3x - x^2}\, dx$. **[NCERT Part-II, Page 264-265]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of equations $2x + 3y + z = 0$, $3x - 2y - z = 5$, $5x - 4y - 2z = 9$. **[NCERT Part-I, Page 94-95]**

33. Find the vector and Cartesian equations of a line passing through the point $(1, 2, -4)$ and perpendicular to the lines **[Conceptual Application]**

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and

$$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}).$$

OR

Find the image of the point with position vector $(2\hat{i} - \hat{j} + 5\hat{k})$ in the line **[Conceptual Application]**

$\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$.

34. Using integration, find the area of the triangle *ABC*, the coordinates of whose vertices are *A*(4, 1), *B*(6, 6) and *C*(8, 4). **[Conceptual Application]**

35. Let $f: N \rightarrow N$, *N* is the set of natural numbers, be defined as $f(x) = x + 1$, if *x* is odd and $f(x) = x - 1$, if *x* is even. Show that '*f*' is a bijective function. **[NCERT Part-I, Page 7]**

OR

Let set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and *R* be a relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d) \in A \times A$. Show that *R* is an equivalence relation. Also obtain equivalence class $[(3, 7)]$.

[NCERT Part-I, Page 2, 4]

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Two boys are flying kites on the terrace of their respective houses. The straight strings of kites are in the form of skew lines and their equations are given by **[Conceptual Application]**

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + \hat{k})$$

and $$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

(*i*) Find a vector perpendicular to both the strings.

(*ii*) State the condition so that strings intersect each other.

(*iii*) Find the shortest distance between the strings.

OR

(*iii*) Along which direction vectors are the two strings?

Case Study - 2

37. There are two shops in a village named Rama general store and Green general store. Rama general store, stores 30 tin of pure mustard oil and 40 tin of adulterated mustard oil. While Green general store, stores 50 tin of pure mustard oil and 60 tin of adulterated mustard oil. Rani wants to purchase one tin of mustard oil from any of the shops. **[Conceptual Application]**

(*i*) Find the probability of getting an adulterated mustard oil tin from Green general store.

(*ii*) Find the probability of purchasing a pure tin of mustard oil.

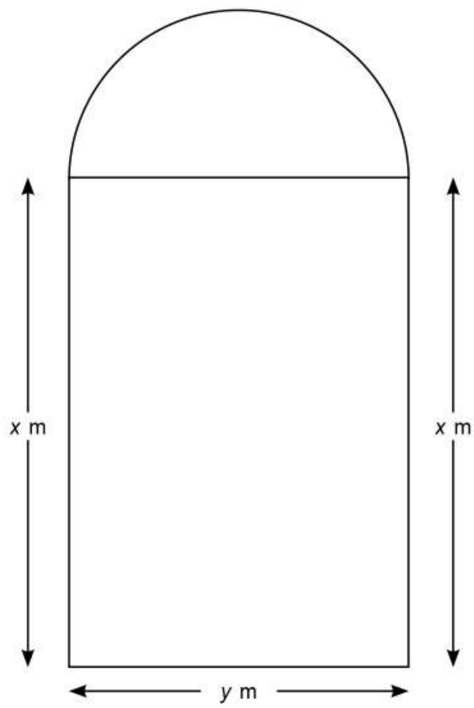
(*iii*) If the tin of mustard oil purchased is pure, what is the probability that it was purchased from Rama general store?

OR

(*iii*) What is the probability of purchasing a pure mustard oil tin from a Green general store or adulterated mustard oil tin from Rama general store?

Case Study - 3

38.



A person wants to purchase a 3 bedroom flat in Noida. When he went to have a look at the flat, he noticed a window in the form of a rectangle surmounted by semicircular shape. The perimeter of window is found to be 10 m. If x m is length of rectangular portion of window and y m is its breadth and semicircular portion is on breadth, then **[Conceptual Application]**

(*i*) find the area of the window in terms of y.

(*ii*) find the value of y for which area of the window is maximum.

SOLUTIONS

1. (*d*) As, $\text{ar}(\Delta ABC) = 4\ \text{ar}(\Delta DEF)$

$$\text{ar}(\Delta DEF) = \frac{1}{2}\begin{vmatrix} 0 & 6 & 1 \\ 5 & 4 & 1 \\ 3 & 10 & 1 \end{vmatrix} = \frac{1}{2}\ [0 - 6\ (5 - 3) + 1\ (50 - 12)]$$

$$= \frac{1}{2}[-12 + 38] = 13 \text{ sq units}$$

$\therefore$ $\text{ar}(\Delta ABC) = 4 \times 13 = 52$ sq units

As, area of the Δ formed by joining the mid-points of the sides of a given Δ is $\frac{1}{4}$ of the area of given triangle.

2. (*b*) $|3A| = 3^3|A| = k|A| \Rightarrow k = 27$

3. (*d*) $$A^2 = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A^2 + 3I_2 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

4. (*c*) $|A| = 3(0 - 1) + 1(0 - 1) + 2(0 - 4) = -3 - 1 - 8 = -12$

$$|A^{-1}| = \frac{1}{|A|} = -\frac{1}{12}$$

5. (*c*) $$|2\vec{a} + \vec{b} + \vec{c}\,|^2 = \left(2\vec{a} + \vec{b} + \vec{c}\right)\cdot\left(2\vec{a} + \vec{b} + \vec{c}\right)$$

$$= 4\vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{c} + 4\vec{a}\cdot\vec{b} + 4\vec{a}\cdot\vec{c} + 2\vec{b}\cdot\vec{c}$$

$$= 4|\vec{a}\,|^2 + |\vec{b}\,|^2 + |\vec{c}\,|^2 + 0 + 0 + 0$$

$$= 4 + 1 + 1 = 6$$

$\Rightarrow$ $|2\vec{a} + \vec{b} + \vec{c}\,| = \sqrt{6}$.

