3 SAMPLE PAPER

[Easy Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(*iii*) *In* ***Section A****, Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B****, Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E****, Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions****, carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If A and B are two matrices such that $AB = A$ and $BA = B$ then B^2 is equal to **[Conceptual Application]**

(*a*) B (*b*) A (*c*) 1 (*d*) 0

2. The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is **[Conceptual Application]**

(*a*) $\frac{3\pi}{5}$ (*b*) $-\frac{\pi}{10}$ (*c*) $\frac{\pi}{10}$ (*d*) $\frac{7\pi}{5}$

3. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then $x =$ **[NCERT Part-I, Page 77]**

(*a*) 3 (*b*) ± 3 (*c*) ± 6 (*d*) 6

4. If $\vec{a}$ and $\vec{b}$ represent the diagonals of a rhombus, then **[Conceptual Application]**

(*a*) $\vec{a} \times \vec{b} = \vec{0}$ (*b*) $\vec{a} \cdot \vec{b} = 0$

(*c*) $\vec{a} \cdot \vec{b} = 1$ (*d*) $\vec{a} \times \vec{b} = \vec{a}$

5. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right) = y^3$, is **[NCERT Part-II, Page 302]**

(*a*) $\frac{1}{2}$ (*b*) 2 (*c*) 3 (*d*) 4

6. $\int \frac{\sin^2 x}{\cos^4 x} dx =$ **[NCERT Part-II, Page 241]**

(*a*) $\frac{1}{3}\tan^2 x + C$ (*b*) $\frac{1}{2}\tan^2 x + C$ (*c*) $\frac{1}{3}\tan^3 x + C$ (*d*) None of these

7. In the interval (1, 2), function $f(x) = 2|x - 1| + 3|x - 2|$ is **[NCERT Part-I, Page 152-153]**

(*a*) increasing (*b*) decreasing (*c*) constant (*d*) None of these

8. If the function $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k \quad , & x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is **[NCERT Part-I, Page 105]**

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2

9. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin|x|\, dx$ is equal to **[Conceptual Application]**

(*a*) 1 (*b*) 2 (*c*) –1 (*d*) –2

10. If matrix $A = [a_{ij}]_{2\times2}$, where $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$, then A^2 is equal to **[NCERT Part-I, Page 36]**

(*a*) I (*b*) A (*c*) O (*d*) $-I$

11. If $|\vec{a}| = |\vec{b}|$, then $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) =$ **[Conceptual Application]**

(*a*) $|\vec{a}|$ (*b*) $2|\vec{a}|$ (*c*) 0 (*d*) $|\vec{a}| - |\vec{b}|$

12. The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 square units. The value of k will be **[NCERT Part-I, Page 82]**

(*a*) 9 (*b*) ±3 (*c*) –9 (*d*) 6

13. The angle between the lines $\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-5}{1} = \frac{z-4}{2}$ is **[NCERT Part-II, Page 383-384]**

(*a*) $\cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$ (*b*) $\cos^{-1}\left(\frac{5}{8\sqrt{3}}\right)$ (*c*) $\cos^{-1}\left(\frac{1}{5}\right)$ (*d*) None of these

14. The probability that a leap year will have 53 Fridays or 53 Saturdays is **[Conceptual Application]**

(*a*) $\frac{2}{7}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{4}{7}$ (*d*) $\frac{1}{7}$

15. The point at which the maximum value of $x + y$, subject to the constraints $x + 2y \leq 70$, $2x + y \leq 95$, $x, y \geq 0$ is obtained, is **[Conceptual Application]**

(*a*) (30,25) (*b*) (20,35) (*c*) (35,20) (*d*) (40,15)

16. If a line makes angle α, β, γ with the axes respectively then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$ **[NCERT Part-II, Page 377-378]**

(*a*) –2 (*b*) –1 (*c*) 1 (*d*) 2

17. If the angle between the vectors $x\hat{i} + 3\hat{j} - 7\hat{k}$ and $x\hat{i} - x\hat{j} + 4\hat{k}$ is acute, then x lies in the interval **[NCERT Part-II, Page 356]**

(*a*) (–4, 7) (*b*) [–4, 7] (*c*) R – [–4, 7] (*d*) R – (4, 7)

18. The general solution of differential equation $\frac{dy}{dx} = e^{x+y}$, is **[NCERT Part-II, Page 306-307]**

(*a*) $e^x + e^{-y} = C$ (*b*) $e^x + e^y = C$ (*c*) $e^{-x} + e^y = C$ (*d*) $e^{-x} + e^{-y} = C$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. Assertion (*A*): The inverse of matrix $A = \begin{bmatrix} 43 & 1 & 6 \\ 35 & 7 & 4 \\ 17 & 3 & 2 \end{bmatrix}$ does not exist. **[NCERT Part-I, Page 90]**

Reason (*R*): The inverse of singular matrix is not possible.

20. Assertion (*A*): $f(x) = e^x$ is an increasing function, $\forall\, x \in R$. **[NCERT Part-I, Page 153]**

Reason(*R*): If $f'(x) \le 0$, then $f(x)$ is an increasing function.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$. **[Conceptual Application]**

OR

Show that $\sin^{-1}\dfrac{5}{13} = \tan^{-1}\dfrac{5}{12}$. **[Conceptual Application]**

22. Find the values of 'a' so that the function $f(x)$ defined by $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ may be continuous at $x = 0$. **[NCERT Part-I, Page 105]**

23. If a vector makes angles α, β and γ with OX, OY and OZ respectively, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$. **[NCERT Part-II, Page 358]**

OR

Find the foot of the perpendicular from the point (0, 2, 3) on the line $\dfrac{x+3}{5} = \dfrac{y-1}{2} = \dfrac{z+4}{3}$.

[Conceptual Application]

24. Find the intervals in which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ is (*i*) increasing (*ii*) decreasing. **[NCERT Part-I, Page 153]**

25. Find a vector of magnitude 9, which is perpendicular to both the vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int x\sqrt{\dfrac{a^2 - x^2}{a^2 + x^2}}\,dx$ **[Integrated Question]**

27. A black and a red die are rolled together. Find the conditional probability of obtaining a sum of 8 given that the red die resulted in number less than 4. **[NCERT Part-II, Page 408]**

OR

A town has two fire extinguishing engines functioning independently. The probability of availability of each engine, when needed, is 0.95. What is the probability that **[Conceptual Application]**

(*i*) neither of them is available when needed?

(*ii*) an engine is available when needed?

(*iii*) exactly one engine is available when needed?

28. Evaluate: $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} dx.$ **[NCERT Part-II, Page 252-253]**

OR

The area of an expanding rectangle is increasing at the rate of 48 cm^2/s. The length of the rectangle is always equal to square of breadth. At what rate, the length is increasing at the instant when breadth is 4.5 cm? **[NCERT Part-I, Page 147-148]**

29. Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y.$ **[NCERT Part-II, Page 322-323]**

OR

Solve: $\frac{dy}{dx} = \cos(x + y) + \sin(x + y).$ **[NCERT Part-II, Page 306-307]**

30. Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$. **[NCERT Part-II, Page 166]**

31. Evaluate: $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx.$ **[NCERT Part-II, Page 273-274]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Using integration, find the area of the region bounded by the line $2y = x + 8$, x-axis and the lines $x = 2$ and $x = 4$. **[Conceptual Application]**

33. Show that the function $f: N \to N$ given by, $f(n) = n - (-1)^n$ for all $n \in N$ is a bijective. **[NCERT Part-I, Page 7]**

OR

Let $f: N \cup \{0\} \to N \cup \{0\}$ be defined by **[NCERT Part-I, Page 7]**

$f(n) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-1, & \text{if } n \text{ is odd} \end{cases}$. Show that f is a bijective.

34. Find the length of the perpendicular from the point (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. **[Conceptual Application]**

OR

Find the shortest distance between the following pair of lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ **[NCERT Part-II, Page 386-387]**

and $\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

35. Solve the following system of linear equations using inverse of matrix, **[NCERT Part-I, Page 94-95]**

$$x + 3y + 4z = 8$$
$$2x + y + 2z = 5$$
$$5x + y + z = 7$$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. A poster is to be formed for a company advertisement. The top and bottom margins of the poster should be 9 cm and the side margins should be 6 cm. Also, the area for printing the advertisement should be 864 cm^2. **[Conceptual Application]**

(*i*) If a cm be the width and b cm be the height of the poster then what will be the area of poster in terms of a and b?

(*ii*) Find the relation between a and b.

(*iii*) Find the area of poster in terms of b.

OR

(*iii*) Find the value of a and b for which the area of poster is minimum.

Case Study - 2

37. A class XII student appearing for the Pre-board exam was asked to attempt the following question on LPP. **[NCERT Part-II, Page 397-398]**

Let $Z = 22x + 18y$ be the objective function

Subject to constraints

$360x + 240y \leq 5760, x + y \leq 20, x, y \geq 0$

(*i*) For the constraint $360x + 240y \leq 5760$, state with reason whether feasible solution contains the origin or not?

(*ii*) Find the corner points of feasible region.

(*iii*) Find the values of x and y at which maximum of Z occurs.

OR

(*iii*) Find the maximum value of Z.

Case Study - 3

38. A doctor is to visit a patient. From the past experience, it is known that the probability that he will come by cab, metro, bike or any other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively. **[NCERT Part-II, Page 425]**

(*i*) When the doctor arrives late , what is the probability that he comes by metro?

(*ii*) When the doctor arrives late, what is the probability that he comes by other means?

SOLUTIONS

1. (*a*) Now here $AB = A$ and $BA = B$

Consider $BA = B,$

Post multiplying by matrix B, we get

$$BAB = B^2$$

$\Rightarrow$ $B(AB) = B^2$

$\Rightarrow$ $BA = B^2$ $[\because AB = A]$

$\Rightarrow$ $B = B^2$ $[\because BA = B]$

2. (*b*) We need to find the value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$

$$\sin^{-1}\left(\cos\frac{33\pi}{5}\right) = \sin^{-1}\left(\cos\left(\frac{30\pi+3\pi}{5}\right)\right) = \sin^{-1}\left(\cos\left(6\pi+\frac{3\pi}{5}\right)\right)$$

$$= \sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{10}\right)\right)$$

$$= -\frac{\pi}{10} \qquad \left\{\because \sin^{-1}(\sin x) = x, \text{ for } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right\}$$

$\because$ $\sin^{-1}\left(\cos\frac{33\pi}{5}\right) = -\frac{\pi}{10}$

3. (*c*) $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow$ $2x^2 - 40 = 18 + 14 \Rightarrow 2x^2 = 72$

$\Rightarrow$ $x^2 = 36 \Rightarrow x = \pm 6$

4. (*b*) We know that the diagonals $(\vec{a}$ and $\vec{b})$ of a rhombus are perpendicular.

Therefore, their dot product is zero.

$\Rightarrow$ $\vec{a}\cdot\vec{b} = 0$

5. (*b*) Degree = 2

6. (*c*) Let $I = \int \frac{\sin^2 x}{\cos^4 x}\,dx = \int \tan^2 x \cdot \sec^2 x\,dx$

Let $\tan x = t$

$\Rightarrow$ $\sec^2 x\,dx = dt$

$$I = \int t^2 dt = \frac{t^3}{3} + C = \frac{\tan^3 x}{3} + C$$

7. (*b*) $f(x) = 2|x-1| + 3|x-2|$ in $(1, 2)$

when $x \in (1, 2)$, then $f(x) = +2(x-1) - 3(x-2)$

$$= 2x - 2 - 3x + 6$$

$$= -x + 4$$

$$f'(x) = -1$$

$\because$ $f'(x) < 0$

$\Rightarrow f(x)$ is decreasing.

8. (*d*) The function is continuous. So

$$\lim_{x \to 1} f(x) = f(1)$$

$$\Rightarrow \quad \lim_{x \to 1}\left(\frac{x^2 - 1}{x - 1}\right) = f(1)$$

$$\Rightarrow \quad \lim_{x \to 1}(x + 1) = k \quad \Rightarrow \quad 1 + 1 = k$$

$$\Rightarrow \quad k = 2$$

9. (*b*) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin|x|dx$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$= \int_{-\frac{\pi}{2}}^{0} \sin(-x)\,dx + \int_{0}^{\frac{\pi}{2}} \sin x\,dx$$

$$= -\int_{-\pi/2}^{0} \sin x\,dx + \int_{0}^{\pi/2} \sin x\,dx$$

$$= [\cos x]_{-\pi/2}^{0} - [\cos x]_{0}^{\pi/2}$$

$$= 1 - 0 - 0 + 1 = 2$$

10. (*a*) $A = [a_{ij}]_{2 \times 2}$

$$a_{11} = 0,\ a_{12} = 1,\ a_{21} = 1,\ a_{22} = 0$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