6. (*a*) $$\vec{r} = \frac{5(\vec{a} - 2\vec{b}) - 3(2\vec{a} + \vec{b})}{2}$$

$$= \frac{5\vec{a} - 10\vec{b} - 6\vec{a} - 3\vec{b}}{2} = \frac{-\vec{a} - 13\vec{b}}{2}$$

7. (*b*) $|AB| = |2I| \Rightarrow |A||B| = 8$

$\Rightarrow$ $|B| = \frac{8}{4} = 2$

8. (*d*) Degree is not defined, as equation cannot be written as polynomial of derivatives.

9. (*a*) $$\int_0^{\pi}|\cos x\,|\,dx = \int_0^{\pi/2}\cos x\,dx - \int_{\pi/2}^{\pi}\cos x\,dx$$

$$= \Big[\sin x\Big]_0^{\pi/2} - \Big[\sin x\Big]_{\pi/2}^{\pi}$$

$$= \left[\sin\frac{\pi}{2} - \sin 0\right] - \left[\sin\pi - \sin\frac{\pi}{2}\right] = 2$$

10. (*c*) If $f(x)$ continuous at $x = 0$, then

$$\lim_{x \to 0} f(x) = f(0)$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{e^{x+\lambda} - e^{\lambda}}{x} = e^4$$

$$\Rightarrow \quad e^{\lambda} \lim_{x \to 0} \frac{e^x - 1}{x} = e^4 \Rightarrow e^{\lambda} = e^4$$

$$\Rightarrow \quad \lambda = 4$$

11. (*a*) As for (0, 0), $0 + 0 - 2 \le 0$, true, but feasible region does not contain (0, 0).

12. (*d*)
$$\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a} \cdot \vec{b}\right)^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$\Rightarrow \quad 225 = (5)^2 |\vec{b}|^2 \Rightarrow |\vec{b}|^2 = 9 \Rightarrow |\vec{b}| = 3$$

13. (*b*)
$$Z_{(2,\,5)} = Z_{(6,\,3)}$$

$$\Rightarrow \quad 2p + 5q = 6p + 3q$$

$$\Rightarrow \quad 4p = 2q \Rightarrow 2p = q$$

Comparing with $p = kq$, we get $k = \frac{1}{2}$

14. (*b*)

15. (*a*)
$$\frac{2x - 1}{3} = \frac{2 - y}{5};\ z = 4$$

$$\Rightarrow \quad \frac{x - \frac{1}{2}}{\frac{3}{2}} = \frac{y - 2}{-5} = \frac{z - 4}{0}$$

$$\Rightarrow \quad \frac{x - \frac{1}{2}}{3} = \frac{y - 2}{-10} = \frac{z - 4}{0}$$

DR's are 3, –10, 0

DC's are $\frac{3}{\sqrt{109}}, \frac{-10}{\sqrt{109}}, 0$

16. (*d*) DC's are $\cos\alpha$, $\cos\beta$, $\cos\gamma$.

$$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow \quad 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$

$$\Rightarrow \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

17. (*b*)
$$P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$$
$$= 0.4 + 0.3 - 0.12 = 0.7 - 0.12 = 0.58$$

18. (*d*)
$$\frac{dy}{dx} = 2x \cdot e^{x^2 - y} = 2xe^{x^2} \cdot e^{-y}$$

$$\Rightarrow \quad \int e^y\,dy = \int 2x\,e^{x^2}\,dx$$

$$= \int e^t\,dt = e^t + C \qquad \left|\begin{array}{l}\text{Let } x^2 = t \\ \Rightarrow 2x\,dx = dt\end{array}\right.$$

$$\Rightarrow \quad e^y = e^{x^2} + C$$

19. (*d*) Assertion (A) is false, as for $0 \in R$(co-domain) there is no pre-image in domain. Reason (R) is true.

20. (*b*) Assertion $R'(x) = 6x + 36$

$$R'(5) = 30 + 36 = 66, \text{ true}$$

Both (A) and (R) are true and (R) is not the correct explanation of (A).

21. Consider

$$y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$= \tan^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\cot\frac{x}{2}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]$$

$$\Rightarrow \quad y = \frac{\pi}{2} - \frac{x}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}.$$

OR

We have,

$$x = at^2 \Rightarrow \frac{dx}{dt} = 2at$$

$$y = 2bt \Rightarrow \frac{dy}{dt} = 2b$$

(*i*) $$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{2b}{2at} = \frac{b}{at}$$

(*ii*) $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{b}{at}\right)$$

$$= \frac{b}{a}\cdot\left(-\frac{1}{t^2}\cdot\frac{dt}{dx}\right)$$

$$= -\frac{b}{at^2}\cdot\frac{1}{2at} = -\frac{b}{2a^2t^3}$$

22.

$$f(x) = x^3 - 3x^2 + 6x - 100$$

$$f'(x) = 3x^2 - 6x + 6$$

$$= 3(x^2 - 2x + 2) = 3[(x-1)^2 + 1]$$

$$f'(x) \neq 0, \text{ for any } x \in R \text{ as } (x-1)^2 + 1 > 0$$

Hence, no critical points for the function.