11. (*c*) $|\vec{a}| = |\vec{b}|$ (given)

Now, $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b}$

$$= |\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} - |\vec{b}|^2$$

$$= |\vec{a}|^2 - |\vec{b}|^2$$

$$= |\vec{a}|^2 - |\vec{a}|^2$$

$$= 0$$

12. (*b*) We know that, area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$\therefore \quad \Delta = \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$

$\pm 9 = \frac{1}{2}[-3(-k) - 0 + 1(3k)]$

$\Rightarrow \quad \pm 18 = 3k + 3k = 6k$

$\therefore \quad k = \pm\frac{18}{6} = \pm 3$

13. (*a*) The given lines are:

$$\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4} \quad \text{...}(i)$$

$$\frac{x+1}{1} = \frac{y-5}{1} = \frac{z-4}{2} \quad \text{...}(ii)$$

The dr's of line (*i*) are proportional to 3, 5, 4

and dr's of line (*ii*) are proportional to 1, 1, 2

Let $<a_1, b_1, c_1> \equiv <3, 5, 4>$ and $<a_2, b_2, c_2> \equiv <1, 1, 2>$

Let 'θ' be the angle between lines (*i*) and (*ii*)

$$\therefore \quad \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{3+5+8}{\sqrt{9+25+16} \cdot \sqrt{1+1+4}}$$

$$= \frac{16}{5\sqrt{2} \cdot \sqrt{6}} = \frac{16}{10\sqrt{3}} = \frac{8}{5\sqrt{3}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

14. (*b*) We know that a leap year has 366 days.

(i.e., $7 \times 52 + 2$) = 52 weeks and 2 extra days.

The sample space for these 2 extra days is given below:

S = {(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}

There are 7 cases.

$\therefore \quad n(S) = 7$

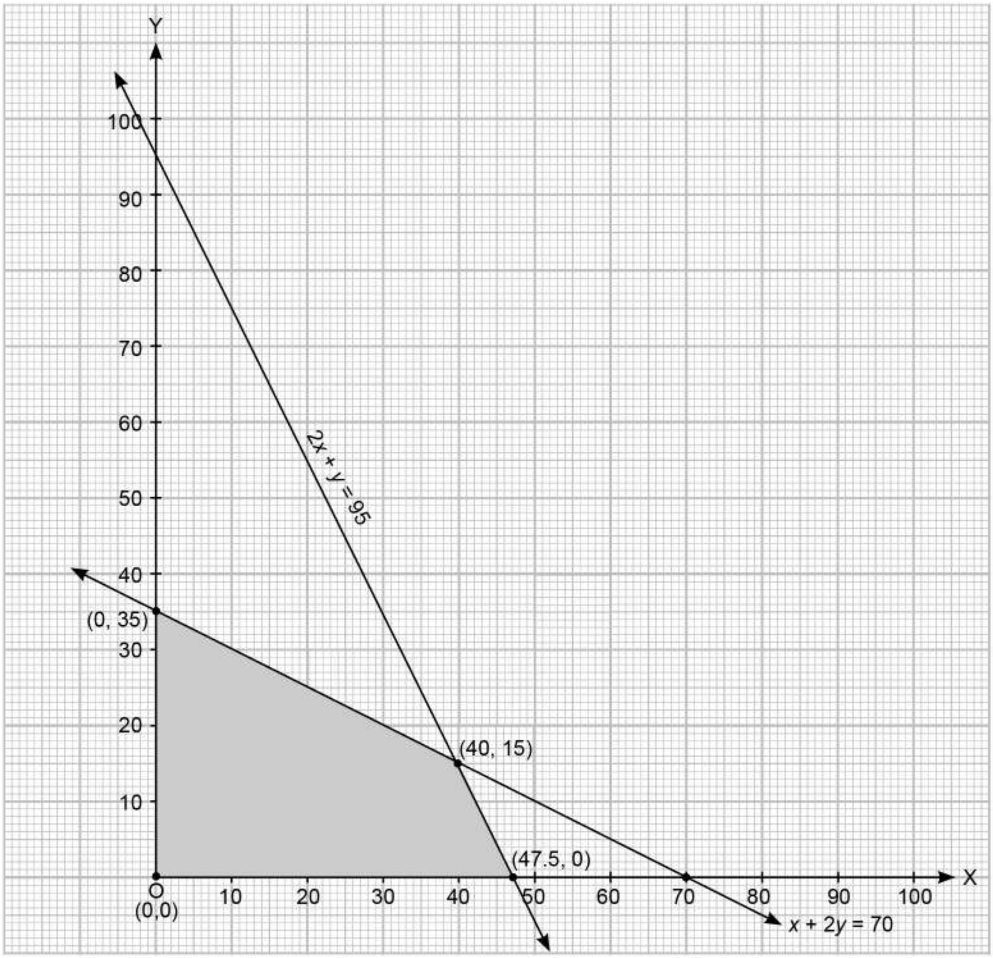
Let E be the event that a leap year has 53 Fridays or 53 Saturdays.

i.e. $\quad n(E) = 3$

$$\therefore \quad p(E) = \frac{n(E)}{n(S)} = \frac{3}{7}$$

Hence, the probability that a leap year has 53 Fridays or 53 Saturdays is $\frac{3}{7}$.

15. (*d*) Plotting the given inequation, we notice shaded portion is feasible solution.



The coordinates of corner points are:

(0, 0), (47.5, 0), (40, 15), (0, 35)

Corner point	Values of $Z = x + y$
(0, 0)	0
(47.5, 0)	47.5
(40, 15)	55 ← Maximum
(0, 35)	35

So, maximum Z occurs at (40, 15).

16. (*b*) Let $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow \quad \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$$

$$\Rightarrow \quad 1 + \cos 2\alpha + 1 + \cos 2\beta + 1 + \cos 2\gamma = 2$$

$$\Rightarrow \quad 3 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2$$

$$\Rightarrow \quad \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

17. (*c*) Let $\vec{a} = x\hat{i} + 3\hat{j} - 7\hat{k}$ and $\vec{b} = x\hat{i} - x\hat{j} + 4\hat{k}$

Since θ is acute, then $\cos\theta > 0$

$$\Rightarrow \quad \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} > 0$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} > 0$$

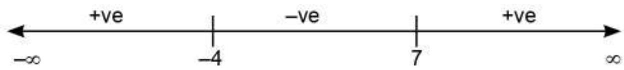
$$\Rightarrow \quad \left(x\hat{i} + 3\hat{j} - 7\hat{k}\right)\cdot\left(x\hat{i} - x\hat{j} + 4\hat{k}\right) > 0$$

$$\Rightarrow \quad x^2 - 3x - 28 > 0$$

$$\Rightarrow \quad (x-7)(x+4) > 0$$

$$\Rightarrow \quad x - 7 > 0, x + 4 > 0 \text{ or } x - 7 < 0, x + 4 < 0$$

$$\Rightarrow \quad x > 7 \quad \text{or } x < -4$$



So, $x \in (-\infty, -4) \cup (7, \infty)$ or $x \in R - [-4, 7]$

18. (*a*)

$$\frac{dy}{dx} = e^{x+y} \Rightarrow \frac{dy}{dx} = e^x \cdot e^y$$

$$\Rightarrow \quad \frac{dy}{e^y} = e^x \cdot dx$$

Integrating both sides, we get

$$\int e^{-y}dy = \int e^x dx$$

$$\Rightarrow \quad -e^{-y} = e^x + C_1$$

$$\Rightarrow \quad e^x + e^{-y} = C, \text{ where } C = -C_1$$

19. (*a*) **Assertion (*A*):** The inverse of $A = \begin{bmatrix} 43 & 1 & 6 \\ 35 & 7 & 4 \\ 17 & 3 & 2 \end{bmatrix}$ does not exist.

Since $|A| = 0 \Rightarrow A$ is a singular matrix.

So A^{-1} does not exist.

Reason (*R*): The inverse of singular matrix is not possible.

20. (*c*) Since $f(x) = e^x$ is an increasing function, $\forall\, x \in R$

$$f'(x) = e^x \geq 0$$

but if $f'(x) \leq 0$, then $f(x)$ will be a decreasing function.

A is true but *R* is false.

21. $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

$$= \tan^{-1}\left\{\tan\frac{\pi}{3}\right\} - \cot^{-1}\left\{-\cot\left(\frac{\pi}{6}\right)\right\}$$

$$= \frac{\pi}{3} - \cot^{-1}\left\{\cot\left(\pi - \frac{\pi}{6}\right)\right\}$$

$$= \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$$

OR

Let $\sin^{-1}\frac{5}{13} = \theta \Rightarrow \sin\theta = \frac{5}{13}$

$\Rightarrow$ $$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{\frac{5}{13}}{\sqrt{1-\frac{25}{169}}} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12} \Rightarrow \theta = \tan^{-1}\frac{5}{12}$$

$\therefore$ $$\sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12}$$

22. The function $f(x)$ will be continuous at $x = 0$, if

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0}\frac{\sin^2 ax}{x^2} = 1 \qquad [\because f(0) = 1]$$

$\Rightarrow$ $$a^2 \lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)^2 = 1$$

$\Rightarrow$ $a^2(1)^2 = 1 \Rightarrow a = \pm 1$

Thus, $f(x)$ will be continuous at $x = 0$, if $a = \pm 1$.

23. Let l, m, n be the direction cosines of the given vector. Then,

$l = \cos\alpha, m = \cos\beta, n = \cos\gamma$.

Now, $$l^2 + m^2 + n^2 = 1$$

$\Rightarrow$ $$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$\Rightarrow$ $$(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$$

$\Rightarrow$ $$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

OR

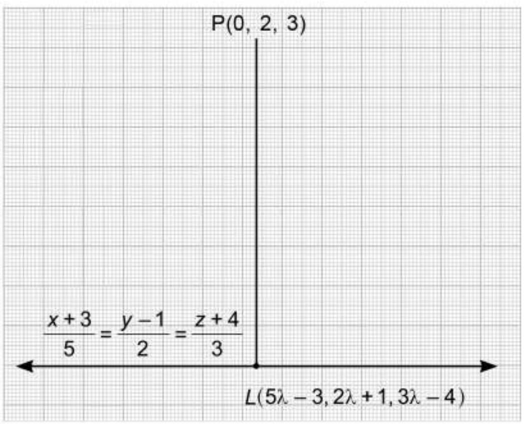
Let L be the foot of the perpendicular drawn from the point P(0, 2, 3) to the given line

The coordinates of a general point on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ are given by

$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$ (say)

or, $x = 5\lambda - 3, y = 2\lambda + 1, z = 3\lambda - 4$.

Let the coordinates of L be $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$. Therefore, direction ratios of PL are proportional to $5\lambda - 3 - 0, 2\lambda + 1 - 2, 3\lambda - 4 - 3$ i.e $5\lambda - 3, 2\lambda - 1, 3\lambda - 7$.



Direction ratios of the given line are proportional to 5, 2, 3.

But, PL is perpendicular to the given line.

$\therefore\ 5(5\lambda - 3) + 2(2\lambda - 1) + 3\,(3\lambda - 7) = 0 \quad \Rightarrow \quad \lambda = 1$

Putting $\lambda = 1$ in $(5\lambda - 3,\ 2\lambda + 1,\ 3\lambda - 4)$, the coordinates of L are $(2, 3, -1)$.

24. We have

$$f(x) = 2x^3 + 9x^2 + 12x + 20$$

$$\therefore \quad f'(x) = 6x^2 + 18x + 12 = 6(x^2 + 3x + 2)$$

(*i*) For $f(x)$ to be increasing, we must have

$$f'(x) > 0$$

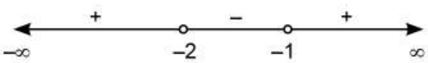
$$\Rightarrow \quad 6\,(x^2 + 3x + 2) > 0$$

$$\Rightarrow \quad (x^2 + 3x + 2) > 0 \qquad [\because 6 > 0 \text{ and } 6(x^2 + 3x + 2) > 0 \quad \therefore x^2 + 3x + 2 > 0]$$

$$\Rightarrow \quad (x + 1)\,(x + 2) > 0$$

$$\Rightarrow \quad x < -2 \text{ or } x > -1$$

$$\Rightarrow \quad x \in (-\infty, -2) \cup (-1, \infty)$$



Signs of f′(x) for different values of x

So, $f(x)$ is increasing on $(-\infty, -2) \cup (-1, \infty)$

(*ii*) For $f(x)$ to be decreasing, we must have

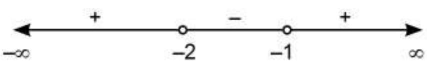
$$f'(x) < 0$$

$$\Rightarrow \quad 6(x^2 + 3x + 2) < 0$$

$$\Rightarrow \quad (x^2 + 3x + 2) < 0 \qquad [\because 6 > 0 \text{ and } 6(x^2 + 3x + 2) < 0 \ \therefore x^2 + 3x + 2 < 0]$$

$$\Rightarrow \quad (x + 1)\,(x + 2) < 0$$

$$-2 < x < -1$$



Signs of f′(x) for different values of x

So, $f(x)$ is decreasing on $(-2, -1)$.