23.

$$\sin\left[\cos^{-1}\left(\cos\frac{7\pi}{4}\right)\right] = \sin\left[\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$= \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

OR

For domain $-1 \leq 3x + 4 \leq 1$

$\Rightarrow$ $-5 \leq 3x \leq -3$

$\Rightarrow$ $-\frac{5}{3} \leq x \leq -1$

Domain $= \left[-\frac{5}{3}, -1\right]$

24. Consider $\int \cos x(\text{cosec } x + 1)dx = \int (\cot x + \cos x)dx$

$$= \log|\sin x| + \sin x + C$$

25. We have, $f(x) = \sin 3x$

$\Rightarrow$ $f'(x) = 3 \cos 3x$ ...(*i*)

For critical points, $f'(x) = 0$

$\Rightarrow$ $\cos 3x = 0$

$\Rightarrow$ $3x = \frac{\pi}{2}, \frac{3\pi}{2}$

So, $x = \frac{\pi}{6}$ and $\frac{\pi}{2}$

Case I: When $0 < x < \frac{\pi}{6}$ i.e. $0 < 3x < \frac{\pi}{2}$

In this case, we have

$$\cos 3x > 0$$

$\Rightarrow$ $3 \cos 3x > 0$

$\Rightarrow$ $f'(x) > 0$

$\therefore f(x)$ is increasing on $\left(0, \frac{\pi}{6}\right)$.

Case II: When $\frac{\pi}{6} < x < \frac{\pi}{2}$ i.e. $\frac{\pi}{2} < 3x < \frac{3\pi}{2}$

In this case, we have

$$\cos 3x < 0$$

$\Rightarrow$ $3 \cos 3x < 0$

$\Rightarrow$ $f'(x) < 0$

$\therefore f(x)$ is decreasing on $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$.

26. Consider $e^{x+y} = e^x + e^y$ ...(*i*)

Differentiating w.r.t. x, we get

$$e^{x+y} \cdot \left[1 + \frac{dy}{dx}\right] = e^x + e^y \cdot \frac{dy}{dx}$$

$\Rightarrow$ $e^{x+y} + e^{x+y}\left(\frac{dy}{dx}\right) = e^x + e^y \cdot \left(\frac{dy}{dx}\right)$

$\Rightarrow$ $[e^{x+y} - e^y]\frac{dy}{dx} = e^x - e^{x+y}$

$\Rightarrow$ $(e^x + e^y - e^y) = \frac{dy}{dx} = e^x - e^x - e^y$ [from (*i*)]

$\Rightarrow$ $e^x \frac{dy}{dx} = -e^y$

$\Rightarrow$ $\frac{dy}{dx} = -\frac{e^y}{e^x}$

$\Rightarrow$ $\frac{dy}{dx} = -e^{y-x}$

27.

$$P(A) = \frac{80}{100} = \frac{4}{5}, \quad P(B) = \frac{90}{100} = \frac{9}{10}$$

$$P(\overline{A}) = \frac{1}{5}, P(\overline{B}) = \frac{1}{10}$$

$$P(\text{same fact}) = P(AB \text{ or } \overline{A}\,\overline{B})$$

$$= \frac{4}{5} \times \frac{9}{10} + \frac{1}{5} \times \frac{1}{10}$$

$$= \frac{36+1}{50} = \frac{37}{50}$$

74% cases they are likely to state same fact.

28.

$$y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$$

$\Rightarrow$ $y = u + v$, where $u = (\sin x)^{\tan x}$ and $v = (\cos x)^{\sec x}$

Now, $$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad ...(i)$$

Now, $$u = (\sin x)^{\tan x}$$

$\Rightarrow$ $$\log u = \tan x \cdot \log \sin x$$

Differentiating both sides w.r.t. 'x', we get

$$\frac{1}{u}\frac{du}{dx} = \tan x \times \frac{\cos x}{\sin x} + \log(\sin x) \times \sec^2 x$$

$\Rightarrow$ $$\frac{du}{dx} = (\sin x)^{\tan x}[1 + \sec^2 x . \log(\sin x)]$$

Now, $$v = (\cos x)^{\sec x}$$

$\Rightarrow$ $$\log v = \sec x \cdot \log(\cos x)$$

$\Rightarrow$ $$\frac{1}{v}\frac{dv}{dx} = \sec x \times \frac{(-\sin x)}{\cos x} + \log(\cos x) \times \sec x . \tan x$$

$\Rightarrow$ $$\frac{dv}{dx} = (\cos x)^{\sec x} . \sec x . \tan x[\log \cos x - 1]$$

$\therefore$ From (i), we get

$$\frac{dy}{dx} = (\sin x)^{\tan x}[1 + \sec^2 x \cdot \log(\sin x)] + (\cos x)^{\sec x} \cdot \sec x \cdot \tan x[\log(\cos x) - 1]$$

OR

$$(1 - y^2)(1 + \log x)dx + 2xy\, dy = 0$$

$\Rightarrow$ $$2xy\, dy = -(1 - y^2)(1 + \log x)\, dx$$

$\Rightarrow$ $$-\frac{2y}{1-y^2}dy = \frac{1+\log x}{x}dx$$

Integrating both sides, we get

$$-\int \frac{2y}{1-y^2}dy = \int \frac{1+\log x}{x}dx$$

$\Rightarrow$ $$\int \frac{1}{t}dt = \int u\, du$$

[In I^st integral, Put $1 - y^2 = t$ $\Rightarrow -2y dy = dt$ | In II^nd integral, Put $1 + \log x = u$ $\Rightarrow \frac{1}{x}dx = du$]

$\Rightarrow$ $$\log |t| = \frac{u^2}{2} + C$$

$\Rightarrow$ $$\log|1 - y^2| = \frac{(1+\log x)^2}{2} + C \quad ...(i)$$

When $x = 1$, $y = 0$, then from (i), we get

$$\Rightarrow \quad \log|1| = \frac{1}{2}(1+\log 1)^2 + C$$

$$\Rightarrow \quad 0 = \frac{1}{2}\times 1 + C$$

$$\Rightarrow \quad C = -\frac{1}{2}$$

Substituting in (i), we get

$\log|1-y^2| = \frac{1}{2}(1+\log x)^2 - \frac{1}{2}$ is required solution.

29. Consider, $\displaystyle\int \frac{\sin\theta}{(5-\sin^2\theta)(2-\cos^2\theta)}d\theta = \int \frac{\sin\theta}{(4+\cos^2\theta)(2-\cos^2\theta)}d\theta$

| Let $\cos\theta = t$
| $\Rightarrow -\sin\theta\, d\theta = dt$

$$= -\int \frac{1}{(4+t^2)(2-t^2)}dt$$

$$= -\frac{1}{6}\int \frac{(4+t^2)+(2-t^2)}{(4+t^2)(2-t^2)}dt$$

$$= -\frac{1}{6}\left[\int \frac{1}{2-t^2}dt + \int \frac{1}{4+t^2}dt\right]$$

$$= -\frac{1}{6}\left[\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right| + \frac{1}{2}\tan^{-1}t\right] + C$$

$$= -\frac{1}{12\sqrt{2}}\log\left|\frac{\sqrt{2}+\cos\theta}{\sqrt{2}-\cos\theta}\right|$$

$$-\frac{1}{12}\tan^{-1}(\cos\theta) + C$$

OR

$$\int_0^{3/2} |x\sin\pi x|dx$$

For $0 < x < 1$, $\quad x\sin\pi x > 0$;

For $1 < x < \frac{3}{2}$, $\quad x\sin\pi x < 0$

$$\therefore \quad \int_0^{3/2} |x\sin\pi x|dx = \int_0^1 x\sin\pi x\, dx + \int_1^{3/2} (-x\sin\pi x)dx \quad ...(i)$$

Now, $\displaystyle\int \underset{①}{x}.\underset{②}{\sin}\,\pi x\, dx = x.\left(\frac{-\cos\pi x}{\pi}\right) - \int 1.\left(\frac{-\cos\pi x}{\pi}\right)dx = -\frac{x}{\pi}\cos\pi x + \frac{1}{\pi^2}\sin\pi x$

From(i), we get

$$\int_0^{\frac{3}{2}} |x\sin\pi x|dx = \left[-\frac{x}{\pi}\cos\pi x + \frac{1}{\pi^2}\sin\pi x\right]_0^1 - \left[-\frac{x}{\pi}\cos\pi x + \frac{1}{\pi^2}\sin\pi x\right]_1^{\frac{3}{2}}$$

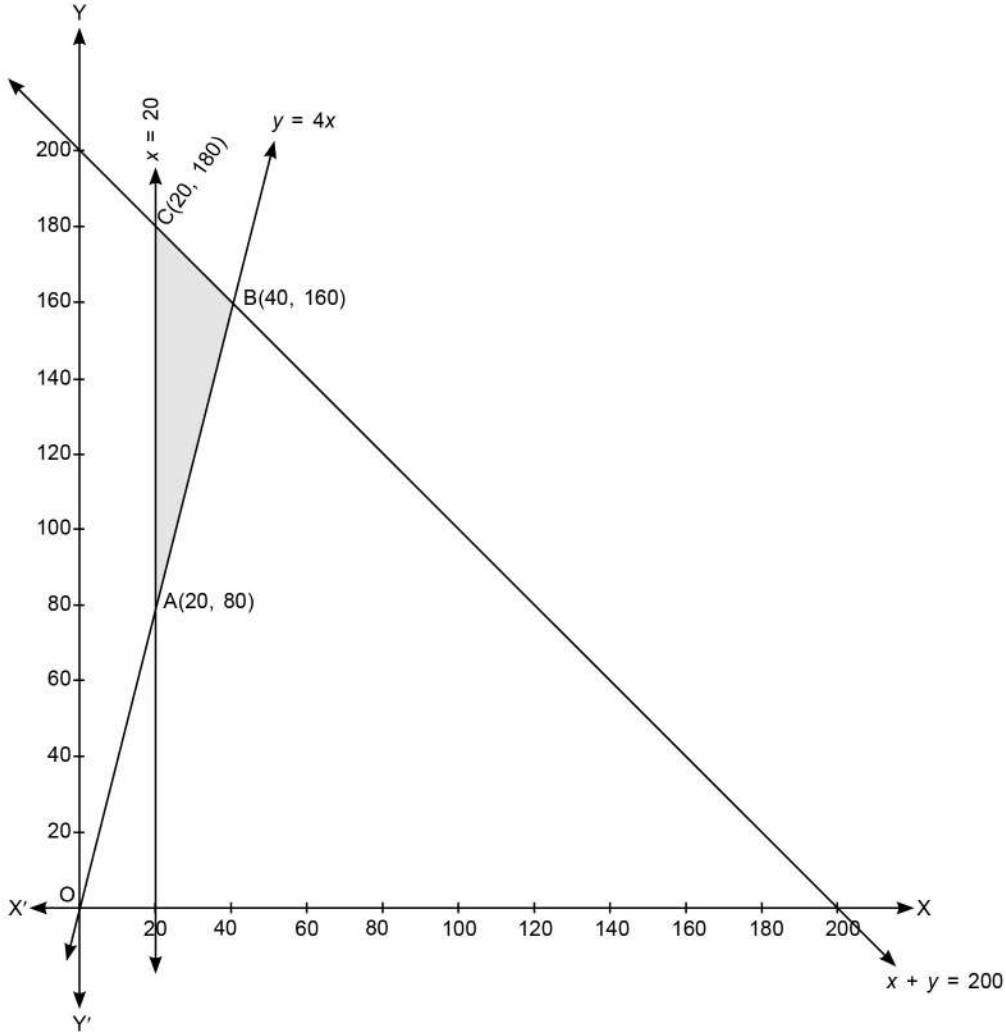
$$= \left(-\frac{1}{\pi}\cos\pi + \frac{1}{\pi^2}\sin\pi\right) - (0) - \left(-\frac{3}{2\pi}\cos\frac{3\pi}{2} + \frac{1}{\pi^2}\sin\frac{3\pi}{2}\right) + \left(-\frac{1}{\pi}\cos\pi + \frac{1}{\pi^2}\sin\pi\right)$$