25. We have, $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= (2 - 3)\hat{i} - (-8 + 6)\hat{j} + (4 - 2)\hat{k} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\Rightarrow \quad |\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + 2^2 + 2^2} = 3$$

$$\therefore \quad \text{Required vector} = 9\left\{\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}\right\} = \frac{9}{3}\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right) = -3\hat{i} + 6\hat{j} + 6\hat{k}.$$

26. Putting $x^2 = t$, and $2x\,dx = dt$ or, $dx = \dfrac{dt}{2x}$, we get

$$I = \int x\sqrt{\frac{a^2 - t}{a^2 + t}}\,\frac{dt}{2x} = \frac{1}{2}\int\sqrt{\frac{a^2 - t}{a^2 + t}}\,dt = \frac{1}{2}\int\sqrt{\frac{a^2 - t}{a^2 + t} \times \frac{a^2 - t}{a^2 - t}}\,dt$$

$$\Rightarrow \quad I = \frac{1}{2}\int\frac{a^2 - t}{\sqrt{a^4 - t^2}}\,dt = \frac{1}{2}\int\frac{a^2}{\sqrt{a^4 - t^2}}\,dt - \frac{1}{2}\int\frac{t\,dt}{\sqrt{a^4 - t^2}}$$

$$\Rightarrow \quad I = \frac{1}{2}a^2\int\frac{1}{\sqrt{(a^2)^2 - t^2}}\,dt + \frac{1}{4}\int\frac{-2t}{\sqrt{a^4 - t^2}}\,dt$$

$$\Rightarrow \qquad I = \frac{1}{2}a^2 \sin^{-1}\left(\frac{t}{a^2}\right) + \frac{1}{4}\int \frac{du}{\sqrt{u}}, \qquad \left| \begin{array}{l} \text{where } a^4 - t^2 = u \\ \Rightarrow \quad -2t\, dt = du \end{array} \right.$$

$$\Rightarrow \qquad I = \frac{1}{2}a^2 \sin^{-1}\left(\frac{t}{a^2}\right) + \frac{1}{4}\left(\frac{u^{1/2}}{1/2}\right) + C$$

$$\Rightarrow \qquad I = \frac{1}{2}a^2 \sin^{-1}\left(\frac{t}{a^2}\right) + \frac{1}{2}\sqrt{a^4 - t^2} + C = \frac{1}{2}a^2 \sin^{-1}\left(\frac{x^2}{a^2}\right) + \frac{1}{2}\sqrt{a^4 - x^4} + C$$

27. A: getting sum of 8

A : getting number less them 4 on red die.

A = {(2, 6), (3, 5), (4, 4) (5, 3), (6, 2)}

B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 0), (5, 3), (6, 1), (6, 2), (6, 3)}

$A \cap B$ = {(5, 3), (6, 2)}

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{n(B)} = \frac{n(A \cap B)}{n(A)} = \frac{2}{18} = \frac{1}{9}.$$

OR

Let A denote the event that first engine is available when needed and B, the event that second engine is available when needed. The, $P(A) = P(B) = 0.95$.

(*i*) Required probability $= P(\overline{A} \cap \overline{B})$

$= P(\overline{A})\, P(\overline{B})$ [$\because$ A, B are independent]

$= (0.05) \times (0.05) = 0.0025$

(*ii*) Required probability $= P(A \cup B)$

$= 1 - P(\overline{A})\, P(\overline{B})$ [$\because$ A, B are independent]

$= 1 - (0.05)\,(0.05) = 0.9975$

(*iii*) Required probability $= P(A \cap \overline{B}) + P(\overline{A} \cap B) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

$= P(A) + P(B) - 2P(A) \times P(B) = 0.95 + 0.95 - 2 \times 0.95 \times 0.95 = 0.095$

28. Let $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + \dfrac{A}{(x-4)} + \dfrac{B}{(x-5)} + \dfrac{C}{(x-6)}$ ...(*i*)

[As degree of numerator and degree of denominator is same and coefficients of x^3 is '1' in each case]

Then, $(x-1)(x-2)(x-3) = (x-4)(x-5)(x-6) + A(x-5)(x-6) + B(x-4)(x-6) + C(x-4)(x-5)$ ...(*ii*)

Putting $x = 4$, 5 and 6 successively in (*ii*), we obtain

$A = 3$, $B = -24$ and $C = 30$

Substituting values of A, B and C in (*i*), we obtain

$$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + \frac{3}{x-4} - \frac{24}{x-5} + \frac{30}{x-6}$$

$$\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}\, dx = \int 1 \cdot dx + 3\int \frac{1}{x-4}\, dx - 24\int \frac{1}{x-5}\, dx + 30\int \frac{1}{x-6}\, dx$$

$$= x + 3\log|x-4| - 24\log|x-5| + 30\log|x-6| + C$$

OR

Let the length of the rectangle be l and its breadth b at any time t.

Then $\quad l = b^2 \Rightarrow A = l \cdot \sqrt{l} = l^{3/2}$

$$\frac{dA}{dt} = \frac{3}{2}\sqrt{l} \cdot \frac{dl}{dt}$$

$$\Rightarrow \quad 48 = \frac{3}{2} \times b \times \frac{dl}{dt} \quad (\because \sqrt{l} = b)$$

$$\Rightarrow \quad \left.\frac{dl}{dt}\right]_{b\,=\,4.5\text{ cm}} = \frac{2 \times 48}{3 \times 4.5} = \frac{320}{45} = 7.11 \text{ cm/s}$$

29. The given differential equation can be written as

$$\sec^2 y \frac{dy}{dx} + x\frac{\sin 2y}{\cos^2 y} = x^3 \Rightarrow \sec^2 y \frac{dy}{dx} + 2x \tan y = x^3 \quad ...(i)$$

Let $\tan y = v \Rightarrow \sec^2 y \dfrac{dy}{dx} = \dfrac{dv}{dx}$

From (i), we get $\quad \dfrac{dv}{dx} + (2x)v = x^3 \quad ...(ii)$

This is a linear differential equation of the form $\dfrac{dv}{dx} + P(x)v = Q(x)$, where $P(x) = 2x$ and $Q(x) = x^3$.