$$= \frac{1}{\pi} + 0 - 0 + 0 + \frac{1}{\pi^2} + \frac{1}{\pi} + 0 = \frac{2}{\pi} + \frac{1}{\pi^2} = \frac{2\pi+1}{\pi^2}$$

30. Plotting the constraints on graph

$x + y \leq 200$, $x \geq 20$, $y \geq 4x$

$x, y \geq 0$, we notice shaded portion is feasible solution.



Possible points for maximum Z are $A(20, 80)$, $B(40, 160)$, $C(20, 180)$

Corner Points	$Z = 1000x + 600y$	**Values**	
$A(20, 80)$	20000 + 48000	68000	
$B(40, 160)$	40000 + 96000	1,36,000	←Maximum
$C(20, 180)$	20000 + 108000	1,28,000	

Z is maximum for $B(40, 160)$ i.e. $x = 40$, $y = 160$

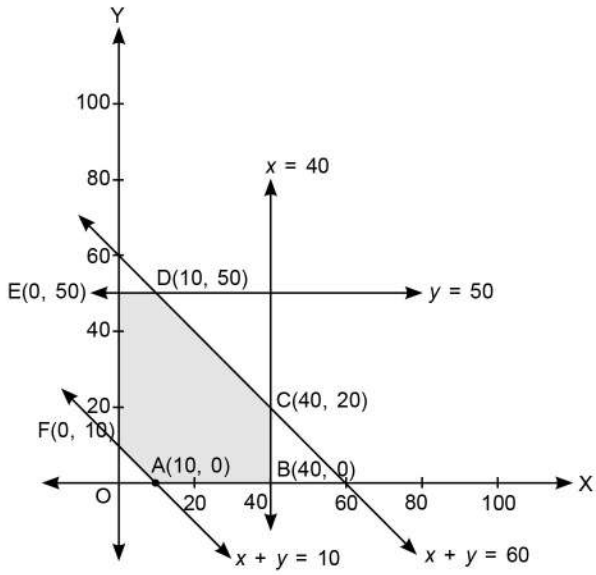
Maximum, $Z = 1{,}36{,}000$

OR

Plotting the inequations

$x, y \geq 0$, $x + y \leq 60$, $x + y \geq 10$,

$x \leq 40$, $y \leq 50$ on the graph, we notice shaded portion is feasible solution.



Possible points for minimum Z are $A(10, 0)$, $B(40, 0)$, $C(40, 20)$, $D(10, 50)$, $E(0, 50)$, $F(0, 10)$.

Corner points	$Z = 3x + 4y + 370$	**Values**	
$A(10, 0)$	30 + 0 + 370	400	←Minimum
$B(40, 0)$	120 + 0 + 370	490	
$C(40, 20)$	120 + 80 + 370	570	
$D(10, 50)$	30 + 200 + 370	600	
$E(0, 50)$	0 + 200 + 370	570	
$F(0, 10)$	0 + 40 + 370	410	

Z is minimum for $A(10, 0)$ i.e. $x = 10$, $y = 0$.

Minimum, $Z = 400$

31. Consider, $\int \sqrt{5 - 3x - x^2}\,dx = \int \sqrt{\left(\frac{\sqrt{29}}{2}\right)^2 - \left(x + \frac{3}{2}\right)^2}\,dx$

$$= \frac{x + \frac{3}{2}}{2}\sqrt{5 - 3x - x^2} + \frac{29}{8}\sin^{-1}\left(\frac{x + \frac{3}{2}}{\frac{\sqrt{29}}{2}}\right) + C$$

$$= \frac{2x + 3}{4}\sqrt{5 - 3x - x^2} + \frac{29}{8}\sin^{-1}\left(\frac{2x + 3}{\sqrt{29}}\right) + C$$

$$-[x^2 + 3x - 5]$$
$$= -\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 5\right]$$
$$= \frac{29}{4} - \left(x + \frac{3}{2}\right)^2$$

32. $$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A = -\frac{1}{1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \quad ...(i)$$

Consider equations

$$2x + 3y + z = 0$$

$$3x - 2y - z = 5 \quad \Rightarrow \quad -3x + 2y + z = -5$$

$$5x - 4y - 2z = 9$$

Matrix equation is

$$\begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 9 \end{bmatrix}$$

$\Rightarrow$ $$A'X = B$$

Solution is $$X = (A')^{-1}B$$

$$= (A^{-1})'B$$

$$= \begin{bmatrix} 0 & -2 & -1 \\ 1 & 9 & 5 \\ -2 & -23 & -13 \end{bmatrix}\begin{bmatrix} 0 \\ -5 \\ 9 \end{bmatrix} \quad \text{[From } (i)\text{]}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 + 10 - 9 \\ 0 - 45 + 45 \\ 0 + 115 - 117 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$\Rightarrow x = 1, y = 0, z = -2$ is required solution.

33. Let line through the point (1, 2, –4) be $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'\vec{m}$, λ' is a scalar. ...(i)

Line (*i*) is perpendicular to lines

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

$\therefore$ $$(3\hat{i} - 16\hat{j} + 7\hat{k})\cdot\vec{m} = 0 \text{ and } (3\hat{i} + 8\hat{j} - 5\hat{k})\cdot\vec{m} = 0$$

$\Rightarrow$ $$\vec{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$\therefore$ From (*i*)

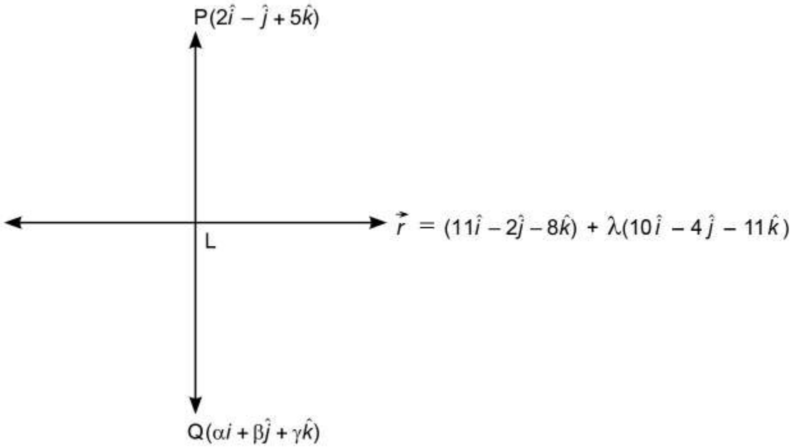
line is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'(24\hat{i} + 36\hat{j} + 72\hat{k})$

or $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda''(2\hat{i} + 3\hat{j} + 6\hat{k})$, where $\lambda'' = 12\lambda'$

For Cartesian equation, line passes through the point (1, 2, –4) and DR's are <2, 3, 6>

$\therefore$ Cartesian equations of line are: $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+4}{6}$

OR



Let $Q((\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ be image of $P\left(2\hat{i} - \hat{j} + 5\hat{k}\right)$ in the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$.

Then PQ is perpendicular to line and L is mid-point of PQ.

General point on line is

$$\vec{r} = (11+10\lambda)\hat{i} + (-2-4\lambda)\hat{j} + (-8-11\lambda)\hat{k} \quad ...(i)$$

$$\overrightarrow{PL} = (11+10\lambda-2)\hat{i} + (-2-4\lambda+1)\hat{j} + (-8-11\lambda-5)\hat{k}$$

$$= (9+10\lambda)\hat{i} + (-4\lambda-1)\hat{j} + (-11\lambda-13)\hat{k}$$

If PL is perpendicular to line, then

$$10(9+10\lambda) - 4(-4\lambda-1) - 11(-11\lambda-13) = 0$$

$$\Rightarrow \quad 90 + 100\lambda + 16\lambda + 4 + 121\lambda + 143 = 0$$

$$\Rightarrow \quad 237\lambda = -237 \Rightarrow \lambda = -1$$

Substituting in (*i*)

Position vector for foot of perpendicular is $L\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ ...(*ii*)

Also, position vector of L (mid-point of PQ) is

$$\left\{\left(\frac{2+\alpha}{2}\right)\hat{i} + \left(\frac{-1+\beta}{2}\right)\hat{j} + \left(\frac{5+\gamma}{2}\right)\hat{k}\right\} \quad ...(iii)$$