$\therefore \quad \text{I.F.} = e^{\int 2x\,dx} = e^{x^2}$

Solution is $\quad ve^{x^2} = \int x^3 e^{x^2} dx$

$$ve^{x^2} = \frac{1}{2}\int t e^t dt + C, \text{ where } t = x^2$$

$$ve^{x^2} = \frac{1}{2}(t-1)e^t + C$$

$e^{x^2} \tan y = \dfrac{1}{2}(x^2-1)e^{x^2} + C$, which gives the required solution.

OR

Let $x + y = v$. Then, $\quad 1 + \dfrac{dy}{dx} = \dfrac{dv}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{dv}{dx} - 1$

Putting $x + y = v$ and $\dfrac{dy}{dx} = \dfrac{dv}{dx} - 1$ in the given differential equation, we get

$$\frac{dv}{dx} - 1 = \cos v + \sin v$$

$$\Rightarrow \quad \frac{dv}{dx} = 1 + \cos v + \sin v$$

$$\Rightarrow \quad \frac{1}{1 + \cos v + \sin v} dv = dx \quad \text{[By separating the variables]}$$

$$\Rightarrow \quad \int \frac{1}{1 + \cos v + \sin v} dv = \int 1 \cdot dx \quad \text{[On integrating]}$$

$$\Rightarrow \int \frac{1}{1 + \dfrac{1 - \tan^2\left(\frac{v}{2}\right)}{1 + \tan^2\left(\frac{v}{2}\right)} + \dfrac{2\tan\left(\frac{v}{2}\right)}{1 + \tan^2\left(\frac{v}{2}\right)}} dv = x + C$$

$$\Rightarrow \quad \int \frac{\sec^2\left(\frac{v}{2}\right)}{2\left(1+\tan\frac{v}{2}\right)}\,dv = x + C$$

$$\Rightarrow \quad \log\left|1+\tan\left(\frac{v}{2}\right)\right| = x + C$$

$$\Rightarrow \quad \log\left|1+\tan\left(\frac{x+y}{2}\right)\right| = x + C, \text{ which is the required solution.}$$

30. We have

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$

$$\Rightarrow \quad f'(x) = 6x^2 - 42x + 36$$

The critical points of $f(x)$ are given by $f'(x) = 0$.

Now, $f'(x) = 0 \quad \Rightarrow \quad 6x^2 - 42x + 36 = 0 \Rightarrow 6(x-1)(x-6) = 0 \Rightarrow x = 1, 6.$

Thus, $x = 1$ and $x = 6$ are the possible points of local maxima or minima.

Now, we test the function at each of these points.

We have, $f''(x) = 12x - 42$

At $x = 1$: We have,.

$$f''(1) = 12 - 42 = -30 < 0$$

So, $x = 1$ is a point of local maximum.

The local maximum value is $f(1) = 2 - 21 + 36 - 20 = -3$

At $x = 6$: We have,

$$f''(6) = 12(6) - 42 = 30 > 0$$

So $x = 6$ is a point of local minimum.

The local minimum value is $f(6) = 2(6)^3 - 21(6)^2 + 36 \times 6 - 20 = -128.$

31. Let $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}\,dx$. Then,

$$I = \int_0^{\pi/2} \left\{\frac{1}{1+e^{\sin x}} + \frac{1}{1+e^{-\sin x}}\right\} dx \qquad \left[\because \int_{-a}^{a} f(x)\,dx = \int_0^a \{f(x) + f(-x)\}\,dx\right]$$

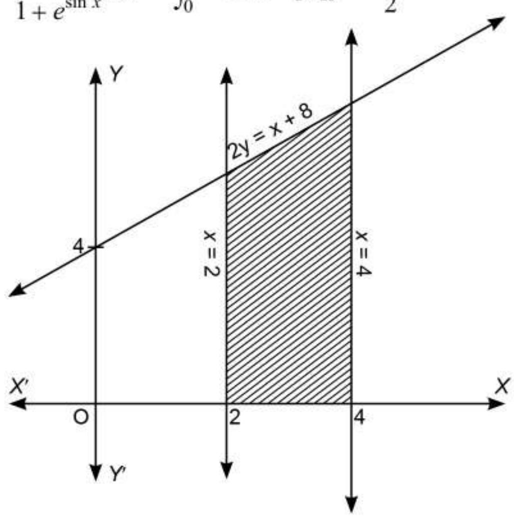
$$I = \int_0^{\pi/2} \left\{\frac{1}{1+e^{\sin x}} + \frac{e^{\sin x}}{1+e^{\sin x}}\right\} dx = \int_0^{\pi/2} \frac{1+e^{\sin x}}{1+e^{\sin x}}\,dx = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

32. Required area $= \int_2^4 y\,dx$

Now $2y = 8 + x \Rightarrow y = \frac{8+x}{2}$

So area $= \int_2^4 \left(\frac{8+x}{2}\right) dx$

$$= \frac{1}{2}\left[8x + \frac{x^2}{2}\right]_2^4 = 11 \text{ sq units.}$$



33. We have

$$f(n) = n - (-1)^n \text{ for all } n \in N$$

$$\Rightarrow \qquad f(n) = \begin{cases} n-1, \text{ if } n \text{ is even} \\ n+1, \text{ if } n \text{ is odd} \end{cases}$$

Injectivity: Let n, m be any two even natural numbers. Then,

$$f(n) = f(m) \Rightarrow n - 1 = m - 1 \Rightarrow n = m$$

If n, m are any two odd natural numbers. Then,

$$f(n) = f(m) \Rightarrow n + 1 = m + 1 \Rightarrow n = m.$$

Thus in both the cases, $f(n) = f(m) \Rightarrow n = m$.

If one is even other is odd:

Let n is even and m is odd, then $n \neq m$. Also $f(n)$ is odd and $f(m)$ is even. So, $f(n) \neq f(m)$.

Let n is odd and m is even then $n \neq m$. Also $f(n)$ is even and $f(m)$ is odd. So, $f(n) \neq f(m)$

Thus, $\quad n \neq m \Rightarrow f(n) \neq f(m)$.

So, f is an injective map.

Surjectivity: Let n be an arbitrary natural number.

If n is an odd natural number, then there exists an even natural number $n + 1$ such that

$$f(n + 1) = n + 1 - 1 = n$$

If n is an even natural number, then there exists an odd natural number $(n - 1)$ such that

$$f(n - 1) = n - 1 + 1 = n$$

Thus, every $n \in N$ has its pre-image in N. So, $f : N \to N$ is a surjection.

Hence, $f : N \to N$ is a bijection.

OR

Injection: Let $n, m \in N \cup \{0\}$.

If n and m are even, then

$$f(n) = f(m) \Rightarrow n + 1 = m + 1 \Rightarrow n = m$$

If n and m are odd, then

$$f(n) = f(m) \Rightarrow n - 1 = m - 1 \Rightarrow n = m$$

Thus, in both case, we have

$$f(n) = f(m) \Rightarrow n = m.$$

If n is odd and m is even, then $f(n) = n - 1$ is even and $f(m) = m + 1$ is odd. Therefore,

$$n \neq m \Rightarrow f(n) \neq f(m).$$

Similarly, if n is even and m is odd, then $f(n) = n + 1$ is odd and $f(m) = m - 1$ is even. So,

$$n \neq m \Rightarrow f(n) \neq f(m).$$

Hence, f is an injection.

Surjection: Let n be an arbitrary element of $N \cup \{0\}$.

If n is an odd natural number, there exist an even natural number $n - 1 \in N \cup \{0\}$ (domain) such that

$f(n-1) = n - 1 + 1$
$= n.$

If n is an even natural number, then there exists an odd natural number $n + 1 \in N \cup \{0\}$ (domain) such that
$f(n+1) = n + 1 - 1$
$= n.$

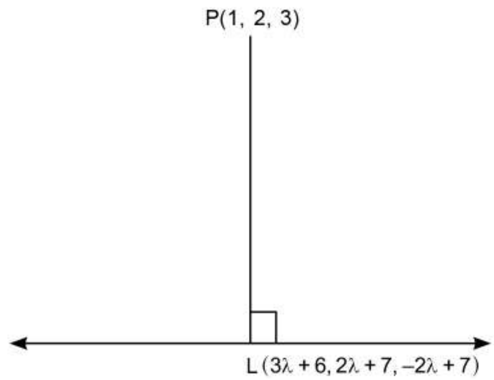
Also, $f(1) = 0$.

Thus, every element of $N \cup \{0\}$ (co-domain) has its pre-image in $N \cup \{0\}$ (domain). So, f is an onto function.

So, $f : N \cup \{0\} \to N \cup \{0\}$ is a bijection.

34. Let L be the foot of the perpendicular drawn from the point $P(1, 2, 3)$ to the given line. The coordinates of a general point on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ are given by

$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$ (say) or, $x = 3\lambda + 6$, $y = 2\lambda + 7$, $z = -2\lambda + 7$



Let the coordinates of L be $(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$ ...(*i*)

The direction ratios of PL are proportional to $3\lambda + 6 - 1, 2\lambda + 7 - 2, -2\lambda + 7 - 3$ or, $3\lambda + 5, 2\lambda + 5, -2\lambda + 4$.
The direction ratios of the given line are proportional to 3, 2, –2.

Since PL is perpendicular to the given line. Therefore,

$\therefore\ 3(3\lambda + 5) + 2(2\lambda + 5) + (-2)(-2\lambda + 4) = 0 \Rightarrow \lambda = -1.$

Putting $\lambda = -1$ in (*i*), we obtain the coordinates of L as (3, 5, 9)

$\therefore\ PL = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2} = 7$ units

Hence, the required length of the perpendicular is 7 units.

OR

The given vector equations of the lines are:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) \quad ...(i)$$

and

$$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k}) \quad ...(ii)$$

Comparing (*i*) and (*ii*) with $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$

$$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k},\ \vec{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Shortest distance, $d = \left|\dfrac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i}+2\hat{j}-\hat{k}$$

$$\vec{a_2}-\vec{a_1} = \hat{i}+2\hat{j}+2\hat{k}$$

$$d = \left[\frac{(-\hat{i}+2\hat{j}-\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{|-\hat{i}+2\hat{j}-\hat{k}|}\right]$$

$$= \left|\frac{-1+4-2}{\sqrt{(-1)^2+2^2+(-1)^2}}\right| = \frac{1}{\sqrt{6}} \text{ units}$$

35. The above system of simultaneous linear equations can be written in matrix form as

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

or, $AX = B$, where $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

Its solution is $X = A^{-1}B$.

Now, $|A| = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{vmatrix} = 1\,(1-2) - 3\,(2-10) + 4(2-5) = -1 + 24 - 12 = 11 \neq 0$

So, A^{-1} exists.

Let C_{ij} be the cofactor of a_{ij} in $A = [a_{ij}]$. Then,

$C_{11} = -1$, $C_{12} = 8$, $C_{13} = -3$, $C_{21} = 1$, $C_{22} = -19$, $C_{23} = 14$, $C_{31} = 2$, $C_{32} = 6$, and $C_{33} = -5$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -1 & 8 & -3 \\ 1 & -19 & 14 \\ 2 & 6 & -5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|}\, adj\ A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

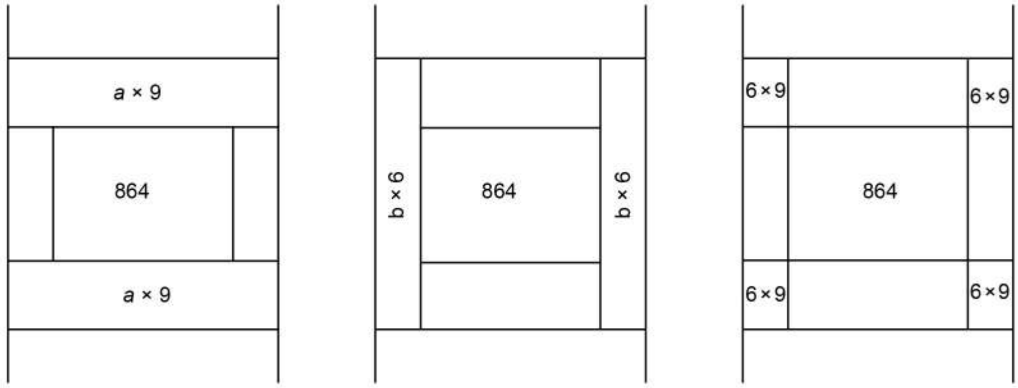
Thus, the solution of the system of equations is given by

$$X = A^{-1}B = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8+5+14 \\ 64-95+42 \\ -24+70-35 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 11 \\ 11 \\ 11 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow \quad x = 1, y = 1$ and $z = 1$.