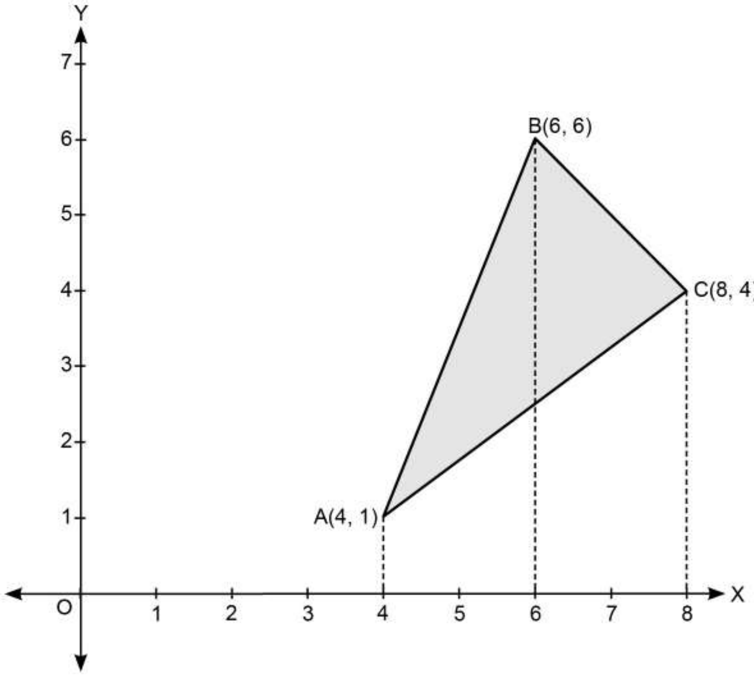
$\therefore$ From (*ii*) and (*iii*), we get

$$\frac{2+\alpha}{2} = 1, \ \frac{-1+\beta}{2} = 2, \ \frac{5+\gamma}{2} = 3$$

$\Rightarrow \alpha = 0, \beta = 5, \gamma = 1$

$\therefore$ Position vector of image is $Q\left(5\hat{j} + \hat{k}\right)$.

34. Plotting the points $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$ on graph, we notice, we have to find shaded area.



$$\text{Area } (\Delta ABC) = \int_4^6 y_{AB}\,dx + \int_6^8 y_{BC}\,dx - \int_4^8 y_{AC}\,dx$$

$$\text{Equation of } AB : y - 1 = \frac{6-1}{6-4}(x-4) \Rightarrow y - 1 = \frac{5}{2}(x-4)$$

$$\Rightarrow \qquad y = \frac{5}{2}x - 10 + 1 \Rightarrow y = \frac{5}{2}x - 9$$

$$\text{Equation of } BC : y - 6 = \frac{4-6}{8-6}(x-6) \Rightarrow y - 6 = -1(x-6)$$

$$\Rightarrow \qquad y - 6 = -x + 6 \Rightarrow y = -x + 12$$

$$\text{Equation of } AC : y - 1 = \frac{4-1}{8-4}(x-4) \Rightarrow y - 1 = \frac{3}{4}(x-4)$$

$$\Rightarrow \qquad y = \frac{3}{4}x - 3 + 1 \Rightarrow y = \frac{3}{4}x - 2$$

$$\therefore \qquad \text{Area} = \int_4^6 \left(\frac{5}{2}x - 9\right)dx + \int_6^8 (-x+12)\,dx - \int_4^8 \left(\frac{3}{4}x - 2\right)dx$$

$$= \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[-\frac{x^2}{2} + 12x\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$$

$$= [(45 - 54) - (20 - 36)] + [(-32 + 96) - (-18 + 72)] - [(24 - 16) - (6 - 8)]$$

$$= [-9 + 16] + [64 - 54] - [8 + 2]$$

$$= 7 + 10 - 10$$

$$= 7 \text{ sq units}$$

35. Given function $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd.} \\ x-1, & \text{if } x \text{ is even.} \end{cases}$

For one-one:

(*i*) Let $x_1, x_2 \in N$ and x_1, x_2 are both even. Then,

$f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$

(*ii*) Let $x_1, x_2 \in N$ and x_1, x_2 are both odd. Then,

$f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$

(*iii*) Let $x_1, x_2 \in N$ and x_1 is even and x_2 is odd. Then $x_1 \neq x_2$.

Also, $f(x_1) = x_1 - 1$ (odd) and $f(x_2) = x_2 + 1$ (even) $\Rightarrow f(x_1) \neq f(x_2)$

So, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

(*iv*) We can also take x_1 as odd and x_2 as even and show as above $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

Hence, function 'f' is one-one.

For onto: Let $y \in N$ (co-domain)

If y is even

$\Rightarrow y = x + 1 \Rightarrow x = y - 1 \in N$ (domain)

$f(y - 1) = y - 1 + 1 = y$

If y is odd

$\Rightarrow y = x - 1 \Rightarrow x = y + 1 \in N$ (domain)

$f(y + 1) = y + 1 - 1 = y$

$\therefore$ For every $y \in N$ (co-domain), there exists $x \in N$ (domain) such that $y = f(x)$. Hence, function is onto.

$\therefore$ f is both one-one and onto and hence bijective.

OR

For reflexive: Let for $(a, b) \in A \times A$, $(a, b)\, R\, (a, b)$.

Now, $(a, b)\, R\, (a, b) \Rightarrow a + b = b + a$, true. Hence, R is reflexive.

For symmetric: Let for $(a, b), (c, d) \in A \times A$, $(a, b)\, R\, (c, d)$

Now, $(a, b)\, R\, (c, d) \Rightarrow a + d = b + c \Rightarrow b + c = a + d$

$\Rightarrow c + b = d + a \Rightarrow (c, d)\, R\, (a, b)$

$(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for $(a, b), (c, d) \in A \times A$

Hence, R is symmetric.

For transitive: Let for $(a, b), (c, d), (e, f) \in A \times A$, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$.

Now, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

$\Rightarrow a + d = b + c$ and $c + f = d + e$

$\Rightarrow a + d + c + f = b + c + d + e$

$\Rightarrow a + f = b + e \Rightarrow (a, b)\, R\, (e, f)$

As for $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f) \Rightarrow (a, b)\, R\, (e, f)$ for $(a, b), (c, d), (e, f) \in A \times A$

Hence, R is transitive.

As R is reflexive, symmetric and transitive, so R is an equivalence relation.

If $(a, b) \in$ equivalence class $[(3, 7)]$ then

$(a, b)\, R\, (3, 7) \Rightarrow a + 7 = b + 3$

e.g. $(1, 5) \in [(3, 7)]$

$\therefore$ Equivalence class $[(3, 7)] = \{(1, 5), (2, 6), (3, 7), (4, 8), (5, 9)\}$

36. Strings are represented by

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + \hat{k})$$

and

$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$$

(*i*) Vector perpendicular to both the strings is $\vec{n} = (\hat{i} - 2\hat{j} + \hat{k}) \times (7\hat{i} - 6\hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

(*ii*) Strings will intersect if shortest distance between strings is zero.

(*iii*) Here, $\vec{a_1} = 3\hat{i} + 5\hat{j} + 7\hat{k}$, $\vec{b_1} = \hat{i} - 2\hat{j} + \hat{k}$

and $\vec{a_2} = -\hat{i} - \hat{j} - \hat{k}$, $\vec{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\vec{a_2} - \vec{a_1} = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\text{Shortest distance} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{-16 - 36 - 64}{\sqrt{16 + 36 + 64}} \right| = \left| \frac{-116}{\sqrt{116}} \right|$$

$$= \sqrt{116} \text{ units} = 2\sqrt{29} \text{ units}$$

OR

(*iii*) Strings are along direction vectors, $\vec{b_1} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$

37.

Shop	Pure (P) mustard oil tins	Adulterated (A) mustard oil tins
Rama general store (*R*)	30	40
Green general store (*G*)	50	60

(*i*) Probability of getting adulterated mustard oil tin from Green general store

$$= P(G) \cdot P(A/G) = \frac{1}{2} \times \frac{60}{110} = \frac{3}{11}$$

(*ii*) Probability of purchasing pure tin mustard oil $= P(R) \cdot P(P/R) + P(G) \cdot P(P/G)$

$$= \frac{1}{2} \times \frac{30}{70} + \frac{1}{2} \times \frac{50}{110} = \frac{1}{2}\left(\frac{3}{7} + \frac{5}{11}\right)$$

$$= \frac{1}{2}\left(\frac{68}{77}\right) = \frac{34}{77}$$

(*iii*) Probability of tin of pure mustard oil purchased from Rama general store

$$= \frac{\frac{1}{2} \times \frac{30}{70}}{\frac{34}{77}} = \frac{3}{14} \times \frac{77}{34} = \frac{33}{68}$$

OR

(*iii*) Probability of purchasing a pure mustard oil tin from Green general store or adulterated mustard oil tin from Rama general store

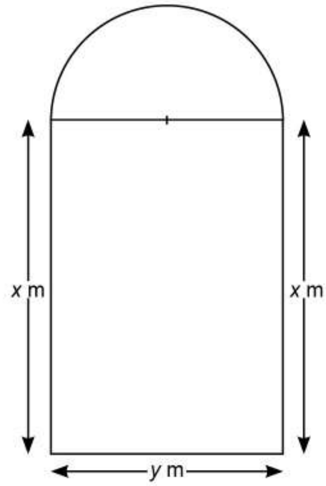
$$= P(G) \cdot P(P/G) + P(R) + P(A/R) = \frac{1}{2} \times \frac{5}{11} + \frac{1}{2} \times \frac{4}{7} = \frac{1}{2}\left(\frac{5}{11} + \frac{4}{7}\right) = \frac{1}{2} \times \frac{79}{77} = \frac{79}{154}.$$

38. (*i*) Perimeter = 10 m

$$\Rightarrow \quad 2x + y + \pi \cdot \frac{y}{2} = 10$$

$$\Rightarrow \quad 4x + 2y + \pi y = 20$$

$$\Rightarrow \quad 4x + (2 + \pi)y = 20 \qquad ...(i)$$



$$\text{Area } (A) = xy + \frac{1}{2}\pi\frac{y^2}{4} = xy + \frac{1}{8}\pi y^2$$

$$A = y\left[\frac{20 - (2 + \pi)y}{4}\right] + \frac{1}{8}\pi y^2$$

$$= [5y - \frac{1}{4}(2 + \pi)y^2 + \frac{1}{8}\pi y^2]$$

$$= [5y - \frac{1}{8}(4 + 2\pi - \pi)y^2]$$

$$= [5y - \frac{1}{8}(4 + \pi)y^2]$$

(*ii*)
$$\frac{dA}{dy} = 5 - \frac{2}{8}(\pi + 4)\, y = 5 - \frac{1}{4}(\pi + 4)y$$

For maximum area, $\frac{dA}{dy} = 0$

$$\Rightarrow \quad 20 - (4 + \pi)y = 0 \quad \Rightarrow \quad y = \frac{20}{4 + \pi}$$

$$\frac{d^2A}{dy^2} = -\frac{1}{4}(4 + \pi)$$

Now, $$\left.\frac{d^2A}{dy^2}\right]_{y = \frac{20}{4+\pi}} = \frac{-(\pi + 4)}{4} < 0$$

$\therefore$ For $y = \frac{20}{4 + \pi}$ m, area is maximum.