36. (*i*)



Let A be the area of the poster, then

$$A = 864 + 2(a \times 9) + 2(b \times 6) - 4(6 \times 9)$$
$$= 864 + 18a + 12b - 216$$
$$= 648 + 18a + 12b$$

(*ii*) $A = a.b$

$\Rightarrow \quad 648 + 18a + 12b = ab$

$\Rightarrow \quad ab - 18a = 648 + 12b$

$\Rightarrow \quad a(b - 18) = 648 + 12b$

$\Rightarrow \quad a = \dfrac{648 + 12b}{b - 18}$

(*iii*)
$$A = a \cdot b$$
$$= \left(\frac{648 + 12b}{b - 18}\right) \cdot b$$
$$= \frac{648b + 12b^2}{b - 18}$$

OR

(*iii*)
$$A'(b) = \frac{(b - 18)(648 + 24b) - (648b + 12b^2)}{(b - 18)^2}$$
$$= \frac{12(b^2 - 36b - 972)}{(b - 18)^2}$$

For minimum, consider $A'(b) = 0$

$\Rightarrow \quad b^2 - 36b - 972 = 0$

$\Rightarrow \quad b^2 - 54b + 18b - 972 = 0$

$\Rightarrow \quad b(b - 54) + 18(b - 54) = 0$

$\Rightarrow \quad (b - 54)(b + 18) = 0$

$\Rightarrow \quad b + 18 = 0 \quad$ or $b - 54 = 0$

$b = -18$ or $b = 54$

b is height, therefore can't be negative.

$$b = 54$$

$$A''(b) = 12\left[\frac{(b - 18)^2(2b - 36) - (b^2 - 36b - 972)\,2(b - 18)}{(b - 18)^4}\right]$$

Now, $\quad A''(54) > 0$

So, A is minimum when $b = 54$.

and $$a = \frac{648 + 12 \times 54}{54 - 18} = 36$$

So, area of poster is minimum when $a = 36$ cm and $b = 54$ cm.

37. (*i*) The given constraint is $360x + 240y \leq 5760$

Putting $x = 0, y = 0$ in above inequation, we observe that $360 \times 0 + 240 \times 0 = 0 \leq 5760$, is true.

So, the region containing the origin represents the solution of given constraint.

(*ii*) The given constraints are $360x + 240y \leq 5760$ or $3x + 2y \leq 48$,

$x + y \leq 20, x \geq 0, y \geq 0$.

$3x + 2y \leq 48$

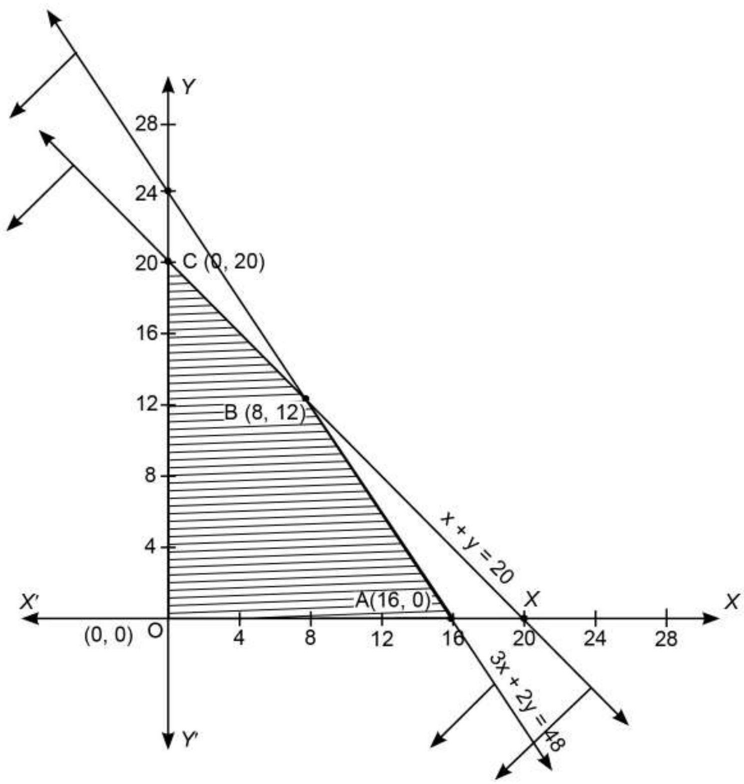
Consider corresponding equation $3x + 2y = 48$

x	0	16	8
y	24	0	12

$x + y \leq 20$

Consider corresponding equation $x + y = 20$

x	0	20	8
y	20	0	12



On plotting the points we notice shaded region is feasible solution.

The corner points are: (0, 20), (16, 0), (8, 12) and (0, 0)

(*iii*)

Points	$Z = 22x + 18y$	Values
(0, 20)	$22 \times 0 + 18 \times 20$	360
(16, 0)	$22 \times 16 + 18 \times 0$	352
(8, 12)	$22 \times 8 + 18 \times 12$	392
(0, 0)	$22 \times 0 + 18 \times 0$	0

Z is maximum for (8, 12) i.e. $x = 8, y = 12$

OR

(*iii*) Maximum value of $Z = 22 \times 8 + 18 \times 12$

$= 392$

38. Consider the following events:

E = doctor visit the patient late.

A_1 = doctor comes by cab

A_2 = doctor comes by metro

A_3 = doctor comes by bike

A_4 = doctor comes by other means

$$P(A_1) = 0.3,\ P(A_2) = 0.2$$

$$P(A_3) = 0.1,\ P(A_4) = 0.4$$

$$P(E/A_1) = 0.25,\ P(E/A_2) = 0.3$$

$$P(E/A_3) = 0.35,\ P(E/A_4) = 0.1$$

(*i*)

$$P(A_2/E) = \frac{P(A_2)\cdot P(E/A_2)}{\Sigma P(A_i)\, P(E/A_i)}$$

$$= \frac{(0.2)\,(0.3)}{(0.3)\,(0.25) + (0.2)\,(0.3) + (0.1)\,(0.35) + (0.4)\,(0.1)}$$

$$= \frac{0.06}{0.21}$$

$$= \frac{2}{7}$$

(*ii*)

$$P(A_4/E) = \frac{P(A_4)\cdot P(E/A_4)}{\Sigma P(A_i)\cdot P(E/A_i)}$$

$$= \frac{(0.4)\times(0.1)}{(0.3)(0.25) + (0.2)(0.3) + (0.1)(0.35) + (0.4)(0.1)}$$

$$= \frac{0.04}{0.21}$$

$$= \frac{4}{21}$$